When Does Aftermarket Monopolization Soften Foremarket Competition

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Abstract

This paper investigates firms’ abilities to tacitly collude when these firms each monopolize a proprietary aftermarket. When firms’ aftermarkets are isolated from foremarket competition, they cannot tacitly collude more easily than single product firms do. However, when their aftermarket power is contested by foremarket competition as equipment owners view new equipment as a substitute for their incumbent firm’s aftermarket product, the monopoly profit is sustainable among a larger number of firms. More strikingly, as long as existing customers have a shorter market life expectancy than incoming customers, for any discount factor, supranormal profits are sustainable among arbitrarily many firms each selling ex ante identical products. These results suggest the importance of distinguishing between two types of aftermarket power which are often considered to be qualitatively the same. Conditions under which introduction of aftermarket competition hinders firms’ ability to tacitly collude are characterized.

Key Words: Constrained Aftermarket Power, Tacit Collusion, Aftermarket Competition

JEL Codes: L12, L13, L41

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1 Introduction

In 1992, the U.S. Supreme Court ruled in favor of eighteen independent service organizations (ISOs) which sued Kodak for its refusal to sell to them replacement parts for servicing Kodak’s photocopiers and micrographics equipment.¹ The Supreme Court decision has drawn the attention of — and generated much debate among — legal scholars and economists. The questions of particular interest to economists are, (i) whether firms like Kodak which do not have substantial market power in the equipment market are able to exercise their power in the related proprietary aftermarkets, and (ii) whether these firms can earn substantial overall industry profits and cause significant consumer injury.

Borenstein, Mackie-Mason, and Netz (1995, 2000) show that equipment manufacturers tend to set supranormal prices in their proprietary aftermarkets even when the equipment market is competitive and customers have perfect foresight and are fully aware of the life-cycle cost. The idea is that as long as firms cannot commit to future prices, they will be tempted to raise the price of the aftermarket service as soon as they have established an installed base from sales in the equipment market. Shapiro and Teece (1994) and Shapiro (1995) argue, however, that installed-base opportunism is unlikely if equipment manufacturers are concerned about long-term reputation and can provide protection to customers through long-term contracts.

While these studies provide different answers to question (i) concerning equipment sellers’ abilities to exercise their aftermarket power, the authors largely agree on question (ii) that as long as the equipment market is competitive, firms with monopolized proprietary aftermarkets cannot earn supranormal profits. The idea is that even if equipment manufacturers can hold up their customers in the aftermarkets, competition in the equipment market would induce them to rebate these profits through the offers of lower equipment prices. This argument implies that competition in the equipment market disciplines equipment sellers with proprietary aftermarkets the same way it disciplines single product firms which do not have aftermarket power.

To demonstrate the relevance of proprietary aftermarkets to firm profits, this paper investigates equipment sellers’ ability to collude when these firms each monopolize a proprietary aftermarket. I explicitly distinguish between two types of aftermarket power: unconstrained aftermarket power and constrained aftermarket power. A firm’s aftermarket power is said to be unconstrained if the firm’s

¹Details of the case are available, for example, in Hay (1993). Borenstein et al. (2000) observed that over twenty antitrust cases were under legal process against manufacturers at the time their paper was published.
aftermarket product is for enhancing the functionality of the equipment and its established customers do not consider new equipment of a different brand as a substitute for the firm’s aftermarket product. In this case, firms’ aftermarket are isolated from foremarket competition. For example, hotel owners who may charge high prices for room service and mini-bar items enjoy unconstrained aftermarket power because guests in need of late night refreshment do not consider renting another hotel room as a substitute. On the other hand, a firm’s aftermarket power is said to be constrained if the firm’s aftermarket product is for restoring the functionality of the equipment and its established customers consider new equipment of a different brand as a substitute for the firm’s aftermarket product. For example, printer manufactures only enjoy constrained aftermarket power because their existing printer owners consider a compatible replacement cartridge and a brand new printer as substitutes.

It is shown that when firms’ aftermarket power is unconstrained, firms’ ability to sustain supranormal profits is no different from that of single-product firms. Ironically, when their aftermarket power is constrained by foremarket competition, the monopoly profit becomes sustainable among a larger number of firms. More strikingly, as long as existing customers have a shorter market life expectancy than incoming customers, for any discount factor, supranormal profits are sustainable among arbitrarily many firms each selling ex ante identical products. Such contrast suggests it is important to distinguish between unconstrained aftermarket power and constrained aftermarket power which are usually considered to be qualitatively the same.

My analysis exploits the temporal structure of customers’ demands for the equipment and aftermarket products and the potentially substitutability between these two products for existing equipment owners. I consider (potentially many) oligopolistic firms competing in the equipment market, each of them the sole provider in the equipment’s aftermarket. New customers arrive in the market every period, each staying for multiple periods. Each customer purchases the equipment in the first period of his/her market life and the aftermarket product in the future period(s). Products offered by different firms are ex ante homogeneous to consumers in the sense that if consumers anticipate to pay the same total price for different firms’ equipment and aftermarket services, then consumers in the first period of

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2 Kühn and Padilla (1996) study a durable good monopolist who also sells a nondurable. A consumers views the durable and nondurable as substitutes as long as he has not purchased the durable but once he has purchased the durable, he has no more demand for either product. In our setup, by contrast, consumers view the equipment and aftermarket product as substitutes only after they have purchased the equipment.

3 In the main model, consumers stay in the market for two periods. In the extended model in Section 7, consumers exit the market with a certain probability each period and can potentially stay in the market for any number of periods.
their market life value these firms’ products equally.

It is well known that in a single-product market, if a firm deviates by undercutting the market price, it will capture the entire industry profit for one period. However, by triggering a price war, it will lose its share of profits from all future demands. This leads to the result that the industry can sustain any profit between zero and the monopoly profit if the number of firms is no larger than a critical value, but if the number of firms exceeds this critical value, then the unique equilibrium outcome is zero profit. In an overlapping generations model in which consumers live multiple periods, firms will have the same ability to sustain collusion as single product firms if a deviating firm is able to steal the entire industry’s life-cycle profits from one generation of customers before losing its share of profits from all future generations of customers.

When firms enjoy unconstrained aftermarket power, there is no competition between the foremarket and aftermarket products. As a result, an established customer can derive utility only from the aftermarket product provided by his equipment manufacturer but not from any other firm’s equipment or aftermarket product. As a result, firms can charge their established customers up to their reservation value for the aftermarket product both in a collusive equilibrium and on the punishment path on which firms earn zero profits from future generations of customers. If any firm undercuts the equipment price, it will capture the entire industry profit from equipment sale to one generation of consumers because consumers expect each firm to set the same aftermarket price on the punishment path. Following the price cut, the deviating firm will continue to capture the monopoly profit in the aftermarket from these consumers it steals. This implies that a deviating firm is able to steal the entire industry profit from one generation of customers before losing its share of the profits from all future generations. This explains why unconstrained aftermarket power does not facilitate tacit collusion.

When firms enjoy constrained aftermarket power, however, since existing equipment owners view new equipment of any brand as a perfect substitute for their equipment’s aftermarket service, firms’ aftermarket power is still subject to competition from the equipment market. For example, if it cost a printer owner more to buy a replacement cartridge than a brand new printer, then the printer owner would buy a new printer instead of a replacement cartridge when his existing cartridge runs out. In a collusive outcome, firm profits from each generation of customers come from aftermarket sales as well as equipment sales which take place in different periods. First consider the case where both the equipment and aftermarket product sales are profitable yet the equipment is sold at a price substantially
higher than that of the aftermarket product so that undercutting the equipment price to steal only new customers is a more profitable deviation than undercutting the service price to steal both new customers and competitors’ established customers. By undercutting the equipment price, a deviating firm is able to capture the entire industry’s equipment sales revenue from the incoming generation of customers. However, it will not be able to capture the entire industry’s equilibrium aftermarket sales revenue from the new customers it steals, for the reason that aftermarket sales take place with a time lag. By the time the deviating firm sells aftermarket services to these customers, the price war in the equipment market will have begun. Since existing equipment owners consider new equipment and aftermarket service as substitutes, the price war in the equipment market will bring down the aftermarket price. It remains true that the deviating firm loses its share of profits from all future generations of customers. The fact that the deviating firm is unable to capture the entire industry profit from one generation of customers before losing its profits from future generations of customers explains why tacit collusion is generally easier to sustain among firms possessing proprietary aftermarkets than among single-product firms.

As the number of firms becomes sufficiently large, successful tacit collusion necessarily entails selling the equipment at a loss and relying on aftermarket sales for overall profitability. Otherwise the benefit from stealing competitors’ equipment-market shares will eventually dominate a firm’s future equilibrium profits. Now suppose firms sell both the equipment and aftermarket supply at the same price, where the equipment serves as a loss leader, yet firms each earn a positive life-cycle profit from every generation of customers. Selling both the equipment and aftermarket supply below the marginal cost of equipment can be profitable so long as it costs less to produce the aftermarket supply than the equipment. Since the equipment is priced below cost, any deviation to undercut the equipment price necessarily leads to an immediate loss. Furthermore, in the following period, the price war will bring down both the equipment and aftermarket prices to a level that the overall profit from future customers’ life-cycle consumption is zero. As a result, the deviating firm will not be able to sell aftermarket supply at the collusive equilibrium price. Most importantly, since the equipment and aftermarket supply are priced at the same level, a deviating firm trying to steal new customers will inevitably induce its competitors’ established customers to abandon their usable equipment and purchase equipment from the deviating firm anew. This deepens the deviating firm’s up-front loss from equipment sale. I formally show in Section 7 that as long as existing customers have a shorter market life expectancy than new customers do, a positive industry profit can be supported among any number of firms.
In most existing aftermarket analyses, the competitiveness of the equipment market is treated as unaffected by the existence of aftermarkets. Moreover, whenever the related equipment market is unconcentrated, it is presumed to be competitive. Although my analysis leaves open the possibility that under intense competition in the equipment market aftermarket profits may be rebated to customers, my findings caution that diffusion of market shares in the equipment market does not by itself warrant competition among firms if these firms possess constrained aftermarket power.

Given firms’ unusual ability to tacitly collude when they possess constrained aftermarket power, I also investigate introduction of competition into equipment sellers’ aftermarkets as a remedy. It is shown that aftermarket competition limits firms’ equipment sellers’ ability to tacitly collude if equipment cost is relatively high, industry is relatively unconcentrated, or firms can offer bundles.

2 Related Literature

Studies of competition among firms which possess aftermarket power are by now quite voluminous. Here I review some recent contributions that I did not cover in the Introduction. Chen and Ross (1999) show that when aftermarkets of repair are monopolized, manufacturers can use price discrimination to more efficiently serve a market in which customers use the equipment with different intensities and thus value the equipment differently. Aftermarket monopolization allows firms to charge a low price in the primary market and ensures that high-intensity, high-valuation customers pay more in the aftermarket. Introducing competition into the aftermarket removes the industry’s ability to price-discriminate and thus may lead to lower consumer welfare. A result of somewhat similar spirit is presented by Carlton and Waldman (2001), who point out that if the equipment is priced above marginal cost, either due to monopoly power or brand switching costs, then it is inefficient to have a competitive aftermarket for maintenance. This is because customers tend to repair the equipment when it is socially efficient to replace it with a new one. Carlton (2001) also argues that in the absence of scale effects, it is unlikely for monopolization of the aftermarket (through for e.g., refusal to deal) to be harmful to customers.

By focusing on how market structures may impact firms’ abilities to tacitly collude, I find that constrained aftermarket power can cause significant consumer injury but unconstrained aftermarket power cannot. My result that an industry can achieve supranormal profits even in the presence of arbitrarily many firms, each selling *ex-ante* homogeneous products in the equipment market challenges Shapiro (1995) and Chen, Ross, and Stanbury (1998) offer detailed reviews of earlier aftermarket theories.
the conventional wisdom that when market concentration is low, the market outcome can be presumed to be competitive.

Ellison (2005) provides an interesting theory for high add-on prices that seeks to explain firms’ incentives to conceal high add-on prices from customers. He analyzes static duopolist firms, each selling a low-quality and a high-quality good, who serve customers of different price elasticities, valuation for qualities, and brand preferences. The high-quality good can be interpreted as the low-quality good coupled with an upgrade add-on. Customers of low price elasticity are assumed to also value quality more. When firms compete by posting two prices, in a separating equilibrium, customers of low price elasticity purchase the high quality good and those with high price elasticity purchase the low-quality good. When firms compete by posting only the price of the low-quality good, customers infer that firms will charge a high fixed markup for a quality upgrade. In the latter case, due to the fixed markup, a firm that wants to cut prices to attract customers is forced to offer equal price cuts for both goods. Such equal price cuts on high-quality and low-quality goods tend to attract disproportionately more price sensitive customers who are less likely to purchase the high-quality good and thus are less profitable. As a result, when firms compete by posting one price instead of two, their incentives to cut prices are weakened and they each earn higher profits.

In a model with naïve customers who systematically underestimate their aftermarket consumption and are not aware of their bias, Gabaix and Laibson (2006) also study firms’ strategy of hiding the aftermarket price from consumers. They demonstrate why competitive forces may fail to incentivize firms to undercut the industry’s high aftermarket prices and inform the naïve customers of its competitors’ high aftermarket prices. The rationale is that once a firm has educated a particular group of customers, this group will exert an effort to avoid purchasing any firm’s expensive aftermarket product, including the deviating firm’s. Therefore, no firm will blow the whistle.

My aftermarket theory differs from those by Ellison and Gabaix and Laibson in several significant ways. First, in their analyses, the high aftermarket prices are supported by firms’ concealment of these prices at the time of equipment sale while in my theory, the high aftermarket prices are publicly observable. Second, in their analyses, high aftermarket prices impact firms’ profitability only when products are heterogeneous.\(^5\) In contrast, in my analysis, aftermarket monopolization may impact firms’ profitability even when many firms sell homogeneous products. Moreover, the distinction between

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\(^5\)By extending Gabaix and Laibson’s model with behavioral consumers to a dynamic setting, Miao (2006) shows that duopoly firms can earn overall positive profits by exploiting naïve consumers even when products are homogenous.
unconstrained aftermarket power and constrained aftermarket power is important in my analysis while it is not in Ellison’s analysis. My assumption that consumers are fully rational and forward looking also distinguishes my analysis from Gabaix and Laibson’s. Therefore, our theories are complementary, each suitable for different types of aftermarkets.

Morita and Waldman (2004) show that, by monopolizing the maintenance market as well as the primary market, a durable-goods monopolist can commit not to cut product price after having sold it to the customers with the highest willingness to pay. The commitment is credible because cutting product price will harm the monopolist’s profit from maintenance. When consumers anticipate the monopolist’s lack of temptation to cut price in the future, early adopters are willing to pay a higher price for the durable good. As a result, monopolization of maintenance market enhances a durable-goods monopolist’s profit like leasing contracts do. In the current paper, in contrast, constrained aftermarket power impacts firm profits by softening competition among firms instead of helping an individual firm overcome the Coase conjecture. In fact, firms’ time-inconsistency problem faced by a durable-goods seller is absent in my model because of my simplifying assumptions of constant inflow of new demands and homogenous unit demands.

The literature has also studied brand switching cost [first introduced by Klemperer (1987a, 1987b)] as a potential source of aftermarket power. Padilla (1995) and Anderson, Kumar, and Rajiv (2004) show that switching costs make tacit collusion harder to sustain. This is because brand switching costs protect a deviating firm in the punishment phase. Since consumers do not have to incur a brand switching cost in my analysis, this effect is absent in my model. Another difference is that in these authors’ models, firms each sell one single product to both new and existing customers and always charge them the same price. In my setting, firms sell equipment to new customers but induce existing customers to purchase only the aftermarket product which cost less to produce than the equipment. For tacit collusion to be sustainable among a large number of firms, it requires that these firms offer the equipment as a loss-leader and earn more than enough profit to compensate for this loss in the sales of the aftermarket product. In the switching cost literature, since new and existing customers pay the same price and the marginal costs of the goods sold to the new and existing customers are identical, profitable loss-leader strategies are impossible.

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6Klemperer (1987a) informally argues that, when monitoring is imperfect, switching costs may facilitate tacit collusion. The idea is that when there are brand-specific switching costs, to successfully steal competitors’ customers, a firm has to offer a large price cut, which is necessarily more easily detected by competitors.
This paper is closely related to the literature on multimarket contact (see Bernheim and Whinston, 1990). Bernheim and Whinston’s analysis is based on markets which open simultaneously. However, if markets instead open sequentially, then collusive firms have weaker incentives to deviate, knowing that stealing business in one market will lead to punishment in multiple markets. In the current paper, since equipment owners view new equipment as a substitute for aftermarket product/service, a firm deviating in the equipment market will be punished in both the equipment market and aftermarket. As a result, firms’ incentive to deviate from collusion is weakened. Moreover, the particular way in which the foremarket and aftermarkets are links allows profitable tacit collusion to be sustainable among arbitrarily many equipment sellers with constrained aftermarket power, which is impossible in a standard model of multimarket contact.

Finally, this paper is also related to the broader literature that studies outcomes sustained by tacit collusion in order to evaluate market performance in various other contexts. For instance, Bernheim and Whinston (1990) analyze how multi-market contracts affect firms’ ability to sustain high prices; Ausubel and Deneckere (1987), Gul (1987), and Dutta, Matros, and Weibull (2003) analyze how firms use tacit collusion to sustain high prices for durable goods; Nocke and White (forthcoming) show that vertical integration of a single firm can help all firms in an industry sustain tacit collusion; and Bernhardt and Chambers (forthcoming) show that profit sharing with workers allow firms to tacitly collude more effectively when demand is uncertain. Also see Ivaldi, Jullien, Rey, Seabright, and Tirole (2003) for an excellent review of theories built on the assumption that firms tacitly collude.

3 Environment

There are $n \geq 2$ infinitely lived sellers who each produce two products, the foremarket product and the aftermarket product, at constant marginal costs $C$ and $c$, where $0 \leq c < C$. Throughout the paper, I use printer – containing an initial cartridge – as a working example of the foremarket product, and replacement cartridge as a working example of the aftermarket product, but my analysis applies more broadly to markets in which refill supplies or maintenance and repair services of equipment are a significant part of the business.

Consumers arrive in overlapping generations. In each period, a continuum of consumers of measure one enter the market and each of them stays in the market for two periods. Firms and consumers

\footnote{In Section 7, I check the robustness of my findings and draw more general conclusions by modifying the model to...}
have a common discount factor $\delta \in (0, 1)$. A consumer in the first period of his market life is called a new consumer. A consumer in the second period of his market life is called an established customer (or sometimes existing customers) if he already owns a printer. Every consumer demands up to one functional printer in each period of his life, where a functional printer is either a brand new printer or an old one with a compatible replacement cartridge installed. Each printer produced by any firm provides a new consumer a utility of $U$. Cartridges are firm specific in the sense that a cartridge produced by firm $i$ is compatible only with firm $i$’s printer but not with any other firm’s. An established customer values a cartridge compatible with the printer he owns at $U$ but an incompatible cartridge has no value to him. A new printer of any brand is valued by an established customer identically to a cartridge compatible with his existing printer. So although firms have market power in the cartridge market, they may still face competition from the printer market. For this reason, I call firms’ market power in the cartridge markets constrained aftermarket power.

I assume that the production of printers and cartridges is socially efficient:

$$C + \delta c < (1 + \delta)U. \quad (E)$$

Note that although established customers only value the cartridges of the same brand as their printers, products produced by different firms are ex ante homogeneous to new customers.

Firms individually maximize the discounted values of their profits. Established customers maximize their instantaneous consumer surplus and new customers maximize the discounted value of their lifetime consumer surpluses.

In each period $t \in \mathbb{N}$, each firm $i \in \{1, 2, \ldots, n\}$ simultaneously announces the price of its printer $P_{i,t}$ and if a firm has established customers, it announces the price of its cartridge $p_{i,t}$. In other words, I assume that when a firm sells its printer to a customer, it cannot commit to the future price of a cartridge to this customer. In each period, after the prices are announced, new and established customers make their purchase decisions.

Throughout the paper, I restrict my attention to symmetric, stationary subgame perfect Nash equilibria. For this reason, the time and firm subscripts for prices are dropped for ease of exposition.

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8 This assumption is commonly adopted in the durable goods and switching costs literatures. This assumption is more reasonable in a more realistic setting where consumers’ demand for, and/or the production cost of, the cartridge is uncertain, and/or the quality of the cartridge is not verifiable. In section 6 we discuss how our findings are modified if firms are able to commit to future aftermarket price at the time of equipment sale.
4 Unconstrained Aftermarket Power Does Not Facilitate Tacit Collusion

The main focus of this paper is in markets in which firms possess constrained aftermarket power. However, in this section, I modify the main model to eliminate potential competition between the foremarket and aftermarket products. More specifically, I assume that a customer derives utility from the equipment only in the first period of his market life. In the second period of his market life, if the customer already owned the equipment, then he can derive utility from the aftermarket product provided by his equipment manufacturer but not from any new equipment. For that reason, we say equipment sellers enjoy unconstrained aftermarket power in this case. Because of the change in the nature of the aftermarket products, I call them add-ons. Let the utility from the add-on be $V > c$ which in general is different from $U$, but can also be equal to $U$ by coincidence. Without facing competition from the equipment market, firms can charge their established customers up to $p = V$ regardless of the condition in the equipment market.

In a zero-profit equilibrium, which is also assumed to be the punishment path of tacit collusion, firms necessarily charge $p^C = V$ for the add-on; otherwise a firm can generate a positive profit by raising its add-on price. The zero-profit condition $\left( P^C - C \right) + \delta \left( p^C - c \right) = 0$ further implies that $P^C = C - \delta (V - c)$.

Now suppose firms tacitly collude on a price pair $(P, p)$. Expecting to pay $p \in [c, V]$ for the add-on in the second period of their market life, new customers are willing to pay up to $U + \delta (V - p)$ for the equipment. For firms to earn positive profits from a customer’s life-cycle demands, it is required that $P - C + \delta (p - c) > 0$. For any $p \in [c, V]$ and $P \in (C - \delta (p - c), U + \delta (V - p)]$, the discounted value of a firm’s share of the industry profit is $\frac{P - C + \delta (p - c)}{n(1 - \delta)}$. If any firm deviates, in the following period, all firms which have established customers will charge $V$ for the add-on. Given that every firm will fully extract their established customers’ surplus on the punishment path, new customers will not accept any deviating offer with $P' > U$. However, by charging any price $P' < \min \{U, P\}$, the deviating firm will be able to capture the entire generation of incoming customers to whom the firm will charge $V$ for the add-on in the following period. Moreover, the deviating firm will also immediately raise its price to its 1/n established customers to $V$. This implies that a deviating firm is able to earn a deviation profit arbitrarily close to $(\min \{U, P\} - C) + \delta (V - c) + (V - p) / n$. It will then lose its share of profits from
all future generations. Therefore, it is incentive compatible for firms to charge the equilibrium prices if
and only if\(^9\)
\[
\frac{P - C + \delta (p - c)}{n (1 - \delta)} \geq \left( \min \{ U, P \} - C \right) + \delta (V - c) + \frac{V - p}{n}.
\]
(1)
By raising \(p\) and lowering \(P\) while keeping the equilibrium profit \(\pi = P - C + \delta (p - c)\) constant, firms
can lower both the deviation profit from the incoming generation of customers and the deviation profit
from the established customers. Therefore, to most effectively sustain any profit level, firms will set
\(p = V\). If firms set \(p = V\), then consumers will purchase the equipment only if \(P \leq U\). Plugging \(p = V\)
and \(P \leq U\) back into the incentive constraint, tacit collusion is sustainable if and only if
\[
\frac{P - C + \delta (V - c)}{n (1 - \delta)} \geq P - C + \delta (V - c),
\]
which is equivalent to
\[
n \leq \frac{1}{1 - \delta}.
\]
I summarize the analysis in this section as follows:

**Lemma 1** If equipment sellers possess unconstrained aftermarket power, then any per-generation in-
dustry profit
\[
\pi \in [0, \pi^M]
\]
is sustainable if
\[
n \leq \frac{1}{1 - \delta}.
\]
Otherwise, the unique equilibrium profit is zero.

According to Lemma 1, the condition under which firms possessing unconstrained aftermarket power
can earn supranormal profits is the same as the condition under which single product firms can earn
supranormal profits, namely, \(n \leq \frac{1}{1 - \delta}\). When firms’ aftermarket power is not contested by the equipment
market, the onset of a price war in the foremarket does not prevent the deviating firm from selling its
add-on to the customers it has stolen at the equilibrium price. In other words, a deviating firm can

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\(^9\)In the incentive constraint, I do not include the deviating firm’s equilibrium profit from established customers in the
period of deviation, \((p - c)/n\). By including that on both sides of the incentive constraint, the incentive constraint would
have been equivalently written as
\[
\frac{p-c}{n} + \frac{P-C+\delta(p-c)}{n(1-\delta)} \geq \left( \min \{ U, P \} - C \right) + \frac{V-c}{n} + \delta (V - c).
\]
capture the entire industry profit from one generation of customers, just as in the case of a single-product market, before losing the profits from all future generations of customers. This explains why unconstrained aftermarket power does not facilitate tacit collusion.

5 Tacit Collusion Facilitated by Constrained Aftermarket Power

Now we return to the main model in which firms possess constrained aftermarket power. My main objective in this section is to identify, for all $\delta \in (0,1)$ and for all $n \geq 2$, the range of steady state per-generation industry profits that can be supported by tacit collusion. We assume that following any deviation from a collusion outcome, firms revert forever to a SPE play path in which they earn zero profits from each generation of consumers.

5.1 Punishment Path: Zero-Profit Equilibrium

Let $(P,p)$ be an arbitrary printer-cartridge price pair and $\pi$ be the profit the industry earns from a generation of customers, which I call per-generation industry profit. First, in any equilibrium in which firms earn zero profit from each consumer’s life-cycle demands, $p = P$ must hold for the following reasons. Suppose $p > P$. Then no cartridges would be sold and the per-generation industry profit would be $\pi = P - C$. Zero profit would imply $P = C$. In this case, a firm could earn a positive profit by lowering its cartridge price to some $p' \in (c,C)$. Next, if $p < P$, then a firm could raise its profit above zero by charging its established customers a higher price $p'' \in (p,P)$ for the cartridge. The deviating firm’s established customers will continue to purchase its cartridge because a new printer costs more.

Furthermore, in any zero-profit equilibrium all established customers purchase a compatible cartridge. Since $p = P$, if some established customers purchased new printers, then some firms could earn a positive profit by lowering the cartridge price by an infinitesimal amount to induce these established customers to purchase the cartridge instead of the printer. Let $p^C$ denote the common price in a zero-profit equilibrium. Then $(p^C - C) + \delta (p^C - c) = 0$, or

$$c < p = P = p^C \equiv \frac{C + \delta c}{1 + \delta} < C. \quad (2)$$

It is easy to verify that no firm has an incentive to deviate. While firms earn zero profit overall, the aftermarket price is above marginal cost: $(C + \delta c) / (1 + \delta) > c$. This equilibrium is qualitatively similar
to that characterized in Borenstein et al. (2000). The main difference is that the demand for the aftermarket product is downward sloping in Borenstein et al. (2000) so there is consumer injury in their equilibrium but not in mine.

One noteworthy observation at this stage is that while firms earn zero profits from each generation of customer, their per-period profits are positive beginning with the second period:

$$2 \frac{C + \delta c}{1 + \delta} - C - c = \frac{(1 - \delta)(C - c)}{1 + \delta}.$$ 

This profit is exactly offset by the loss incurred in the first period of the game so that firms indeed earn zero profit overall:

$$\left( \frac{C + \delta c}{1 + \delta} - C \right) + \frac{\delta}{1 - \delta} \left( \frac{(1 - \delta)(C - c)}{1 + \delta} \right) = 0.$$ 

5.2 The Most Effective Collusive Prices

In my analysis, I assume that any deviation from tacit collusion is punished by all firms reverting to the zero-profit equilibrium prices \( P = p = \frac{C + \delta c}{1 + \delta} \) as stated in (2) forever, where the common zero-profit price for printer and cartridge is below the cost of a printer but above the cost of a cartridge, \( c < \frac{C + \delta c}{1 + \delta} < C \). Maintaining this assumption on the punishment path of tacit collusion, I define the most effective collusive prices as follows:

**Definition 1** For any given per-generation industry profit \( \pi \), a printer-cartridge price pair \((P, p)\) that yields the per-generation industry profit \( \pi \) are the most effective collusive prices if and only if they minimize the deviation payoff.

While any given per-generation industry profit may be achieved by many combinations of printer and cartridge prices, it is obvious that if the most effective collusive prices fail to sustain this per-generation industry profit, then there exists no other price pair which can support such profit, as the alternative prices necessarily lead to a higher deviation payoff. Therefore, for the purpose of characterizing the set of industry profits sustainable by tacit collusion, there is no loss of generality in assuming that firms always adopt the most effective collusive prices. For this reason, I adopt this assumption throughout.

I begin the derivation of the most effective collusive prices by pointing out one intuitive observation:

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10 Equilibria with this property have also been reported in the switching cost literature.

11 This may not constitute the maximal punishment. Therefore, our results might be strengthened if we required firms to implement the maximal punishment.
Lemma 2  The most effective collusive prices must satisfy $p \leq P$.

Proof. Suppose instead that $p > P$ in equilibrium. Every established customer would strictly prefer purchasing a new printer to purchasing a replacement cartridge. So no cartridges would be sold and the per-generation industry profit would be $\pi = (P - C) + \delta (P - C)$. By cutting the printer price below $P$, a deviating firm could steal all the new and established customers.

Suppose firms instead coordinate on the common printer and cartridge price $p'$, where

$$p' = P - \frac{\delta (C - c)}{1 + \delta} < P < p,$$

so that cartridges will be sold in equilibrium. One can verify that the equilibrium per-generation industry profit would remain at $(P - C) + \delta (P - C)$. However, it would now require a deviating firm to cut the printer price below $p'$ to steal the new and established customers. This would lower the deviation profit and weaken the incentives to deviate. ■

It is clear that for established customers to be willing to purchase the cartridge, the cartridge price must not exceed their reservation value $U$. However, new customers may still purchase the printer even if its price exceeds their reservation value, as long as they expect to earn a positive surplus from the consumption of the cartridge. Suppose consumers expect to pay $p \leq U$ for the cartridge and earn a surplus of $(U - p)$ in the second period of their life. Then they are willing to pay up to $U + \delta (U - p)$ for the printer. In other words, for both the printer and cartridge to be purchased, it is necessary that $p \leq U$ and $P \leq U + \delta (U - p)$. Suppose firms collude on the price pair $(P, p)$ such that the industry is earning a profit of $\pi \equiv (P - C) + \delta (p - c) > 0$ from each generation of customers. In the steady state, by staying on the equilibrium path, each firm will earn a discounted profit of

$$\frac{\pi}{n (1 - \delta)} = \frac{(P - C) + \delta (p - c)}{n (1 - \delta)}$$

from customers entering the market in the current and all the future periods. Note that a profit of $(p - c) / n$ which comes from the established customers who already purchased the printer in the previous period is excluded from this expression.

Now consider a firm’s deviation payoff. First, look at the case where $p < P$. Since consumers are rational, they can anticipate both the printer and cartridge prices to become $C + \delta c$ according to (2) in the period following a unilateral deviation. Because consumers can purchase either the printer or the cartridge at the same price once the price war begins, ownership of an old printer does not affect the
second-period consumer surplus. This implies that the deviating firm has to cut the printer price below $U$ to attract new consumers, or otherwise these consumers will respond to the deviation by abstaining from consumption for one period. Summing up, the deviating firm can attracts an entire generation of new customers by setting a printer price $P'$ arbitrarily close to but below $\min \{P, U\}$.

Since $\min \{P, U\} > p$, the deviating firm can also simultaneously raise the cartridge price up to $P'$ without losing its measure $1/n$ of established customers or inducing them to purchase its new printer which costs more than the cartridge. This leads to an instantaneous deviation profit arbitrarily close to

$$\min \{P, U\} - C + \frac{\min \{P, U\} - p}{n}.$$  

If the firm cuts the printer price further so that $P'$ is arbitrarily close to but less than $p$, then it also attracts a measure $(n - 1)/n$ of established customers from its competitors. By doing so, it will earn an instantaneous profit arbitrarily close to $(2n - 1) (p - C) / n$. Note that the deviating firm has to lower its cartridge price to $P'$ as well to avoid having its existing customers replace its old printer with a new one. However, since $P'$ is arbitrarily close to $p$, the latter price cut does not affect the deviation profit.

Whether the deviating firm undercuts $\min \{P, U\}$ or $p$, the new consumers it attracts will continue to purchase the cartridge from it at the price of $p^C = (C + \delta c) / (1 + \delta)$ in the following period, allowing it to earn an additional discounted profit of $\delta (C - c) / (1 + \delta)$. The deviating firm does not earn additional profits from the competitors’ existing customers it has attracted because they will leave the market in the following period.

Due to the ensuing price war, the deviating firm will not earn any more profit from future generations of customers. This gives rise to the following incentive constraint for firms to stay collusive:  

$$\frac{\pi}{n(1 - \delta)} \geq \max \left\{ \min \{P, U\} - C + \frac{\min \{P, U\} - p}{n} + \delta \frac{(C - c)}{1 + \delta}, \frac{(2n - 1)(p - C)}{n} + \delta \frac{(C - c)}{1 + \delta} \right\}, \text{ for } p < P. \quad (3)$$

Next, look at the case where firms collude by setting $P = p$. When a deviating firm undercuts the equilibrium printer price, it attracts the whole generation of new customers as well as all the established customers of its competitors. By cutting the price of its cartridge by the same infinitesimal amount it can avoid inducing its own established customers to purchase its new printer. Therefore, the incentive constraint becomes

$$\frac{\pi}{n(1 - \delta)} \geq \frac{(2n - 1)(p - C)}{n} + \delta \frac{(C - c)}{1 + \delta}, \text{ for } p = P. \quad (4)$$

Recall that the profit from the established customers who already purchased the printer in the previous period is excluded from both sides of the inequality sign.
To most effectively collude, for any given per-generation industry profit level $\pi$ that the firms target to achieve, firms choose a price pair $(P, p)$ satisfying $(P - C) + \delta (p - c) = \pi$ such that the deviation profit is minimized. This transforms the identification of the most effective collusive prices into the following problem of minimizing a firm’s deviation payoff:

$$
\min_{(p, P)} \{ D \} = \begin{cases} 
\max \left\{ \frac{(n+1)\min(P,U) - p - nC}{n} + \delta \frac{(C-c)}{1+\delta}, \frac{(2n-1)(p-C)}{n} + \delta \frac{(C-c)}{1+\delta} \right\} & \text{if } p < P, \\
\frac{(2n-1)(p-C)}{n} + \delta \frac{(C-c)}{1+\delta} & \text{if } p = P, \\
(P - C) + \delta (p - c) = \pi, & \text{subject to } p \leq P.
\end{cases} \quad (5)
$$

The following proposition characterizes the most effective collusive prices that solves Problem (5):

**Proposition 1** Let $\tilde{\pi} = U + \delta \frac{(n+1)U+(n-1)C}{2n} - C - \delta c$. (i) If $U > C$, then $\delta (C - c) < \tilde{\pi} < \pi^M$, and the most effective prices are

$$(P, p) = \begin{cases} 
\left( \frac{\pi + C + \delta c}{1+\delta}, \frac{\pi + C + \delta c}{1+\delta} \right) & \text{if } \pi \in (0, \delta (C - c)), \\
\left( \frac{2n\pi + (2n - \delta (n-1))C + 2n\delta c}{2n + n\delta + \delta}, \frac{2n\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta} \right) & \text{if } \pi \in [\delta (C - c), \tilde{\pi}), \\
\left( \pi + \frac{(2n - \delta (n-1))C + 2n\delta c - (n+1)U}{2n}, \frac{(n+1)U+(n-1)C}{2n} \right) & \text{if } \pi \in [\tilde{\pi}, \pi^M],
\end{cases} \quad (6)
$$

(ii) If $U \leq C$, then $\pi^M \leq \tilde{\pi} \leq \delta (C - c)$, and, for all $\pi \in [0, \pi^M]$, the most effective collusive prices are

$$(P, p) = \left( \frac{\pi + C + \delta c}{1+\delta}, \frac{\pi + C + \delta c}{1+\delta} \right).$$

**Proof.** See Appendix. Figure A1 is included to enhance the exposition of the proof. ■

Here I discuss some properties of the most effective collusive prices as derived in Proposition 1. First, when the industry intends to support a relatively low profit level, as measured by $\pi < \delta (C - c)$, firms will charge the same price for both the printer and cartridge. These prices are both below the marginal cost of a printer:

$$\frac{\pi + C + \delta c}{1+\delta} < C \Leftrightarrow \pi < \delta (C - c).$$

The advantage of setting identical prices both below the marginal cost of the printer is that when a firm deviates by undercutting the printer price, it necessarily attracts its competitors’ existing customers to abandon their old printers and buy new ones from the firm, forcing the firm to incur an immediate loss on every printer sold to these established customers. Since these established customers will leave the market in the following period, the deviating firm is unable to recoup this loss. The net loss on
competitors’ established customers will offset some of the deviation profit the deviating firm obtains by capturing the printer sale to a whole generation of the new customers.

As firms try to support a larger per-generation industry profit, as measured by \( \pi \in [\delta (C - c), \bar{\pi}) \), the printer and cartridge prices necessarily have to be raised above the marginal cost of a printer. In this case, it becomes profitable to steal competitors’ established customers. Any given \( \pi \) can be achieved by either a low \( p \) with a high \( P \) or a high \( p \) with a low \( P \). The first option will lead to a high deviation payoff from undercutting just the printer price; the second option will lead to a high deviation payoff from undercutting both the printer and cartridge prices. Since the deviating firm is free to choose either option to deviate, the deviation incentive is minimized when firms post prices such that a deviating firm feels indifferent between undercutting the printer price and undercutting both prices. The equalization of deviation payoffs happens at

\[
(P, p) = \left( \frac{2n\pi + (2n - \delta (n - 1)) C + 2n\delta c}{2n + n\delta + \delta}, \frac{(n + 1) \pi + 2nC + (n + 1) \delta c}{2n + n\delta + \delta} \right).
\]

As the targeted per-generation industry profit is set higher, the printer price will be pushed beyond consumers’ reservation value for a printer, \( U \). This happens when \( \pi > \bar{\pi} \). In this case, a deviating firm has to discretely lower the printer price to below \( U \) in order to steal new customers. Furthermore, the deviating firm can also choose to undercut the cartridge price in order to steal its competitors’ existing customers. The deviation profit from cutting the printer price to just below \( U \) still decreases in \( p \) because the deviating firm will simultaneously raise its cartridge price from \( p \) to just below \( U \), and a lower \( p \) allows the deviating firm to raise the cartridge price by a greater amount. By the same logic adopted in the previous paragraph, the price pair

\[
(P, p) = \left( \frac{\pi + (2n - \delta (n - 1)) C + 2n\delta c - (n + 1) \delta U}{2n}, \frac{(n + 1) U + (n - 1) C}{2n} \right)
\]

is chosen such that a deviating firm is indifferent between the two deviation options. Note that \( p \) is independent of \( \pi \) for \( \pi \geq \bar{\pi} \).

In the case where \( C \geq U \), \( \pi^M \) is sufficiently small that it falls below \( \delta (C - c) \). Therefore, any feasible profit is most effectively supported by identical printer and cartridge prices.

5.3 Characterization of Profits Sustainable by Tacit Collusion

Building on Proposition 1, I proceed to characterize the set of per-generation profits sustainable by tacit collusion. In the discussion following Proposition 1, I pointed out that when the industry tacitly colludes
on the most effective collusive prices, the firm choosing to deviate either has no choice but to undercut
the cartridge price since \( p = P \), as when \( \pi < \delta (C - c) \), or is indifferent between undercutting \( \min \{ P, U \} \)
and undercutting \( p \), as when \( \pi \geq \delta (C - c) \). Therefore, the characterization of the equilibrium profit set
can be accomplished by plugging (6) into (4), assuming that that the deviating firm always undercut
the cartridge price. The following theorem deals with the more complicated case when \( U > C \).

**Theorem 1** Suppose \( C < U \). Then \( \pi^M > \delta (C - c) \). In this case, there exist \( \hat{n}_1, \hat{n}_2, \) and \( \hat{n}_3 \), where

\[
\frac{1}{1 - \delta} < \hat{n}_1 < \hat{n}_2 < \hat{n}_3,
\]

such that the following hold. (i) If the number of firms is no larger than \( \hat{n}_1 \), then any per-generation
industry profit \( \pi \in \left[ 0, \pi^M \right] \) can be supported by tacit collusion. (ii) If the number of firms is in the
interval \((\hat{n}_1, \hat{n}_2]\), then any per-generation industry profit

\[
\pi \in \left[ 0, \frac{1}{1 + \delta} \left( \frac{1 - \delta}{2(1 - \delta)(n + 1)} \right) \left( \frac{U - C}{1 + \delta} \right) \right] 
\]

\[
\cup \left[ \frac{1}{2n} \left( \frac{1 - \delta}{2(1 - \delta)} \right) \left( \frac{n - 1}{n + 1} \right) \frac{1 - \delta}{1 + \delta} \frac{(C - c)}{1 + \delta}, \pi^M \right]
\]

can be supported by tacit collusion. (iii) If the number of firms is in the interval \((\hat{n}_2, \hat{n}_3]\), then any per-generation
industry profit

\[
\pi \in \left[ 0, \frac{1}{(n + 1)} \frac{1 - \delta}{2(1 - \delta)} \left( \frac{n - 1}{n + 1} \right) \frac{1 - \delta}{1 + \delta} \frac{(C - c)}{1 + \delta} \right]
\]

can be supported by tacit collusion. (iv) If the number of firms exceeds \( \hat{n}_3 \), then any per-generation
industry profit

\[
\pi \in \left[ 0, \frac{\delta}{2(n - 2)} \frac{(1 - \delta)(n - 1)}{(C - c)} \left( \frac{1}{n - 2} \right) \right]
\]

can be supported by tacit collusion.

**Proof.** See Appendix. ■
Figure 1 depicts the set of per-generation industry profits that can be supported by tacit collusion in the case where \( C < U \). This corresponds to the characterization reported in Theorem 1.

**Figure 1: Set of collusive per-generation industry profits, \( C < U \)**

The curves \( \pi = \delta (C - c) \) and \( \pi = \tilde{\pi} \) in Figure 1 divide the set of feasible profits into three regions as they are divided into three cases in Proposition 1: \( [0, \delta (C - c)) \), \( \pi \in [\delta (C - c), \tilde{\pi}) \), and \( \pi \in [\tilde{\pi}, \pi^M] \). The per-generation profits in different regions are most effectively supported by prices with different expressions reported in the proposition.

Recall that when firms possess unconstrained aftermarket power, or if they sell a single product, tacit collusion can support up to the monopoly profit whenever \( n \leq 1/(1 - \delta) \), but firms necessarily earn zero profit otherwise. According to Theorem 1, when firms possess constrained aftermarket powers, the full set of feasible profits is sustainable among a larger number of firms, up to \( n = \hat{n}_1 \) if \( U > C \), up to \( n = \frac{1+\delta}{1-\delta} \frac{2(U-C) + \delta(C-c)}{2(1+\delta)(U-C) + \delta(C-c)} \) if \( U \in \left( \frac{(2+\delta)C+\delta c}{2(1+\delta)}, C \right) \), and for any number of firms if \( U \in \left( \frac{C+\delta c}{1+\delta}, \frac{(2+\delta)C+\delta c}{2(1+\delta)} \right] \). Moreover, even when the number of firms exceeds these values, the industry can still maintain a positive profit when firms possess constrained aftermarket power.

Now I provide some intuition as to why it is easier to sustain tacit collusion when firms possess aftermarket power. When firms each possess constrained aftermarket power, the profit from each generation of customers is split into two parts. The first part comes from the sale of printers and the
remainder comes from the sale of cartridges which takes place one period later. Suppose for the time being \( p^C < p < P \leq U \) and that the printer price is sufficiently larger than the cartridge price so that the deviating firm (weakly) prefers stealing only new customers’ business. By undercutting the printer price, the deviating firm captures the entire industry’s printer sale from the incoming new customers. However, when it sells its cartridge to these customers in the following period, the price war will already have begun and consequently caused the cartridge price to drop to \( p^C \). As a result, the deviating firm is unable to capture the entire industry’s life-cycle profit from a generation of customers before it loses its equilibrium profits from all future generations of consumers. This comparison makes clear that constrained aftermarket power facilitates tacit collusion.

When the industry targets a profit higher than \( \tilde{\pi} \), the profit is most effectively supported by setting the printer price above consumers’ per-period reservation value, i.e., \( P > U \). When \( P > U \), there is another effect that limits the incentive to deviate. Because consumers are rational and anticipate a price war upon observing a deviation, a deviating firm has to cut the printer price below the reservation value, which is discretely below the printer price, to attract new consumers. If the deviating firm set any price above the reservation value, new customers would choose not to consume for one period. In other words, the equilibrium profit increases in \( P \) over \((U, U + \delta(U - p))\) but the deviation profit does not. This explains why, as illustrated in Figure 1, when \( n \in (\hat{n}_1, \hat{n}_2] \), it is possible to sustain high but not moderate profits.

Firms can earn a positive per-generation industry profit by charging the same price for the printer and cartridge, with the common price set below the marginal cost of the printer. This happens when the targeted profit is sufficiently modest \( (\pi \leq \delta(C - c)) \). In this case, the printer serves as a loss leader and firms rely on the sale of cartridges to earn an overall positive profit. Such a pricing strategy further weakens the incentive to deviate. When a firm deviates by undercutting the common price for the printer and cartridge, it has to incur an immediate loss to serve the demand of all the customers it attracts which includes competitors’ established customers. The deviating firm can recoup the up-front loss on the printer sale to the new consumers by selling to them the cartridge in the following period, although at a lowered price. What makes deviation particularly inefficient and unprofitable is the fact that the deviating firm has to produce a new printer for every established customer of its competitors at a loss, who otherwise would have bought a cartridge instead, yet these customers will leave the market in the following period. It is due to the fear of such inefficient switching that profitable tacit collusion
becomes sustainable among arbitrarily many firms.

The following corollary characterizes the profit set sustainable by tacit collusion in the simpler case of \(U \leq C\):

**Corollary 1** Suppose \(U \leq C\). If \(U \in \left( \frac{(2+\delta)C+\delta c}{2(1+\delta)}, C \right]\), then \(\pi^M \in \left( \frac{\delta(C-c)}{2}, \delta(C-c) \right]\). In this case, there exists \(\hat{n}_4 \geq \hat{n}_3\) such that (i) if \(n \leq \hat{n}_4\), then any per-generation industry profit \(\pi \in [0, \pi^M]\) can be supported by tacit collusion. (ii) If \(n > \hat{n}_4\), then any per-generation industry profit

\[
\pi \in \left[ 0, \frac{\delta(1-\delta)(n-1)(C-c)}{2(n(1-\delta)-1)} \right]
\]

can be supported by tacit collusion.

If \(U \in \left( C+\delta c, \frac{(2+\delta)C+\delta c}{2(1+\delta)} \right]\), then \(\pi^M \in \left( 0, \frac{\delta(C-c)}{2} \right]\). In this case, for all \(n\), any per-generation industry profit \(\pi \in [0, \pi^M]\) can be supported by tacit collusion.

**Proof.** See Appendix. \(\blacksquare\)

The first part of Corollary 1 shows that as the monopoly profit falls below \(\delta(C-c)\), which happens when \(U \leq C\), the number of firms among which the monopoly profit can be support increases to \(\hat{n}_4\). The second part of the corollary points out that if the monopoly profit falls below \(\frac{\delta(C-c)}{2}\), which happens when \(U \leq \frac{(2+\delta)C+\delta c}{2(1+\delta)}\), then the monopoly profit is sustainable among any number of firms.

Another corollary of Theorem 1 is that profitable tacit collusion is sustainable even when there are arbitrarily many firms.

**Corollary 2** For all \(\delta \in (0, 1)\), as \(n\) approaches infinity, the set of per-generation industry profit sustainable by tacit collusion converges to \(\left[ 0, \min\{\pi^M, \frac{\delta(C-c)}{2}\} \right]\).

**Proof.** When \(U \leq \frac{(2+\delta)C+\delta c}{2(1+\delta)}\) so that \(\pi^M \leq \frac{\delta(C-c)}{2}\), according to Corollary 1, \(\pi^M\) remains the upper bound of the sustainable profit as \(n\) goes to infinity. When \(U > \frac{(2+\delta)C+\delta c}{2(1+\delta)}\) so that \(\frac{\delta(C-c)}{2} < \pi^M\), according to Theorem 1 and the first part of Corollary 1, for sufficiently large \(n\), the upper bound of the sustainable profit becomes \(\frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)}\), where \(\lim_{n \to \infty} \frac{\delta(n-1)(1-\delta)(C-c)}{2(n(1-\delta)-1)} = \frac{\delta(C-c)}{2}\). \(\blacksquare\)

It is important to note that the assumption that established customers have no future cartridge demands is not crucial for profitable tacit collusion to be sustainable among any number of firms. I show in Section 7 with an extended model that profitable tacit collusion remains sustainable among arbitrarily many firms as long as new customers have a longer market life expectancy than existing
customers. Put differently, this striking result holds as long as serving existing customers is not as profitable as serving new ones.

6 Discussion

6.1 Aftermarket Competition

In this section, I investigate how introduction of competition into firms’ aftermarkets impacts these firms’ ability to tacitly collude. I assume that in a competitive aftermarket, \( p = c \). One can interpret this as being achieved by having sufficiently many independent sellers selling the cartridge compatible with each printer or enforced by regulation. Knowing that they only have to pay \( c \) for the cartridge and are able to gain a surplus of \((U - c)\) in the second period of their market life, new consumers are willing to pay up to \( U + \delta (U - c) \) for a printer. As a result, firms may still collude on a printer price \( P \in (C, U + \delta (U - c)) \). In this setting, since firms only earn profits from printer sales, the per-generation industry profit becomes \( \pi = P - C \). The discounted value of the stream of profits to a firm is \((P - C)/n(1 - \delta)\), where \( n \) is the number of firms. Zero-profit pricing means \( P = C \).

For a deviating firm to attract new customers, it is necessary that the deviation price \( P' \) is less than \( P \). However, that may not be sufficient. Notice that upon seeing a price cut, rational consumers will anticipate a price war to follow in which printers are sold at \( P = C \). In other words, if a new customer abstains from consumption for one period, in the following period, he will enjoy a consumer surplus of \((U - C)\). Therefore, a deviating price \( P' \) will attract new customers only if

\[
U - P' + \delta (U - c) \geq 0 + \delta (U - C),
\]

or

\[
P' \leq U + \delta (C - c) .
\]

Summing up, a deviating firm can gain an instantaneous profit arbitrarily close to \((\min \{ P, U + \delta (C - c) \} - C)\). Therefore, the condition for the collusive outcome to be sustainable is

\[
\frac{P - C}{n(1 - \delta)} \geq \min \{ P, U + \delta (C - c) \} - C. \tag{7}
\]

Analyzing incentive constraint (7) for various parameter values leads to the following characterization:
Lemma 3 Suppose the cartridge price is exogenously fixed at $c$. (i) If $U \leq C$, then if $n \leq \frac{1}{1-\delta}$, then any per-generation industry profit $\pi \in [0, \pi^M]$ is sustainable by tacit collusion; otherwise, firms necessarily earn zero profit.

(ii) If $C < U$, then if $n \leq \frac{1}{1-\delta}$, then any per-generation industry profit $\pi \in [0, \pi^M]$ is sustainable by tacit collusion; if

$$n \in \left( \frac{1}{1-\delta}, \frac{U - C + \delta (U - c)}{1 - \delta (U - C + \delta (C - c))} \right),$$

then any per-generation industry profit $\pi \in [n (1 - \delta) (U - C + \delta (C - c)), \pi^M]$ is sustainable by tacit collusion; otherwise, firms necessarily earn zero profit.

Proof See Appendix. ■

According to Lemma 3, when $U \leq C$, firms with competitive aftermarkets cannot tacitly collude more easily than single-product firms. However, when $C < U$, even if the aftermarkets are competitive, it is still easier for the equipment sellers to sustain tacit collusion than for single-product firms.

Now study how introducing aftermarket competition affects equipment sellers’ ability to tacitly collude. It will be shown that, if $C$ is larger than $U$ or if $C$ is only moderately less than $U$, then, regardless of the number of firms, introduction of aftermarket competition always weakens equipment sellers’ ability to tacitly collude. However, if $C$ is sufficiently small, then aftermarket competition weakens equipment sellers’ ability to tacitly collude if and only if market concentration is relatively low.

For our purpose of illustrating this point, we only compare firms’ abilities to sustain the monopoly profit. According to Lemma 3, it is clear that when $C \geq U$, it is easier to sustain tacit collusion among firms with constrained aftermarket power than among firms with competitive aftermarkets because firms with competitive aftermarkets cannot tacitly collude more easily than single product firms. Now recall from the proof of Theorem 1 [condition (30)] that when $C < U$, the monopoly profit is sustainable among equipment sellers with constrained aftermarket power if and only if

$$\frac{U - C + \delta (U - c)}{n (1 - \delta)} \geq \frac{(2n - 1) (n + 1)}{2n^2} (U - C) + \frac{\delta (C - c)}{1 + \delta}. \quad (8)$$

On the other hand, plugging $P = U + \delta (U - c)$ into (7) implies that firms with competitive aftermarkets can sustain the monopoly profit if and only if

$$\frac{U - C + \delta (U - c)}{n (1 - \delta)} \geq U - C + \delta (C - c). \quad (9)$$

The main finding of this subsection is summarized in the following proposition:
Proposition 2  The monopoly profit is sustainable among a larger set of discount factors when firms possess constrained aftermarket than when the aftermarket are competitive if and only if

(i) \( C > C \equiv (1 - \bar{\gamma}) U + \bar{\gamma} c \), where \( \bar{\gamma} = \frac{9\sqrt{17} + 55}{206} \approx 0.447 \), or

(ii) \( C \in (c, C] \) and \( n > \hat{n}(C) \), for some \( \hat{n}(C) \in (2, \infty) \).

Proof See Appendix.

Proposition 2 provides some interesting insights on how aftermarket competition affects equipment sellers' profit potentials. Aftermarket competition limits firms' ability to tacitly collude when the production cost of the equipment is relatively high compared to that of the aftermarket product (i.e., \( C > C \in (c, U) \)) or when the equipment market is relatively unconcentrated (i.e., \( C \in (c, C] \) and \( n > \hat{n}(C) \)). However, it also suggests the interesting possibility that equipment sellers may welcome entry of independent providers of their aftermarket products when the production costs of the equipment and aftermarket products are sufficiently similar and the equipment market is relatively concentrated (i.e., \( C \in (c, C] \) and \( n < \hat{n}(C) \)). Nevertheless, as more equipment sellers enter the market (so that \( n > \hat{n}(C) \)), they will have the incentive to force aftermarket competitors out of the market.

In the following section we show that if firms can offer bundles, however, then introduction of aftermarket competition always hinders firms' ability to tacitly collude.

6.2 Long-term Contracts/Bundling

Firms' ability to commit to future prices can significantly impact market outcomes. For example, commitment to future prices can solve the hold-up problem arising from brand switching cost and help a durable-good monopolist overcome the Coase conjecture (Farrell and Shapiro 1988, p. 123). In this subsection, I analyze how the (potential) use of long-term contracts/bundling impacts equipment sellers' ability to tacitly collude. When a firm sells a bundle to a customer, it provides the consumer a printer (with initial cartridge included) in the first period of the consumer's market life and commits to providing him the replacement cartridge in the second period of his market life at no additional charge. I call this bundling although literally selling the printer and cartridge in a bundle is less efficient than committing to providing the replacement cartridge one period later, as the former requires the firm to incur the production cost of the replacement cartridge one period before the customer utilizes it. When firms are allowed to bundle, they are free to bundle or not to bundle both on and off equilibrium.
Allowing firms to offer bundles affect their ability to tacitly collude in two ways. First, if firms offer bundles in equilibrium, then the deviating firm is unable to raise its price to its established customers. This lowers the deviation profit, making tacit collusion easier to sustain. On the other hand, since established consumers’ demands are already satisfied, a deviating firm also does not have to worry about attracting undesirable established customers of its competitors’. Even if bundles are not offered in equilibrium, a deviating firm can still use a bundle to steal new customers without attracting competitors’ established customers. This can increase the deviation profit, making tacit collusion harder to sustain.13

Given that allowing firms to offer bundles enables the deviating firm to focus on stealing new customers, tacit collusion is not sustainable among infinitely many firms anymore. However, in reality, bundling may not be feasible in some markets simply because consumers’ future demand and/or firms’ production costs of the aftermarket product are uncertain, and/or the quality of the aftermarket product is nonverifiable. Most importantly, allowing firms to offer bundles will not affect the main finding that equipment sellers can tacitly collude more easily than single product firms. In fact, we will show that allowing firms with constrained aftermarket power to offer bundles renders the monopoly profit sustainable among a larger number of firms. Moreover, if equipment sellers can offer bundles, then it is unambiguously easier for firms with constrained aftermarket power to tacitly collude than for firms with competitive aftermarkets.

First note that when the aftermarkets are competitive, allowing firms to sell bundles does not affect their ability to tacitly collude. Recall that when firms do not offer bundles, consumers are willing to pay for a printer at any price \( P \in (C, U + \delta(U - c)) \), anticipating \( p = c \). If a deviating firm offers a printer, it attracts new customers only if \( P' \leq \min\{P, U + \delta(C - c)\} \), leading to an instantaneous deviation profit of \( \min\{P, U + \delta(C - c)\} - C \). By selling a bundle instead, firms can charge an extra amount \( \delta c \), i.e., selling the bundle at any price \( P_B \in (C + \delta c, (1 + \delta)U) \). This does not affect the set of firms’ equilibrium profits. If a deviating firm offers a bundle, the bundle attracts new customers if and only if both \( P_B' < P + \delta c \) and \( (1 + \delta)U - P_B' \geq \delta(U - C) \) hold. Therefore, the deviation profit is

\[
\min\{(P + \delta c), U + \delta C\} - (C + \delta c)
\]

which is identical to the deviation profit when the firm deviates by selling a printer. This prove that,

---

13 Bundling is somewhat similar to offering consumers a long-term contract. Dana and Fong (2006) show that long-term contracts unambiguously facilitate tacit collusion in markets where firms do not possess aftermarket power.
even when equipment sellers with competitive aftermarkets are allowed to offer bundles, their ability to tacit collude is still characterized by Lemma 3.

Now suppose firms with constrained aftermarket power sell bundles at the price $P_B = U + \delta U (> U)$ in order to target the monopoly profit. Also suppose that if any firm deviates, they revert to the zero profit equilibrium in Section 5.1 forever. The fact that the deviating firm cannot raise its cartridge price to its established customers in the period of deviation guarantees that firms with constrained aftermarket power can tacitly collude as easily as firms with competitive aftermarkets do. In fact, since firms with constrained aftermarket power are not restricted to set $p = c$, it is easier for them to sustain tacit collusion than for firms with competitive aftermarkets. The argument is as follows. Suppose a firm deviates by offering a bundle.$^{14}$ Because new consumers foresee the printer price and cartridge price to both drop to $(C + \delta c)/(1 + \delta)$ and they have the option of abstaining from consumption for one period, a deviating firm can attract new customers if and only if the deviating price for the bundle is no larger than $U + [\delta (C + \delta c)/(1 + \delta)]$. Therefore, the deviation profit is$^{15}$

$$\left( U - C \right) + \delta \frac{C - c}{1 - \delta}. $$

In other words, tacit collusion is sustainable if and only if

$$\frac{(U - C) + \delta (U - c)}{n (1 - \delta)} \geq (U - C) + \delta \frac{(C - c)}{1 + \delta}. \quad (10)$$

Comparing (10) with (8) and noticing that $\frac{[2n-1](n+1)}{2n^2} > 1$, it follows that the monopoly profit is easier to sustain if firms offer bundles. Comparing (10) with (9) and noticing that $\frac{1}{1+\delta} < 1$, it follows that if firms can offer bundles, then tacit collusion is easier to sustain among firms with constrained aftermarket power than among firms with competitive aftermarkets.

### 7 Extension: Generalized Flow of Consumers

In the main body of my analysis, I assume that consumers live exactly two periods and exit the market with certainty afterward. A more realistic model would allow consumers to potentially stay in the market for longer and not to exit in such an abrupt manner. In this section, I modify the main model

---

$^{14}$It can be easily checked that deviating by offering a printer will never be more profitable.

$^{15}$If a firm deviates by offering a printer instead, then the offer attracts new customers if and only if the printer is priced below $U$. In the following period, the deviating firm will earn a profit of $\frac{C - c}{1 - \delta}$ from each of the customers it attract. This leads to the same deviation profit.
to allow for these features. By doing so, I demonstrate that the facilitation of tacit collusion owing to firms’ aftermarket power is a general property that extends to markets wherein consumers exhibit a exit rate between zero and one. The extended model to be presented here includes, as special cases, the model analyzed in the previous sections as well as markets where the exit of established consumers exhibits a constant exit rate property. It will also be clear from my presentation in this section that the striking result that tacit collusion can be sustained among arbitrarily many firms applies as long as the market life expectancies of established customers are lower than that of new customers.

7.1 Model Modification

I first describe the entry and exit of consumers in periods starting from the second period. In every period \( t \geq 2 \), a measure one of consumers arrives. New consumers remain in the market in the following period with probability \( \theta \). All established customers, regardless of when they arrived, remain in the market in the following period with probability \( \phi \in [0, \theta] \). To ensure that the market arrives at a steady state in the second period, I assume that \( \theta + \phi = 1 \) and that in the first period, \( 1/\theta \) new customers enter the market. It is clear that starting from the second period, there are a measure one of new consumers and a measure one of established customers in every period. Note that this model captures two polar cases: (i) \( \theta = \phi = 0.5 \) (constant exit rate) and (ii) \( \theta = 1, \phi = 0 \) (the main model). When \( \theta = \phi \), new customers and established customers have the same market life expectancy. When \( \theta > \phi \), established customers have a shorter market life expectancy than new customers do.

7.2 Zero Profit Equilibrium

Following the same procedure as in Section 3, I can compute the zero profit equilibrium prices. With the zero profit condition

\[
(P - C) + \delta \theta \left[ 1 + \frac{\phi \delta}{1 - \delta \phi} \right] (P - c) = 0
\]

and the requirement that \( P = p \), I can pin down the zero profit equilibrium prices to be

\[
P^C = p^C = \frac{(1 - \delta \phi) C + \delta \theta c}{1 - \delta \phi + \delta \theta}.
\]
7.3 Tacit Collusion

By serving both the formarket and aftermarket, a monopolist could earn from each generation of consumers

\[ \pi^M = U - C + \delta \theta \frac{(U - c)}{1 - \delta \phi}. \]

As in Section 3, if the cartridge price were fixed at \( c \), possibly caused by competitive supply in aftermarket, then collusive pricing and full (consumer) surplus extraction could be sustained in equilibrium if and only if \( n \leq (1 - \delta)^{-1} \).

I focus the remainder of this section on deriving the maximum steady-state profit firms can sustain through tacit collusion, without imposing a fixed cartridge price. The tacit collusion I consider is supported by trigger strategies in which firms revert to the zero profit equilibrium pricing (11) as soon as any firm deviates. I prove the following result:

**Proposition 3** (i) For all \( (\theta, \phi) \in [0,1]^2 \) such that \( \phi = 1 - \theta \) and \( \phi \leq \theta \), there exists \( \tilde{n} > \frac{1}{1 - \delta} \) such that any per-generation industry profit \( \pi \in [0, \pi^M] \) is sustainable if and only if \( n \leq \tilde{n} \). Moreover, (ii) for all \( n \geq 2 \), any per-generation industry profit

\[ \pi \in \left[0, \min\{\pi^M, \frac{\delta (\theta - \phi)(C - c)}{2(1 - \delta \phi)}\}\right] \]

can be supported by tacit collusion.

**Proof.** See Appendix. □

Proposition 3 shows that aftermarket power allows a larger number of firms to sustain any profit between zero and the monopoly profit, whether established customers have a shorter market life expectancy or not. This is because regardless of the rates at which customers exit the market, it remains true that after a deviating firm steals the new generation of customers from its competitors, it will not be able to charge these customers the equilibrium cartridge price in the following period. In other words, aftermarket power still prevents a deviating firm from stealing the entire industry profit from one generation of customers.

However, since \( \frac{\delta (\theta - \phi)(C - c)}{2(1 - \delta \phi)} > 0 \) if and only if \( \theta > \phi \), the proposition above also clarifies the fact that the sustainability of profitable tacit collusion among arbitrarily many firms of any discount factor relies on the property that established customers exit at a higher hazard rate than new customers, i.e., established customers have a shorter market life expectancy. This property can be a consequence of
consumers’ market lifetimes being finite or due to the fact that the products are targeted to consumers of a particular age group (e.g. entry level printers targeted to college students).

To see why the industry can sustain positive profits as long as established customers have shorter market life expectancies than new customers, suppose the industry tacitly collude on some identical printer and cartridge prices whereby firms earn a small but positive expected equilibrium profit in serving each generation of customers’ market lifetime demands. Because the printer and cartridge are priced at the same level, a deviating firm necessarily attracts the other firms’ established customers when it tries to steal new customers. The deviating firm has to provide new equipment to every established customer it steals, just as it does to new customers. It is true that the deviating firm is able to steal the entire industry profit from the new customers. However, as established customers demand fewer cartridges in their life time than new customers, the profit from an established customer it steals is strictly less than the profit from a new customer. Because of this, one can always find a low enough, yet positive, equilibrium profit from each generation of customers whereby a deviating firm takes a loss on the established consumers it steals and this loss dominates the profit it earns on its stolen new customers.

8 Conclusion

In this paper, I illustrate how equipment sellers with monopolized aftermarkets can use their constrained aftermarket power to soften competition in the equipment market. The time lag between foremarket consumption and aftermarket consumption and the substitutability between the foremarket and the aftermarket products for established customers prevents a deviating firm from capturing the entire industry profit from a generation of customers before losing the profits from all future generations of customers. This remains true even if a firm can deviate by offering a bundle, as long as consumers are rational and can anticipate a price war upon seeing a price cut. I believe this competition softening effect is very general. According to my analysis, it is important to distinguish aftermarket power originating from monopolization of addons from aftermarket power originating from monopolization of refills/maintenances; only the latter source of aftermarket power softens competition among firms.

I also prove the stronger result that when firms possess constrained aftermarket power, a supranormal industry profit is sustainable among any number of firms and for any discount factor. This result hinges on the assumptions that it is inefficient to sell foremarket and aftermarket products in a bundle and
that established customers have a shorter market life expectancy. Although these assumptions can be justified in many real-world settings, there may be other extraneous factors that can prevent a large number of firms from tacitly colluding. For one, the monitoring of deviation may become difficult when the number of firms becomes sufficiently large.

As in analyses of tacit collusion among durable goods sellers (e.g., Ausubel and Deneckere (1987), Gul (1987), and Dutta, Matros, and Weibull (2003)), consumers’ understanding of firms’ collusive strategies, which gives them the ability to anticipate a price war upon seeing a price cut, plays a role in our analysis. Unlike in these analyses, however, such role in the current paper is only quantitative but not qualitative. Even if consumers are not sophisticated enough to anticipate a price war upon seeing a price cut, it remains true that when a firm deviates in the equipment market, both the equipment price and aftermarket price will fall in the following period. It also remains true that when the printer and cartridge are priced identically below the marginal cost of the printer, the firm to steal new customers will be forced to at the same time steal other firms’ established customers. Therefore, whether consumers understand firms’ collusive strategies or not, when firms possess constrained aftermarket power, the monopoly profit is sustainable among more firms and supranormal profits are sustainable among any number of firms.

One obvious and important extension for future research is to allow for downward-sloping demand functions. While I believe my analysis provides useful insights on how constrained aftermarket power impacts firm profitability and consumer welfare, my model with unit demands is not suited to the analysis of the overall welfare of the market. In my analysis, consumers always consume both the equipment and aftermarket products so the first best is always achieved; any consumer injury caused by aftermarket power is captured by the industry as profit.
Appendix

**Proof of Proposition 1.** By substituting the rearranged constraint

\[ P = \pi + C - \delta (p - c) \]  \hspace{1cm} (12)

into the minimization problem (5), the latter can be rewritten as:

\[
\min_{p \in [0, \bar{p}]} D (p, \pi) = \begin{cases} 
\max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \} & \text{if } p < \bar{p}, \\
\max f_3 (p) & \text{if } p = \bar{p},
\end{cases} \]

(13)

where \( \bar{p} \equiv \frac{\pi + C + \delta c}{1 + \delta} \) and

\[
f_1 (p, \pi) = \frac{(n + 1) (\pi + C - \delta (p - c)) - p - nC}{n} + \frac{\delta (C - c)}{1 + \delta},
\]

\[
f_2 (p) = \frac{(n + 1) (\pi + C - \delta (p - c)) - p - nC}{n} + \frac{\delta (C - c)}{1 + \delta},
\]

\[
f_3 (p) = \frac{(n + 1) (\pi + C - \delta (p - c)) - p - nC}{n} + \frac{\delta (C - c)}{1 + \delta}.
\]

Notice that

\[
\frac{\partial f_1}{\partial p} = -\frac{(n + 1) \delta + 1}{n} < \frac{\partial f_2}{\partial p} = -\frac{1}{n} < 0 < \frac{\partial f_3}{\partial p} = \frac{2n - 1}{n}.
\]  \hspace{1cm} (14)

Let \( p = \hat{p}_{12} \) solve \( f_1 (p, \pi) = f_2 (p) \), \( p = \hat{p}_{13} \) solve \( f_1 (p, \pi) = f_2 (p) \), and \( p = \hat{p}_{23} \) solve \( f_2 (p) = f_3 (p) \). It can be verified that

\[
\hat{p}_{12} = \frac{\pi - U + C + \delta c}{\delta},
\]

(15)

\[
\hat{p}_{13} = \frac{(n + 1) \pi + 2nC + (n + 1) \delta c}{2n + n \delta + \delta},
\]

(16)

\[
\hat{p}_{23} = \frac{(n + 1) U + (n - 1) C}{2n}.
\]

(17)

By applying (14), we can also obtain that

\[
f_1 (p, \pi) < f_2 (p) \quad \text{if and only if } p > \hat{p}_{12},
\]

\[
f_1 (p, \pi) < f_3 (p) \quad \text{if and only if } p > \hat{p}_{13},
\]

\[
f_2 (p) < f_3 (p) \quad \text{if and only if } p > \hat{p}_{23}.
\]  \hspace{1cm} (18)

Next, it can be verified that \( f_1 = f_2 = f_3 \) and \( \hat{p}_{12} = \hat{p}_{13} = \hat{p}_{23} \) if and only if

\[
\pi = \bar{\pi} \equiv U + \delta \frac{(n + 1) U + (n - 1) C}{2n} - C - \delta c.
\]
Since

\[ \frac{\partial \hat{p}_{23}}{\partial \pi} = 0 < \frac{\partial \hat{p}_{13}}{\partial \pi} = \frac{n + 1}{2n + \delta n + \delta} < \frac{\partial \hat{p}_{12}}{\partial \pi} = \frac{1}{\delta}, \]

\[ \hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23} \quad \text{if } \pi < \hat{\pi}, \]

\[ \hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23} \quad \text{if } \pi \geq \hat{\pi}. \]  \hspace{1cm} (19)

It can be verified that

\[ \hat{\pi} = \frac{\delta (n - 1)}{2n} (U - C), \]

\[ \hat{\pi} - \delta (C - c) = \frac{2n + \delta (n + 1)}{2n} (U - C). \]

So,

\[ \delta (C - c) < \hat{\pi} < \pi^M \quad \text{if } C < U, \]

\[ \delta (C - c) \geq \hat{\pi} \geq \pi^M \quad \text{if } C \geq U. \]  \hspace{1cm} (20)

(i) First consider the case that \( C < U. \)

(i.i) Also suppose for now \( \pi < \hat{\pi}. \) Then according to (19), \( \hat{p}_{12} < \hat{p}_{13} < \hat{p}_{23}. \) Applying (18), it follows that

\[ \max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \} = \begin{cases} f_2 (p) & \text{if } p < \hat{p}_{12}, \\ f_1 (p, \pi) & \text{if } p \in [\hat{p}_{12}, \hat{p}_{13}), \\ f_3 (p) & \text{if } p \geq \hat{p}_{13}. \end{cases} \]

According to (14),

**Lemma A1** If \( \pi < \hat{\pi}, \) then \( \max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \} \) is decreasing in \( p \) for \( p < \hat{p}_{13} \) and increasing in \( p \) for \( p \geq \hat{p}_{13}. \)

If \( \bar{p} < \hat{p}_{13}, \) which holds if and only if

\[ \frac{\pi + C + \delta c}{1 + \delta} < \frac{(n + 1) \pi + 2nC + (n + 1) \delta c}{2n + n\delta + \delta} \]

\[ \Leftrightarrow \quad \pi < \delta (C - c), \]

then

\[ f_3 (\bar{p}) \leq \max \{ \min \{ f_1 (\bar{p}, \pi), f_2 (\bar{p}) \}, f_3 (\bar{p}) \} \]

\[ = \min \{ f_1 (\bar{p}, \pi), f_2 (\bar{p}) \} \]

\[ \leq \min \{ f_1 (p, \pi), f_2 (p) \}. \]
The equality is implied by (18) and (19) and the second inequality follows Lemma A1. So, the deviation profit $D(p, \pi)$ is minimized at $p = \bar{p}$.

Figure A1 provides a graphical illustration of the identification of the most effective collusive prices for the case of $C < U$ which covers the sub-cases considered in parts (i.ii)-(i.iii), although the formal proof does not utilize the figure. The deviation profit $D(p, \pi)$ is depicted by bolded lines in the figure.

**Figure A1: Deviation Profit $D(p, \pi)$, $C < U$**

(i.ii) Now look at the case where $\pi < \tilde{\pi}$ and $\hat{p}_{13} \leq \bar{p}$; the latter inequality holds if and only if $\pi \geq \delta (C - c)$. According to Lemma A1, $\max \{ \min \{ f_1(p, \pi), f_2(p) \}, f_3(p) \}$ is minimized at $p = \hat{p}_{13}$. Since $f_3(\hat{p}_{13}) < f_3(\bar{p})$, as implied by $f_3(\cdot)$ being increasing,

$$\arg \min_{p \in [0, \bar{p}]} D(p, \pi) = \arg \min_{p \in [0, \bar{p}]} \max \{ \min \{ f_1(p, \pi), f_2(p) \}, f_3(p) \} = \hat{p}_{13}.$$
(i.iii) Now look at the case where $\pi \geq \hat{\pi}$. According to (19), $\hat{p}_{12} \geq \hat{p}_{13} \geq \hat{p}_{23}$. Applying (18), it follows that

$$
\max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \} = \begin{cases} 
    f_2 (p, \pi) & \text{if } p < \hat{p}_{23}, \\
    f_3 (p) & \text{if } p \in [\hat{p}_{23}, \hat{p}_{12}],
\end{cases}
$$

which is decreasing in $p$ for $p < \hat{p}_{23}$ and increasing in $p$ for $p \geq \hat{p}_{23}$. Thus, $\max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \}$ is minimized at $p = \hat{p}_{23}$. Since $\pi \geq \hat{\pi} > \delta (C - c)$, it also follows that $\hat{p}_{23} < \bar{p}$; the latter and the fact that $f_3 (\cdot)$ is increasing imply $f_3 (\hat{p}_{23}) < f_3 (\bar{p})$. Therefore,

$$
\arg \min_{p \in [0, \bar{p}]} D (p, \pi) = \arg \min_{p \in [0, \bar{p}]} \max \{ \min \{ f_1 (p, \pi), f_2 (p) \}, f_3 (p) \} = \hat{p}_{23}.
$$

(ii) Now, consider the case where $U \leq C$. In this case, for all $\pi \in [0, \pi^M]$, $\pi \leq \delta (C - c)$. Therefore, we can apply part (i.i) of the proof to establish that

$$
\arg \min_{p \in [0, \bar{p}]} D (p, \pi) = \bar{p}.
$$

Finally, the corresponding printer prices are easily obtained by using (12). This completes the proof of the proposition. ■

**Proof of Theorem 1.** From the proof of Proposition 1, we can see that when firms charge the most effective collusive prices, a deviating firm is either forced to undercut the cartridge price (when $p = P$) or indifferent between undercutting the printer price and undercutting the cartridge price (when $p < P$). In other words, given that we assume that firms post the most effective collusive prices, the deviation profit is always

$$
f_3 (p) = \frac{(2n - 1) (P - C)}{n} + \frac{\delta (C - c)}{1 + \delta}.
$$

This result will be applied repeated in this proof.

Since $(U - C) + \delta (U - c) > \delta (C - c)$ if and only if $U > C$ and $(U - C) + \delta (U - c) > \frac{\delta (C - c)}{2}$ if and only if $U > \frac{(2 + \delta) C + \delta c}{2 (1 + \delta)}$, we have

$$
\begin{align*}
\pi^M &> \delta (C - c) \quad \text{if } U > C, \\
\pi^M &\in \left( \frac{\delta (C - c)}{2}, \delta (C - c) \right) \quad \text{if } U \in \left( \frac{(2 + \delta) C + \delta c}{2 (1 + \delta)}, C \right], \\
\pi^M &\in \left( 0, \frac{\delta (C - c)}{2} \right) \quad \text{if } U \in \left( \frac{C + \delta c}{1 + \delta}, \frac{(2 + \delta) C + \delta c}{2 (1 + \delta)} \right].
\end{align*}
\tag{21}
$$

(i) The first part of the theorem focuses on the case of $U > C$, i.e., $\pi^M > \delta (C - c)$.

Suppose for now the industry targets a per-generation industry profit of $\pi \leq \delta (C - c)$. Applying Proposition 1, the deviation profit is minimized at $P = p = \bar{p} = \frac{\pi + C + \delta c}{1 + \delta}$. So, for $\pi \leq \delta (C - c)$, firms’
incentive constraint reduces to
\[ \frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n} \left( \frac{\pi + C + \delta c}{1 + \delta} - C \right) + \delta \frac{C - c}{1 + \delta}, \] (22)
which can be rewritten as
\[ 2(n(1-\delta) - 1) \pi \leq \delta (n - 1) (1 - \delta) (C - c). \] (23)
This incentive constraint is obviously satisfied if \( n \leq \frac{1}{1-\delta}. \) And for \( n > \frac{1}{1-\delta}, \) it is easier to satisfy with a lower \( \pi. \) Therefore, the incentive constraint is satisfied for all \( \pi \leq \delta (C - c) \) if it is satisfied at \( \pi = \delta (C - c), \) i.e.,
\[ 2(n(1-\delta) - 1) \delta (C - c) \leq \delta (n - 1) (1 - \delta) (C - c) \]
\[ \iff n \leq \frac{1+\delta}{1-\delta} = \hat{n}_3. \]
And for \( n > \hat{n}_3, \) the set of sustainable profits is characterized by (23). By now we have established the following lemma:

**Lemma A2** For all \( n \leq \hat{n}_3, \) any profit \( \pi \in [0, \delta (C - c)] \) can be supported by tacit collusion. For all \( n > \hat{n}_3, \) any profit
\[ \pi \in \left[0, \frac{\delta (n - 1) (1 - \delta) (C - c)}{2(n(1-\delta) - 1)} \right] \] (24)
can be supported by tacit collusion.

Next, suppose the industry targets a per-generation industry profit of \( \pi \in [\delta (C - c), \tilde{\pi}]. \) According to Proposition 1, the most effective collusive prices are \((P, p) = \left( \frac{2n\pi + (2n-\delta(n-1))(2n\delta c + (n+1)n\pi + 2nC + (n+1)\delta c)}{2n + n\delta + \delta}, \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + n\delta + \delta} \right).\) Therefore, tacit collusion is sustainable if and only if
\[ \frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n} \left( \frac{(n+1)\pi + 2nC + (n+1)\delta c}{2n + (n+1)\delta} - C \right) + \delta \frac{(C - c)}{1 + \delta}, \] (25)
which can be rewritten as
\[ \left( \frac{(2n-1)(n+1)}{2n + (n+1)\delta} - \frac{1}{1-\delta} \right) \pi \leq \frac{(n-1)((n+1)\delta + 1)\delta (C - c)}{(1 + \delta)(2n + (n+1)\delta)} \] (26)
The incentive constraint is always satisfied if
\[ \frac{(2n-1)(n+1)}{2n + (n+1)\delta} \leq \frac{1}{1-\delta} \]
\[ \iff n \leq \frac{(1 + 2\delta) + \sqrt{4\delta^2 - 4\delta + 9}}{4(1-\delta)}. \]
When \( n \) exceeds this critical value, the incentive constrain is easier to satisfy with a lower \( \pi \). Therefore, it is satisfied for all \( \pi \in [\delta (C - c), \bar{\pi}] \) if it is satisfied at \( \pi = \bar{\pi} \), i.e.,

\[
\left( \frac{(2n-1)(n+1)}{2n+(n+1)\delta} - \frac{1}{1-\delta} \right) \left( U + \delta \frac{n+1}{2n}(n-1)C - C - \delta c \right) \leq \frac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},
\]

which can be rewritten as

\[
n \leq \frac{(\delta+1)(U-C+2\delta(U-c))}{\sqrt{(\delta+1)^2(U-C+2\delta(U-c))^2+8(1-\delta^2)(U-C+U\delta-c\delta)(U-C)}} \equiv \hat{n}_1.
\]

For \( n > \hat{n}_1 \), the sustainable profit is bounded from above according to (26):

\[
\pi \leq \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)}.
\]

Besides, to support any \( \pi \geq \delta (C - c) \), it is also necessary that

\[
\left( \frac{(2n-1)(n+1)}{2n+(n+1)\delta} - \frac{1}{1-\delta} \right) \delta (C - c) \leq \frac{(n-1)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2n+(n+1)\delta)},
\]

which can be verified to be equivalent to

\[
n \leq \hat{n}_3.
\]

Summing up, we have:

**Lemma A3** For \( n \leq \hat{n}_1 \), any profit \( \pi \in [\delta (C - c), \bar{\pi}] \) is sustainable by tacit collusion. For \( n \in (\hat{n}_1, \hat{n}_3] \), any profit

\[
\pi \in \left[ \delta (C - c), \frac{(n-1)(1-\delta)((n+1)\delta+1)\delta(C-c)}{(1+\delta)(2(1-\delta)n^2-(1+2\delta)n-1)} \right]
\]

is sustainable by tacit collusion.

To support \( \pi \in [\bar{\pi}, \pi^M] \), according to Proposition 1, the most effective collusive prices are

\[
(P, p) = (\pi + \frac{(2n-\delta(n-1))C+2n\delta c-(n+1)\delta U}{2n}, \frac{(n+1)U+(n-1)C}{2n}).
\]

Therefore, the incentive constraint is

\[
\frac{\pi}{n(1-\delta)} \geq \frac{(2n-1)}{n} \left( \frac{n+1}{2n}(n-1)C - C \right) + \frac{\delta (C - c)}{1+\delta} = \frac{(2n-1)(n+1)}{2n} \frac{(U-C)}{2n^2} + \frac{\delta (C - c)}{1+\delta}.
\]

(29)

This is easier to satisfy with higher \( \pi \) because the deviation profit is independent of \( \pi \). In other words, (29) is satisfied for all \( \pi \in [\bar{\pi}, \pi^M] \) if it is satisfied at \( \pi = \bar{\pi} \), i.e.,

\[
\frac{1}{n(1-\delta)} \left( U + \frac{\delta(n+1)(U+(n-1)C - C - \delta c)}{2n} \right) \geq \frac{(1-\delta)(2n-1)(n+1)(U-C)}{2n} + \frac{4n(1-\delta)(C-c)}{1+\delta}.
\]
which can be verified to be equivalent to
\[ n \leq \hat{n}_1. \]

Besides, to support any \( \pi \leq \pi^M = (1 + \delta) U - C - \delta c \), it is necessary that
\[
\frac{(1 + \delta) U - C - \delta c}{n (1 - \delta)} \geq \frac{(2n - 1) (n + 1)}{2n^2} (U - C) + \delta \frac{(C - c)}{1 + \delta},
\]
which can be rewritten as
\[
n \leq \frac{1}{(1 + \delta)} \left( \frac{(1 + 3\delta)(U - C) + 2\delta(C - c)}{2n(1 - \delta)} \right)^2 \left( \frac{U - C}{n} + \delta \frac{(C - c)}{1 + \delta} \right) \equiv \hat{n}_2.
\]

It can be verified that the (29) is easier to satisfy for smaller \( n \). This, with the facts that (29) is easier to satisfy for larger \( \pi \) and that \( \hat{\pi} < \pi^M \), implies that \( \hat{n}_1 < \hat{n}_2 \). Summing up, we have:

**Lemma A4** For \( n \leq \hat{n}_1 \), any profit \( \pi \in [\hat{\pi}, \pi^M] \) is sustainable by tacit collusion. For \( n \in (\hat{n}_1, \hat{n}_2) \), then any
\[
\pi \in \left[ \frac{(1 - \delta)(2n - 1)(n + 1)(U - C)}{2n^2} + \frac{\delta n(1 - \delta)(C - c)}{1 + \delta}, \pi^M \right]
\]
is sustainable by tacit collusion.

Next, we show that \( \frac{1}{1 - \delta} < \hat{n}_1 \) and \( \hat{n}_2 < \hat{n}_3 \). Recall that the per-generation industry profit \( \hat{\pi} \) can be supported by setting \( P = U \) and \( p = \frac{(n + 1)(U + (n + 1)C)}{2n^2} \) if and only if \( n \leq \hat{n}_1 \). At \( \pi = \hat{\pi} \) and \( n = \frac{1}{1 - \delta} \), the difference between the equilibrium profit and the deviation profit is
\[
\frac{1}{n(1 - \delta)} \left( U + \delta \frac{(n + 1)U + (n - 1)C}{2n} - C - \delta c \right) - \left( \frac{(2n - 1)(n + 1)}{2n^2} (U - C) + \delta \frac{(C - c)}{1 + \delta} \right) \bigg|_{n=(1-\delta)^{-1}}
\]
\[
= \left( U + \delta \frac{(n + 1)U + (n - 1)C}{2n} - C - \delta c \right) - \left( \frac{(2n - 1)(n + 1)}{2n^2} (U - C) + \delta \frac{(C - c)}{1 + \delta} \right) \bigg|_{n=(1-\delta)^{-1}}
\]
\[
= \frac{(1 + \delta)(\delta n(n + 1) - (n - 1)(U - C) + 2\delta^2 n^2(C - c))}{2n^2 (1 + \delta)^2} \bigg|_{n=(1-\delta)^{-1}}
\]
\[
= \delta \frac{(1 + \delta)(U - C) + 2\delta C - c)}{2(1 + \delta)} > 0.
\]

In other words, the per-generation industry profit \( \hat{\pi} \) can be supported among more than \( (1 - \delta)^{-1} \) firms; so \( \hat{n}_1 > (1 - \delta)^{-1} \).

Next, \( \hat{n}_2 < \hat{n}_3 \) is established by the fact that the per-generation industry profit \( \pi^M \) can be supported
among \( \hat{n}_2 \) firms but cannot be supported among \( \hat{n}_3 \equiv (1 + \delta)/(1 - \delta) \) firms, as implied by

\[
\frac{\pi}{n(1 - \delta)} - \left( \frac{(2n - 1)}{n} \right) \left( \frac{(n + 1)U + (n - 1)C}{2n} - C \right) + \delta (C - c) = \frac{1}{(1 + \delta)^2} \left( (2 + \delta)(n + 1) \right) \left( \frac{1 + \delta}{1 - \delta} \right) - \frac{1}{2} \left( \frac{1 + \delta}{1 - \delta} \right) C - \frac{1}{2} \delta (C - c) \leq 0.
\]

Now, we are ready to summarize the characterization of the set of equilibrium profits that tacit collusion can support for the case that \( C < U \). By applying Lemmas A2-A4, for all \( n \leq \hat{n}_1 \), any per-generation industry profit in \( [0, \delta(C - c)] \cup (\delta(C - c), \hat{\pi}] \cup (\hat{\pi}, \pi^M] = [0, \pi^M] \) can be supported by tacit collusion; this proves part (i.i) of the theorem. By once again applying Lemmas A2-A4, the set of sustainable per-generation industry profits for \( n \in (\hat{n}_1, \hat{n}_2] \) is

\[
[0, \delta(C - c)] \cup \left[ \delta(C - c), (\pi^M_{(n-1)})(\pi^M_{(n+1)} + (\delta(C - c)) \cup \left[ \left( \frac{1}{2} \frac{1 + \delta}{1 - \delta} \right) \left( \frac{1 + \delta}{1 - \delta} \right) U - C \right) + \delta (C - c) \right] \cup \left[ \frac{(1 - \delta)(2n - 1)(n + 1)(U - C)}{2n} + \frac{\delta n(1 - \delta)(C - c)}{1 + \delta}, \pi^M \right] = \left[ 0, \pi^M \right].
\]

This proves part (i.ii) of the theorem. Similarly, according to Lemmas A2-A4, for \( n \in (\hat{n}_2, \hat{n}_3] \), the set of sustainable \( \pi \) is

\[
[0, \delta(C - c)] \cup \left[ \delta(C - c), (\pi^M_{(n-1)})(\pi^M_{(n+1)} + (\delta(C - c)) \cup \left[ \left( \frac{1}{2} \frac{1 + \delta}{1 - \delta} \right) \left( \frac{1 + \delta}{1 - \delta} \right) U - C \right) + \delta (C - c) \right] \cup \left[ \frac{(1 - \delta)(2n - 1)(n + 1)(U - C)}{2n} + \frac{\delta n(1 - \delta)(C - c)}{1 + \delta}, \pi^M \right] = \left[ 0, \pi^M \right].
\]

This proves part (i.iii) of the theorem. Finally, the range of sustainable \( \pi \) as listed in part (i.vi) of the theorem for \( n > \hat{n}_3 \) follows immediately Lemma A2.

(ii) & (iii) Now we prove the second and third parts of the theorem concerning the case that \( U \leq C \). First, the value of \( \pi^M \) follows immediately (21). When \( U \leq C \), for any \( \pi \in [0, \pi^M] \), according to (20) in the proof of Proposition 1, \( \pi \leq \hat{\pi} \leq \delta(C - c) \). According to Proposition 1, the most effective way to support the profit levels is to set \( P = p = \frac{\pi C + \delta c}{1 + \delta} \) for all \( \pi \in [0, \pi^M] \). Therefore, tacit collusion is sustainable for all \( \pi \in [0, \pi^M] \) if

\[
\frac{\pi^M}{n(1 - \delta)} \geq \frac{(2n - 1)}{n} \left( \frac{\pi^M + C + \delta c}{1 + \delta} - C \right) + \delta \frac{C - c}{1 + \delta},
\]

which is equivalent to

\[
n(2(1 + \delta)(U - C) + \delta(C - c)) \leq \frac{1}{1 - \delta} \left( 2(U - C) + \delta(C - c) \right).
\]

This incentive constraint is satisfied for all \( n \) if \( 2(1 + \delta)(U - C) + \delta(C - c) \leq 0 \), i.e.,

\[
U \leq \frac{(2 + \delta)C + \delta c}{2(1 + \delta)}.
\]
This proves part (iii) of the theorem.

For \( U \in (\frac{(2+\delta)C+c}{2(1+\delta)}, C] \), (32) is satisfied if and only if

\[
n \leq \hat{n}_4 = \frac{1 + \delta}{1 - \delta} \frac{2(U - C) + \delta (C - c)}{2(1 + \delta) (U - C) + \delta (C - c)}.
\]  

(33)

Since \( U \leq C \),

\[
\hat{n}_4 - \hat{n}_3 = \frac{1 + \delta}{1 - \delta} \frac{2(U - C) + \delta (C - c)}{2(1 + \delta) (U - C) + \delta (C - c)} - \frac{1 + \delta}{1 - \delta} = \frac{2\delta (1 + \delta) (C - U)}{(1 - \delta) (2(1 + \delta) (U - C) + \delta (C - c))} > 0.
\]

This proves part (ii.i) of the theorem. If \( n < \hat{n}_4 \), then, according to part (i) of this proof, (24) characterizes the range of per-generation industry profit sustainable. This proves part (ii.ii) of the theorem and completes the proof for the theorem. \( \blacksquare \)

**Proof of Lemma 3** (i) First, consider the case that \( U \leq C \). For all \( P \in (C, U + \delta (U - c)] \), \( P \leq U + \delta (C - c) \). As a result, (7) is equivalent to

\[
\frac{P - C}{n (1 - \delta)} \geq P - C,
\]

which holds if and only if

\[
n \leq \frac{1}{1 - \delta}.
\]

(ii) Now, suppose \( U > C \). For all \( \pi = P - C \in (0, U - C + \delta (C - c)] \), \( P \leq U + \delta (C - c) \). So once again tacit collusion is sustainable if and only if

\[
n \leq \frac{1}{1 - \delta}.
\]

For \( \pi = P - C \in (U - C + \delta (C - c), U - C + \delta (U - c)] \), \( P > U + \delta (C - c) \). So tacit collusion is sustainable if and only if

\[
\frac{\pi}{n (1 - \delta)} \geq U + \delta (C - c) - C,
\]  

(34)

which is the easiest to satisfy when \( \pi = \pi^M \). Therefore, the necessary condition for sustainability of some profits is

\[
n \leq \left[ \frac{\pi^M}{(1 - \delta) (U - C + \delta (C - c))] \right] = \frac{1}{1 - \delta} \frac{U - C + \delta (U - c)}{U - C + \delta (C - c)}.
\]  

(35)

If (35) is satisfied, then (34) provides the lower bound on the sustainable profit as \( \pi \geq n (1 - \delta) (U - C + \delta (C - c)) \). \( \blacksquare \)
Proof of Proposition 2 Define $\gamma = (U - C) / (U - c)$. Further define

\[
F(\delta, n, \gamma) = \frac{\gamma + \delta}{n(1 - \delta)},
\]

\[
G_{MA}(\delta, n, \gamma) = \frac{(2n - 1)(n + 1)}{2n^2} \gamma + \frac{\delta}{1 + \delta} (1 - \gamma),
\]

\[
G_{CA}(\delta, \gamma) = \gamma + \delta (1 - \gamma).
\]

Therefore, (8) and (9) are equivalent to $F(\delta, n, \gamma) \geq G_{MA}(\delta, n, \gamma)$ and $F(\delta, n, \gamma) \geq G_{CA}(\delta, \gamma)$.

It is obvious that if $U \leq C$, i.e., $\gamma \leq 0$, then $G_{MA}(\delta, n, \gamma) < G_{CA}(\delta, \gamma)$ and thus the monopolist profit is sustainable for a wider range of discount factors with monopolized aftermarkets than with competitive aftermarkets. Therefore, in the remainder of the proof, we only consider the case of $C \in (c, U)$, i.e., $\gamma \in (0, 1)$.

Note that

\[
F(0, n, \gamma) = \frac{\gamma}{n} < G_{CA}(0, \gamma) = \gamma < G_{MA}(0, n, \gamma) = \frac{(2n - 1)(n + 1)}{2n^2} \gamma,
\]

\[
\frac{\partial F(\delta, n, \gamma)}{\partial \delta} = \frac{1 + \gamma}{n(1 - \delta)^2} > 0,
\]

and

\[
\frac{\partial^2 F(\delta, n, \gamma)}{\partial \delta^2} G_{MA}(\delta, n, \gamma) = -2 \frac{(1 - \gamma)}{(1 + \delta)^3} < \frac{\partial^2 F(\delta, n, \gamma)}{\partial \delta^2} G_{CA}(\delta, \gamma) = 0 < \frac{\partial^2 F(\delta, n, \gamma)}{\partial \delta^2} F(\delta, n, \gamma) = 2 - \frac{1 + \gamma}{n(1 - \delta)^3}.
\]

Equations (36) and (38) imply that each pair of these curves at most cross each other once in $\delta \in (0, 1)$.

Since

\[
\lim_{\delta \to 1} F(\delta, n, \gamma) = \infty,
\]

\[
\lim_{\delta \to 1} G_{MA}(\delta, n, \gamma) = \frac{(2n - 1)(n + 1)}{2n^2} \gamma + \frac{(1 - \gamma)}{2} < \infty,
\]

\[
\lim_{\delta \to 1} G_{CA}(\delta, \gamma) = 1 + \gamma < \infty,
\]

there exists unique $\tilde{\delta}_{MA}(n, \gamma) \in (0, 1)$ and unique $\tilde{\delta}_{CA}(n, \gamma) \in (0, 1)$ at which $F(\delta, n, \gamma)$ intersects with $G_{MA}(\delta, n, \gamma)$ and $F(\delta, n, \gamma)$ intersects with $G_{CA}(\delta, \gamma)$ respectively and (8) is satisfied if and only if $\delta \geq \tilde{\delta}_{MA}(n, \gamma)$ and (9) is satisfied if and only if $\delta \geq \tilde{\delta}_{CA}(n, \gamma)$.

Lemma A5 Suppose $\tilde{\delta}_{MA}(n, \gamma) \leq \tilde{\delta}_{CA}(n, \gamma)$. Then $\tilde{\delta}_{MA}(n', \gamma) < \tilde{\delta}_{CA}(n', \gamma)$ for all $n' > n$.

Suppose $\tilde{\delta}_{MA}(n, \gamma) \geq \tilde{\delta}_{CA}(n, \gamma)$. Then $\tilde{\delta}_{MA}(n', \gamma) > \tilde{\delta}_{CA}(n', \gamma)$ for all $n' < n$.

Proof Let $\tilde{\delta}(n, \gamma) \in (0, 1)$ solve $G_{MA}(\delta, n, \gamma) = G_{CA}(\delta, \gamma)$. It follows from (36) and (38) that $G_{MA}(\delta, n, \gamma) > G_{CA}(\delta, \gamma)$ for $\delta \in (0, \tilde{\delta})$ and $G_{MA}(\delta, n, \gamma) < G_{CA}(\delta, \gamma)$ for $\delta \in (\tilde{\delta}, 1)$. Suppose $F$
intersects with $G_{MA}$ or $G_{CA}$ at some $\delta < \bar{\delta}$. Since $G_{MA} > G_{CA}$ for $\delta < \bar{\delta}$, $F$ must first intersect with $G_{CA}$ as $\delta$ increases. Since they intersect only once, $F$ must intersect with $G_{MA}$ at above $G_{CA}$ which can take place only at some $\delta_{MA} \leq \bar{\delta}$. That $F$ increases in $\delta$ and $G_{CA} < G_{MA}$ for $\delta < \bar{\delta}$ imply that $\hat{\delta}_{CA} \leq \delta_{MA}$. By extending this logic to the other possibility that $F$ intersects with either $G_{MA}$ or $G_{CA}$ at some value larger than $\bar{\delta}$, we can conclude that there are only two possibilities: $\hat{\delta}_{MA} (n, \gamma) \leq \hat{\delta}_{CA} (n, \gamma) \leq \bar{\delta}$ or $\hat{\delta}_{MA} (n, \gamma) \geq \hat{\delta}_{CA} (n, \gamma) \geq \bar{\delta}$.

Suppose $\hat{\delta}_{MA} (n, \gamma) \leq \hat{\delta}_{CA} (n, \gamma) \leq \bar{\delta}$. If $n' > n$, then $F(\delta, n', \gamma) > F(\delta, n, \gamma)$ and $G_{MA} (\delta, n', \gamma) > G_{MA} (\delta, n, \gamma)$. Following the upward shifts of both $F$ and $G_{MA}$, $F$ intersects with $G_{MA}$ and $G_{CA}$ to the left of $\bar{\delta}$. Therefore, $\hat{\delta}_{MA} (n', \gamma) < \hat{\delta}_{CA} (n', \gamma)$.

A similar logic implies that if $\hat{\delta}_{MA} (n, \gamma) \geq \hat{\delta}_{CA} (n, \gamma) \geq \bar{\delta}$, then $\hat{\delta}_{MA} (n', \gamma) > \hat{\delta}_{CA} (n', \gamma) > \bar{\delta}$ for $n' > n$. This completes the proof of Lemma A5.

When $n = 2$, (8) and (9) are reduced to

$$\frac{\gamma + \delta}{2 (1 - \delta)} \geq \frac{9}{8} \gamma + \frac{\delta}{1 + \delta} (1 - \gamma),$$

(39)

$$\frac{\gamma + \delta}{2 (1 - \delta)} \geq \gamma + \delta (1 - \gamma).$$

(40)

These inequalities can be rewritten as

$$\delta \geq \hat{\delta}_{MA} (2, \gamma) \equiv \frac{1}{\gamma + 12} \left( \sqrt{36 \gamma + 41 \gamma^2 + 4 + 2 - 6 \gamma} \right),$$

$$\delta \geq \hat{\delta}_{CA} (2, \gamma) \equiv \frac{1}{4 (1 - \gamma)} \left( \sqrt{8 \gamma^2 + 1 + 1 - 4 \gamma} \right).$$

It can be verified that $\hat{\delta}_{MA} (2, \gamma) < \hat{\delta}_{CA} (2, \gamma)$ if and only if $\gamma > \bar{\gamma} \equiv \frac{9 \sqrt{17} + 55}{206} \approx 0.447$. Applying Lemma A5, we know that $\hat{\delta}_{MA} (n, \gamma) < \hat{\delta}_{CA} (n, \gamma)$ for all $n \geq 2$ and $\gamma > \bar{\gamma}$, i.e., $C > \bar{C} \equiv (1 - \bar{\gamma}) U + \bar{\gamma} c$.

Suppose $\gamma \leq \frac{9 \sqrt{17} + 55}{206}$, i.e., $C \leq \bar{C}$. Since

$$\lim_{n \to \infty} G_{MA} (\delta, n, \gamma) - G_{CA} (\delta, \gamma) = \frac{1 - \gamma}{1 + \delta} < 0.$$

Therefore, there exists $\hat{n} (\gamma) > 2$ such that for all $n > \hat{n} (\gamma)$, $\hat{\delta}_{MA} (n, \gamma) < \hat{\delta}_{CA} (n, \gamma)$ and for all $n < \hat{n} (\gamma)$, $\hat{\delta}_{MA} (n, \gamma) > \hat{\delta}_{CA} (n, \gamma)$.

When $\gamma = 1$, i.e., $C = c$. In this case,

$$G_{MA} (\delta, n, \gamma) - G_{CA} (\delta, \gamma) = \frac{n - 1}{2 \bar{n}^2} > 0.$$
Therefore, $\tilde{\delta}_{MA}(n, \gamma) > \delta_{CA}(n, \gamma)$. This completes the proof that $\tilde{\delta}_{MA} < \tilde{\delta}_{CA}$ if and only if the condition in the proposition is satisfied. ■

**Proof of Proposition 3** First, we prove that any profit ranging from zero to the monopoly profit can be supported among a larger number of firms than $1/(1 - \delta)$. For this purpose, we do not have to fully characterize the conditions under which profitable tacit collusion is sustainable. Instead, we will just derive *sufficient conditions* for sustainability of tacit collusion.

Given a print-cartridge price pair $(P, p)$, where $p \leq P$, the equilibrium per-generation industry profit will be

$$\pi = (P - C) + \delta \theta \frac{P - c}{1 - \delta \phi},$$

which can be rewritten as

$$P = C + \pi - \delta \theta \frac{P - c}{1 - \delta \phi}. \quad (41)$$

If firms charge the same price for the printer and cartridge, then

$$P = p = \bar{p} = \frac{(1 - \delta \phi) (\pi + C) + \delta \theta c}{1 + \delta (\theta - \phi)}. \quad (42)$$

Note that, however, the steady state profit each firm earns *per-period* will be $(P + p - c) / n$ instead of $\pi / n$. This is because in every period starting from the second period, each firm will serve a measure $1/n$ of established and a measure $1/n$ of new customers. Therefore, the discounted profit of each firm, inclusive of profit from its established customers is

$$\frac{P - C + p - c}{n (1 - \delta)} = \frac{C + \pi - \delta \theta \frac{P - c}{1 - \delta \phi} - C + p - c}{n (1 - \delta)} = \frac{\pi + \frac{1 - \delta \theta - \delta \phi}{1 - \delta \phi} (P - c)}{n (1 - \delta)}. \quad (43)$$

If a firm deviates by setting the printer price arbitrarily close to but less than $\min \{P, U\}$, then it will capture a measure of $\frac{1 - \delta \theta - \delta \phi}{1 - \delta \phi}$ of new consumers, earning from them an immediate profit of $(\min \{P, U\} - C)$ and, in all future periods, a discounted profit of $\delta \theta \left( \frac{P - c}{1 - \delta \phi} \right)$, based on the expectation that a price war will begin in the following period. When the firm deviates, it will also raise its cartridge price to arbitrarily close to $\min \{U, P\}$ and earn from its $1/n$ established customers an immediate profit arbitrarily close to $(\min \{U, P\} - c) / n$ [instead of $(p - c) / n$ as in equilibrium] and a discounted future
of $\frac{\delta \phi}{n} \left( \frac{p_c - c}{1 - \delta \phi} \right)$. This gives rise to a deviation profit of

$$
\begin{align*}
\min \{ P, U \} - C + \frac{(p - c)}{n} + \delta \left( \theta + \phi \right) \left( \frac{p_c - c}{1 - \delta \phi} \right) \\
\leq (P - C) + \frac{(p - c)}{n} + \delta \left( \theta + \phi \right) \left( \frac{p_c - c}{1 - \delta \phi} \right) \\
= \frac{(n + 1) P - nC - c}{n} + \delta \left( \theta + \phi \right) \left( \frac{C - c}{1 - \delta \phi + \delta \theta} \right) \\
= \frac{(n + 1)}{n} \left( C + \pi - \delta \phi \left( \frac{p - c}{1 - \delta \phi} \right) \right) - C - \frac{c}{n} + \delta \left( \theta + \phi \right) \left( \frac{C - c}{1 - \delta \phi + \delta \theta} \right) \equiv g_1(p, \pi).
\end{align*}
$$

The inequality is trivial, the first equality follows (11), and the second equality follows (41).

If the deviating firm instead sets the printer price arbitrarily close to but less than $p$, then it will earn a profit of $(p - C)$ from the new consumers and a profit of $(\frac{p_c - c}{n})$ from its own established customers. It will also steal a measure $(1 - \frac{1}{n})$ of established customers from its competitors, earning from them a profit of $(1 - \frac{1}{n})(p - C)$. The deviating firm will also earn a discounted profit of $\delta \phi \left( \frac{p_c - c}{1 - \delta \phi} \right)$ from the new customers and a discounted profit of $\delta \phi \left( \frac{C - c}{1 - \delta \phi + \delta \theta} \right)$ from the established customers.

Therefore, such deviation leads to a profit of

$$
g_2(p) = \left( 2 - \frac{1}{n} \right) (p - C) + \frac{p - c}{n} + \delta \left( \theta + \phi \right) \left( \frac{C - c}{1 - \delta \phi + \delta \theta} \right).
$$

Therefore, for $p < P$ (i.e., $p < \bar{p}$), the deviation profit is no larger than $\max \{ g_1(p, \pi), g_2(p) \}$. Suppose $P = p$, then the deviation profit is necessarily $g_2(p)$. Summing up, the deviation profit does not exceed

$$\hat{D}(p, \pi) = \begin{cases} 
\max \{ g_1(p, \pi), g_2(p) \} & \text{if } p < \bar{p}, \\
g_2(p) & \text{if } p = \bar{p}.
\end{cases}
$$

Let $p = \tilde{p}_{12}$ solve $g_1(p, \pi) = g_2(p)$. It can be verified that

$$\tilde{p}_{12} = \frac{1 - \delta \phi}{2n (1 - \delta \phi + \delta \theta (n + 1))} \left( (n + 1) \pi + 2nC + \frac{(n + 1) \delta \theta c}{1 - \delta \phi} - \delta \phi (n - 1) \left( \frac{C - c}{1 + \delta (\theta - \phi)} \right) \right). \quad (44)
$$

Also, we know that $g_1$ is decreasing in $p$ and $g_2$ is increasing in $p$ and thus

$$\max \{ g_1(p, \pi), g_2(p) \} = \begin{cases} 
g_1(p, \pi) & \text{if } p \leq \tilde{p}_{12}, \\
g_2(p) & \text{if } p > \tilde{p}_{12},
\end{cases}
$$

and $\max \{ g_1(p, \pi), g_2(p) \}$ is minimized at $p = \tilde{p}_{12}$.

By comparing (42) with (44), it can be verified that

$$\tilde{p}_{12} < \bar{p} \text{ if and only if } \pi > \frac{\delta (\theta - \phi)(C - c)}{1 - \delta \phi}. \quad (45)$$
First suppose the industry targets some \( \pi \geq \frac{\delta(\theta - \phi)(C - c)}{1 - \delta \phi} \) so that \( \tilde{p}_{12} \leq \tilde{p} \). In this case,

\[
\hat{D}(p, \pi) = \begin{cases} 
  g_1(p, \pi) & \text{if } p < \tilde{p}_{12}, \\
  g_2(p) & \text{if } p \in [\tilde{p}_{12}, \tilde{p}],
\end{cases}
\]

and \( \hat{D}(p, \pi) \) is minimized at \( p = \tilde{p}_{12} \). Suppose that firms support this profit by setting \( p = \tilde{p}_{12} \) and setting \( P \) according to (41). In this case, the sufficient condition for sustainability of tacit collusion is

\[
\frac{\pi + \frac{1 - \delta \theta - \delta \phi}{1 - \delta \phi} (\tilde{p}_{12} - c)}{n (1 - \delta)} \geq \hat{D}(\tilde{p}_{12}, \pi).
\]

By plugging (44) into \( \hat{D}(\tilde{p}_{12}, \pi) \), we can show that

\[
\frac{\pi + \frac{1 - \delta \theta - \delta \phi}{1 - \delta \phi} (\tilde{p} - c)}{n (1 - \delta)} - \hat{D}(\tilde{p}_{12}, \pi) = \left(1 - \delta \phi\right) K_1 \pi + \frac{\delta K_2 (C - c)}{(1 - \delta) n (2 - 2\delta \phi + \delta \theta) + (1 + \delta (\theta - \phi)) (n (2 - 2\delta \phi + \delta \theta) + \delta \theta)}
\]

where

\[
K_1 = (1 + 2\delta) n + 1 - 2 (1 - \delta) n^2
\]

\[
K_2 = (1 - \delta) (\theta - \phi) \delta \theta n^2 - (\theta + 3\phi - \theta \delta + 2\delta \phi + 2\theta^2 \delta - 3\delta \phi^2 - \theta \delta \phi - 2) n
\]

\[- (\theta - \phi - \theta^2 \delta^2 - \theta \delta + \theta^2 \delta + \delta \phi^2 + \theta \delta^2 \phi)\]

By plugging \( \theta = 1 - \phi \) into \( K_2 \) and recalling that \( \phi \leq 0.5 \), we have

\[
K_2 = (1 - \delta) (1 - 2\phi) (n - 1) ((n + 1) \delta (1 - \phi) + 1) \geq 0.
\]

Moreover, it can be verified that

\[
K_1 \geq 0 \quad \iff \quad n \leq \tilde{n}_1 = \frac{\sqrt{4\delta^2 - 4\delta + 9} + 2\delta + 1}{4 (1 - \delta)}.
\]

In other words, if \( \theta \geq \phi \), then any profit in the range \( \left[\frac{\delta(\theta - \phi)(C - c)}{1 - \delta \phi}, \pi^M\right] \) is sustainable for all \( n \leq \tilde{n}_1 \), and \( \tilde{n}_1 > \frac{1}{1 - \delta} \) because

\[
\tilde{n}_1 - \frac{1}{1 - \delta} = \frac{\sqrt{4\delta^2 - 4\delta + 9} + 2\delta + 1 - 4}{4 (1 - \delta)} = \frac{\sqrt{4\delta^2 - 4\delta + 9} - (3 - 2\delta)}{4 (1 - \delta)}
\]

\[
= \frac{\sqrt{4\delta^2 - 4\delta + 9} - \sqrt{4\delta^2 - 12\delta + 9}}{4 (1 - \delta)} > 0.
\]
In the case that \( \theta = \phi \), the lower bound of \( \frac{\delta(\theta - \phi)(C - c)}{1-\delta\phi}, \pi^M \) is 0. Then the proof of part (i) of the proposition can be completed by choosing \( \tilde{n} = \tilde{n}_1 \).

Now, consider the case that \( \theta > 0.5 > \phi \) so that \( \frac{\delta(\theta - \phi)(C - c)}{1-\delta\phi} > 0 \). Suppose the industry targets some \( \pi < \frac{\delta(\theta - \phi)(C - c)}{1-\delta\phi} \). In this case \( \tilde{p} < \tilde{p}_{12} \) and

\[
\hat{D}(p, \pi) = \begin{cases} 
g_1(p, \pi) & \text{if } p < \tilde{p}, 
g_2(p) & \text{if } p = \tilde{p}, \end{cases}
\]

where \( g_2(\tilde{p}) < g_1(\tilde{p}, \pi) \) because \( \tilde{p} < \tilde{p}_{12} \). Since \( g_1(p, \pi) \) is decreasing, \( \hat{D}(p, \pi) \) is minimized at \( p = \tilde{p} \).

Suppose the industry sets \( P = p = \tilde{p} \) to support the targeted \( \pi \). In this case, the sufficient condition for sustainability of tacit collusion is

\[
\frac{2\tilde{p} - (C + c)}{n(1-\delta)} \geq \left(2 - \frac{1}{n}\right)(\tilde{p} - C) + \frac{(\tilde{p} - c)}{n} + \frac{\delta(\theta + \phi)(C - c)}{1-\delta\phi + \delta\theta}. \tag{46}
\]

By plugging (42) and \( \theta = 1 - \phi \) into (46), the latter can be simplified as

\[
n \left(\pi - \frac{\delta(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}\right) \leq \frac{1}{1 - \delta} \left(\pi - \frac{\delta(1 - \delta)(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}\right). \tag{47}
\]

For \( \pi \in \left(\frac{\delta(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}, \frac{\delta(\theta - \phi)(C - c)}{1-\delta\phi}\right) \), (47) can be further rewritten as

\[
n \leq \tilde{n}_2 = \frac{1}{1 - \delta} \frac{2(1 - \delta\phi)\pi - \delta(1 - \delta)(1 - 2\phi)(C - c)}{2(1 - \delta\phi)\pi - \delta(1 - 2\phi)(C - c)}.
\]

It can be verified that \( \phi < 0.5 \) implies

\[
\frac{2(1 - \delta\phi)\pi - \delta(1 - \delta)(1 - 2\phi)(C - c)}{2(1 - \delta\phi)\pi - \delta(1 - 2\phi)(C - c)} > 1.
\]

Therefore,

\[
\tilde{n}_2 > \frac{1}{1 - \delta}.
\]

For \( \pi \in \left(\frac{\delta(1 - \delta)(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}, \frac{\delta(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}\right) \), (47) holds for all \( n \) because the LHS of the equation is negative while its RHS is positive for all \( n \). For \( \pi \in \left[0, \frac{\delta(1 - \delta)(1 - 2\phi)(C - c)}{2(1 - \delta\phi)}\right] \), (47) becomes

\[
n \geq \frac{1}{1 - \delta} \frac{\delta(1 - \delta)(1 - 2\phi)(C - c) - 2(1 - \delta\phi)\pi}{\delta(1 - 2\phi)(C - c) - 2(1 - \delta\phi)\pi} - \frac{\delta(1 - 2\phi)(C - c) - 2(1 - \delta\phi)\pi}{(1 - \delta)} < 1,
\]

which is always satisfied. So, in the case that \( \theta > 0.5 > \phi \), any profit \( \pi \in [0, \pi^M] \) can be supported by tacit collusion for all \( n \leq \min\{\tilde{n}_1, \tilde{n}_2\} \), where \( \min\{\tilde{n}_1, \tilde{n}_2\} > 1/(1 - \delta) \). Therefore, we can complete the proof of part (i) of the proposition by setting \( \tilde{n} = \min\{\tilde{n}_1, \tilde{n}_2\} \).
Note that (47) can be rewritten as

\[ 2 (1 - \delta \phi) ((1 - \delta) n - 1) \pi \leq \delta (1 - \delta) (n - 1) (1 - 2 \phi) (C - c). \]  

(48)

This holds for all \( n \leq 1 / (1 - \delta) \) and when \( n > 1 / (1 - \delta) \), it hold if and only if

\[
\pi \leq \frac{\delta (1 - \delta) (n - 1) (1 - 2 \phi) (C - c)}{2 (1 - \delta \phi) ((1 - \delta) n - 1)} = \frac{\delta (1 - \delta) (n - 1) (\theta - \phi) (C - c)}{2 (1 - \delta \phi) ((1 - \delta) n - 1)}.
\]

\[
\frac{d}{dn} \left( \frac{\delta (1 - \delta) (n - 1) (\theta - \phi) (C - c)}{2 (1 - \delta \phi) ((1 - \delta) n - 1)} \right) = \frac{-\delta^2 (1 - \delta) (\theta - \phi) (C - c)}{2 (1 - \delta \phi) (n (1 - \delta) - 1)^2} < 0.
\]

As \( n \) approaches infinity, the upper bound on \( \pi \) becomes

\[
\lim_{n \to \infty} \frac{\delta (1 - \delta) (n - 1) (\theta - \phi) (C - c)}{2 (1 - \delta \phi) ((1 - \delta) n - 1)} = \frac{\delta (\theta - \phi) (C - c)}{2 (1 - \delta \phi)}.
\]

Therefore, for all \( n \geq 2 \), any \( \pi \in \left[ 0, \frac{\delta (\theta - \phi) (C - c)}{2 (1 - \delta \phi)} \right] \) can be supported by tacit collusion. Obviously, \( \pi \) cannot exceed \( \pi^M \) as well. This completes the proof of part (ii) of the proposition. ■
References


