

**San Jose State University**

---

**From the Selected Works of Yeung-Nan Shieh**

---

1996

## Effects of Wage Discrimination on Employment and Firm's Location

Yeung-Nan Shieh, *San Jose State University*



**SELECTEDWORKS™**

Available at: [http://works.bepress.com/yeung-nan\\_shieh/31/](http://works.bepress.com/yeung-nan_shieh/31/)

**Title:** Effects of wage discrimination on employment and firm's location  
**Author(s):** [Yeung-Nan Shieh](#)  
**Source:** [Southern Economic Journal](#). 62.3 (Jan. 1996): p768.  
**Document Type:** Article

**Full Text:** COPYRIGHT 1996 Southern Economic Association  
<http://www.utc.edu/Outreach/SouthernEconomicAssociation/southern-economic-journal.html>

Full Text:

## I. Introduction

In his famous book, *The Economics of Welfare* [6], Pigou showed graphically that the third degree price discrimination does not change total output of a monopoly if the demand curves in two separate markets are linear. Later, Robinson [7] confirmed Pigou's proposition mathematically. Recently, Ekelund, Higgins and Smithson [2], Mai and Shih [4] extended Pigou-Robinson's analysis to the hiring of labor by a monopsony. They showed that the wage discrimination doesn't change total employment of a monopsony if the supply curves of labor in two separate markets are linear, and that Pigou-Robinson's proposition applies to the input markets. However, their analysis is based on the traditional non-spatial setting in which transportation cost and location decision are insignificant and negligible. It would be interesting and important to investigate the effect of wage discrimination on employment in a spatial world.

The purpose of this paper is to investigate the effects of wage discrimination on total employment and plant location of a monopsony in the Weber-Moses triangle. It will be shown that in the spatial economy the Pigou-Robinson proposition holds if the plant location is predetermined. However, if the plant location is a choice variable, the wage discrimination may change total employment even if the supply curves are linear. This indicates that location decision and transportation cost play an important role in the determination of wage discrimination on total employment.

## II. The Basic Model

Our analysis is based on the well-known Weber-Moses triangular model with the following assumptions:

(a) A monopsonist employs a single input (labor) located at two separate markets, A and B, to produce a single output which is sold in a monopolistic market C. The Weber-Moses triangle in Figure 1 depicts the location problem of the firm, [9; 5]. In Figure 1, the distances  $a$  and  $b$  and the angle  $[\pi]/2$  [greater than]  $[\alpha]$  [greater than]  $0$  are known, and  $[\theta]$  [less than]  $[\alpha]$ . The distance between the plant and the

output market,  $h$ , is held constant, thus all points along the arc  $IJ$  are the only ones which are considered as possible plant locations for the monopsony.(1) The plant location is determined if the value of  $[\Theta]$  is chosen.

(b) The production function is specified as:

$$q = f(L) = f([L_{\text{sub.1}}] + [L_{\text{sub.2}}]), [f_{\text{sub.L}}] \text{ [greater than] } 0, [f_{\text{sub.LL}}] \text{ [less than] } 0 \quad (1)$$

where  $[L_{\text{sub.1}}]$  is located at  $A$  and  $[L_{\text{sub.2}}]$  is located at  $B$ .

(c) The workers charge f.o.b. prices for labor and the producer charges c.i.f. price for output. The firm has monopsony power and faces two upward sloping supply curves at sources  $A$  and  $B$ , i.e.,

$$[w_{\text{sub.1}}] = [a_{\text{sub.1}}] + [b_{\text{sub.1}}][L_{\text{sub.1}}], [w_{\text{sub.2}}] = [a_{\text{sub.2}}] + [b_{\text{sub.2}}][L_{\text{sub.2}}] \quad (2)$$

where  $[a_{\text{sub.1}}]$ ,  $[a_{\text{sub.2}}]$ ,  $[b_{\text{sub.1}}]$  and  $[b_{\text{sub.2}}]$  are constant.

(d) The cost of hiring a worker at the plant is the wage rate at source plus the cost of transporting one unit of labor to the plant, i.e.,

$$[c_{\text{sub.1}}] = [w_{\text{sub.1}}] + [ts_{\text{sub.1}}], [c_{\text{sub.2}}] = [w_{\text{sub.2}}] + [ts_{\text{sub.2}}] \quad (3)$$

where  $t$  is the constant transportation rate of labor.(2) By the law of cosines the distance variables  $[s_{\text{sub.1}}]$  and  $[s_{\text{sub.2}}]$  can be defined as:

$$[s_{\text{sub.1}}] = [([a_{\text{sup.2}}] + [h_{\text{sup.2}}] - 2ah \cos [\Theta])^{\text{sup.1/2}}] \quad (4)$$

$$[s_{\text{sub.2}}] = [([b_{\text{sup.2}}] + [h_{\text{sup.2}}] - 2bh \cos([\Alpha] - [\Theta]))^{\text{sup.1/2}}]. \quad (5)$$

The price of output at the plant is the market price minus the cost of transporting one unit of output from the plant to the market,

$$p - rh \quad (6)$$

where  $p = p(q) = p[f(L)]$ , and  $r$  is the constant transportation rate of output.

(e) The objective of the firm is to choose the profit-maximizing employment and location.

It is worth mentioning that the inclusion of distance and transportation rate constitutes the major point of departure from the non-spatial model.

To investigate the effect of wage discrimination on employment, according to Silberberg [8] and Mai and Shih [4], we consider the case in which the discriminating monopsony is posited to maximize profits subject to a constraint that  $w_1 - w_2 = k$  where  $k$  is a parameter. The advantage of this approach is that, via the comparative statics, we can see how total employment changes when  $k$  moves from zero to the optimal value. The Lagrangean for this problem is

$$V = (p - r_h)f(L) - [c_1][L_1] - [c_2][L_2] + [\lambda](k - [w_1] + [w_2]) \quad (7)$$

where  $[\lambda]$ ,  $[L_1]$ ,  $[L_2]$  and  $[\theta]$  are choice variables.

Differentiating  $V$  with respect to  $[\lambda]$ ,  $[L_1]$ ,  $[L_2]$  and  $[\theta]$  yields the following first-order conditions.

$$[V_{\lambda}] = k - [w_1] + [w_2] = 0 \quad (8)$$

$$[V_{L_1}] = MRP - r_h[f_{L_1}] - ([w_1] + [ts_1]) - [b_1][L_1] - [\lambda][b_1] = 0 \quad (9)$$

$$[V_{L_2}] = MRP - r_h[f_{L_2}] - ([w_2] + [ts_2]) - [b_2][L_2] + [\lambda][b_2] = 0 \quad (10)$$

$$[V_{\theta}] = -[ts_1][\theta][L_1] - [ts_2][\theta][L_2] = 0 \quad (11)$$

where  $MRP = (p + [p_q]q)[f_{L_1}]$  is the marginal revenue product,  $[s_1][\theta] = ah([s_1])^{sup.-1/2} \sin [\theta] \geq 0$ ,  $[s_2][\theta] = -bh([s_2])^{sup.-1/2} \sin([\alpha] - [\theta]) \leq 0$ . If the second-order sufficient conditions are satisfied, i.e.,  $D$  (the bordered Hessian determinant)  $\leq 0$  and  $[D_{22}]$  (the bordered-preserving principal minor of order 2)  $\geq 0$ , equations (8)-(11) can be solved for  $[L_1]$ ,  $[L_2]$ ,  $[\theta]$  and  $[\lambda]$  in terms of  $k$  and  $[\Gamma] = (r, t, a, b, [\alpha], h)$ , where  $[\Gamma]$  is a vector of the remaining parameters. This yields

$$[L_1] = [L_1](k, [\Gamma]), [L_2] = [L_2](k, [\Gamma]), [\theta] = [\theta](k, [\Gamma]), [\lambda] = [\lambda](k, [\Gamma]). \quad (12)$$

This completes the model which constitutes our basic framework.

### III. Effects of Wage Discrimination

We are now in a position to examine the effects of wage discrimination. First, we examine the case in which the plant location is predetermined, i.e.,  $[\theta]$  is constant. To derive the effect of wage discrimination on total employment, we totally

differentiate (8) - (10) with respect to  $[\Lambda]$ ,  $[L_{\text{sub}.1}]$ , and  $[L_{\text{sub}.2}]$ , and use Cramer's rule to yield

$$[(\Delta)[L_{\text{sub}.1}]/(\Delta)k]_{\text{sub}.[\Theta]} = [\Theta]_{\text{sub}.0} = (-1/[D_{\text{sub}.2}])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])(MRP[\text{prime}] - rh[f_{\text{sub}.LL}]) - 2[b_{\text{sub}.1}][b_{\text{sub}.2}] \quad (13)$$

$$[(\Delta)[L_{\text{sub}.2}]/(\Delta)k]_{\text{sub}.[\Theta]} = [\Theta]_{\text{sub}.0} = (1/[D_{\text{sub}.2}])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])(MRP[\text{prime}] - rh[f_{\text{sub}.LL}]) - 2[b_{\text{sub}.1}][b_{\text{sub}.2}] \quad (14)$$

$$[(\Delta)[L_{\text{sub}.1}]/(\Delta)k]_{\text{sub}.[\Theta]} = [\Theta]_{\text{sub}.0} = [(\Delta)[L_{\text{sub}.1}]/(\Delta)k]_{\text{sub}.[\Theta]} = [\Theta]_{\text{sub}.0} + [(\Delta)[L_{\text{sub}.1}]/(\Delta)k]_{\text{sub}.[\Theta]} = [\Theta]_{\text{sub}.0} = 0 \quad (15)$$

$$[D_{\text{sub}.2}] = -(MRP[\text{prime}] - rh[f_{\text{sub}.LL}])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])^{\text{sup}.2} + 2[b_{\text{sub}.1}][b_{\text{sub}.2}] \quad (16)$$

where [Mathematical Expression Omitted], and  $[D_{\text{sub}.2}]$  [greater than] 0 if the second-order condition is satisfied. In other words, the wage discrimination will not change total employment. This indicates that the Pigou-Robinson proposition holds when the plant location is predetermined.

Next, we turn to the case in which the plant location is a decision variable. Totally differentiating (9)-(11) and applying Cramer's rule, we obtain

$$[\Delta][\Theta]/(\Delta)k = (1/D)t([s_{\text{sub}.1}[\Theta]] - [s_{\text{sub}.2}[\Theta]])[2[b_{\text{sub}.1}][b_{\text{sub}.2}] - (MRP[\text{prime}] - rh[f_{\text{sub}.LL])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])}] \text{ [less than] } 0 \quad (17)$$

$$[\Delta][L_{\text{sub}.1}]/(\Delta)k = (1/D)\{[2[b_{\text{sub}.1}][b_{\text{sub}.2}] - (MRP[\text{prime}] - rh[f_{\text{sub}.LL])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])][H_{\text{sub}.[\Theta]}[\Theta]] + [ts_{\text{sub}.2}[\Theta]]([b_{\text{sub}.1}][ts_{\text{sub}.2}[\Theta]] + [b_{\text{sub}.2}][ts_{\text{sub}.1}[\Theta]])]\} \quad (18)$$

$$[\Delta][L_{\text{sub}.2}]/(\Delta)k = (-1/D)\{[2[b_{\text{sub}.1}][b_{\text{sub}.2}] - (MRP[\text{prime}] - rh[f_{\text{sub}.LL])([b_{\text{sub}.1}] + [b_{\text{sub}.2}])][H_{\text{sub}.[\Theta]}[\Theta]] + [ts_{\text{sub}.1}[\Theta]]([b_{\text{sub}.1}][ts_{\text{sub}.2}[\Theta]] + [b_{\text{sub}.2}][ts_{\text{sub}.1}[\Theta]])]\} \quad (19)$$

[Mathematical Expression Omitted]

$$D = [t^{\text{sup}.2}([b_{\text{sub}.1}][s_{\text{sub}.2}[\Theta]] + [b_{\text{sub}.2}][s_{\text{sub}.1}[\Theta]])^{\text{sup}.2} + ([b_{\text{sub}.1}] + [b_{\text{sub}.2}])[2[b_{\text{sub}.1}][b_{\text{sub}.2}] - ([b_{\text{sub}.1}] + [b_{\text{sub}.2}])(MRP[\text{prime}] - rh[f_{\text{sub}.LL])][H_{\text{sub}.[\Theta]}[\Theta]]] \quad (21)$$

where  $[H_{\text{sub}.[\Theta]}[\Theta]] = -[ts_{\text{sub}.1}[\Theta]][\Theta][L_{\text{sub}.1}] - [ts_{\text{sub}.2}[\Theta]][\Theta][L_{\text{sub}.2}]$ , and  $D$  [less than] 0, if the second-order sufficient conditions are satisfied. Clearly,  $[\Delta][\Theta]/(\Delta)k$  [less than] 0, but the sign of  $[\Delta]L/(\Delta)$

k is in general ambiguous. In other words, the wage discrimination will move the plant location toward the labor market with higher wage rate, and may change total employment of a monopsony.

The effect of wage discrimination on employment is, perhaps, surprising. According to the Pigou-Robinson proposition, in the non-spatial economy total employment is unchanged by discrimination if the supply curves are linear. But the above result shows that the wage discrimination may change total employment in the spatial economy. The difference results are due to the location effect.

The location effect of wage discrimination on employment can be depicted more clearly by using the SMFC-DMFC (simple monopsonist's marginal factor cost curve-discriminating monopsonist's marginal factor cost curve) approach, [2; 3]. In Figure 2,  $MRP - rh[f.sub.L]$  shows the marginal revenue product minus the marginal transportation cost of output. SMFC and DMFC show the marginal factor cost plus the marginal transportation cost of inputs confronting a simple monopsony and that of confronting a discriminating monopsony. The intersection of  $MRP - rh[f.sub.L]$  and SMFC determines a simple monopolist's total employment, and the intersection of  $MRP - rh[f.sub.L]$  and DMFC determines a discriminating monopolist's total employment. In the case where the plant location is predetermined ( $[\Theta] = [\Theta].sub.0$ ), SMFC is identical to DMFC(3), so total employment is unchanged by wage discrimination, i.e.,  $[L.sub.d] = [L.sub.s]$ . However, in the case where the plant location is a choice variable, the wage discrimination will move the plant location toward the labor market A. The change of plant location will change the marginal transportation cost of  $[L.sub.1]$  and  $[L.sub.2]$ . If the change in marginal transport cost of  $[L.sub.1]$  is greater than that of  $[L.sub.2]$ , i.e.,  $(1/[b.sub.1])[ts.sub.1[\Theta]]$  [greater than]  $-(1/[b.sub.2])[ts.sub.2[\Theta]]$ , DMFC will shift to the right(4) and intersect  $MRP - rh[f.sub.L]$  such that total employment increase, i.e.,  $[L'prime].sub.d$  [greater than]  $[L.sub.s]$ , under wage discrimination.(5)

#### IV. Concluding Remarks

We have attempted to show how the introduction of space and location in the wage discrimination model may alter the well-established Pigou-Robinson proposition. Assume that two supply curves are linear. We have shown that if the plant location is predetermined, the wage discrimination will not change total employment of a monopsony. In this case, the Pigou-Robinson proposition holds. We have also shown that if the plant location is treated as a decision variable, the wage discrimination will move the plant location toward the labor market with higher wage rate, and may change employment of a monopsony. This indicates that the Pigou-Robinson proposition can not be applied to the spatial economy. The upshot of this analysis is that firm's location decision and transportation costs have important influence on the effect of wage discrimination on employment.

Yeung-Nan Shieh San Jose State University San Jose, California

I am grateful to a referee for stimulating criticism and helpful suggestions. All errors remain with the author.

1. For simplicity, as did Moses [5], we assume that  $h$  is held constant. However, our basic results remain unaffected if  $h$  is a choice variable.
2. Following Weber [9] and Moses [5], we assume that "inputs are sold f.o.b.", [5, 260], i.e., the same wage at each source and the firm bears the transportation cost. In the ease where workers absorb the transportation cost, i.e., the c.i.f. pricing. The supply of labor can be specified as:

$$[L.sub.i] = -[m.sub.i] + [n.sub.i]([w[prime].sub.i] - [ts.sub.i]).$$

Rearranging this equation, we obtain

$$\begin{aligned} [w[prime].sub.i] &= ([m.sub.i]/[n.sub.i]) + (1/[n.sub.i])[L.sub.i] + [ts.sub.i] \\ &= ([a.sub.i] + [b.sub.i][L.sub.i]) + [ts.sub.i] \\ &= [w.sub.i] + [ts.sub.i] \end{aligned}$$

where  $[w[prime].sub.i]$  is the wage paid by the firm at source regardless of the transportation cost,  $[a.sub.i] = ([m.sub.i]/[n.sub.i])$ ,  $[b.sub.i] = (1/[n.sub.i])$  and  $i = 1, 2$ . Thus, the labor cost would be

$$[C.sub.i] = [w[prime].sub.i][L.sub.i] = [c.sub.i][L.sub.i].$$

Clearly, this is identical to the one in the f.o.b. pricing. It can also be shown that the profit function of the f.o.b. pricing is equivalent to that of the c.i.f. pricing. This result is consistent with Beckmann and Ingene's in the spatial monopoly market. Beckmann and Ingene showed that "the profit maximizing problems involved (in the f.o.b., pricing and the c.i.f. pricing) are mathematically equivalent when the demand functions are linear" [1, 327].

3. To verify this proposition, in the case of simple monopsony, we combine equations (2) and (3) and obtain

$$L = [L.sub.1] + [L.sub.2] = (1/[b.sub.1][b.sub.2])[b.sub.2](c - [a.sub.1] - [ts.sub.1]) + [b.sub.1](c - [a.sub.2] - [ts.sub.2]).$$

Note that  $[c.sub.1] = [c.sub.2] = c$  when the simple monopsony hires workers from both markets. Solving for  $c$  and defining the cost as  $C = cL$ , the SMFC function would

be

$$SMFC = [1/([b.sub.1] + [b.sub.2])][2[b.sub.1][b.sub.2]L + ([a.sub.1] + [ts.sub.1])[b.sub.2] + ([a.sub.2] + [ts.sub.2])[b.sub.1]].$$

In the case of discriminating monopsony, the cost functions for two markets are  $[C.sub.1] = [c.sub.1][L.sub.1]$  and  $[C.sub.2] = [c.sub.2][L.sub.2]$ . Differentiating  $[C.sub.1]$  and  $[C.sub.2]$  with respect to  $[L.sub.1]$  and  $[L.sub.2]$ , we obtain

$$[DMFC.sub.1] = [a.sub.1] + [ts.sub.1] + 2[b.sub.1][L.sub.1], [DMFC.sub.2] = [a.sub.2] + [ts.sub.2] + 2[b.sub.2][L.sub.2].$$

Solving for  $[L.sub.1]$  and  $[L.sub.2]$  and summing horizontally, the DMFC function would be

$$DMFC = [1/([b.sub.1] + [b.sub.2])][2[b.sub.1][b.sub.2]L + ([a.sub.1] + [ts.sub.1])[b.sub.2] + ([a.sub.2] + [ts.sub.2])[b.sub.1]].$$

It is clear that the SMFC curve is identical to the DMFC curve if the plant location is given.

4. At the given level of  $L$ , the shift of DMFC, as a result of moving plant location toward market A, is

[Mathematical Expression Omitted],

as

[Mathematical Expression Omitted].

5. To avoid cluttering the diagram, we only demonstrate the case of  $(1/[b.sub.1])[ts.sub.1[Theta]]$  [greater than]  $-(1/[b.sub.2])[ts.sub.2[Theta]]$ . However, it is easy to show that the wage discrimination will shift DMFC to the left and decrease total employment if  $(1/[b.sub.1])[ts.sub.1[Theta]]$  [less than]  $-(1/[b.sub.2])[ts.sub.2[Theta]]$ . It is also easy to see that DMFC and total employment are unchanged by wage discrimination if  $(1/[b.sub.1])[ts.sub.1[Theta]] = -(1/[b.sub.2])[ts.sub.2[Theta]]$ .

## References

1. Beckmann, Martin J. and Charles A. Ingene, "The Profit Equivalence of Mill and Uniform Pricing Policies." *Regional Science and Urban Economics*, 6, 1976, 327-29.
2. Ekelund, Robert B. Jr., Richard S. Higgins and Charles W. Smithson, "Can Discrimination Increase Employment: A Neoclassical Perspective." *Southern*



Economic Journal, January 1981, 664-73.

3. Formby John P., Stephen K. Layson and W. James Smith, "Discriminatory Changes in Employment: Comment." *Southern Economic Journal*, October 1982, 550-55.

4. Mai, Chao-cheng and Jun-ji Shih, "Employment Effect Under Monopsony Wage Discrimination." *Southern Economic Journal*, July 1982, 242-45.

5. Moses, Leon M., "Location and The Theory of Production." *Quarterly Journal of Economics*, May 1958, 259-72.

6. Pigou, Arthur C. *The Economics of Welfare*. London: Macmillan, 1922.

7. Robinson, Joan. *The Economics of Imperfect Competition*. 2nd edition. New York: St. Martin's Press, 1969. First Edition was published in 1933.

8. Silberberg, Eugene, "Output Under Discriminating Monopoly: A Revisited." *Southern Economic Journal*, July 1970, 84-87.

9. Weber, Alfred. *Theory of Location of Industries*. Chicago: University of Chicago Press, 1929.

#### Abstract:

The effects of wage discrimination on total employment and plant location of a monopsony in the Weber-Moses triangle are examined. Findings indicate that location decision and transportation cost are significant factors in determining wage discrimination on total employment. This is because in the spatial economy, wage discrimination does not change total employment of a monopsony if the supply curves of labor in two separate markets are linear, as Pigou and Robinson proposed, if the plant location is predetermined. However, the wage discrimination may change total employment even if the supply curves are linear, if plant location is a choice variable.

#### Source Citation (MLA 7<sup>th</sup> Edition)

Shieh, Yeung-Nan. "Effects of wage discrimination on employment and firm's location." *Southern Economic Journal* 62.3 (1996): 768+. *General OneFile*. Web. 31 July 2013.

#### Document URL

<http://go.galegroup.com/ps/i.do?id=GALE%7CA18019354&v=2.1&u=csunorthridge&it=r&p=ITOF&sw=w>

**Gale Document Number:** GALE|A18019354