Long-Lived Consumers, Intertemporal Bundling, and Tacit Collusion

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Abstract

In a repeated price game with long but finitely-lived consumers, the use of staggered long-term contracts allows firms to earn positive profits for a wider range of discount factors and market structures than without intertemporal bundling. Because consumers anticipate a price war, intertemporal bundling reduces the gains from business-stealing while leaving the cost of the resulting price war is unchanged. Though less empirically relevant, we also show that in a repeated price game with infinitely-lived, and arbitrarily-small, consumers, firms can use a menu of single-period and infinite-length contracts, to earn strictly positive profits for any discount factor or market structure.

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1 Introduction

Intertemporal bundling is a common practice. Serial products, such as newspapers and magazines, and services, such as cellular telephone and DSL internet access, are often sold using multi-period service agreements that require customers to commit to multi-period service agreements.\(^1\) We show that such intertemporal bundling may facilitate tacit collusion. More precisely, we show that if consumers are small and long-lived and then firms competing with one another will be able to sustain higher profits when they are free to intertemporal bundle their products.

Our main result, that intertemporal bundling facilitates tacit collusion, is counterintuitive in two respects. First, it implies that long-lived consumers are worse off than short-lived consumers. Indeed, we find that long-lived consumers would collectively be better off if they could commit to act like short-lived consumers. And second, increasing the time between offers is known to make tacit collusion more difficult. However, it is actually easy to see why intertemporal bundling does not have the same effect. This is because, intertemporal bundling does not restrict the time between offers, only the time between consumers’ purchase decisions. In particular, punishments can still be just as swift and consumers realize this when making their purchase decisions.

To better understand the latter point, and more generally, to better understand the role of intertemporal bundling in our model, it is useful to first ask what would happen if firms were restricted to offer only long-term contracts. For example, if firms must offer twelve-month contracts that begin on January 1st tacit collusion more difficult than if they must offer one-month contracts that begin on the first of each month. This is because intertemporal bundling increases the firms’ short-run deviation profits but not their equilibrium path profits.

Now suppose that firms are restricted to offer twelve-month contracts that begin on the 1st of any month (not just January 1st). If every firm offers a twelve-month contract that

\(^1\)Not all multi-period service agreements are examples of intertemporal bundling. In particular, some multi-period service agreements leave the consumer free to change service providers at anytime, and instead only restrict the firm, either by limiting its ability to increase price or to terminate service.
begins on January 1st, the firms’ deviation profit has not increased. This is because forward-looking consumers fully anticipate that a price war will begin in February, one month after the deviation occurs, so the largest profit a deviator can capture is the entire market surplus for just one month. If the deviator attempts to capture more surplus from consumers, the consumers will forgo their consumption for one month and then purchase from the rival firms at marginal cost. The important intuition is that the deviation profits are proportional to the time between offers, not the length of the contracts.

In fact, intertemporal bundling makes it easier to tacitly collude, not harder. One way to see why this is true is to imagine that firms can temporally segment the market. When the market is temporally segmented, the ability to capture the entire market through a price deviation is removed. For example, suppose firms offer twelve-month contracts and for exogenous reasons consumers’ contract renewals are staggered. Then each month only one twelfth of all consumers are looking to sign a new contract. A deviating firm can increase its short-run profits by lowering its price, but it can only capture one month of surplus from one twelfth of the consumers. Yet the cost of a price cut is the forgone future profits in all twelve segments, so tacit collusion is easier to sustain.

Formally, we show that intertemporal bundling facilitates higher profits even when consumers are not exogenously staggered and even when the duration of firms’ contracts is chosen at the time the contracts are offered. Because we assume all consumers are present from the start of time, when firms offer staggered long-term contracts in equilibrium, they must initially offer consumers incentives to induce them to subsequently stagger their purchases. Despite the short-run costs of these incentives, they facilitate higher profits than can be sustained when firms face short-lived consumers (or when firms are barred from using multi-period contracts).

While they are empirically more plausible, equilibria with staggered long-term contracts are just one of a variety of ways that firms can exploit intertemporal bundling. Moreover, they are not necessarily the most profitable way. In particular, we show that when consumers are infinitely lived, firms can always attain the highest feasible profit by offering a menu of single-period and infinite-length contracts while they may not be able to do so with staggered contracts.
The paper is organized as follows. We begin with a brief review of the related literature. In Section 3 we describe the model and review the set of SPNE when firms face short-lived consumers (or are restricted to one-period contracts). In Section 4, we consider a model with long but finitely-lived consumers and show that intertemporal bundling using overlapping, or staggered, finite-length contracts facilitates tacit collusion. In Section 5 we consider a model with infinitely-lived consumers and show that intertemporal bundling facilitates tacit collusion for any market structure and discount rate, however the equilibrium strategies that support tacit collusion, offering a menu of short and infinite-length contracts in the first period, are not feasible when consumers are finitely lived or of finite size. In Section 6, we consider a model with a finite number of infinitely lived consumers and show that intertemporal bundling using menus of one period and multi-period finite-length contracts facilitates tacit collusion in a way that is robust to deviations by a positive measure of consumers. Section 7 concludes.

2 Literature

Intertemporal bundling is a special case of product bundling. So by analogy, the multi-product bundling literature has suggested several potential rationales for intertemporal bundling.\textsuperscript{2} For example, Adams and Yellen (1976) implies that if consumers’ valuations are negatively correlated across time, firms with market power can reduce the deadweight loss by intertemporal bundling. And Whinston (1990) implies that intertemporal bundling can be a mechanism for leveraging market power into the future.\textsuperscript{3}

A few papers have looked more specifically at intertemporal bundling. DeGraba and Mohammed (1999) studied how a capacity-constrained firm can induce a buying frenzy by first selling only a bundle of multiple products and then only later selling products

\textsuperscript{2}Perhaps the most common given justification for intertemporal bundling is that there are fixed costs of establishing service. While these costs undoubtedly exist in many environments, it is less apparent why these costs must be recouped with service agreements rather than installation fees.

\textsuperscript{3}In a related model, Cabral and Villas-Boas (2005) show that bundling of unrelated products can lead to more homogeneous tastes and thus more intense competition among firms and suggest that this may be a rationale for intertemporal bundling.
individually. In this frenzy, consumers who place high values on one product but low values on the other are forced to purchase a bundle of both products in the first period, because they anticipate that their preferred product will be rationed in the second period. Rationing is a self-fulfilling prophecy when consumers respond to such expectation by rushing to purchase the bundle. While the authors describe this tactic as “intertemporal mixed bundling,” the firm bundles across products, not across time. However, in a closely related paper, Mialon and Chen (2008) consider a version of DeGraba and Mohammed’s model in which consumers make repeat purchases and the monopolist bundles consumption across time.

In the behavioral economics literature, Loewenstein, Donoghue, and Rabin (2003) considered a model of projection bias. Consumers who possess “projection bias” rely too much on their current tastes to estimate their future preferences. The authors conjectured that a firm may use intertemporal bundling to take advantage of consumers with projection bias who currently place a high value on its product. DellaVigna and Malmendier (2004) analyze firm behavior when customers are time inconsistent and partially naïve about it. They show that when a product brings consumers immediate benefit and postponed cost, firms tend to specify a high per usage fee in a subscription contract and when a product brings consumers immediate cost and postponed benefit, firms tend to set a per usage fee below the marginal cost.

Our paper is also related to the literature on multimarket contact. Bernheim and Whinston (1990) argued that multimarket contact can facilitate tacit collusion when the markets are asymmetric in specific ways. Although they do not analyze it, an obvious asymmetry which facilitates tacit collusion is temporally separated markets (e.g., all firms set prices annually, but they set their prices in market A in January, in market B in February, in market C in March, etc.). We argue that intertemporal bundling can be used to endogenously separate consumers into different markets. Once the markets are separated, multimarket contact facilitates tacit collusion because firms cannot steal business simultaneously in all twelve markets, but their rivals can retaliate in all twelve markets.

Finally, there are other papers in which consumers’ anticipation of future retaliatory price wars affects firms ability to sustain high profits. Ausubel and Deneckere (1987) and Gul (1987) consider related oligopoly models of durable-goods pricing and show that the Coase conjecture does not hold with more than one firm because firms find it easier to
tacitly collude as the time between offers shrinks. Consumers who rationally anticipate a price war following a price deviation will have an incentive to ignore the price deviation and wait for the price war to begin, so a deviator will only be able to capture the time value of the consumers’ surplus. Since durable goods can be thought of as a sequence of non-durable goods that have been intertemporally bundled, one can interpret these as models in which consumers are purchasing a service that has been exogenously intertemporally bundled. An important way in which our paper differs is that we allow firms to endogenously chose whether or not to intertemporally bundle their product.

Dutta, Matros, and Weibull (2007) consider a related model in which oligopolists sell to overlapping generations of consumers who demand at most one unit of the good. If consumers live a single period, the model reduces to a simple repeated price game while if they live forever it is equivalent to a durable good problem. They find that all else equal, as consumers live longer (equivalently old customers become more and more important relative to new customers) tacit collusion is more difficult to sustain. The deviation profits increase as the fraction of old consumers grows, because holding prices fixed the total number of old consumers in the market who have not yet purchased the good (because their valuations are below the historical market price) is growing relative to the number of new consumers. This affect does not arise in our model because we assume consumers have homogeneous valuations.

And, in a related model of non-durable goods, Liski and Montero (2006) show that tacit collusion is easier to sustain when firms can offer forward contracts. They assume that there are two contracting stages prior to every consumption period. They consider an equilibrium in which consumers sign contracts in the first contracting period. If firms react to a deviation in the first contracting period by cutting price, no firm has an incentive to deviate because forward looking consumers will wait and purchase in the second contracting period at an even lower price. And since much of the demand was already served in the first contracting period, there is also little incentive to deviate in the second.
3 The Model

We generalize a standard infinitely-repeated, oligopoly price game by assuming that consumers are long lived and that firms can intertemporally bundle their output. We assume \( n \) infinitely-lived firms sell a homogeneous and perfectly divisible product to a continuum of long-lived, homogeneous consumers. The firms have zero unit costs and each consumer’s valuation is \( V \) per unit of the good. Aggregate demand in each period is constant and normalized to one. Firms and consumers have a common, strictly positive discount factor, \( \delta \in (0, 1) \).

Each period, firms simultaneously announce a menu of contracts, which can vary in both price and length, and then consumers either choose a contract from among the firms’ menus of offers or consume their outside option. The outside option for consumers who do not have a prior contract is to consume nothing and derive valuation 0 for that period. The outside option for consumers who have a prior contract is to continue consuming from their existing contract (which they are free to dispose of at any time).

Firms’ one-period contract offers are denoted simply by their price, \( p \). Firms’ multi-period contracts are denoted by price-duration pairs, \( \{P, k\} \), where \( P \) is the present discounted value of the stream of per-period payments specified in the contract and \( k \) is the contract length. However, the exposition of the paper is made easier when we suppose that multi-period contracts require a stream of payments as opposed to a single lump sum payment up front (in particular this makes it easier to compare a multi-period contract to a series of single-period contracts). Since contracts are binding and both firms and consumers share a common discount factor, any two \( k \)-period contracts whose streams of payments have the same present discounted value are equivalent. So we will refer to our contracts as requiring a stream of payments. For example, a finite length, multi-period contract, \( \{P, k\} \), can be written as a requirement to pay a stream of \( k \) identical payments, \( p_k \), where \( P = \sum_{s=1}^{k} \delta^{s-1} p_k \).

Since there are a continuum of consumers, we assume that firms’ strategies depend only on other firms’ actions and on the fraction of consumers choosing each contract, not on the actions of an individual consumer. This means that we do not need to specify firm’s strategies following a deviation by an individual consumer.
An important benchmark is the case in which intertemporal bundling is not feasible. If the firms are restricted to make one-period offers, the SPNE of the game are well-known (see, for example, Tirole, 1988).

**Claim 1.** When intertemporal bundling is not feasible, then i) if \( n \leq \frac{1}{1-\delta} \) then any level of profit between zero and the monopoly profit is sustainable in a symmetric SPNE, and ii) if \( n > \frac{1}{1-\delta} \), the unique sub-game perfect Nash equilibrium outcome is zero profits (marginal cost pricing).

Note that Claim 1 holds regardless of how long consumers live. That is, the presence of long-lived consumers alone does not facilitate tacit collusion.

## 4 Finitely-Lived Consumers

We begin by considering a model in which consumers live for a finite number of periods, denoted by \( l \). Each period new consumers enter the market replacing the consumers whose lives are ending, so the number of consumers in the market and the distribution of their remaining lives is stationary over time). In order to assure that the number of consumers and the distribution of their remaining lifetimes is stationary, is constant from the start of the game, we assume that in period 1 there are an equal number of consumers in each generation, so a fraction \( \frac{1}{l} \) of the consumers live for one period, another fraction \( \frac{1}{l} \) of the consumers live for two periods, and so on. We also normalize the total number of consumers in period 1 to 1.

In an overlapping generations model with finitely-lived consumers, a natural equilibrium to consider is one in which consumers purchase lifetime contracts. That is, in each period after period 1, firms offer new customers \( l \) period contracts. Of course, in period 1, in order for all consumers to be able to purchase lifetime contracts, firms must offer menus of contracts of length 1 to \( l \) since the consumers in period 1 have remaining lifetimes that vary between 1 and \( l \).

The following definition of a *Staggered Contract Equilibrium* (SCE) captures this idea. Note that in our definition we do not restrict a *Staggered Contract Equilibrium* to be an
equilibrium in which all consumers purchase only lifetime contracts, but in our analysis of finitely lived consumers we will consider only Staggered Contract Equilibria with lifetime contracts.

**Definition.** Staggered Contract Equilibrium: *In each period after the initial period firms offer only k-period contracts \( \left( \sum_{s=1}^{l} \delta^{s-1} p, k \right) \) and a fraction 1/k of all consumers purchase these contracts and divide their purchases equally across the firms. In period 1 firms offer a menu of contracts including one-period contract at a price \( p_1 \), a two-period contract \( (p_1 + \delta p, 2) \), and an \( j \)-period contract \( (p_1 + \sum_{s=2}^{j} \delta^{s-1} p, j) \), \( \forall j \leq k \) and divide their purchases equally across the firms and their contracts. Following any firm deviation, firms offer one-period, marginal-cost contracts forever. Following any deviation by a consumer, firms and other consumers continue to play their equilibrium path strategies.*

**Proposition 1.** With finitely-lived consumers, a Staggered Contract Equilibrium with lifetime contracts exists if and only if

\[
n \leq \frac{1}{1 - \delta} \sum_{j=1}^{l} \delta^{j-1}. \tag{1}
\]

The range of industry profit levels that can be supported in a Staggered Contract Equilibrium with lifetime contracts is

\[
\left[ 0, \frac{V}{1 - \delta} \right] \cup \left\{ 0 \right\} \cup \left[ \frac{n\delta V}{\sum_{j=1}^{l} \delta^{j-1}}, \frac{n \delta}{n - 1} \frac{\delta}{1 - \delta} V \right] \quad \text{if } n \in \left( \frac{1}{1 - \delta}, \frac{1}{1 - \delta} \sum_{j=1}^{l} \delta^{j-1} \right),
\]

and \( \left\{ 0 \right\} \) otherwise.

**Proof:** Clearly a necessary condition for all consumers (including those who live just a single period) to purchase the equilibrium contracts in period 1 is that their consumer surplus is non-negative, or \( p_1 \leq V \) and \( p_1 + \sum_{s=2}^{j} \delta^{s-1} p \leq V \sum_{s=1}^{j} \delta^{s-1} \) for all \( j = 2, \ldots, l \). Similarly, a necessary condition for all consumers to purchase the equilibrium contracts in every other period is \( \sum_{s=1}^{l} \delta^{s-1} p \leq \sum_{s=1}^{l} \delta^{s-1} V \), or \( p \leq V \). Given any \( p_1 \leq V \) and \( p \leq V \) it is clear all consumers will purchase the lifetime contracts intended for them.
Given that every consumer purchases the contract intended for them, in Period 1, each firm’s equilibrium path profit is
\[
\frac{1}{n} \left[ p_1 + \delta \frac{p}{1 - \delta} \right].
\] (2)

Since consumers anticipate a price of zero following any deviation, a deviation will only be accepted if it is more attractive to the consumer than signing another firm’s one-period contract at the price \(p_1\) and waiting for the ensuing price war. Therefore, the highest profit a deviating firm can earn in Period 1 is \(p_1\), and no profitable deviation exists in Period 1 if
\[
\frac{1}{n} \left[ p_1 + \delta \frac{p}{1 - \delta} \right] \geq p_1,
\] (3)
or equivalently,
\[
p_1 \leq \frac{1}{n - 1} \delta \frac{p}{1 - \delta}.
\] (4)

In Period 2 and beyond, each firm’s equilibrium path profit is
\[
\frac{1}{n} \frac{p}{1 - \delta},
\] (5)
which includes revenues of
\[
\frac{1}{n} \sum_{h=1}^{l-1} \sum_{j=1}^{h} \delta^{j-1} \frac{nl}{p}
\]
from consumers who have already committed to multi-period contracts signed in the past. If a firm deviates, consumers have two alternatives to accepting the offer. They can wait and not buy anything and then enjoy the ensuing price war, or they can sign a rival’s \(l\)-period contract.

When \(\sum_{j=1}^{l} \delta^{j-1} p < V\), then consumers’ best alternative to accepting a deviating offer is to sign a rival’s \(l\)-period contract, so the most profitable deviation for a firm is a to offer an \(l\)-period contract at a slightly lower price than the equilibrium price, so no profitable deviation exists if
\[
\frac{1}{l} \sum_{j=1}^{l} \delta^{j-1} p + \sum_{h=1}^{l-1} \sum_{j=1}^{h} \delta^{j-1} \frac{nl}{p} \leq \frac{1}{n} \frac{p}{1 - \delta} = \sum_{h=1}^{l} \frac{1}{1 - \delta \frac{nl}{p}}.
\] (6)

Substituting
\[
\frac{1}{1 - \delta} = \sum_{j=1}^{h} \delta^{j-1} + \frac{\delta^{h}}{1 - \delta}
\]
into the right hand side, equation (6) can be rewritten as

\[ \frac{1}{l} \sum_{j=1}^{l} \delta^{j-1} p + \sum_{h=1}^{l-1} \sum_{j=1}^{h} \delta^{j-1} \frac{p}{nl} \leq \sum_{h=1}^{l} \left[ \sum_{j=1}^{h} \delta^{j-1} + \frac{\delta^{h}}{1-\delta} \right] \frac{p}{nl}. \]

or

\[ \frac{1}{l} \sum_{j=1}^{l} \delta^{j-1} p \leq \left[ \sum_{j=1}^{l} \delta^{j-1} + \sum_{h=1}^{l} \delta^{h} \frac{1}{1-\delta} \right] \frac{p}{nl} = \sum_{j=1}^{l} \frac{\delta^{j-1} p}{1-\delta nl}, \] (7)

which is satisfied for all \( p \in \left[ 0, \frac{V}{\sum_{j=1}^{l} \delta^{j-1}} \right] \), if and only if \( n \leq \frac{1}{1-\delta} \).

When \( \sum_{j=1}^{l} \delta^{j-1} p \geq V \), or

\[ p \in \left[ \frac{V}{\sum_{j=1}^{l} \delta^{j-1}}, V \right], \]

consumers’ best alternative accepting a deviating offer is to wait one period, so the most profitable deviation for a firm is to offer one-period contract at a price just below \( V \), and no profitable deviation exists if

\[ \frac{1}{l} V + \sum_{h=1}^{l-1} \left( \sum_{j=1}^{h} \frac{\delta^{j-1} p}{nl} \right) \leq \frac{1}{n} \frac{p}{1-\delta}. \] (8)

Substituting

\[ \frac{1}{1-\delta} = \sum_{j=1}^{h} \frac{\delta^{j-1}}{1-\delta} + \frac{\delta^{h}}{1-\delta} \]

into the right hand side, equation (8) can be rewritten as

\[ \frac{V}{l} \leq \sum_{j=1}^{l} \frac{\delta^{j-1}}{nl} \frac{p}{1-\delta}. \] (9)

which is satisfied for all \( p \in \left[ \frac{V}{\sum_{j=1}^{l} \delta^{j-1}}, \frac{V}{l} \right] \). This interval exists as long as

\[ n \leq \sum_{j=1}^{l} \delta^{j-1} \frac{1}{1-\delta}. \]

We can now describe the second period prices that can be supported as a \textit{Staggered Contract Equilibrium}. First, when \( n \leq \frac{1}{1-\delta} \), then (9) is satisfied for all \( p \in \left[ \frac{V}{\sum_{j=1}^{l} \delta^{j-1}}, \frac{V}{l} \right] \).
Combining this with the earlier result, it follows that when \( n \leq \frac{1}{1-\delta} \), any second period price, \( p \in [0,V] \) can be supported in a \emph{Staggered Contract Equilibrium}. Second, when \( \sum_{j=1}^{l} \frac{\delta_j}{1-\delta} \geq n > \frac{1}{1-\delta} \), any second period price satisfying

\[
p \in \left[ \frac{nV(1-\delta)}{\sum_{j=1}^{l} \delta_j-1}, V \right]
\]

can be supported in a \emph{Staggered Contract Equilibrium}. Finally, when \( n > \sum_{j=1}^{l} \frac{\delta_j}{1-\delta} \), no positive profit \emph{Staggered Contract Equilibrium} exists.

The \emph{Staggered Contract Equilibrium} industry profit,

\[
\left[ p_1 + \frac{\delta}{1-\delta}p \right],
\]

is the highest when \( p = V \) and \( p_1 \) is set at the highest feasible price. Since \( p_1 \) must satisfy (4) and \( p_1 \leq V \), the upper bound on \( p_1 \) is

\[
\frac{1}{n-1} \frac{\delta}{1-\delta} V
\]

or \( V \), whichever is smaller, and the upper bound on \( p \) is \( V \). So the upper bound on the industry profit is \( V + \frac{\delta}{1-\delta} V = \frac{V}{1-\delta} \) when \( n \leq \frac{1}{1-\delta} \) and \( \frac{1}{n-1} \frac{\delta}{1-\delta} V + \frac{\delta}{1-\delta} V = \frac{n}{n-1} \frac{\delta}{1-\delta} V \) when \( \frac{1}{1-\delta} < n \leq \sum_{j=1}^{l} \frac{\delta_j}{1-\delta} \).

The industry profit is lowest when \( p_1 = 0 \) and \( p \) is set at the lower bound. Clearly the lower bound on industry profit is equal to 0 when \( n < \frac{1}{1-\delta} \). When \( \sum_{j=1}^{l} \frac{\delta_j}{1-\delta} \geq n > \frac{1}{1-\delta} \), the lower bound on each firm’s profit is obtained when \( p_1 = 0 \) and \( p = \frac{nV(1-\delta)}{\sum_{j=1}^{l} \delta_j-1} \) and is equal to

\[
\frac{n\delta V}{\sum_{j=1}^{l} \delta_j-1}.
\]

Note that \( l \) affects the lower bound on equilibrium profits, and whether or not a \emph{Staggered Contract Equilibrium} exists, but not the upper bound on industry profits when it is sustainable.

Intuitively, the big advantage of staggered contracts is that they relax the incentive constraint in Period 2 and beyond. Since customers correctly anticipate the price will fall to zero following a deviation profit, the firms’ deviation profit can be no larger than \( V \)
per customer, or \( V/l \) in total (since only \( 1/l \) new customers are available in each period). Part of the profit forgone by the deviator is all of the future profit from the firm’s share of these \( 1/l \) consumers future renewals. This tradeoff (\( V/l \) now versus \( V/ln \) forever) within the current cohort of customers is the same as the tradeoff that a deviating firm faces when there is no intertemporal bundling. However, when contracts are staggered, the deviating firm also forgoes all the renewal profits associated with the other \((l-1)/l \) consumers who are currently locked in to existing contracts. Hence, intertemporal bundling through staggered subscription contracts weakens firms’ incentives to deviate from a collusive outcome.

For \( n = 2 \) and \( l = 12 \), the incentive constraint is \( \delta \geq \hat{\delta} \) where \( \hat{\delta} \approx 0.293 \).\(^4\) That is, when the discount factor lies between \( .293 \) and \( .5 \) tacit collusion in a duopoly is feasible when firms use intertemporal bundling (in a Staggered Contract Equilibrium) but not otherwise.

Comparing Claim 1 and Proposition 1 we can also see that intertemporal bundling using staggered contracts helps the industry sustain tacit collusion when the number of firms is between \( \frac{1}{1-\delta} \) and \( \sum_{j=1}^{l} \frac{\delta^{j-1}}{1-\delta} \). To get an idea of the power of staggering contracts, suppose, for example, that \( \delta = 0.9 \) and \( l = 12 \). In this case, when intertemporal bundling is not feasible, tacit collusion can be sustained in an industry with up to ten firms. But when firms can offer staggered contracts, tacit collusion can be sustained with up to 71 firms (\( \sum_{j=1}^{12} \frac{(0.9)^{j-1}}{1-(0.9)} \approx 71.76 \)).

The following proposition establishes that Staggered Contract Equilibria not only can increase firms’ profits relative to simple tacit collusion, but when they exist (that is, when they can be used to sustain strictly positive profits), the most profitable Staggered Contract Equilibrium achieves the theoretical upper bound for industry profit in any symmetric SPNE.

**Proposition 2.** For \( n \leq \frac{1}{1-\delta} \) the industry profit that can be supported in any symmetric SPNE is at most \( \frac{V}{1-\delta} \). For \( n > \frac{1}{1-\delta} \), the industry profit that can be supported in any symmetric SPNE is at most \( \frac{n}{n-1} \delta \frac{V}{1-\delta} \).

\(^4\)Note that \( \delta = \hat{\delta} \) solves

\[
2 = \sum_{j=1}^{12} \frac{\delta^{j-1}}{1-\delta} = \frac{1 - \delta^{12}}{(1 - \delta)^2}.
\]

\[
0 = \delta^{12} + 2\delta^2 - 4\delta + 1.
\]
**Proof:** When \( n \leq \frac{1}{1-\delta} \), clearly the highest sustainable profit is \( \frac{V}{1-\delta} \), since this can be supported with single period contracts and static Nash reversion and since the firms are capturing the entire surplus.

When \( n \geq \frac{1}{1-\delta} \), let \( \pi \) denote each firm’s equilibrium profit and \( \pi^D \) denote the equilibrium first-period deviation profit of an individual firm. Let \( F \) denote the set of contracts offered to consumers in equilibrium which include consumption in period 1. Let \( f \) be an element of \( F \) and \( P_f \) the associated equilibrium price.

Suppose \( \min_{f \in F} P_f \geq V \). This implies that after observing a deviation, consumers’ best outside option is to abstain from consumption for one period and then purchase at a price of 0 every period afterwards. So, \( \pi^D = V \). However in any equilibrium it must be the case that \( \pi \geq \pi_D \), so \( \pi \geq \pi^D = V \), and since \( n > 1/(1-\delta) \), this implies

\[
n\pi \geq nV > \frac{V}{1-\delta}.
\]

which is impossible since industry profit cannot exceed total surplus. So it follows that in any SPNE equilibrium \( \min_{f \in F} P_f < V \).

When \( \min_{f \in F} P_f < V \), it follows after observing a deviation consumers’ best outside option is to purchase the least expensive contract available for \( \min_{f \in F} P_f \) and then purchase at a price of 0 every period afterwards. So

\[
\pi^D = \min_{f \in F} P_f, \tag{13}
\]

that is, the highest profit that a deviating firm can earn is what consumers pay for their outside option.

Note that equilibrium consumer surplus must be at least equal to \( V - \min_{f \in F} P_f \) since this is the surplus consumers get from purchasing the lowest priced available contract and consuming only in period 1, so

\[
\frac{V}{1-\delta} - n\pi \geq V - \min_{f \in F} P_f, \tag{14}
\]

or

\[
n\pi \leq \min_{f \in F} P_f + \frac{\delta}{1-\delta} \frac{V}{1-\delta} \tag{15}
\]
In any equilibrium, $\pi_D \leq \pi$, so (13) and (15) imply
\[
\min_{f \in F} P_f \leq \frac{1}{n} \left( \min_{f \in F} P_f + \delta \frac{V}{1 - \delta} \right).
\] (16)

or
\[
\min_{f \in F} P_f \leq \frac{\delta}{n - 1} \frac{V}{1 - \delta}.
\] (17)

Inequalities (15) and (17) together imply that
\[
n\pi \leq \frac{\delta}{n - 1} \frac{V}{1 - \delta} + \delta \frac{V}{1 - \delta}
\]
or
\[
n\pi \leq \frac{n}{n - 1} \frac{\delta V}{1 - \delta}.
\]

Note that Proposition 2 holds regardless of how long consumers live, and in particular, holds even if consumers are infinitely lived.

A *Staggered Contract Equilibrium* is easier to support as \(l\) gets larger and consumers live longer. Most importantly, as \(l\) increases, the range of market structures for which a *Staggered Contract Equilibrium* (with life-time contracts) exists also grows. In addition, as \(l\) increases, the range of profits that can be supported expands (the lower bound on the range falls). This can be seen in Figure 1, which shows the industry profits that can be sustained in a *Staggered Contract Equilibrium*.

The region \(A_0\) denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, whether or not intertemporal bundling is feasible. The region \(A_0 \cup A_1\) denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms can intertemporally bundle and choose to play a *Staggered Contract Equilibrium* and consumers live \(x\) periods. The region \(A_0 \cup A_1 \cup A_2\) denotes the set of equilibrium profits that are sustainable when firms play *Staggered Contract Equilibrium* and consumers live \(y > x\) periods.\(^5\)

A natural question is what happens as \(l\) approaches \(\infty\). The next section of the paper looks at a model with infinitely-lived consumers and considers *Staggered Contract Equilibria*.

\(^5\)Note that the number of firms, \(n\), is an integer, but for ease of illustration Figure 1 treats \(n\) as any positive real number.
Figure 1: The Impact of Consumers’ Lifetimes on Feasible Profits in Staggered Contract Equilibria: $x < y$

in that context. While Staggered Contract Equilibrium do not achieve the theoretical upper bound even in that model when $l$ is very large, we characterize another equilibria which does achieve the theoretical upper bound even when $l$ goes to $\infty$.

5 Infinitely-Lived Consumers

In this section we assume consumers are infinitely lived. We begin by revisiting Staggered Contract Equilibria. When consumers are infinitely lived, Staggered Contract Equilibria still exist, but Staggered Contract Equilibria have finite-length contracts so we now consider Staggered Contract Equilibrium with an arbitrary finite contract length, $k$, which is strictly less than consumers’ lifetimes. The following proposition follows immediately from Propositions 1 and 2.
Proposition 3. With infinitely-lived consumers, a Staggered Contract Equilibrium exists and earns the highest possible industry profits if and only if

\[ n < \frac{1}{(1 - \delta)^2}. \]  

(18)

The range of industry profit levels that can be supported in a Staggered Contract Equilibrium is

\[ \left[ 0, \frac{V}{1 - \delta} \right] \quad \text{if } n \leq \frac{1}{1 - \delta}, \]

\[ \{0\} \cup \left( n\delta(1 - \delta)V, \frac{n}{n - 1} \frac{\delta}{1 - \delta}V \right) \quad \text{if } n \in \left( \frac{1}{1 - \delta}, \frac{1}{(1 - \delta)^2} \right), \]

and \(\{0\}\) otherwise.

Note that the highest profit that can be sustained is the same as in Proposition 1 (i.e., for finite-lived consumers), but the range of market structures and discount factors for which this profit can be supported is larger, and the range of profit levels that can be supported is larger. Specifically, the lower bound on profit and the lower bound on the range of market structures is the limit as \(l\) goes to infinity of the range of profits and market structures that could be supported with finite-length contracts:

\[ \lim_{k \to \infty} \frac{n\delta V}{\sum_{j=1}^{k} \delta^{j-1}} = n\delta(1 - \delta)V, \]  

(19)

and

\[ \lim_{k \to \infty} \sum_{j=1}^{k} \frac{\delta^{j-1}}{1 - \delta} = \frac{1}{(1 - \delta)^2}. \]  

(20)

Note also that the range of market structure are open intervals since we consider only staggered contracts of finite length \(k\).

We now show that it is possible for firms to sustain positive industry profits even when \(n\) is larger than the upper bound specified in Proposition 3.

Consider the following subgame perfect equilibrium strategies, which we call a Dual Contract Equilibrium. In period 1, every firm offers consumers two contracts: a one-period contract with a price \(p_1\) and an infinite-length contract with a price of \(p_1\) in period 1 and \(p\) in every period thereafter. In subsequent periods, firms only offer infinite-length contracts at
a per period price of $p$. On the equilibrium path, consumers ignore the one-period contracts and only purchase the firms’ infinite-length contracts in period 1, and they divide their purchases equally across the $n$ firms. Off the equilibrium path, if a firm deviates, then consumers purchase a one-period contract from one of the deviator’s competitors, or from the deviator if its one period price is below $p_1$, and in every subsequent period every firm offers only one-period contracts with a price of 0.

We call these strategies *dual-contract* strategies because consumers are offered two contracts yet only purchase one. That is, the firms are using contract offers on the equilibrium path to alter deviation incentives even those these contracts are never accepted by consumers on the equilibrium path.

Clearly consumers’ strategies are optimal. Consumers cannot affect firms’ prices, so they simply maximize their surplus given firms’ strategies. In period 1, they are indifferent between the one-period contract and the infinite-length contract (and if the one-period contract price were slightly higher they would strictly prefer the infinite-length contract), so purchasing the infinite-length contract is optimal. And purchasing the cheapest one-period contract from any firm is clearly optimal following a deviation.

Firms’ equilibrium path strategies are optimal as long as no profitable deviation exists in period 1. We can ignore deviations that begin in subsequent periods since on the equilibrium path all consumers sign infinite-length contracts in period 1. If any firm deviates in period 1, consumers anticipate that the deviation will start a price war, so consumers have the option to purchase a one-period contract from a non-deviating firm at a price of $p_1$ in period 1 and a one-period contract at a price of 0 in every subsequent period. So the greatest profit that a deviating firm can earn is $p_1$, and therefore the firm prefers its equilibrium strategy as long as

$$
\frac{1}{n}p_1 + \frac{1}{n} \sum_{i=2}^{\infty} \delta^{i-1} p \geq p_1, \tag{21}
$$

which is clearly satisfied for $p_1$ sufficiently close to zero. We can now prove the following proposition.

**Proposition 4.** When consumers are infinitely lived, a Dual Contract Equilibrium exists
for all $\delta$ and $n$. The range of industry profit levels that can be supported is

$$[0, \frac{V}{1 - \delta}]$$

if $n \leq \frac{1}{1 - \delta}$,

$$[0, \frac{n - 1 - \delta}{n - 1} V]$$

if $n > \frac{1}{1 - \delta}$,

so for all $\delta$ and $n$, a Dual Contract Equilibrium exists which achieves the highest feasible profits in any SPNE.

**Proof:** We have already shown that for any $p_1$ and $p$ a strictly positive profit Dual Contract Equilibrium exists as long as (21) is satisfied. From (21) it follows that given $p$, any value of $p_1$ less than $\frac{1}{n-1} \frac{\delta}{1-\delta} p$ satisfies the constraint. When $n \leq \frac{1}{1 - \delta}$, this implies any $p_1 = p$ can be supported, so the range of profit that can be supported is clearly $[0, \frac{V}{1 - \delta}]$.

When $n > \frac{1}{1 - \delta}$, (21) is satisfied for any $p \in [0, V]$ and any $p_1 \leq \frac{1}{n-1} \frac{\delta}{1-\delta} p$, so the highest industry profit that can be supported in a Dual Contract Equilibriums is $\frac{1}{n-1} \frac{\delta}{1-\delta} V + \frac{\delta}{1-\delta} V$, or $\frac{n}{n-1} \frac{\delta}{1-\delta} V$. It follows that the range of profit that can be supported when $n \geq \frac{1}{1 - \delta}$ is $[0, \frac{n}{n-1} \frac{\delta}{1-\delta} V]$.

Finally, Proposition 2 implies that when $p = V$ and $p_1 = \frac{1}{n-1} \frac{\delta}{1-\delta} p$, for any market structure and discount factor, the Dual Contract Equilibrium achieves the highest feasible profit sustainable across all SPNE. □

Figure 2 graphically compares the range of industry profits that can supported by Staggered Contract Equilibria to the range of industry profits that can be supported by Dual Contract Equilibria when when consumers are infinitely lived.

The region $B_0$ denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when intertemporal bundling is not feasible; and the region $B_0 \cup B_1$ denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms can intertemporally bundle and choose to play a Staggered Contract Equilibrium (letting the contract length, $k$, be arbitrarily large).

As is shown in Figure 2, when $n \in (\frac{1}{1-\delta}, \frac{1}{(1-\delta)^2})$, only industry profit levels above $n\delta (1-\delta) V$ are sustainable in a Staggered Contract Equilibrium. Lower prices (and lower
associated profits) cannot be supported by a *Staggered Contract Equilibrium* because firms will strictly prefer to offer a slightly lower priced \( k \)-period contract. Because the candidate equilibrium price is so low, consumers will accept this contract rather than wait for the price war to begin.\(^6\)

In Figure 2, the region \( B_0 \cup B_1 \cup B_2 \) denotes the set of industry profits that are sustainable in any SPNE, and in particular, in a *Dual Contract Equilibria*.

The *Dual Contract Equilibria* characterized in this section are noteworthy because they can be used to support any feasible equilibrium profit levels. However, the existence of these equilibria relies on two strong assumptions. First, it relies trivially on the fact that consumers are infinitely lived. And second, it relies on the assumption that consumers are

\(^6\)However, lower profit levels can easily be sustained using variants of *Staggered Contract Equilibria*. Most simply, by mixing between the zero profit equilibrium and any *Staggered Contract Equilibrium* using a common correlation device, any profits in the region below \( B_1 \) can be sustained.
arbitrarily small. When consumers are finite sized (or can act in a coordinated way), a Dual Contract Equilibrium does not exist because following a deviation by a consumer, firms have an incentive to lower their price to capture that consumers’ business because every other consumer is locked in to an infinite-length contract.

When consumers are finitely lived or finite sized, firms may still be able to tacitly collude by offering a menu of contracts, but the long-term contract must have finite length. In the next section we look at equilibria that come arbitrarily close to sustaining the highest achievable profits when consumers are finite sized.

6 Finite Number of Consumers

When there are a finite number of consumers, even if they are infinitely lived, the strategies considered in the previous section are no longer an equilibrium. The problem is that it is not credible for firms to ignore deviations by individual consumers. However, firms can reduce their incentive to respond to a consumer deviation by using finite-length contracts. When contracts are finite, a firm loses the future profits associated with contract renewals when it cuts price in response to a deviation by a single consumer. We now define a class of equilibria which are capable of achieving a profit level that is arbitrarily close to the theoretical upper bound on profits described by Proposition 2 as long as the number of consumers is sufficiently large.

Definition. Intermittent Discount Dual Contract Equilibrium: Firms offer a menu of contracts every $k$ periods beginning in period 1 and offer no contracts in the interim. The menu consists of a one-period discount contract at a price $p_1$ and an $k$-period contract at a price $p_1$ in the first period of the contract and a price $p > p_1$ in every period thereafter. Consumers accept the $k$-period contracts. Following a deviation by any firm, consumers purchase the one-period discount contracts and firms revert to one-period contracts at marginal cost for the rest of the game. Following a deviation by a consumer, firms and other consumers continue to play their equilibrium strategies and the deviating consumer reverts to equilibrium path play at the first available opportunity.
The following proposition is proved in the Appendix:

**Proposition 5.** Let $m$ be the number of homogeneous consumers (or equivalently, let $1/m$ be the measure of each consumer.) For all $\delta$ and $n$, there exist an $m$ sufficiently large such that an Intermittent Discount Dual Contract Equilibrium with strictly positive industry profit exists. The range of industry profit levels that can be supported in an Intermittent Discount Dual Contract Equilibrium is

$$[0, \frac{V}{1-\delta}] \quad \text{if } n < \frac{1}{1-\delta},$$

$$[0, \frac{n \delta}{n - 1 - \delta V}] \quad \text{if } n \geq \frac{1}{1-\delta}. \quad (23)$$

The proposition demonstrates that intertemporal bundling facilitates tacit collusion even when consumers are finite sized. Because the long-term contracts are renewed every $k$ periods, firms have an incentive to ignore consumers who deviate by signing a short term contract as long as consumers are not too large. Of course, if there were only one consumer, or if all consumers could coordinate their actions, then firms would never find it profitable to ignore a consumer deviation and intertemporal bundling would not facilitate tacit collusion.

### 7 Staggered Contract Equilibria versus Intermittent Discount Dual Contract Equilibria

When consumers are small and infinitely lived, simple Dual Contract Equilibria can be used to support any profit level that can be supported in the set of all Subgame Perfect Equilibria. But these assumptions are strong. Staggered Contract Equilibrium and Intermittent Discount Dual Contract Equilibria may both exist when these assumptions are relaxed.

In our paper, we have emphasized Staggered Contract Equilibria over Intermittent Discount Dual Contract Equilibria for two reasons. First, Staggered Contract Equilibria are more descriptive of what firms actually do. The menus of short and long term contracts characterized by Intermittent Discount Equilibria, in which the short term contract is never signed on the equilibrium does not match firm behavior.
More importantly, Staggered Contract Equilibria are more profitable in a model of finitely lived consumers with overlapping generations. In this case Intermittent Discount Dual Contract Equilibrium are ineffective. Since new consumers are arriving every period, in order to eliminate the incentive to deviate firms would need to offer a short term contract every period, but then consumers would strictly prefer the short term contract to the long term contract. Intermittent Discount Dual Contract Equilibrium also exist in a models with finite-lived consumers, but they are effective only when consumers live concurrently and not in overlapping generations. It is easy to see that in this case Intermittent Discount Dual Contract Equilibrium achieve the upper bound of feasible profits (though the range of market structures and discount factors for which this is true is limited by the length of consumers lives). However, in this case Staggered Contract Equilibria are ineffective.

In other words, Staggered Contract Equilibria outperform Intermittent Discount Dual Contract Equilibria in models that seem more empirically relevant, while Intemittent Discount Dual Contract Equilibria outperform Staggered Contract Equilibria in models of finitely-lived consumers which seem less empirically relevant.

The one remaining issue is to verify that Staggered Contract Equilibria are robust to relaxing the assumption that consumers are arbitrarily small. In an earlier version of the paper we analyzed Staggered Contract Equilibria with finite consumers. We showed that as long as several consumer deviations are necessary before a firm would find it profitable to deviate from its equilibrium strategy, that is, as long as consumers are not too large, then Staggered Contract Equilibria still exist. Firms respond to a deviation by any consumer by offering the consumer a contract that expires at the same time as the equilibrium contract that the consumer previously declined. Following a deviation, other consumers expect the consumer to sign that contract, so it is not possible for a single consumer to alter the behavior of other consumers by deviating.\footnote{Formally this argument relies on consumers choosing weakly dominated strategies off the equilibrium path. However it is straight forward to generalize firms’ strategies so that consumers who deviate bear a positive cost in every period in which they deviate. In an earlier version of our paper, we constructed Staggered Contract Equilibria’ in undominated strategies in which firms’ equilibrium path price $p$ is strictly less than $V$ per period, and firms respond to a consumer deviation by offering contracts at prices higher than the equilibrium path price. In this equilibrium, a consumer who deviates by signing an one-period discount contract consumption bears a strictly positive opportunity cost.}
8 Conclusion

In this paper, we demonstrated that intertemporal bundling may help soften competition by facilitating tacit collusion. By offering a menu of multi-period contracts of different lengths, firms break up the market into multiple segments each open at different points of time. Tacit collusion is easier to sustain because a deviating firm can steal business only in one market segment at one point of time but will be punished in every market segments.

Our results relied on several strong assumptions. We assumed that consumers are small and unable to act collectively. If a single consumer represented a large portion of a firm’s business, or if a large measure of consumers could act collectively, firms’ ability to tacitly collude would be diminished. We also assumed that price deviations were observable, whether or not consumers made purchases, though we argued that a variant of our results would still hold if this assumption were relaxed.

More notably, we assumed that consumers are forward looking and are able to forecast a price war when they observe a deviation by any firm. In some ways our results strongly depend on this assumption that consumers are forward looking and understand firm behavior. In particular, if consumers do not anticipate a price war upon seeing a deviation, then a deviating firm can capture all of the future profit from consumers currently in the market by offering them lifetime contracts, so when consumers are naïve, intertemporal bundling makes sustaining tacit collusion with single period contracts more difficult. In a companion paper, Dana and Fong (2008), we analyze a model in which some consumers are naïve. Comparing equilibria in which firms use only single period contracts on the equilibrium path, when sufficiently many consumers are naïve, the highest feasible profit increases when intertemporal bundling is banned. Such a ban eliminates the temptation to deviate to lifetime contracts. However, in the absence of such a ban, we find that the highest feasible equilibrium profits are higher in equilibrium in which firms offer staggered subscription contracts on the equilibrium path than in equilibrium in which firms do not. In other words, intertemporal bundling can facilitate collusion even when some consumers are naïve.

Finally, our analysis relied on the implicit assumption that a firm’s price cut is observed by its competitors even when consumers do not purchase at the lower price. This assumption
guarantees that a deviating firm cannot steal more than one period of the surplus from all the available consumers. If it tries to steal more, all consumers will wait for the ensuing price war. However, we believe our results still hold when price cuts are only observed if they actually steal business. Consumers will accept any offer below the equilibrium path price if they anticipate that no one else will accept it, and they will reject any offer if they think other consumers will accept it and waiting for the price war to begin is more attractive. So a symmetric mixed strategy equilibrium should exist, at least in a model with finite consumers. In equilibrium, following a deviation consumers mix between accepting and rejecting, and so price deviations will attract some business. Of course, if consumers are arbitrarily small, price deviations will attract an arbitrarily small portion of the market. Alternatively, it should be possible to construct an asymmetric pure strategy equilibrium in which exactly one consumer accepts the offer and the rest wait for the price war to begin. Since the price war will not begin otherwise, the designated consumer will always accept a lower price. In either case, intertemporal bundling will still make supporting positive profits easier because it limits deviating firms ability to capture all of the available consumer surplus for more than a single period.

While we characterized several equilibrium strategies that soften competition, we think the Staggered Contract Equilibria are the most realistic and most empirically relevant. In these equilibria firms write overlapping subscription contracts with their consumers similar to the contracts used in the cellular telephone industry.

Our paper has some important empirical implications. First, we predict that firms’ margins may be sensitive to the expected lifetime of consumers and to the feasibility of long-term contracts. If consumers are not long-lived, then firms will be unable to soften competition using long-term contracts. Similarly, we predict firms’ margins may be sensitive to the ability of consumers to forecast their demands over time. Absent predicable demand, long-term contracts would be inefficient and unable to facilitate tacit collusion.

While we show intertemporal bundling facilitates tacit collusion, it is important to add that there are also many efficiency explanations for intertemporal bundling. Subscription contracts reduce transactions costs, facilitate firm production planning (as do other advance purchase contracts), and speed delivery time for consumers.
9 Appendix

Proof of Proposition 5:

Proving the existence of an *Intermittent Discount Dual Contract Equilibrium* when consumers are finite requires that we show that the firms’ strategies are optimal on the equilibrium path as well as off the equilibrium path after a single consumer deviates. We must also show that consumers equilibrium strategies are optimal.

We first look at firms’ equilibrium strategies, both in the first period of their $k$-period contracts, as well as in the remaining periods, and both on the equilibrium path and following a consumer deviation.

*Periods* $1, k+1, 2k+1, 3k+1, \ldots$

Since a firm’s $k$-period contracts include a first period discount of $p - p_1$, and these contracts are signed every $k$ periods, a firm’s equilibrium profit beginning in periods $1, k+1, 2k+1, 3k+1, \text{etc.}$, that is, beginning in period $ak+1$ where $a$ is any non-negative integer, is

$$\pi_{ak+1} = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1}p - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k}(p-p_1).$$

If any firm deviates in these periods, consumers have the option to purchase a one period contract from a non-deviating firm at a price of $p_1$ in the current period and at a price of 0 in every subsequent period, because the deviation will start a price war. So the highest profit a deviating firm can earn is $p_1$. This means the firm prefers its equilibrium strategy as long as

$$\frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1}p - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k}(p-p_1) \geq p_1. \quad (A-1)$$

This is clearly satisfied for $p_1 = 0$ (and for some $p_1 > 0$ sufficiently close to zero).

Also, it is clear that $\pi_1$ is increasing in $k$ and as $k$ approaches infinity $\pi_1$ approaches

$$\frac{1}{n} \sum_{i=2}^{\infty} \delta^{i-1}p + \frac{1}{n}p_1, \quad (A-2)$$
or
\[ \delta \frac{p}{1 - \delta} + \frac{1}{n} p_1. \tag{A-3} \]
Therefore, there exists a \( k \) sufficiently large such that (A-1) is satisfied if and only if
\[ p_1 < \frac{1}{n - 1} \delta \frac{p}{1 - \delta}. \tag{A-4} \]
Finally, these strategies are optimal even when a consumer deviates in the past because past consumer deviations do not affect payoffs in these periods (unless a firm has also deviated).

**All other periods**

In these periods firms’ actions have no impact on profits as long as consumers play their equilibrium strategies, so it is easy to see that their actions maximize profits.

If a consumer deviates by choosing a one-period contract (or not purchasing at all) in the period \( ak + 1 \), firms ignore the deviation in each of the \( k - 1 \) subsequent periods as long as
\[ \delta^{k-s} \pi_1 \geq \frac{1}{m} V, \forall s = 1, ..., k - 1. \tag{A-5} \]
Clearly, if this set of conditions holds for \( s = 1 \), then it holds for all \( s = 1, 2, ..., k - 1 \) because the gain from a deviation remains the same, but the present value of the cost increases as it draws nearer. So (A-5) holds for all \( s \) as long as
\[ \delta^{k-1} \pi_1 \geq \frac{1}{m} V. \tag{A-6} \]
This condition clearly holds as long as \( m \) is sufficiently large. Indeed, (A-6) implicitly defines an upper bound, \( k^*(m) \), on the contract length for all \( m \). It clearly follows that as \( m \to \infty \), \( k^*(m) \to \infty \).

**Equilibrium Profits**

Clearly the equilibrium profits are highest if \( p = V \). In this case, the equilibrium profits are
\[ \pi_1 = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1} V - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)+k} (V - p_1), \tag{A-7} \]
So the highest sustainable equilibrium profits are defined by (A-7) evaluated at \( k^*(m) \). As \( m \to \infty \) and \( k^*(m) \to \infty \) this profit level approaches
\[ \frac{1}{n} p_1 + \frac{1}{n} \delta \frac{V}{1 - \delta}. \tag{A-8} \]
From (A-4), we know that the equilibrium profit is the highest when \( p_1 \) approaches \( \frac{1}{n-1} \delta \frac{V}{1-\delta} \). In this case, each firm’s profit approaches \( \frac{1}{n-1} \delta \frac{V}{1-\delta} \) and the industry profit approaches \( \frac{n}{n-1} \delta \frac{V}{1-\delta} \).

**Consumer deviations**

Finally, it is clear that no individual consumer has an incentive to deviate. Following a deviation, all other consumers are locked into \( k \)-period contracts and firms have no incentive to deviate from the equilibrium path play so the deviating consumer is weakly worse off because

\[
V - p_1 + \delta^i S \leq V - p_1 + \sum_{i=1}^{k-1} \delta^i (V - p) + \delta^k S,
\]

where \( S \) denotes the consumer’s equilibrium path surplus (beginning in a contracting period).
References


