Tax Planning, Imbalance and Production

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INTRODUCTION

Ten years ago it was unusual to find mainstream corporate tax departments who would buy tax-sheltering ideas. Today, with the tax department viewed as a profit center, it is rare to find a major corporation that does not use them (Avi-Yonah, 2004).

Such understatement of income creates inequity and inefficiency. The inequity is due to the fact that taxpayers vary in their access to tax sheltering activities. Hence, planning is used to reduce the tax burden of some taxpayers (who tend to be among the wealthiest in the society) at the expense of others. Equity considerations are not the focus of this paper.

Inefficiency is usually attributed to non-economic incentives to enter into various transactions to minimize taxes created by tax planning opportunities, leading to inefficient allocation of resources, and to the added complexity of greater compliance and administrative costs (see, e.g., Slemrod and Yitzhaki 2002).

This paper intends to shed light on yet another form of inefficiency caused by the current tax system’s vulnerability to tax planning: the non-optimal production level.

The above statement may seem counterintuitive, because it is well known that income tax rates do not affect the level of production. The firm maximizes its profit by setting its output so that the marginal cost of production (the cost of producing an additional unit of output) equals price. Lowering the tax rate on a firm’s profit does not
change its marginal cost and therefore does not change its output level (Mankiw 2001, p. 297).

Two complimentary factors drive this paper’s counterintuitive result. First, tax planning, such as using financial instruments to manipulate the tax code, does not merely reduce effective tax rates but often has another distressing attribute: it imposes lower tax rates on gains than on losses by accelerating the deduction of losses while deferring recognition of gains, and by reporting gains as capital and losses as ordinary. This, in turn, increases the government’s share in losses while decreasing its share in gains.¹ This tax asymmetry has been termed “imbalance” in the literature, and described as a case in which the government’s gain-loss ratio is lower than one (e.g., Schizer, 2004; Bankman, 1995).²

The second factor that drives this paper’s result is price uncertainty. Unlike firms that operate in a deterministic environment where profit is secured (always positive or

¹ Restrictions on the deduction of losses work in the opposite direction. See Campisano and Romano (1981), Auerbach (1983, 1986), Altshuler and Auerbach (1990). Only in knife-edge cases would the two effects exactly offset each other and the government gain/loss ratio equal one. The direction of the imbalance may differ across industries and across taxpayers, raising additional inefficiency and distributive concerns. Our thesis; namely, that tax asymmetry distorts production level, applies to both directions of asymmetry. In this paper we focus on the case where the gain/loss ratio is below one.

² “Properly structured, these transactions ensure the deduction of losses at high rates and the recognition of gains at low rates, or, what amounts to the same thing, the acceleration of loss recognition and deferral of gain recognition” (Bankman 1995, p. 787).
zero) as in the world of microeconomics textbooks; in reality, a firm may find itself in states of nature of gains or losses.

Tax asymmetry in a world of price uncertainty adds another inefficiency beyond the excess burden caused by symmetric taxation. At any tax rate, asymmetry distorts production output. A low asymmetric tax may be more distortive than a high symmetric tax rate. Asymmetric taxation affects the firm’s choice of projects as well as level of production, causing it to move away from the social optimum.

This paper is novel in analyzing the combined effect of tax asymmetry and uncertainty on production. Many have commented that tax planning leads to inefficient allocation of resources, but its effects on production have not been discussed, let alone proved using a formal model.

The paper is organized as follows. The first section lays out our model, considering two cases. First, it considers the case where the tax system is symmetric, showing that the socially-optimal production level is attained. Second, it considers the case of asymmetric taxation, and provides a proof that the output level is distorted and

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4 For example, contributors to the literature on mal-taxation of financial instruments usually discuss its effect on individuals’ decisions regarding their portfolios of financial assets.
over-production takes place. The second section discusses the relationship between effective tax rates on gains and on losses. Finally we offer some concluding remarks discussing potential policy implications.

THE MODEL

Consider a price-taking risk-neutral (hence maximizing expected net profits) firm, which produces a commodity whose price is a continuous random variable $\tilde{P}$. We assume $\tilde{P}$ to accept values in the interval $[P, \bar{P}]$, and $\tilde{P}$ to have a density function $f(p)$. The firm determines its output $Q$ at time 0, production is completed at time 1, when the price $\tilde{P}$ is realized and transactions take place. The firm’s technology of production gives rise to a cost function $C(Q)$ which satisfies: $0 < C(Q), 0 < C'(Q), 0 < C''(Q)$. For simplicity we assume that the production cost is not a random variable.

We denote by $E(\tilde{P})$ the expected value of random variable $\tilde{P}$:

$$[1] \quad E(\tilde{P}) = \int_p^\bar{p} pf(p)dp$$

and denote by $\tilde{\pi}(p,Q)$ the random variable of the profit before tax:

$$[2] \quad \tilde{\pi}(p,Q) = \tilde{P}Q - C(Q)$$

Expressing the profit in expected values:

$$[3] \quad E(\tilde{\pi}(p,Q)) = E(\tilde{P})Q - C(Q)$$

We next make two assumptions:
(i) When the price is $P^*$, namely, the minimal value of the support of $\hat{P}$, the firm loses. Mathematically expressed: $P^*Q - C(Q) < 0$, for any $Q$.

(ii) We further assume that for any quantity the firm produces, if it is paid the expected price, it will make a profit. Mathematically expressed:

$$E(\hat{P})Q - C(Q) > 0$$

In fact, this assumption is stronger than what we need for the proof to hold. It would be enough to assume that if, when the firm produces at its optimal level of production and the price it is paid is the expected price, the firm will make a profit. This is a very reasonable assumption, because if even at the expected price the firm will not make a profit, it is not likely to produce at all.

Introducing income tax into our model, we assume, first, a symmetric tax system.

**The Symmetric Tax case**

When the government shares equally in gains and losses under uncertainty, the tax has no incentive effect. In the context of production this is manifested by the well-known fact that under the standard assumption regarding the shape of the production cost function, profit is maximized when marginal cost equals the price, which here is the expected price. That is:

$$\max_{\hat{Q}} ([E(\hat{P})Q - C(Q)](1 - \tau))$$

is uniquely attained for $\hat{Q}$ such that

$$E(\hat{P}) - C'(\hat{Q}) = 0$$
We are looking for a $\hat{Q}$ for which $C'(\hat{Q})$ will be equal to the expected price. We see this by differentiating equation (3). $Q$ is, in effect, a bet placed by the firm, because we assume that the firm has to decide on its output level before the price is known. The firm needs to know what the optimal bet to place is.

We find the firm’s optimal production quantity, $\hat{Q}$, by maximizing this expression, namely, finding the $Q$ where the derivative is zero.

The uniqueness follows from our assumption that $C''(Q) > 0$. The marginal cost function is strictly increasing and therefore there is only one point, the maximum, where it intersects with the horizontal line.

Turning next to the asymmetric tax case, we assume that when the firm has a profit it pays tax at an effective rate of $t_1$ and when it suffers a loss it is compensated by the tax authority at an effective rate of $t_2$.

**The Asymmetric Tax Case**

In this paper we assume that due to tax planning, asymmetry favors the taxpayer; namely, that the government’s gain/loss ratio is below one.

Our theorem holds for any tax rates as long as the effective tax rate on gains is lower than the effective tax rate on losses. The general case of asymmetry (in favor of the taxpayer) is *transformable* into a simplified notation according to which $t_2 = t > 0$, namely a positive effective tax rate on losses, while $t_1 = 0$. Our results remain unchanged as long as $0 \leq t_1 < t_2$. See discussion in the next section.
The *expected* after-tax profit for $Q$, denoted as $EN(Q)$, taking into account taxes (on profits) and subsidies (on losses), is the following:

$$EN(Q) = E(\tilde{P}Q - C(Q)) - t \cdot E(\min(0, \tilde{P}Q - C(Q)))$$

where the second term is the subsidy on the expected loss.\(^5\)

There is a (pre-tax) loss, for any given $Q$, when $pQ - C(Q) < 0$. Dividing this by $Q$ we see that when $p < \frac{C(Q)}{Q}$, there is a loss.

For a given $Q$, the price range where there is a (pre-tax) *loss* is:

$$P < P \leq \frac{C(Q)}{Q} = P(Q)$$

Thus [3], [5], [6] and the definition of expectation imply that the expected after tax profit is given by

$$EN(Q) = E(\tilde{P})Q - C(Q) - t \int_{p}^{P(Q)} (pQ - C(Q)) f(p) dp$$

Next we differentiate [7] to find $Q^*$ which is the output level that gives the firm the maximum expected after-tax profit. We show that under asymmetric taxation where the gain/loss ratio is below one, $\hat{Q} < Q^*$. Namely, $Q^*$ will be greater than $\hat{Q}$, the socially optimal level.

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\(^5\) Note that $\min(0, \tilde{P}Q - C(Q))$ is negative, therefore, $-t \cdot E(\min(0, \tilde{P}Q - C(Q)))$ is positive, namely, a subsidy.
Proof

The following is based on the Leibniz Integral Rule:

Lemma 1 Let

\[ H(Q) = \int_{P}^{P(Q)} F(p, Q) \, dp \]

then

\[ H'(Q) = P'(Q) \cdot F(P(Q), Q) + \int_{P}^{P(Q)} \frac{\partial F(p, Q)}{\partial Q} \, dp \]

Theorem

\[ \hat{Q} < Q^* \]

Proof. Differentiating [7] to find the maximizing \( Q^* \) and using Lemma 1. In this case, for all \( Q \), \( P(Q) = \frac{C(Q)}{Q} \), and \( F(p, Q) = (pQ - C(Q))f(p) \). Hence, \( F(P(Q), Q) = 0 \).

Thus

\[ [8] \quad E N'(Q^*) = E(\tilde{P}) - C'(Q^*) - t \int_{P}^{P(Q)} (p - C'(Q^*))f(p) \, dp = 0 \]

So

\[ [9] \quad E(\tilde{P}) - t \int_{P}^{P(Q)} pf(p) \, dp = C'(Q^*) - C'(Q^*)t \int_{P}^{P(Q)} f(p) \, dp \]

Denote \( \int_{P}^{P(Q)} f(p) \, dp = \Delta \).

Now \( E(\tilde{P})Q^* - C(Q^*) > 0 \), by assumption ii, hence \( E(\tilde{P}) > P(Q^*) \). Hence,

\[ [10] \quad t \int_{P}^{P(Q)} pf(p) \, dp < t \int_{P}^{P(Q)} E(\tilde{P})f(p) \, dp = tE(\tilde{P})\Delta \]

So from (9) and (10)
\[ E(\tilde{P})(1-t\Delta) < C'(\hat{Q}')(1-t\Delta). \]

Note that \( t\Delta < 1 \). Thus

\[ C'(\hat{Q}) = E(\tilde{P}) < C'(\hat{Q}'). \]

But \( C'(Q) \) is monotone increasing. Hence \( \hat{Q} < Q' \). □

**THE GENERALITY OF THE MODEL**

To assume that tax planning allows taxpayers not to pay tax at all is an implausible assumption. It is more plausible to assume that tax planning may result in a higher effective tax rate for losses than for gains, due to various tax planning techniques such as deferral of recognition of gains for tax purposes, the acceleration of loss recognition, and manipulation of the difference between ordinary and capital gains tax rates.

As mentioned above, the general case of asymmetric taxation was transformed in our model into a simplified notation according to which gains are not taxed while losses are subject to a positive effective tax rate denoted by \( t \). In this section we will show that maximizing net profit when the effective tax rate on gains, \( t_1 \), is smaller than the effective tax rate on losses, \( t_2 \), is like maximizing the net profit when gains are subject to zero tax and losses are subject to positive tax, given by \( \frac{t_2 - t_1}{1 - t_1} \), namely, subsidized. In doing this we will take a closer look at the \( t \) that was used in our model.
Consider the optimization problem of [5] when $0 < t_1 < t_2$, that is

\[ \max_{\tilde{Q}} \left[ E(\tilde{P}Q - C(Q))(1-t_1) - t_2 \cdot E(\min(0, \tilde{P}Q - C(Q))) \right] \]

**Proposition**

$Q^*$ solves [11] if $Q^*$ solves [5] for $t = \frac{t_2 - t_1}{1 - t_1}$.

**Proof**

When $0 < t_1 < t_2$, the optimization problem is formulated as:

\[ EN(Q, t_1, t_2) = E(\tilde{P}Q - C(Q))(1-t_1) - t_2 \cdot E(\min(0, \tilde{P}Q - C(Q))) \]

Because $t_2$ is larger than $t_1$, we write it as $t_2 = t_1(1 + \alpha)$.

Denote, by $E^+$ that part of the expectations of $\tilde{\pi}(p, Q)$ which is positive, namely the part which represents expected gains, and by $E^-$, the negative part, that is the part that represents expected losses. Then,

\[ EN(Q, t_1, t_2) = E^+(1-t_1) + E^- - t_1(1 + \alpha)E^- \]

Hence,

\[ EN(Q, t_1, t_2) = (1-t_1)E(\tilde{P}Q - C(Q)) - t_1\alpha E^- \]

Dividing by $1-t_1$\(^6\), we conclude that maximizing $EN(Q, t_1, t_2)$ is the same as maximizing

\[ EN(Q, 0, \frac{t_2 - t_1}{1 - t_1}) \].

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\(^6\) As long as $1-t_1$ is positive, which seems like a plausible assumption.
CONCLUDING REMARKS

Our paper is based on an assumption regarding asymmetric taxation. By asymmetric, we mean a different effective tax rate for gains and losses, also known in the literature as the government’s gain/loss ratio being different from one.

Various attributes of the current income tax system cause asymmetry. Some of them, such as restrictions on the deduction of losses, work in one direction, driving the gain/loss ratio above one; while the other, namely tax planning, works in the opposite direction.

We argue that this asymmetry entails inefficiency costs, and attempt to prove its distortive effect on production. Our thesis applies to both directions of asymmetry. In this paper we have focused on the case where the gain/loss ratio is below one, and proved its distortive effect on production.

In order to draw policy implications from this model, the policymaker needs to know what happens in reality. Is the gain/loss ratio above one as is often assumed in the literature, especially that which discusses loss restrictions, or is it below one, as other papers, especially those that discuss tax planning, assume? Or, is it exactly one, namely, that the restrictions on one side and tax planning on the other, exactly offset each other?

If the gain/loss ratio is above one, which generally leads to under-production,\(^7\) we might want to adjust the restrictions on the deduction of losses, or we should at least be more concerned about the extent to which the tax rates are graduated.\(^8\) We might also

\(^7\) See Eldor, Margaliot, Sulganik & Zilcha (2001); Eldor & Zilcha (2002); Zilcha & Eldor (2004).

\(^8\) For an explanation of how progressive marginal tax rates can render the tax system asymmetric, driving the government’s gain/loss ratio above one, see Warren (1973).
think that financial instruments that allow hedging transactions are in fact efficient because they allow firms to eliminate some, or all, potential loss situations and thus make the tax system symmetric.\(^9\)

If we find that the gain/loss ratio is below one, as is assumed in this paper, we have another clear demonstration of the inefficiency caused by tax planning and/or sheltering.

If we find that, in fact, the gain/loss ratio is one, then we know that we should not change either side of the equation, for example, by curtailing tax planning, without dealing with the other side as well. We also know that output distortion might be the cost of interfering with the balance by changing only one side of the equation. Explaining the importance of maintaining the balance; that is, keeping the government’s gain/loss ratio at one, by proving that tax planning causes over-production, is this paper’s principal goal.

**Policy Implications**

What are the tax policy implications of the proof provided in this paper? First, we see that asymmetric taxation involves inefficiency that takes the form of output distortion. The paper analyzes the case of tax planning that tilts the government’s gain/loss ratio below one, and provides us with a proof of inefficiency caused by tax planning, namely, over-production.

Second, the paper shows that the sort of inefficiency entailed by asymmetric taxation is different from the inefficiency caused by the (symmetric) income tax system, captured by the conventional concept of excess burden.

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\(^9\) See note 7 above.
Third, when we analyze the exact nature of the \( t \) we used in our model, \( \frac{t_2 - t_1}{1 - t_1} \), we see that the size of the distortion does not exclusively depend on absolute tax rates.\(^{10}\) Indeed, as in the case of symmetric taxation, the higher the tax-rate, the greater the distortion. However, for this type of distortion, what matters most is the difference between the tax rate on losses and the tax rate on gains. In fact, there could be more than one set of asymmetric tax rates that create the same \( t \), that is, the same output distortion.

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