Predictive Modeling of the Hypothalamic-Pituitary-Adrenal (Hpa) Axis Response to Acute and Chronic Stress

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Oscillatory dynamics appears to be essential for self-organization and self-regulation in living systems. Oscillations with different periodicity, from millisecond to annual range, are implemented at all levels of organization of living organisms, from the molecular to the biosphere level, occurring in processes as diverse as gene transcription in the cell nuclei to seasonal migrations of species between ecosystems [1, 2]. The hypothalamic-pituitary-adrenal (HPA) axis, a neuroendocrine system involved in maintaining homeostasis in mammalian organisms under physiological conditions and stress [3-5], is no exception to that rule. Cortisol, the HPA axis principal hormone in humans, exhibits complex dynamic behavior with characteristic frequencies: ultradian oscillations, with a period of 20-120 min [4-6] superimposed on circadian oscillations, with a period of about 24 h. The importance of circadian rhythms for adequate functioning of the HPA axis has been recognized for years [7, 8]; in addition, new experimental [9-15] and theoretical [16-27] results offer enough evidence to support the indispensable roles of ultradian oscillatory dynamics of HPA hormones levels for normal physiology. Since ultradian and circadian oscillations operate on different time scales, their effects are manifested in different biological realms. However, being coupled they seem to contribute synergistically to better integration and adaptation of an organism to the unpredictable dynamic environment.

Adequate dynamics of HPA activity is essential for maintaining homeostasis in mammalian organisms. Changes in the detailed dynamics of the HPA axis emerge routinely while the axis copes with a myriad of external stimuli [5, 28]. At the same time, the overall dynamics of the HPA axis is remarkably robust and stable in an organism [29, 30]. Stress and a number of...
The HPA model description

The HPA axis dynamics was emulated using a four-dimensional stoichiometric model presented in Table 1. This low-dimensional model, described in detail in our previous studies [17-19], comprises CRH (corticotropin-releasing hormone), ACTH (adrenocorticotropic hormone), ALDO (aldosterone) and CORT (cortisol) as dynamic variables. Reactions (R1)-(R9) epitomize the following complex pathways: (R1) describes basal CRH production from the hypothalamic paraventricular nucleus; (R2) describes aldosterone production under the renin-angiotensin system control; (R3) describes the CRH stimulated ACTH production from the pituitary; (R4) and (R5) describe ACTH stimulated production of cortisol and aldosterone from the adrenal cortex; (R6) describes the positive feedback actions of cortisol, acting through hippocampal GR to enhance CRH, and consequently ACTH and its own production; (R7) exemplifies cortisol negative feedback through hippocampal MR where both aldosterone and cortisol compete for the same receptor, as well as for hypothalamic and pituitary GR; (R8) and (R9) describe ACTH and cortisol elimination, respectively. Thus, ultradian self-regulation in the model is achieved through the experimentally established positive and negative feedback effects of cortisol on the HPA system via glucocorticoid (GR) and mineralocorticoid receptors (MR) [33-36]. However, these receptors are not included directly, but rather introduced implicitly through reac-

Table 1. A basic model describing self-regulation in the HPA system in humans.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Rate</th>
<th>Rate Constant</th>
<th>(M⁻¹·min⁻¹)</th>
<th>(M⁻²·min⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRH → CRH</td>
<td>r₀</td>
<td>k₀</td>
<td>1.82556 × 10⁻⁹</td>
<td>(R1)</td>
</tr>
<tr>
<td>ALDO → ALDO</td>
<td>rₙ</td>
<td>kₙ</td>
<td>6.0852 × 10⁻¹¹</td>
<td>(R2)</td>
</tr>
<tr>
<td>CRH → ACTH</td>
<td>r₁</td>
<td>k₁</td>
<td>1.82556</td>
<td>(R3)</td>
</tr>
<tr>
<td>ACTH → CORT</td>
<td>r₂</td>
<td>k₂</td>
<td>3.6 × 10⁻²</td>
<td>(R4)</td>
</tr>
<tr>
<td>ACTH → ALDO</td>
<td>r₃</td>
<td>k₃</td>
<td>2.88 × 10⁻⁴</td>
<td>(R5)</td>
</tr>
<tr>
<td>ACTH + 2CORT → 3CORT</td>
<td>r₄</td>
<td>k₄</td>
<td>1.26 × 10¹⁴</td>
<td>(R6)</td>
</tr>
<tr>
<td>ALDO + 2CORT → CORT</td>
<td>r₅</td>
<td>k₅</td>
<td>7.0524 × 10¹²</td>
<td>(R7)</td>
</tr>
<tr>
<td>ACTH → P₁</td>
<td>r₆</td>
<td>k₆</td>
<td>5.346 × 10⁻²</td>
<td>(R8)</td>
</tr>
<tr>
<td>CORT → P₂</td>
<td>r₇</td>
<td>k₇</td>
<td>4.0986 × 10⁻¹</td>
<td>(R9)</td>
</tr>
</tbody>
</table>

illnesses are associated with short- or long-term perturbations of the HPA dynamics, changing the amplitude and/or frequency of HPA hormones discharge and their mean levels. Such changes are observed in many diseases: primary (Addison’s disease) and secondary adrenocortical insufficiency, Cushing’s syndrome, visceral obesity, diabetes, hypertension, osteoporosis and major depression [4, 31].

The aim of this study is to understand how the HPA system adapts to external stimuli by achieving altered dynamic states, and what the possible consequences of the altered HPA axis dynamics are. To this aim, we use mathematical modeling, numerical simulations and dynamical systems theory approaches to investigate the dynamic behavior of the HPA system under acute and chronic perturbation, i.e. stress. To enable a meaningful comparison between different dynamic states, we first define new parameters that characterize these states and use them to investigate self-regulation mechanisms in the HPA axis under acute and chronic stress. We compare the model predictions with experimental observations reported in the literature. Furthermore, we use this analysis to discuss in the Appendix how this new understanding may be practically applied for designing treatment with glucocorticoids. These immunosuppressive drugs, commonly used for acute or chronic treatment of inflammatory diseases [32], may perturb the HPA axis dynamics thereby causing unwanted side-effects.
Predictive modeling of the HPA axis

Individual differences (it is well-established that circadian dynamics under basal conditions is very stable for a person, but can differ significantly between individuals). Daily oscillations in CRH levels, driven by the endogenous circadian clock have been modeled differently by different authors (e.g. [16]).

Methods

Computational methods

Numerical simulations were employed to emulate different acute/chronic stress conditions and test the validity of the model’s response to such simulation designs. In order to find solutions of the set of ordinary differential equations (Table 1), the Gear algorithm [39] for integration of stiff differential equations was used.

Definition of dynamic variables and perturbation parameters

A key feature of the model presented in Table 1 is the spontaneous evolution of ultradian oscillations, arising as a consequence of an intrinsic instability in the model [17-19, 25]. Thus, ultradian oscillations occur under defined conditions, without the need for circadian rhythm driven CRH pulse generation [17-19]. This very important characteristic of the model is in agreement with experimental findings showing that ultradian rhythmicity of cortisol does not cease after surgically disconnecting the hypothalamus from the pituitary [40], leaving the pituitary corticotrophs unstimulated by the hypothalamic CRH.

A practical consequence of this feature of the model is that without circadian oscillations, the stationary state concentration of CRH ([CRH]SS) is a control parameter defined by the ratio of rate constants for CRH production (k0) and consumption (k1) [19]:

\[ [\text{CRH}]_{SS} = \frac{k_0}{k_1} \]  

This function emulates the asymmetry of the 24 h rhythm in humans, with the nocturnal phase lasting 8 hours. The function D is not differentiable at \( t = v \cdot 1440 \), \( v = 1, 2, ... \). However, this function is continuous and no problems were encountered during numerical integration. It affects the inflow rate of CRH into the system, transforming \( k_0 \) in Eq. (1) into \( k_D(t) = k_0 \cdot D(t) \). The multiplier D couples the rate constant of CRH production (k0) to extrinsic circadian regulation (Fig. 1c and d). Consequently, CRH evolution changes from monotonic (Fig. 1a) to oscillatory (Fig. 1c), and the dynamics of ultradian oscillations becomes more complex (Fig. 1b compared to Fig. 1d). Parameters \( d_1 \) and \( d_2 \) in Eq. (5) decouple the mean daily CRH level in the hypothalamic-pituitary portal vessels (governed by \( d_1 \)) from the amplitude of the circadian CRH oscillation (governed by \( d_2 \)). The very nature of function D is not altered by the parameter separation – in essence, it is the same function as was used before [17-19, 25]. However, separation of the parameters enables us to investigate the effect of mean daily CRH levels in the hypothalamic-pituitary portal vessels independently from the effect of daily CRH amplitude on the HPA axis dynamics, thereby facilitating the study of individual differences (it is well-established that circadian dynamics under basal conditions is very stable for a person, but can differ significantly between individuals). Daily oscillations in CRH levels, driven by the endogenous circadian clock have been modeled differently by different authors (e.g. [16]).
the frequencies and amplitudes of ultradian oscillations are no longer constant, but depend on the actual state of the system. Therefore, absolute values of perturbation intensity cannot be compared directly. In order to make a meaningful comparison between different dynamic states possible, we first define new variables and perturbation parameters that take into account the daily variability in HPA hormonal levels.

The mean daily concentration of CRH in the hypothalamic-pituitary portal vessels \( \langle [CRH] \rangle \). Under circadian regulation, the rate of CRH production changes periodically over time, modeled through the effect of function \( D \) on \( k_0 \). Consequently, \([CRH]_{SS} \) (Eq. 6) is no longer constant and becomes a variable whose value repeats itself after 24 h (Fig. 1c). In order to quantify the average daily CRH levels in the hypothalamic-pituitary portal vessels, the mean daily concentration of CRH in the hypothalamic-pituitary portal vessels during 24 h is introduced:

\[
[CRH] = \frac{k_D(t)}{k_1} = \frac{k_0 \cdot [CRH]_{SS}}{k_1} = \frac{k_0 \cdot d_3}{k_1}, \tag{7}
\]

where \( < D > \) is the average value of the function \( D \) over 24 h.

The absolute amplitude \( A_i(X) \). Under the circadian drive, each two successive ultradian oscillations are characterized by different amplitudes (Fig. 1d). Therefore, the simple definition of amplitude as the difference between two successive extreme values is not satisfactory for further use. Hence, the absolute amplitude of the \( i \)-th ultradian oscillation of species \( X \) \( A_i(X) \) was defined as a difference between the concentration maximum \( [X]_{max,i} \) and the arithmetic mean of concentration values of the two nearest minima, before \( [X]_{min-before,i} \) and after \( [X]_{min-after,i} \) the considered maximum:

\[
A_i(X) = [X]_{max,i} - \frac{[X]_{min-before,i} + [X]_{min-after,i}}{2}, \tag{8}
\]
The relative amplitude \( A_{rel} \). The relative amplitude \( A_{rel} \) of species X is defined as the ratio of amplitudes of a selected reference oscillation after \( A_{ref}(X) \) and before \( A_{ref,0}(X) \) a perturbation:

\[
A_{rel} = \frac{A_{ref}(X)}{A_{ref,0}(X)}.
\]

Thus, when \( A_{rel} < 1 \), \( A_{rel} > 1 \) or \( A_{rel} = 1 \), the amplitude of the reference oscillation has decreased, increased or has not changed, respectively.

In connection to the relative amplitude of the reference oscillation, it is convenient to define three reference values of perturbator P designated as \([P]_{low}[P]_{high}\) and \([P]_{min}\). The perturbator can be an internal species (ACTH, CRH, CORT or ALDO) or an externally introduced species that interacts with the internal species. In this study, cortisol usually played this role. \([P]_{min}\) is the concentration of perturbator P for which the relative amplitude \( A_{rel} \) reaches its minimal value. \([P]_{low}\) is the lowest concentration of perturbator P that induces a visible effect on the HPA system’s dynamics (arbitrarily taken to change the amplitude of the reference oscillation for more than \( \pm 2.5\% \)). \([P]_{high}\) is a non-zero concentration of perturbator P for which \( A_{rel} = 1 \). As we shall see later, these values enable us to efficiently compare different dynamic states.

The ultradian and circadian phase angles. The HPA model response to a perturbation is sensitive to the phase angle at which the perturbation is applied. Two phase angles, one with respect to the period of the i-th ultradian oscillation \( T_{u,i} \) and the other with respect to circadian period \( T_c \) \( T_c=24h \) were therefore defined.

The ultradian phase angle \( \Phi_u \). If the ultradian period is the time between two successive minima surrounding the i-th ultradian maximum \( t_{u,\text{min},i} - t_{u,\text{min},i} \), the ultradian phase angle \( \Phi_u \) represents the time passed from the time-point of the first minimum of a perturbed i-th ultradian oscillation \( t_{u,\text{min},i} \) to the time-point of perturbation \( t_{u,p,i} \), (\( t_{u,p,i} - t_{u,\text{min},i} \)) relative to the time-period of ultradian oscillation \( T_{u,i} \):

\[
\Phi_u = \frac{t_{u,p,i} - t_{u,\text{min},i}}{T_{u,i}},
\]

where \( t_{u,\text{min},i} \) is the time-point of its second minimum.

The circadian phase angle \( \Phi_c \). The circadian phase angle \( \Phi_c \) represents the time passed from the time-point of the first minimum of a perturbed circadian oscillation \( t_{c,\text{min},1} \) to the time-point of perturba-

\[
\phi_c = \frac{t_{c,p} - t_{c,\text{min},1}}{T_c}.
\]

Results

An important distinguishing characteristic of stress is its duration. Acute stress is usually defined as an abrupt, short-lasting (seconds to hours timescale) and isolated perturbation, whereas chronic stress is recurring, persisting for several hours a day for weeks, months or longer [41]. Bearing in mind these differences, acute or chronic stress need to be modeled accordingly.

Acute stress

Acute stress was simulated in the form of a single-pulse perturbation, meaning that the computer program stops the integration algorithm at a given time-point, momentarily rising the concentration of the perturbed species for the given amount. The integration proceeded using as initial conditions the new concentration of the perturbing agent, leaving the concentration of other intermediates unaltered and equal to their values just before the algorithm was stopped. Response of the model to perturbations with chief stress hormones, CRH or cortisol, were studied. Both species generated analogous response in the HPA axis. Therefore, we presented here results for perturbations with cortisol only. For comparison with CRH, see references [17-19].

Effect of acute stress intensity and the ultradian phase angle

In the HPA model with ultradian oscillations alone, the response of the HPA system to stress depends critically on the perturbation intensity and the phase angle of the perturbed ultradian oscillation. A detailed analysis of this case can be found in Refs. 17 and 18.

Effect of acute stress intensity and the circadian phase angle

Similar, but somewhat more complex behavior was observed for the model comprising both circadian and ultradian oscillations (Figs. 2 and 3). A stressful stimulus of the same intensity can increase or decrease the amplitude of ultradian oscillations, depending on the phase angle of the reference ultradian oscillation at
Fig. 3 The HPA model response to acute stress elicited during the diurnal/nocturnal phase of the day. In all cases, perturbations are applied at the same ultradian phase angle ($\phi_u = 0.766$). (a) Relative cortisol amplitude ($A_{rel}$) as a function of the exogenous cortisol concentration pulse ($[CORT]$). The perturbations are applied at night, $\phi_c = 0.168$ (■), and daytime, $\phi_c = 0.428$ (○). The horizontal line $A_{rel} = 1$ divides the graphic area into two regions, where the amplitude of the ensuing oscillation is decreased ($A_{rel} < 1$) or increased ($A_{rel} > 1$) in response to acute stress. An intersection between the line $A_{rel} = 1$ and the curve $A_{rel} = f([CORT])$ corresponds to a point at which the concentration of the perturbator does not change the relative amplitude of cortisol. Down-arrows indicate the values ($[P]_{low, night}$, $[P]_{low, day}$, $[P]_{min, day}$) at night, whereas up-arrows denote the corresponding values during daytime ($[P]_{low, day}$, $[P]_{high, day}$, $[P]_{min, day}$). Temporal evolution of cortisol after a perturbation pulse of exogenous cortisol ($0.4 \times 10^{-9}$ mol dm$^{-3}$) applied at the ascending, night-time (b) and descending, daytime (c) phase of the circadian rhythm.
Predictive modeling of the HPA axis

stimulus exerted during the daytime (Fig. 3 a).

Effect of two successive acute stress signals on the HPA system dynamics

Dynamical response of the HPA axis model to two successive cortisol perturbations is presented in Fig. 4. States 1a, 1b, and 1c, is applied at the same ultradian phase angle (\(\phi_u = 0.766\)), during the ascending circadian phase (night for humans, around 04:00). Depending on the intensity of the first perturbation, the response of the HPA system is different: (a) a decrease in CORT amplitude is observed for acute perturbation with [CORT] = 0.075·10\(^{-9}\) mol dm\(^{-3}\); (b) no perturbation, [CORT] = 0 mol dm\(^{-3}\); (c) increase in CORT amplitude is observed for acute perturbation with [CORT] = 0.4·10\(^{-9}\) mol dm\(^{-3}\). Thus, three distinct dynamical states (a, b, c) are induced, characterized with different ultradian dynamics. The new dynamical states, characterized by different ultradian oscillation amplitudes (\(A_{\text{day}}\) (CORT)) at daytime (for humans, around 10:00): \(A_{\text{day}}\) (CORT) = 0.5190·10\(^{-9}\) mol dm\(^{-3}\), \(A_{\text{day}}\) (CORT) = 1.1445·10\(^{-9}\) mol dm\(^{-3}\), and \(A_{\text{day}}\) (CORT) = 5.8210·10\(^{-9}\) mol dm\(^{-3}\). These secondary perturbations, indicated with arrows 2a, 2b and 2c, elicit different responses of the HPA system. (d) Curves A (■), B (★) and C (●) in the comparative diagram describe differential responses of the HPA system’s different initial states a, b, c to described cortisol pulse designs (2a, 2b, 2c), respectively. (e) The sensitivity parameters \([P]_{\text{low}}\), \([P]_{\text{high}}\) and \([P]_{\text{min}}\) depend linearly on the amplitude of the dynamical states (a, b, c) that are established after the first perturbation.
These results demonstrate that the sensitivity of the HPA system to perturbations depends on the history of the system, i.e. on the state dictated by the first perturbation. Furthermore, these results indicate that the amplitude of the perturbed ultradian oscillations is an important determinant of the HPA axis response to stress (Fig. 4e).

**Chronic stress**

Chronic stress can be regarded as a physiological and behavioral state of an organism that has emerged as a response to recurrent homeostasis disturbing challenges that persist for weeks, months or longer [42]. An organism may be exposed to chronic stress continuously, or through cascades of inappropriate requirements whose cumulative effect is achieved by the onset of a new stimulus before the negative effect of the previous one has died out. In our previous study, chronic stress was modeled by applying a sequence of CRH pulses, whose amplitude and time of onset were randomly varied [19]. Intermittent perturbation of the HPA axis dynamics yielded a complex cortisol oscillation pattern, with ultradian oscillations of randomly varying amplitudes and frequencies (Fig. 6 in reference [19]). Different perturbation patterns generated different response patterns, but even though the details were different, one thing was common – chronic stress altered the mean CRH concentration. This is in line with experimental findings showing that chronic stress changes the mean CRH levels [8], triggering further modifications of the HPA axis dynamics, which altogether are often associated with metabolic and psychological impairments [7, 8, 43]. Therefore, we emulate here chronic stress by varying the parameters \( k_0, d_1 \) and \( d_2 \) that define the CRH dynamics in our model. On one hand, an unambiguous relation exists between the parameter \( k_0 \) and the stable stationary state CRH concentration \([\text{CRH}]_{SS}\) (Eq. (6)) and the parameter \( d_1 \) and the mean daily CRH concentration in the hypophyseal-pituitary portal vessels \( <\text{CRH}> \) (Eq. (7)), on the other. Therefore, all results related to chronic stress are presented as a function of the biologically relevant parameters \([\text{CRH}]_{SS}\) and \( <\text{CRH}> \). The parameter \( d_2 \) defines the amplitude of the circadian CRH oscillation (Eq. (5)).

**Effect of chronic stress on ultradian oscillations**

The stationary state value of CRH concentration \([\text{CRH}]_{SS}\) has a decisive effect on the ultradian dynamics of the HPA system (Figs. 1a and b). The corresponding bifurcation diagram, showing how the dynamic state of the HPA system changes depending on the control parameter \([\text{CRH}]_{SS}\) is given in Fig. 5a. In Fig. 5a, the oscillatory states are presented by pairs (minima and maxima) of cortisol concentrations in the ultradian oscillations, whereas single points denote stable stationary states. Obviously, the ultradian cortisol oscillations exist within a defined range of \([\text{CRH}]_{SS}\) values, whereas stable steady states were observed outside this interval.

**Effect of chronic stress on ultradian and circadian dynamics**

In the model with coupled ultradian and circadian rhythms, the mean daily concentration of CRH in the hypophyseal-pituitary portal vessels \( <\text{CRH}> \) represents the chronic stress parameter, since the hormone concentration levels can be changed by chronic stress for longer periods of time or permanently [4, 8]. Alteration of \( <\text{CRH}> \) evoked qualitative alterations in the HPA dynamics (Fig. 5b) that were analogous to the bifurcation pattern observed for ultradian oscillations alone (Fig. 5a) – the oscillations existed only within a certain range of \( <\text{CRH}> \) values, whereas stable steady states were observed outside this interval. However, \( <\text{CRH}> \) affected the dynamics of cortisol release during the daytime (Fig. 5b, squares) differently than during the night (Fig. 5b, circles).

In addition, bifurcation diagrams shown in Fig. 5b illustrate that the amplitude of ultradian cortisol oscillations initially increases and thereafter decreases as a function of \( <\text{CRH}> \) as the control parameter. Hence, continuous elevation of \( <\text{CRH}> \) levels can produce both an increase and a decrease of ultradian cortisol amplitudes and, consequently, opposite responses of the HPA system to stress. This observation has very important implications – as the amplitude of ultradian cortisol oscillations is the measure of the HPA system’s capability to protect itself against acute stress, and the \( <\text{CRH}> \) is the measure of the chronic stress intensity, we can conclude that mild elevation of cortisol levels due to chronic stress may initially exert beneficial effects on the HPA system capacity to cope with acute external perturbations. Still higher stress will reduce the amplitude of the ultradian HPA oscillations, thus reducing the HPA system’s capacity to respond to stress. Eventually, the dynamical regulation capacity of the HPA axis may be lost as the system undergoes a transition to a stable steady state.
Predictive modeling of the HPA axis

Fig. 5  (a) Bifurcation diagram showing the ultradian cortisol dynamics as a function of $[\text{CRH}]_{SS}$. Single points denote stable stationary states, whereas the oscillatory states are presented by pairs (minima and maxima) of cortisol concentrations in the ultradian oscillations. Vertical lines indicate three different dynamic states: (1) at low CRH levels, $[\text{CRH}]_{SS} = 0.82 \cdot 10^{-8} \text{ mol dm}^{-3}$, a low cortisol concentration steady state, denoted as 1 in Fig. 1b, is established. (2) At $[\text{CRH}]_{SS} = 1.00 \cdot 10^{-8} \text{ mol dm}^{-3}$, ultradian cortisol oscillations, denoted as 2 in Fig. 1b, are established. (3) At high CRH levels, $[\text{CRH}]_{SS} = 1.26 \cdot 10^{-8} \text{ mol dm}^{-3}$, a high cortisol concentration steady state, denoted as 3 in Fig. 1b, is established. (b) Bifurcation diagrams showing the ultradian cortisol dynamics as a function of $<[\text{CRH}>$. In a model with circadian regulation, nocturnal (■) and daytime (○) ultradian cortisol oscillations are different. Vertical lines indicate three different dynamic states: (4) at low $<[\text{CRH}> = 0.88524 \cdot 10^{-8} \text{ mol dm}^{-3}$ (basic physiological conditions), ultradian oscillations are observed during the daytime whereas the amplitude of nocturnal ultradian oscillations is very small (this region is magnified in the insert; corresponding CORT time series are shown in Fig. 1d). (5) At $<[\text{CRH}> = 0.97 \cdot 10^{-8} \text{ mol dm}^{-3}$, ultradian oscillations with relatively large amplitudes are observed during daytime and at night (corresponding CRH and CORT time series are shown in Fig. 5c and 5d, respectively). (6) At $<[\text{CRH}> = 1.07 \cdot 10^{-8} \text{ mol dm}^{-3}$, the ultradian oscillatory dynamics is reversed, showing large-amplitude cortisol oscillations at night, while small-amplitude cortisol oscillations during daytime (corresponding CRH and CORT time series are shown in Fig. 5e and 5f). In all cases, $d_2 = 0.957$. 
Effect of CRH circadian amplitude on the HPA system dynamics

Variation of the circadian amplitude of CRH via the control parameter $d_2$ also affects the HPA system’s dynamics (Fig. 6). Parameter $d_2$ multiplies time-dependent, periodic part of the function $D$ (Eq. 5), therefore governing the amplitude of the extrinsic circadian rhythm. As before (Fig. 5b), the daytime cortisol discharge dynamics (Fig. 6a) was differently affected than its discharge dynamics during the night (Fig. 6b). In the absence of the CRH circadian dive ($d_2 = 0$), CRH was continuously secreted (Fig. 6c) and the cortisol discharge dynamics showed properties of ultradian secretion (Fig. 6d). By increasing the circadian CRH ampli-
As expected, cortisol and CRH perturbations inevitably change the dynamics of the examined HPA axis hormones (Figs. 2-7). At present, our model predictions are in quantitative agreement with experimentally measured aldosterone (data not shown) and cortisol levels (Figs. 1-7), whereas for ACTH and CRH, only qualitative agreement is achieved (The concentrations of ACTH and CRH are not yet optimized and deviate from experimentally measured values. This is a consequence of the low-dimensionality of our model – in order to maintain a small number of variables, precursor species are excluded from the present model. Even so, the generality of conclusions reported in this study is not compromised by this discrepancy). Acute stress exerts transient effects on the HPA system dynamics (Figs. 2-4), whereas the effect of chronic stress is long-lasting, shifting the whole HPA system to new dynamic states (Figs. 5-7). These newly achieved dynamic states are characterized by different amplitudes, frequencies and the general appearance of the ultradian oscillations of all considered HPA system hormones.

In line with model predictions, according to which the HPA axis response to stress depends on the ultradian phase (Fig. 2), there are several experimental observations showing that acute stress induces different effects depending on the phase of the ultradian secretion pulse at which the stress has been induced [11, 13-15]. For example, it has been found that the degree of prednisolone-induced rapid inhibition of cortisol was greater when the time after prednisolone injection to pulse onset was longer [11]. This indicates that HPA axis responsiveness to perturbations with prednisolone depends on the ultradian phase of the endogenous cortisol oscillations.

Dependence of corticosterone concentration response to acute stress on the phase (descending/ascending) of the ultradian rhythm at the moment the stress commences has also been observed in in vivo experiments with rats [13-15]. It has been shown in these studies that the same acute stress (5 min white noise stress of 114 dB) applied either in the ascending (secreting) and interpulse phase or during the descending (non-secreting) phase of corticosterone ultradian rhythm evoked significant increase of corticosterone level only when it had been applied during the ascending [13-15] or interpulse [15] phase of ultradian basal corticosterone pulse. No significant response was detected when the

**Effect of the chronic stress on the frequency of ultradian oscillations**

When uncoupled from the circadian rhythm, the frequency of ultradian cortisol oscillations did not change over time (Fig. 1b). Conversely, in the case of coupling between circadian and ultradian oscillations, ultradian oscillation frequency became time-dependent, showing the highest frequency during the circadian peak (Fig. 7, curve 1).

Chronic stress, modeled as before by varying the mean daily concentration of CRH concentration in the hypothalamic-pituitary portal vessels (<[CRH]>), increased the frequency of the ultradian oscillations over the 24 h course (Fig. 7, curves 2 and 3).
stressful stimulus was applied during the descending phase [13-15].

There are also converse reports, showing that intravenous injection of specific dosage of CRH (2 μg) administered to rats during the rising or falling phase of the corticosterone ultradian oscillations failed to induce significant difference between the two rat groups [14]. A possible explanation may be that the dosage tested is not adequate. HPA response to stress depends not only upon timing, i.e. the phase at which the treatment was applied, but also upon the intensity of the perturbation (Fig. 3). Thus, another CRH dosage could have been more appropriate.

Similarly, it has been demonstrated that cortisol (and ACTH) response to intravenously administered prednisolone did not significantly depend on time of day the administration occurred [11]. However, these experiments were performed in the morning and afternoon, but not during the night. In addition, only one prednisolone dose was tested. Therefore, before ruling out the possibility that diurnal dependence of the HPA axis response to prednisolone (or other externally induced glucocorticoids) exists in humans, additional experiments might be useful.

Due to its long duration and/or frequent incidence, chronic stress may alter receptor localization, rates of chemical reactions and transporting processes in different brain regions, such as the hippocampus, amygdala and prefrontal cortex [44-47], causing altered patterns of activity observable by PET (positron emission tomography), fMRI (functional magnetic resonance imaging), MEG (magnetoencephalography) or EEG (electroencephalography). Under certain conditions the allostasis, i.e. dynamic regulatory mechanisms may no longer be efficient, causing allostatic overload [28, 47]. Such changes may be reflected by the bifurcation diagrams shown in Figs. 5 and 6. Allostatic overload of the HPA system, i.e. the incapability of the HPA axis to cope with the “external pressure” may occur in the form of bifurcation points (Figs. 5 and 6). Transition through a bifurcation point may indicate that the dynamic regulatory mechanism has collapsed, which may be possibly interpreted as a condition that leads to disease onset. Such dynamic modifications may be reversible, but may also be irreversible, showing hysteresis. Thus, even though the stress has ceased, the HPA system does not return to its original physiological state but may end up in a new nonequilibrium stationary state.

The HPA system dynamics is tightly related to sleep-regulation. Furthermore, manipulation of the sleep-wake pattern induces subtle changes in the HPA system dynamics. For example, CRH and ACTH administration have been found to impair sleep [48], whereas chronic insomnia is associated with increased secretion and number of ultradian pulses of cortisol and ACTH during the 24 h period [49]. Our model correctly predicts that increase in the mean daily concentration of CRH in the hypothalamic-pituitary portal vessels (<[CRH]> alters the HPA system dynamics (Fig. 5), eventually leading to a radical disruption of the ultradian secretion dynamics. Under such extreme conditions, the ultradian secretion becomes more active during night (Fig. 5f) rather than during daytime. This inverse secretory activity, with more pronounced nocturnal cortisol and ACTH pulses, was also observed in patients suffering from chronic insomnia [49].

Bifurcation diagrams shown in Fig. 5 may be regarded as dynamical explanation for the adverse response of the HPA axis to chronic stress, offering a theoretical account for the apparently contradictory observation reported in the literature that chronic stress can yield both hypo- and hypercortisolism [43].

It is also well known that stress induces changes in the immune system function. This modulation is achieved via the effect of glucocorticoid hormones and other components of the HPA axis [46-48, 50]. Bifurcation analysis may be helpful for understanding why chronic stress alters the immune system function distinctly and in a non-linear fashion [41, 51].

**Conclusion**

Adaptive transformations in HPA axis dynamics following acute and chronic perturbations were studied using a stoichiometric model of the HPA axis. Model predictions were related with experimental and clinical observations reported in the literature. We outline briefly here the most relevant positive correlations between model predictions and real observations.

As expected, acute stress transiently perturbs the HPA axis dynamics. The response of the HPA axis is complex, depending on the intensity and the time of stress onset (Figs. 2-4). Numerical simulations (Figs. 2, 3) reproduced the empirically well established notion that the same stimulus applied at slightly different time points, may induce different, even opposite changes in cortisol levels in the same individual [13-15].
Predictive modeling of the HPA axis

Modeling elucidates why biphasic dynamic regulation of the HPA axis activity, achieved through the coupling of ultradian and circadian regulation of HPA hormone release, is an efficient strategy for stress modulation, providing at the same time robustness and plasticity. Modeling also reveals the boundaries under which this regulatory mechanism is operational (Figs. 5 and 6). Numerical simulations clarify why stress-induced increase in mean daily CRH levels in the hypothalamic-pituitary portal vessels (control parameter $\langle[CRH]\rangle$ in the model) may increase or decrease mean cortisol levels, showing that there is no contradiction in such observations. In this respect, modeling could serve as theoretical background to answer how and under which circumstances chronic stress leads to hyper- or hypocortisolism, opposite states observed in stress-related illnesses such as major depression or posttraumatic stress disorder [43].

Numerical simulations also reveal that ultradian cortisol dynamics may be reversed under chronic stress, causing more active hormone release during the night instead of daytime (Fig. 5f). Such changes in the HPA axis secretory activity may be the primary cause of an array of changes potentially leading to insomnia [49].

Modeling predicts that chronic stress initially increases the amplitude of ultradian cortisol oscillations (Fig. 5a, b). This effect may be beneficial at first – as the amplitude is increased the sensitivity of the HPA axis to acute stress is reduced. Hence, the increasing amplitude of the ultradian cortisol pulsation may act as a protective, tolerance mechanism. Such regulatory mechanism may also explain why we are best fit for interactions/activity during a certain part of the day.

Modeling also predicts that qualitative changes in the HPA axis dynamics may occur under chronic stress (Fig. 5d, f). Such global changes in the HPA axis dynamics may explain how allostatic overload occurs and why under certain conditions, the dynamic regulatory mechanism is not turned on, or why a certain response is not turned off when it is no longer needed.

**Appendix: Using modeling as a tool for designing glucocorticoid pharmacotherapy distribution strategies**

Our results imply that the effect of pharmacotherapy may depend on therapy distribution strategies in a complex fashion. However, our results also suggest that even very intricate behavior of the HPA axis is deterministic in nature, stemming from the intrinsic rhythmicity of the HPA axis [10] and can therefore be modeled and predicted. This means that insights on HPA axis dynamics that are revealed by modeling may be used as cues for designing treatment strategies. In that regard, we wish to address the significance of timing in glucocorticoid therapy.

Significance of circadian timing for administration of glucocorticoid therapy has been acknowledged in patients suffering from asthma [52] or Addison’s disease [53]. Novel strategies of delayed and sustained cortisol release developed for the treatment of these conditions are expected to mimic the circadian cortisol rhythm more accurately, thus reducing the side-effects of the therapy [54, 55]. In rodents, an infusion protocol capable of mimicking corticosterone ultradian rhythm has been developed, enabling ultradian control of cortisol amplitude and frequency in adrenalectomized animals [10]. Some of these aspects have been also recognized in the alternate day therapy with glucocorticoids, where empirical data show that the application of the same dose of glucocorticoids every second morning is more beneficial to patients than if given every day [32, 56, 57].

The concept of dynamics management

The complexity of the HPA axis dynamics requires a comprehensive approach when treating its malfunctions. Alongside with gene manipulation or development of novel, specific drugs, additional avenue emerges, in which the dynamics of the HPA axis is supposed to be manipulated in a relatively simple manner, by stressing the system at the right time (phase angles of oscillations) with the right concentration(s) of perturbing internal species (glucocorticoids, CRH, ACTH, aldosterone etc.). We refer to this approach of controlled change of dynamics towards desired direction by utilizing appropriate perturbation as *dynamic management*. This approach could help redesigning the existing therapeutic procedures (e.g. glucocorticoid administrations), making them more efficient and with less side-effects. If a patient’s condition allows for, before starting any therapy, a screening of the patient’s HPA axis dynamics should be made with high-enough resolution in order for the ultradian oscillations to be sufficiently characterized. This could be particularly important bearing in mind the interpersonal and intrapersonal differences in the HPA axis dynamics [29, 30], i.e. the fact that the initial state of the axis at the time
of the onset of the therapy is very dissimilar, not only among different individuals, but as well within the same individual over the course of time.

**Significance of the initial state of the HPA axis**

The initial dynamical state of the HPA axis during the onset of treatment is a major determinant of the HPA systems response to the applied treatment. In medical treatments with glucocorticoids, these drugs do not necessarily have the same effects when applied to patients in different physiological states, such as milder form of infection, or the state of acute shock. In the first case, well-established doses of glucocorticoids exert their inhibitory effect on cortisol levels, but in the latter, they could fail in doing so. In a case-study on patients with hypercortisolism evoked by the state of circulatory (septic or non-septic) shock, intravenous infusion of dexamethasone failed to suppress cortisol concentration elevated due to circulatory shock, in contrast with the control group of healthy subjects, where dexamethasone infusion exerted its common behavior – the complete suppression of cortisol [58].

Presumably, the circulatory shock induced alteration of the HPA axis dynamics, shifting the axis to a novel non-basal dynamical state, so when the therapy with dexamethasone had been administered, the axis was unable to exert the same effect (suppression) as it does in basal conditions. Given the results presented in this paper, there probably exists a specific dosage of dexamethasone, different than that for basal dynamical state, at which the cortisol suppression within the HPA axis of non-basal dynamics can occur.

The unique interrelation between the initial state and subsequent dynamical HPA response to perturbations can also be employed for making differential diagnoses. A good example is the low-dose dexamethasone-CRH stimulation test, a diagnostic test clinically used to distinguish patients with Cushing’s syndrome from those of pseudo-Cushing’s state [59]. In this study, two groups of patients responded differentially to the test: low-dose dexamethasone administration (low-dose dexamethasone suppression test) suppressed cortisol and ACTH levels in patients with pseudo-Cushing’s states to a larger extent than in those with Cushing’s syndrome, while the Cushing’s syndrome patients showed greater response (cortisol and ACTH elevation) than the ones with pseudo-Cushing’s states to the subsequent CRH administration (CRH stimulation test). Combined, the two tests (two perturbations) evoked a unique response within the two patient groups, distinctive enough to set the criterion for discrimination of the two dysfunctions. Thus, although both conditions are associated with hypercortisolism, their original pathophysiology [60] i.e. initial states are dissimilar, and this difference was reflected in the low-dose dexamethasone-CRH stimulation test with very high specificity, sensitivity and diagnostic accuracy.

As the understanding of the complexity of the HPA axis increases and experimental procedures for tracking and modifying its activity in time advance, it might even be possible to make personal “phase diagrams” of HPA axis response to stress. These diagrams could be used as indicators for designing the most appropriate therapy for each individual patient.

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**References**


