Economic Consequences of Speculative Side Bets: The Case of Naked Credit Default Swaps

Yeon-Koo Che, Columbia University
Rajiv Sethi, Barnard College

Available at: https://works.bepress.com/yeonkoo/29/
Economic Consequences of Speculative Side Bets: The Case of Naked Credit Default Swaps

Yeon-Koo Che∗ Rajiv Sethi†

August 6, 2010

Abstract

We examine the effects of “naked” credit default swaps on equilibrium debt contracts, project choice, and the likelihood of default when investors have heterogeneous beliefs about the future revenues of the borrower. Although such contracts are zero sum side bets, their existence can have important economic consequences. They induce investors who are most optimistic about the future revenues of borrowers, and would therefore be natural purchasers of debt, to sell credit protection instead. This diverts their capital away from potential borrowers and channels it into collateral to support speculative positions. The resulting shift in the terms of lending against borrowers can cause some projects with positive net present value to remain unfunded, or (in the presence of agency problems) to be replaced by riskier projects with negative net present value. It can also result in an increased likelihood of default and the selection of equilibria in which rollover risk is amplified. The efficiency effects of such contracts are generally ambiguous and belief-dependent, although we identify circumstances in which they result in an unambiguous efficiency loss.

∗Department of Economics, Columbia University, and YERI, Yonsei University. Email: yc2271@columbia.edu.
†Department of Economics, Barnard College, Columbia University and the Santa Fe Institute. Email: rs328@columbia.edu.
1 Introduction

Credit default swaps (CDS) are contracts in which one party sells protection to another against a failure (by a third party) to make contractual debt repayments; they are said to be naked if the protection buyer does not also hold the underlying security. Naked credit default swaps are therefore two-sided directional bets with payoffs that net to zero: one party is betting on default while the other is betting against, and there is no requirement that either has an insurable interest or hedging motive. The notional value of credit default swap contracts prior to the financial crisis of 2008 was estimated to be about ten times as great as that of the underlying bonds (Brunnermeier, 2009). Even with the netting out of multilateral positions and the possibility that some naked protection buyers were facing other exposures that were positively correlated with default, there is little doubt that much of this volume was speculative; see Zuckerman (2009) and Lewis (2010) for some spectacular examples.

In May 2010, Germany became the first major economy to prohibit such contracts outright when it announced a unilateral ban on naked credit default swaps on eurozone debt. Other countries are currently considering following suit. Although it is widely accepted that any such restrictions will have major economic repercussions, there is no consensus on whether these effects will be positive or negative on balance. Some have argued that naked credit default swaps should be banned outright on the grounds that they increase volatility, facilitate bear raids, and make default more likely to occur (Soros 2009, Buiter 2009, Münchau 2010, Portes 2010). Others have countered that they result in more complete markets, better aggregation of information and beliefs, and increased bond market liquidity, making it easier for debt to be issued by distressed borrowers (Carney 2009, Jones 2010, Salmon 2010).

One reason for the continuing controversy is that arguments for and against intervention have been expressed informally, without the benefit of a model in which the consequences of regulation can be rigorously examined. Since naked credit default swaps necessarily have a long and a short side and the aggregate payoff nets to zero, it is not clear why their existence should have any effect on the availability and terms of financing or the likelihood of default. And even if such effects do exist, it is not apparent that prohibitions on side bets should be efficiency enhancing rather than efficiency reducing. Our main purpose in this paper is to develop a framework within which such questions can be explored formally, and to provide some answers.

We begin with a simple model in which a borrower seeks to meet a fixed funding requirement by selling bonds to a group of investors with heterogeneous beliefs concerning the debtor’s future income. Within this framework, we consider three regulatory regimes. The benchmark case is that in which no credit derivatives exist. This is compared with a regime in which bondholders can hedge their risk by purchasing protection against default, but investors cannot purchase protection if they do not also
hold the bonds. The third regime allows for unrestricted contracts, including naked credit default swaps.

In the absence of counterparty risk on the swap contract, the first two regimes are equivalent with respect to market fundamentals: the maximum borrowing requirement that can be funded in equilibrium is the same, as is the size of the bond issue and the interest rate (although the set of investors who hold the bonds will generally be different). The third regime, however, results in an equilibrium with entirely different characteristics. For any given borrowing requirement, the bond issue is larger and the price of bonds accordingly lower in equilibrium when investors are permitted to purchase naked credit default swaps. Investors who are most optimistic about the borrower’s future revenue—and would therefore be natural buyers of the bond in the first regime—prefer to sell protection instead, using their cash endowments as collateral. Investors who are most pessimistic about the borrower’s future finances use their cash endowments to purchase naked credit default swaps. Hence the marginal bond buyer is less optimistic in this regime than in the others, resulting in terms that are less favorable to the borrower.

The finding that borrowers find it easier to raise funds and obtain better terms when the use of credit derivatives is restricted does not, of course, imply that such restrictions are efficient. In fact, the analysis of efficiency in the case of heterogeneous priors involves some conceptual difficulties that do not arise in the more standard (common prior) framework. The prior belief on the basis of which efficiency is to be evaluated must in general be specified, and some efficiency claims may hold only for a particular subset of priors.

We address the question of efficiency by extending the baseline model to allow for an endogenous determination of future revenues, which depend on the borrower’s (unobservable) choice between two available projects. One of these is superior to the other with respect to expected revenues, but the inferior project has greater upside potential, which gives rise to a familiar agency problem in the presence of debt financing. The regulatory regime can then affect not only the willingness of investors to fund the firm, but also the equilibrium choice of project. The presence of naked credit default swaps can prevent the funding of the inferior project under certain conditions, but can also prevent the funding of the superior project under others. It is also possible for naked credit default swaps to induce a shift from the superior to the inferior project, resulting in a worse outcome regardless of the beliefs on the basis of which efficiency is being evaluated.

While the baseline model sheds some light on the manner in which the terms and efficiency of financing can be affected by the availability of credit derivatives, it does not deal with one of the major objections to such contracts: the possibility of self-fulfilling bear raids. To address this issue, we extend the model to allow for a mismatch between the maturity of debt and the life of the borrower. This raises the possibility that a
borrower who is unable to meet contractual obligations because of a revenue shortfall can roll over the residual debt, thereby deferring payment into the future. Multiple equilibria arise naturally in this setting. If investors are confident that debt can be rolled over in the future they accept lower rates of interest on current lending, which in turn implies reduced future obligations and allows the debt to be rolled over if necessary. But if investors suspect that refinancing may not be possible, they demand greater interest rates on current debt, resulting in larger future obligations and an inability to refinance if the revenue shortfall is large.

As in the baseline model, we compare the case of no credit derivatives with that in which naked credit default swaps can be purchased, and uncover two effects. First, the equilibrium in which investors are pessimistic about the ability of the borrower to roll over debt involves higher interest rates when credit derivatives are in use than when they are not. That is, the terms of financing are worse (from the perspective of the borrower) conditional on the selection of the pessimistic equilibrium. Second, the pessimistic equilibrium exists for a larger range of initial borrowing requirements when credit derivatives exist than when they do not. In other words, there is a range of initial borrowing requirements such that fears about the ability of the borrower to repay debt can be self-fulfilling if and only if naked credit default swaps are permitted. It is in this precise sense that the possibility of self-fulfilling bear raids can be said to arise when the use of credit derivatives is unrestricted.

Our approach to modeling debt contracts with collateral and heterogeneous priors builds on seminal work on leverage by John Geanakoplos (1997, 2003, 2010). We extend this work in a number of directions. First, we endogenize the total quantity of the asset issued and its equilibrium likelihood of default. Second, we explore the efficiency implications of different regulatory regimes in the presence of agency problems. And third, we examine the problem of rolling over debt when the borrower engages in maturity transformation. In doing so we also draw upon the work of Adrian and Shin (2008) and Holmstrom and Tirole (1997) on agency problems with debt contracts, and that of Calvo (1988), Cole and Kehoe (2000), and Cohen and Portes (2006) on self-fulfilling expectations of default. None of this prior work has explicitly considered the equilibrium effects of credit derivatives, and this constitutes our main contribution to the literature.\footnote{In related research, Bolton and Oehmke (2010) consider the economic consequences of covered (as opposed to naked) credit default swaps. Bondholders who have purchased protection against default have minimal incentives to work with a distressed borrower to restructure debt. While this has often been cited as a source of inefficiency, Bolton and Oehmke argue that the stronger bargaining position of protected creditors can improve the pledgeability of borrower income, making it easier to raise funds \emph{ex ante}. Since our focus here is on naked credit default swaps, we disregard the issue of bankruptcy reorganization and the effects on fundamentals of covered protection.}

The central message of the paper is that the existence of zero sum side bets on default do indeed have important economic repercussions. They induce investors who
are optimistic about the future revenues of borrowers, and would therefore be natural purchasers of debt, to sell credit protection instead. This diverts their capital away from potential borrowers and channels it into collateral to support their speculative positions. The result is that the marginal bond buyer is less optimistic about the borrower’s prospects, and demands a higher interest rate in order to lend. This can result in a greater likelihood of default and the selection of equilibria in which rollover risk is increased. And it can cause some projects with positive net present value to remain unfunded. Hence we find support for the claim that naked credit default swaps have real economic effects that are detrimental to borrowers and increase rollover risk and the likelihood of default, although less decisive support for the argument that these effects are efficiency reducing.

2 Debt Contracts

Consider a borrower who faces an immediate funding requirement of \( b > 0 \) and chooses to finance this by issuing a quantity \( q > 0 \) of one-period bonds, each with unit face value. The price of these bonds (to be determined endogenously) is \( p \). The amount of money available to the borrower to pay its creditors when the bonds mature is given by a random variable \( y \). Since its obligation on the maturity date is \( q \), the debtor will repay \( \min\{y, q\} \) in the aggregate and each bond will accordingly pay \( \min\{y/q, 1\} \). If the realized value of \( y \) is at least \( q \), the bondholders are paid in full; otherwise they recover only part of the face value of their bonds.

There exists a unit mass of investors, each with a cash endowment of one dollar. Investors have heterogeneous beliefs about the distribution of \( y \): an agent with belief \( \theta \) perceives that the repayment ability \( y \) is distributed according to \( G(y|\theta) \) with support \([0,1] \). We assume that an increase in \( \theta \) shifts the distribution in the sense of first order stochastic dominance, with higher values of \( \theta \) corresponding to more optimistic expectations regarding \( y \). The beliefs \( \theta \) are drawn from the interval \([0,1] \) according to the distribution \( F(\theta) \).

Let

\[
\psi(q; \theta) := \int_0^1 \min\left\{\frac{y}{q}, 1\right\} dG(y|\theta)
\]

(1)

denote the expected payoff per unit of face value, as perceived by a bondholder of type \( \theta \). Note that \( \psi \) is decreasing in \( q \), increasing in \( \theta \), and satisfies \( \psi(q; \theta) < 1 \) for any \( q > 0 \). In particular, let

\[
\Psi(\theta) := \psi(1, \theta) = \int_0^1 ydG(y|\theta)
\]
denote the expected value of $y$ as perceived by type $\theta$. Clearly $\Psi(1) < 1$. Note that

$$q\psi(q; \theta) = \int_0^1 \min\{y, q\} dG(y|\theta) \leq \int_0^1 y dG(y|\theta) = \Psi(\theta),$$

with strict inequality for $q < 1$ and equality for $q = 1$.

Since all investors agree that the borrower’s future income is at most equal to 1, this is also the maximum debt obligation that can be undertaken. That is, we assume $q \leq 1$. Obligations exceeding this imply certain default \textit{ex ante}. Although it is conceivable that investors would purchase such bonds at a sufficiently low price, we rule it out on practical grounds.

Finally, let $\theta_m \in (0, 1)$ denote a critical type such that

$$\Psi(\theta_m) = 1 - F(\theta_m).$$

The left-hand side of the equation $\Psi(\theta_m)$ is the borrower’s expected income—and thus the maximum she can promise to repay—as perceived by type $\theta_m$, and the right-hand side is the total cash endowed by agents who are more optimistic than type $\theta_m$. Clearly, the critical value is well-defined, since the LHS is increasing in $\theta_m$, and the RHS is decreasing in $\theta_m$, and varies from one when $\theta = 0$ to zero when $\theta = 1$. It is intuitive, and will be made precise later, that $\Psi(\theta_m)$ is the maximum amount that the borrower can raise.

We now examine the manner in which the terms and limits of borrowing are affected by restrictions on the use of credit default swaps.

### 2.1 Equilibrium without Credit Derivatives

First consider the case in which no credit default swap contracts are available, so investors must choose between bonds and cash. We consider the properties of an equilibrium in which the borrower is able to raise the needed funds, and then identify conditions under which such an equilibrium exists. We assume that agents can convert a dollar of cash into one unit of a consumption good in either period, and simply maximize their (undiscounted) aggregate consumption.

Consider any equilibrium in which

$$pq \geq b.$$  \hspace{1cm} (3)\hspace{1cm}

is satisfied (so the borrower is able to meet the funding requirement). Each agent can purchase $1/p$ units of the bond with her cash endowment. If the agent has belief $\theta$, her expected payoff when the bond matures is $\psi(q; \theta)/p$. Such an agent will purchase bonds if and only if

$$\psi(q; \theta) \geq p.$$
This expected payoff is monotonic in $\theta$, implying that each agent adopts a cutoff strategy such that she purchases the bond if and only if $\theta$ is no less than

$$\hat{\theta}(p, q) := \sup\{\theta \in [0, 1] \mid \psi(q; \theta) \leq p\};$$

(4)

with the convention that $\hat{\theta}(p, q) = 0$ if $\psi(q; \theta) > p$ for all $\theta$. Whenever $\hat{\theta}(p, q) \in (0, 1)$, we must have

$$\psi(q, \hat{\theta}(p, q)) = p.$$  

(5)

Observe that $\hat{\theta}$ is continuous and nondecreasing in $(p, q)$ and that $\hat{\theta}(1, q) = 1$ and $\hat{\theta}(0, q) = 0$ for any $q \in (0, 1)$. Since $q$ units of bond are sold, the bond market clearing condition is

$$1 - F(\hat{\theta}(p, q)) = pq.$$  

(6)

That is, the market clears when the revenue from bond sale (the right side) equals the total cash endowment the above-marginal type of agents spend (the left side). Since $\hat{\theta}$ is nondecreasing in $p$, the left side of (6) is strictly decreasing in $p$. The right side is clearly strictly increasing in $p$. Furthermore, the left side is continuous, is close to one for $p$ sufficiently close to zero and is close to zero for $p$ sufficiently close to one. Hence, for any $q > 0$, there exists a unique $\overline{p}(q) < 1$ that satisfies (6). That is, for any $q > 0$ there exists a unique price $\overline{p}(q) < 1$ that clears the bond market. Moreover, since $\hat{\theta}(p, q)$ is nondecreasing in $(p, q)$, it must be the case that $\overline{p}(q)$ is decreasing: a larger bond issue results in a lower price per unit. Note also that $\overline{p}(q)$ is continuous.

Suppose two different bond issue sizes, $q$ and $q' > q$ can raise revenue of $b$. Then, the borrower would prefer the smaller bond issue size since it would result in a larger amount of net income accruing to the borrower. Hence, the borrower will issue the smallest quantity of bonds consistent with an equilibrium that meets the borrowing requirement, assuming that this requirement can indeed be met. That is, the borrower will choose

$$q^*(b) = \min\{q \in [0, 1] \mid \overline{p}(q)q \geq b\}.$$  

(7)

The following result characterizes the feasible range of funding requirements and the equilibrium bond issue size and price as functions of the funding requirement. All proofs (unless evident from the discussion) are collected in Appendix.

**Theorem 1.** The maximum revenue that can be raised in equilibrium is $b_m = \Psi(\theta_m)$ at $q = 1$. If $b \leq b_m$, then there exists a unique equilibrium in which the borrower meets the funding requirement of $b$ exactly by issuing $q^*(b)$ bonds, each of which is sold at price $p^*(b) := \overline{p}(q^*(b))$. The marginal investor type $\hat{\theta}^*(b) := \hat{\theta}(p^*(b), q^*(b))$. The equilibrium bond issue size $q^*(b)$ rises and price $p^*(b)$ falls as the funding requirement $b$ rises within the feasible range.

The terms of the equilibrium bond contracts are illustrated using an example.
Example 1. Suppose that $G(y|\theta) = y^{\theta+1}$, and $\theta$ is uniformly distributed so $F(\theta) = \theta$ for $\theta \in [0, 1]$. Then for any $q \leq 1$,

$$\psi(q; \theta) = 1 - \frac{q^{\theta+1}}{\theta+2}$$

Figure 1 shows how the price and total revenue vary with $q$. The upper bound for total revenue is $b_m = 0.59$.

![Figure 1 – Bond Price and Total Revenue as Functions of Issue Size](image)

### 2.2 Covered Credit Default Swaps

We now consider equilibrium in the market for debt under the assumption that a CDS can be purchased, but only with a long position in the underlying bonds. Now agents have four choices: they can issue CDSs (using their cash endowment as collateral), they can buy bonds with or without protection, or they can remain in cash.

Let $r$ denote the (credit default) swap spread: the amount paid per unit of face value to insure bonds for one period. Suppose an agent of type $\theta$ sells protection against default for $x$ units of the bond using her cash endowment, together with the payment received from the protection buyer, as collateral. The protection seller is obliged to
cover the losses of the protection buyer, which requires a transfer of
\[
1 - \min \left\{ \frac{y}{q}, 1 \right\} x
\]
when the bonds mature. If \( y \geq q \) then there is no transfer and the protection seller’s payoff (inclusive of the initial cash endowment) is \( 1 + rx \).

We assume that the protection seller is required to hold enough collateral to cover the worst case loss. That is, we rule out the possibility of default by the protection seller.\(^2\) Hence total collateral \( 1 + rx \) must be large enough to cover the transfer of \( x \) when \( y = 0 \), and the number of contracts sold must satisfy
\[
x = \frac{1}{1 - r}.
\] (8)

The protection seller’s expected payoff (assuming she sells the maximum amount that collateral requirements will permit) is
\[
\pi^{CDS} := 1 + rx - x \int_0^1 \left( 1 - \min \left\{ \frac{y}{q}, 1 \right\} \right) dG(y|\theta),
\]
which simplifies, using (1) and (8), to
\[
\pi^{CDS} = \left( \frac{1}{1 - r} \right) \psi(q; \theta).
\] (9)

Now consider a protection buyer. Such an individual is required (by the assumptions of this section) to have an insurable interest and is therefore also a bondholder. Such agents receive a payoff of 1 regardless of whether the bond defaults. Entering this position costs \( p + r \) per unit of the bond, so the cash endowment is sufficient to purchase \( 1/(p + r) \) protected bonds. The agent’s payoff from a dollar invested in this manner is
\[
u^{CDS} := \frac{1}{p + r}.
\] (10)

Notice that this payoff does not depend on the agent’s belief type.

Finally consider an investor of type \( \theta \) who uses her cash endowment to buy the bond without protection. Her payoff is
\[
u^B := \frac{1}{p} \psi(q; \theta).
\] (11)

\(^2\)Note that we are allowing for default by the bond issuer (which is common, since the bonds are backed by physical assets), but not the swap counterparty. Geanakoplos (2010) argues that default on contracts with financial assets serving as collateral is extremely rare under normal conditions, although it is clear that in the absence of massive government intervention such defaults would have occurred during the recent financial crisis. Our focus here is not on crisis conditions, however, and our results are not sensitive to small changes in this margin requirement.
Comparing (9) and (11), it is clear that selling protection and buying the bond without protection are perfect substitutes. In particular, for protection to be sold in equilibrium, (9) must be no less than (11), which implies that \( p + r \geq 1 \). At the same time, if \( p + r > 1 \), then \( u^{CDS} < 1 \), so no agent will buy bonds with protection (remaining in cash will yield a higher certain payoff). Also, \( p + r > 1 \) implies that no agent will buy the bond without protection since selling protection would be a better strategy. Hence

\[
p + r = 1, \tag{12}
\]

and the payoff from a dollar investment in selling protection is

\[
\pi^{CDS} = \frac{1}{p} \psi(q; \theta) = u^B. \tag{13}
\]

That is, in equilibrium, all investors are indifferent between selling protection and buying the bond without protection. Since an agent must earn no less than a dollar to adopt such a strategy, anyone who does so must be of type \( \theta > \hat{\theta}(p, q) \), as defined in (4). And since each agent with \( \theta > \hat{\theta}(p, q) \) either buys \( 1/p \) units of the bond without protection or sells protection to cover purchases of \( x = 1/(1 - r) = 1/p \) units of the bond, the demand for bonds is given precisely by (6), which results in the same market clearing price \( \overline{p}(q) \) for the same quantity \( q \) sold as in the case where no protection is available. The equilibrium spread \( r^*(q) := 1 - \overline{p}(q^*) \) is then determined by (12). Clearly, the borrower chooses the same quantity \( q^*(b) \) as before, as long as \( b \leq b_m \).

This proves the following:

**Theorem 2.** The bond market with covered CDSs has the same equilibrium quantity and price and the same limit on borrowing as the market without credit derivatives.

Even though the fundamental aspects of the equilibrium are completely pinned down, there is a range of indeterminacy with respect to how many and which individuals purchase bonds with protection and how many and which individuals sell protection or buy bonds without protection. Suppose first \( b \leq F(\hat{\theta}^*(b)) \), in which case the mass of individuals with \( \theta < \hat{\theta}^*(b) \) is large enough to absorb the entire supply of bonds. Then any fraction \( \rho \in [0, 1] \) of agents with \( \theta > \hat{\theta}^*(b) \) can sell protection with the remaining fraction \( (1 - \rho) \) buying bonds without protection. This latter group buys mass \( (1 - \rho)(1 - F(\hat{\theta}^*(b))) \) of the bond, with the remaining units being bought by agents with \( \theta < \hat{\theta}^*(b) \). In the extreme, there is an equilibrium in which all bonds are bought without protection, and an equilibrium in which the entire bond issue is bought with protection.

Suppose, on the other hand, that \( b > F(\hat{\theta}^*(b)) \). In this case, the agents with \( \theta < \hat{\theta}^*(b) \) cannot absorb the entire bond supply, so some bonds must be purchased (without protection) by agents with \( \theta > \hat{\theta}^*(b) \). Again, there is some indeterminacy regarding the identity of those who purchase bonds, and the proportion of these who
also buy protection, but market fundamentals are unchanged relative to the case of no credit derivatives.

2.3 Naked Credit Default Swaps

Now suppose that investors may purchase protection without an insurable interest using naked CDSs. The payoffs to those who sell protection, buy bonds with protection, and buy bonds with no protection are as before, given by (9), (10), and (11) respectively. The argument following (11) also applies: for there to be any protection sold, we must have \( r + p \geq 1 \), but if \( r + p > 1 \), there will be no purchase of bonds either with or without protection. Hence we must have \( r + p = 1 \) in equilibrium. Given this, the protection seller’s payoff is given by (13).

Consider the strategy of purchasing protection without holding the bond. If an agent of type \( \theta \) uses her entire cash endowment for this, she can buy \( 1/r = 1/(1 - p) \) units of (naked) credit default swaps, for a payoff of

\[
u^{NCDS} := \left( \frac{1}{1-p} \right) \int_0^1 \left( 1 - \min \left\{ \frac{y}{q}, 1 \right\} \right) dG(y|\theta) = \left( \frac{1}{1-p} \right) \left( 1 - \psi(q; \theta) \right).
\]

A simple computation reveals that this strategy dominates remaining in cash if and only if \( \theta < \hat{\theta}(p,q) \)—that is, if and only if remaining in cash dominates selling protection. Hence no agent in the economy will purchase credit default swaps for the purpose of insuring the bond: all buyers of protection will be speculating on default. This observation simplifies the characterization of equilibrium considerably. All agents with \( \theta < \hat{\theta}(p,q) \) buy naked CDSs and those with \( \theta > \hat{\theta}(p,q) \) either sell protection or purchase uninsured bonds.

Since each agent with \( \theta < \hat{\theta}(p,q) \) buys \( 1/r = 1/(1 - p) \) units of credit protection, the aggregate demand for CDSs is

\[
\frac{1}{1-p} F(\hat{\theta}(p,q)).
\]

Agents with \( \theta > \hat{\theta}(p,q) \) must absorb entire bond issue \( q \) and generate the entire supply of credit protection, so the proportion of individuals who sell protection is \( 1 - F(\hat{\theta}(p,q)) - pq \). Each of these individuals has just enough collateral to insure \( 1/(1-r) = 1/p \) bonds. The aggregate supply of credit default swaps is therefore

\[
\frac{1}{p} \left( 1 - F(\hat{\theta}(p,q)) \right) - q,
\]

and the swap market clearing condition is:

\[
\frac{1}{p} (1 - F(\hat{\theta}(p,q))) = q + \frac{1}{1-p} F(\hat{\theta}(p,q)).
\]
The fact that \( \hat{\theta}(\cdot, \cdot) \) is nondecreasing may be used to establish that there exists a unique bond price \( \tilde{p}(q) \) that satisfies (15), and that \( \tilde{p}(q) \) is continuous and decreasing.

As before, if the funding requirement \( b \) can be met in equilibrium, the borrower will choose the smallest feasible bond issue:

\[
q^{**}(b) = \min\{q \in [0, 1] \mid \hat{p}(q)q \geq b\}.
\]

The associated bond price is \( p^{**}(b) := \tilde{p}(q^{**}(b)) \), the equilibrium swap spread is \( r^{**}(b) := 1 - p^{**}(b) \), and the threshold type is \( \hat{\theta}^{**}(b) = \hat{\theta}(p^{**}(b), q^{**}(b)) \). The following result characterizes the equilibrium outcome with naked credit default swaps in comparison with the cases considered previously.

**Theorem 3.** For any issue size \( q \in (0, 1] \), the equilibrium bond price in the presence of naked CDSs is \( \tilde{p}(q) < \overline{p}(q) \). The maximum revenue that can be raised in equilibrium is \( \tilde{p}(1) < \overline{p}(1) \) at \( q = 1 \). If \( b \leq \tilde{p}(1) \), then there is a unique equilibrium in which the borrower issues \( q^{**}(b) > q^{*}(b) \) bonds at price \( p^{**}(b) < p^{*}(b) \). The equilibrium swap spread is \( r^{**}(b) > r^{*}(b) \). The probability and magnitude of default are higher relative to the case of no credit protection or covered credit default swaps.

The result establishes that for any given issue size, bond prices (and hence also total revenues) are lower when naked CDSs are permitted than when they are not. Figure 2 illustrates, using the same specifications as in Example 1.

Not only are the terms on which financing is available worse when protection can be purchased without an insurable interest, but the range of deficits that can be financed is itself smaller. This happens because the most optimistic investors prefer to sell protection (when they have an option to do so) rather than to buy bonds, which ties up their capital in the form of collateral and makes the marginal bond buyer more pessimistic than would be the case without credit derivatives. As a result, any borrowing requirement that is feasible without credit derivatives either becomes infeasible, or requires a larger bond issue (and hence a higher interest rate).

### 3 Project Choice and Funding Efficiency

What we have shown to this point is that the presence of naked CDSs shifts the terms of lending against borrowers and requires a larger issue size for any given funding requirement. As a result, some projects that may have been funded in the absence of such contracts will not be funded in their presence. The efficiency effects of this are in general ambiguous, especially when investment decisions by firms are subject to agency problems. We now turn to a consideration of these effects.

Suppose that the borrower is a firm facing a choice between two projects with different distributions of returns, each of which depend on the prior belief \( \theta \in [0, 1] \).
Both projects have the same funding requirement $b$, but one is unambiguously superior to the other in the sense that it has a higher expected return regardless of $\theta$. Specifically, suppose that the superior project has a return $y \in [0, 1]$ drawn from a distribution $G(y | \theta)$, as assumed in our earlier analysis. The inferior project has a return drawn from the same support according to a distribution $B(\cdot | \theta)$. Let $\Psi_G(\theta)$ and $\Psi_B(\theta)$ respectively denote their expected returns conditional on $\theta$. We assume that

$$\Psi_G(\theta) > \Psi_B(\theta)$$

for all $\theta \in [0, 1]$; this is the sense in which the “good” project is superior. As before, investors have heterogeneous beliefs given by $\theta \in [0, 1]$ drawn from the distribution $F(\theta)$ which admits positive density $f(\theta)$ for $\theta \in (0, 1)$. Higher values of $\theta$ correspond to more optimistic beliefs about project returns in the sense of first-order stochastic dominance, and this applies to both projects.

The prior belief of the firm (on the basis of which its project choice is made) is given by $\theta_0 \in (0, 1)$; this belief is commonly known among investors. Although the inferior project has a lower expected return, it carries greater upside potential in the following sense: there exists $y^* \in (0, 1)$ such that

$$(G(y | \theta_0) - B(y | \theta_0))(y - y^*) \geq 0,$$
with strict inequality for \( y \neq y^* \). The highest realizations of \( y \) are therefore more likely to occur if the inferior project is chosen. This creates a potential agency problem when the firm’s project selection is unobservable to the investors, which we assume to be the case. As we show below, this agency problem limits the amount of income that the firm can pledge to investors, and thus its capacity to issue debt.\(^3\)

Suppose the firm issues \( q \) units of debt with unit face value. The expected payment to bondholders when the debt matures (from the perspective of the firm) is \( q\psi_j(q;\theta_0) \) if project \( j = G, B \) is expected to be chosen, where \( \psi_j(q;\theta_0) \) is as defined in (1), but with distribution function \( j = G, B \). Assume (for convenience) that a firm that is indifferent between the two projects will choose the superior one. Then, for the firm to have an incentive to choose the good project, it is necessary that

\[
\Psi_G(\theta_0) - q\psi_G(q;\theta_0) \geq \Psi_B(\theta_0) - q\psi_B(q;\theta_0). \tag{18}
\]

This constraint is satisfied if and only if the size of the bond issue is not too great:

**Lemma 1.** There exists \( \hat{q} \in (0, y^*) \) such that (18) holds if and only if \( q \leq \hat{q} \).

Our analysis of bond market equilibrium with and without CDSs now applies with minimal modifications. Recall that, conditional on the superior project being selected, equilibrium outcomes—the bond issue size, price, and the type of the marginal bond buyer—are \((q^*(b), p^*(b), \hat{\theta}^*(b))\) in the absence of credit derivatives, while the equilibrium outcomes in the presence of naked CDS are \((q^{**}(b), p^{**}(b), \hat{\theta}^{**}(b))\). The equilibrium outcomes when the inferior project is selected differ systematically from these in the following manner:

**Lemma 2.** Conditional on the inferior project being selected in equilibrium, bond market outcomes are as follows:

- In the absence of credit derivatives, the equilibrium outcomes \((q_B^*(b), p_B^*(b), \hat{\theta}_B^*(b))\) satisfy \( \hat{\theta}_B^*(b) = \hat{\theta}^*(b) \), \( q_B^*(b) > q^*(b) \), and \( p_B^*(b) < p^*(b) \).
- In the presence of naked CDS, equilibrium outcomes \((q_B^{**}(b), p_B^{**}(b), \hat{\theta}_B^{**}(b))\) satisfy \( \hat{\theta}_B^{**}(b) \in (\hat{\theta}^{**}(b), \hat{\theta}^*(b)) \), \( q_B^{**}(b) > \max\{q^{**}(b), q_B^*(b)\} \), and \( p_B^{**}(b) < \min\{p^{**}(b), p_B^*(b)\} \).

Holding constant the policy regime, funding is costlier to the borrower when the inferior project is selected in equilibrium, for the simple reason that the distribution of future revenues is worse from the perspective of all investors. And when naked CDSs are permitted and the inferior project is selected, the terms are worse for the borrower.

---

\(^3\)This particular moral hazard problem has been studied by Adrian and Shin (2008), and is also closely related to Holmstrom and Tirole (1997). These authors postulate that the bad project involves a private benefit for management, and assume that (16) holds with the private benefit included. The greater upside potential of the bad project, assumed in (17), makes the private benefit an unnecessary part of the story. The contractual implications of these two approaches are essentially the same: they both limit the pledgeability of income and thus limit the size of debt obligations.
relative to both the case of the superior project being selected, and the cases in which
credit derivatives are absent.

We may now characterize the funding decisions of investors and the firm’s choice
of project.

**Creditworthiness of projects.** Whether a given project, good or bad, can be funded
in equilibrium depends on the distribution of investor beliefs as well as on the financing
requirement, \( b \). The funding of the superior project requires that investors be suffi-
ciently optimistic, while the funding of the inferior project requires that they be even
more so. It is convenient to represent investor sentiment by the parameter \( \omega \in [0, 1] \)
such that the distribution \( F(\theta|\omega) \) is differentiable and strictly decreasing in \( \omega \) for all
\( \theta \in (0, 1) \), with \( \lim_{\omega \downarrow 0} F(\theta|\omega) = 1 \) for all \( \theta > 0 \), and \( \lim_{\omega \uparrow 1} F(\theta|\omega) = 0 \) for all \( \theta < 1 \). That is, an increase in \( \omega \) shifts the belief distribution towards greater optimism. With
this parameterization, all equilibrium variables can be expressed as functions of \( (b, \omega) \); for instance, \( q^*(b, \omega) \) and \( p^*(b, \omega) \), etc.

Define

\[ \hat{\omega}_O(b) := \inf \{ \omega \mid q_{B}^{**}(b, \omega) < 1 \}; \]
\[ \hat{\omega}_N(b) := \inf \{ \omega \mid q_{B}^*(b, \omega) < 1 \}; \]
\[ \hat{\omega}_P(b) := \inf \{ \omega \mid q^*(b, \omega) < 1 \}. \]

Since \( q^*(b, \omega) < q_{B}^*(b, \omega) < q_{B}^{**}(b, \omega) \) and they all decline as \( \omega \) rises (whenever they are
well defined), it follows that

\[ 0 < \hat{\omega}_P(b) < \hat{\omega}_N(b) < \hat{\omega}_O(b) < 1. \]

We say that investors are

- **highly optimistic** if \( \omega > \hat{\omega}_O(b) \): In this case, both projects are creditworthy,
even in the presence of naked CDS.
- **moderately optimistic** if \( \hat{\omega}_O(b) > \omega > \hat{\omega}_N(b) \): In this case, the inferior project
is creditworthy in the absence of naked CDS, but not in their presence.
- **neutral** if \( \hat{\omega}_N(b) > \omega > \hat{\omega}_P(b) \): In this case, the superior project is creditworthy
but the inferior project is not, assuming that naked CDSs are not permitted.
- **pessimistic** if \( \hat{\omega}_P(b) > \omega \): In this case, neither project is creditworthy even when
naked CDSs are not permitted.

**Project choice.** Project choice and funding availability depend not only on investor
expectations but also on the severity of agency problems. Recall from our earlier anal-
ysis that the quantity of bonds issued in the absence of credit derivatives is \( q^*(b, \omega) \),
while the corresponding quantity in the presence of naked credit default swaps is
\( q^{**}(b, \omega) > q^*(b, \omega) \). There are three possibilities.
• \( \hat{q} \geq q^{**}(b, \omega) \): In this case, the incentive condition is *always satisfied*, i.e., regardless of whether naked CDSs are permitted or not. Hence, the firm will choose the good project if it can raise \( b \).

• \( \hat{q} < q^{*}(b, \omega) \): In this case, the incentive condition is *never satisfied*. The firm will not choose the superior project even if it could secure the necessary funding. By Lemma 2, \( q^{**}_B(b, \omega) > q^{*}_B(b, \omega) > q^{*}(b, \omega) > \hat{q} \), so the firm will choose the inferior project if it is able to fund it (regardless of the policy regime).

• \( q^{*}(b, \omega) \leq \hat{q} < q^{**}(b, \omega) \): In this case, the incentive condition is *conditionally satisfied*. That is, the superior project will be chosen (provided that it can be funded) if and only if naked CDSs are not permitted. If naked CDSs are allowed, then the inferior project will be selected provided that it can be funded, since by Lemma 2, we have \( q^{**}_B(b, \omega) > q^{**}(b, \omega) > \hat{q} \).

We may now examine the effects of naked credit default swaps on project choice and funding efficiency. Suppose first the naked CDSs are not permitted. It is clear that if investors are pessimistic, the firm cannot obtain the necessary funding for either project. If investors are not pessimistic, then the equilibrium identified in Theorem 1 holds as long as the issue size \( q^{*}(b, \omega) \) does not exceed the critical level \( \hat{q} \) required for the incentive constraint (18). In this case, the firm can obtain \( b \) and will choose the superior project. Note that the investors understand that if the firm issues \( q = q^{*}(b, \omega) \leq \hat{q} \), then it will select the superior project, and given this, it is in the best interests of the firm to issue \( q = q^{*}(b, \omega) \). Issuing \( q > \hat{q} \) would imply the selection of the inferior project, and in this case even if the firm could raise \( b \) it would not benefit from doing so.\(^4\) If \( q^{*}(b, \omega) > \hat{q} \), however, the firm will never select the superior project. Given this, investors will not fund the firm unless they are (at least moderately) optimistic.

In summary, in the absence of naked CDSs, if \( q^{*}(b, \omega) \leq \hat{q} \) and investors are not pessimistic, the superior project is chosen; and if investors are (at least moderately) optimistic and \( q^{*}(b, \omega) > \hat{q} \), then the inferior project is chosen. In all other cases, the firm cannot obtain funding.

Next suppose that naked CDSs are permitted. In this case, the superior project is chosen only if investors are not pessimistic and, more importantly, the amount \( q^{**}(b, \omega) \) of bonds issued in equilibrium is no greater than \( \hat{q} \). Since \( q^{**}(b, \omega) > q^{*}(b, \omega) \), this condition is more difficult to satisfy than in the absence of such derivatives. That is, if \( q^{**}(b, \omega) > \hat{q} \geq q^{*}(b, \omega) \), then the firm will choose the superior project only if

\(^4\)This can be seen as follows. Suppose the firm indeed picks \( q > \hat{q} \) and raises \( b \). Then, the firm’s expected payoff is

\[ \Psi_B(\theta_0) - q\psi_B(q; \theta_0) < \Psi_B(\theta_0) - q^{*}(b)\psi_B(q^{*}(b); \theta_0), \]

where the inequality holds since \( q > q^{*}(b, \omega) \); i.e., the firm’s debt burden decreases with a smaller issue size. The payoff on the right is smaller than that which the firm would earn by issuing \( q^{*}(b, \omega) \) and choosing the superior project, since \( q^{*}(b, \omega) \leq \hat{q} \).
naked CDSs are disallowed. If such contracts were permitted, the firm would choose
the inferior project if it were able to obtain funding, and it will secure funding if and
only if investors are highly optimistic.

Since \( q_B^* (b, \omega) < q_B^{**} (b, \omega) \), the inferior project is less likely to be adopted in the
presence of naked credit default swaps than in their absence. That is, conditional on
failing the incentive constraint (18), the inferior project will be less frequently chosen
in the presence of such contracts.

Based on these considerations, the manner in which the borrower’s project choice is
affected by the regulatory regime under various conditions may be expressed as follows.

**Theorem 4.** Allowing for naked CDSs results in

- a failure to fund the superior project if investors are neutral or moderately opti-
mistic and the incentive constraint is conditionally satisfied,
- a failure to fund the inferior project if investors are moderately optimistic and the
  incentive constraint is never satisfied, and
- a switch from the superior to the inferior project if investors are highly optimistic
  and the incentive constraint is conditionally satisfied.

In all other cases the project choice is independent of the policy regime.

The result is summarized in Table 1, where \( N \) indicates that neither project is
funded, the arrows represent a change in project selection when naked CDSs are per-
mitted, and \( \emptyset \) means the case is not well defined.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( q^*(b, \omega) )</td>
<td>( B )</td>
<td>( B \rightarrow N )</td>
<td>( N )</td>
<td>( N )</td>
</tr>
<tr>
<td>( \hat{q} \in (q^*(b, \omega), q^{**}(b, \omega)] )</td>
<td>( G \rightarrow B )</td>
<td>( G \rightarrow N )</td>
<td>( G \rightarrow N )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( \hat{q} \geq q^{**}(b, \omega) )</td>
<td>( G )</td>
<td>( G )</td>
<td>( G )</td>
<td>( \emptyset )</td>
</tr>
</tbody>
</table>

**Table 1** – Effects of naked CDS on project choice

As noted above, the efficiency implications of naked credit default swaps in this
heterogeneous prior framework depend, in general, on the beliefs used for purposes
of evaluation. We shall examine these implications from the perspective of a neutral
policy maker, although an analogous exercise could be conducted for other cases. A
neutral individual believes that the superior project should be funded, but the inferior
project should not. In this case, efficiency is either unaffected or increased by the
banning of naked credit default swaps in all circumstances except when investors are
moderately optimistic and the incentive constraint is never satisfied. Under these
conditions, allowing such contracts prevents the inferior project from being funded,
and is therefore efficiency increasing. In all other cases, allowing naked credit default
swaps either has no impact on project selection, or causes a failure of funding for the superior project.

An interesting case with unambiguous efficiency implications arises if the incentive constraint is conditionally satisfied and investors are highly optimistic. In this case funding is available regardless of regulatory regime but the project choice is different: the firm will choose the superior project in the absence of credit derivatives but the inferior project in their presence. Regardless of the beliefs on the basis of which efficiency is evaluated, this entails an efficiency loss: even the most optimistic investors would prefer that the superior project be selected.

Naked CDSs allow the market to aggregate the opinions of market participants with greater weight placed on pessimistic views than would otherwise be the case. This can be beneficial under some circumstances: for instance by preventing the funding of inferior projects when investors are moderately optimistic. But against this advantage has to be weighed the efficiency losses that arise when superior projects fail to be funded because agency problems limit a firm’s capacity for issuing debt, and naked credit default swaps force a larger issue size for any given funding requirement. The most dramatic instance of this arises when investors are highly optimistic and the incentive constraint is conditionally satisfied. In this scenario the existence of naked credit default swaps results in an inferior project being funded when their absence would permit the funding of a project that is believed by all market participants to be superior.

Our findings illustrate the difficulty of modifying the regulatory regime in response to evolving market conditions: allowing naked CDSs may be undesirable if investors are neutral or pessimistic, desirable if they are moderately optimistic (depending on whether or not the incentive constraint is easily satisfied) and undesirable again if they are highly optimistic. Given this non-monotonicity, it may be better to decide on the regulatory regime that performs best on average, rather than attempting to track and respond to investor sentiment, especially when there is considerable uncertainty regarding the incentive constraints under which firms are operating.

4 Rollover Risk

One of the key features of debt contracts is that they frequently involve maturity transformation; the term of the loan is too short to enable full repayment without refinancing. This means that the terms of current financing depend on expectations regarding the ability of the borrower to roll over debt when it comes due. Multiple equilibria arise naturally in this setting, and we are interested in the manner in which the use of credit derivatives affects the terms of financing and the set of equilibria.

Consider three periods $T = 0, 1, 2$. In period $T = 0$, the borrower faces a borrowing
requirement $b_0 > 0$, and proposes to finance this by issuing $q_0$ bonds each with unit face value. The bond price $p_0$ is determined by a competitive market in period $T = 0$. In period $T = 1$, the borrower realizes a (possibly negative) budget surplus $y_1$. If $y_1 \geq q_0$, then all bonds are paid in full and no refinancing is necessary. If $y_1 < q_0$, then the borrower must issue a quantity $q_1$ of bonds with unit face value to cover the shortfall of $q_0 - y_1$. Again a competitive market at period $T = 1$ sets the price $p_1$ of the bonds. In period $T = 2$, the budget surplus $y_2$ is realized, and the bond holders are paid $\min\{q_1, y_2\}$.

To focus on the main idea, we make the simplifying assumption that the borrower’s ability to repay is binary: $y_t \in \{0, 1\}$, for $t = 1, 2$. In period $T = 0$, a type $\theta$-agent believes that $y_1 = 1$ with probability $\theta$. As before, $\theta$ is drawn from the distribution $F(\theta)$. In period $T = 1$, there is no belief heterogeneity about the distribution of $y_2$; all investors believe that $y_2 = 1$ with probability $\lambda$ (and $y_2 = 0$ with probability $1 - \lambda$). This common belief assumption plays no essential role; its only purpose is to simplify the analysis. In particular, it implies that in period $T = 1$, there is no market for credit default swaps even if such contracts are permitted. The tree of uncertainty, as faced by an investor of type $\theta$, is shown in Figure 3.

**Figure 3** – The tree of uncertainty faced by an investor of type $\theta$

We assume, as before, that the borrower cannot take on greater debt obligations than could be honored in any state. That is, we assume $q_t \leq 1$, for $t = 1, 2$. As we show below, this constraint will not be binding in equilibrium as long as the initial borrowing requirement $b_0$ is not too large. We also assume that the borrower cannot complete the project if it is unable to roll over the entire debt due when the low income state is realized at $T = 1$. This rules out a partial rollover of debt, in which earlier investors...
are not paid in full but the firm is nevertheless able to raise new funds. This seems reasonable, given that a default entails considerable fixed costs, loss of reputation, and severely restricted access to capital markets.

Before proceeding, it is important to consider why the borrower finances via a sequence of short-term obligations rather than a long-term bond that matures at \( T = 2 \) and therefore avoids rollover risk. There are a number of reasons why firms engage in such maturity transformation, perhaps the most important of which is the inability to credibly pledge income that is realized in the interim stage \( T = 1 \). Income earned well in advance of the maturity date is difficult to monitor, and it is easier for the borrower to divert such resources away from creditors without raising suspicion.\(^5\) This inability to pledge near-term income to service long-term debt implies that the terms available for long-term financing are not generally favorable relative to a sequence of shorter maturity debt. For instance, if the borrower can fully divert her income at \( T = 1 \) in the presence of a long term contract, the loan is effectively backed only by \( T = 2 \) income. Such a contract is dominated by the sequence of short term loans that we consider.\(^6\)

### 4.1 Equilibrium without Credit Derivatives

We start by characterizing the set of equilibria in an economy without credit derivatives, beginning our analysis at period \( T = 1 \). If \( y_1 = 1 \) the initial debt is fully repaid. If \( y_1 = 0 \), the borrower owes \( q_0 \) and must borrow this amount to avoid default. Suppose it does this by issuing an amount \( q_1 \) of new one period bonds, each with unit face value. Recall that it is common belief on the part of the agents that each such bond will have an expected payoff of precisely \( \lambda \) at \( T = 2 \). Hence the equilibrium bond price must satisfy \( p_1 = \lambda \), and the borrower can therefore borrow \( p_1 q_1 = \lambda q_1 \). Since \( q_1 \leq 1 \), the debt can be rolled over if and only if \( q_0 \leq \lambda \). In particular, if \( q_1 > \lambda \), then no refinancing occurs at all, and bondholders are paid nothing.

Now consider period \( T = 0 \). If \( b_0 \leq \lambda \), then there exists a trivial equilibrium in which the borrower issues \( q_0 = b_0 \) at a price \( p_0 = 1 \). This bond is risk-free (since the debt is certain to be rolled over if necessary) and all investors are therefore willing to pay the face value for each unit regardless of their beliefs.

If \( b_0 > \lambda \), then no such equilibrium exists since a debt this large cannot be refinanced if \( y_1 = 0 \). Hence any bonds sold in the initial period will be repaid if and only if \( y_1 = 1 \). If an agent of type \( \theta \) spends her cash endowment of a dollar on purchasing bonds, she will expect to earn \( \theta/p_0 \). Since this strategy is optimal only when this payoff is no less

---

\(^5\) For instance, such diversions are unlikely to be regarded as fraudulent by a bankruptcy court.  
\(^6\) More precisely, a long-term bond with unit face value will be paid in full with probability \( \lambda \) and will pay nothing with probability \( 1 - \lambda \). Thus the borrower can finance only \( b \leq \lambda \), and must issue \( q = b/\lambda \) bonds, each of which will be sold at price \( \lambda \). As we show below, a sequence of short term loans can be obtained on better terms as long as \( \lambda \) is sufficiently small or \( F^{-1}(1-b) \) is sufficiently large.
than a dollar, the agent will purchase bonds if and only if
\[ \theta > \hat{\theta} = p_0. \]  
(19)

Given that the borrower needs to raise \( p_0q_0 = b_0 \), the market clearing condition \( 1 - F(\hat{\theta}) = b_0 \) may be written
\[ 1 - F\left(\frac{b_0}{q_0}\right) = b_0. \]  
(20)

There is a unique bond issue size that satisfies this, given by:
\[ q_0(b_0) = \frac{b_0}{F^{-1}(1 - b_0)}. \]

It is important to notice that \( q_0(b_0) > b_0 \). This means that even when \( b_0 \leq \lambda \) (so an equilibrium with \( q_0 = b_0 \) exists), there can be a second equilibrium with \( q_0(b_0) > \lambda \geq b_0 \) in which investors have pessimistic expectations regarding the borrower’s ability to refinance in the low income state. This pessimistic equilibrium has a lower bond price and requires the borrower to incur a larger debt obligation in order to meet its borrowing requirement. Define \( \hat{b}_0 := q_0^{-1}(\lambda) \). That is, \( \hat{b}_0 \) is a critical borrowing requirement that satisfies
\[ 1 - F\left(\frac{\hat{b}_0}{\lambda}\right) = \hat{b}_0. \]

Clearly, \( \hat{b}_0 < \lambda \). The following result identifies a range of values for the initial borrowing requirement such that a multiplicity of equilibria exists.

**Theorem 5.** If \( b_0 > \lambda \), there exists a unique equilibrium in which the borrower issues \( q_0(b_0) \) bonds with unit face value at price \( p_0 = b_0/q_0(b_0) \). Default occurs if and only if \( y_1 = 0 \). If \( b_0 < \hat{b}_0 \), there is a unique equilibrium in which the borrower issues \( q_0 = b_0 \) bonds with unit face value and unit price, never defaults on these bonds, and rolls over the debt if \( y_1 = 0 \). If \( \hat{b}_0 \leq b_0 \leq \lambda \), then both equilibria exist.

If the initial borrowing requirement is sufficiently low, then investors fully expect that debt will be successfully rolled over in the low income state, and there is a unique equilibrium with zero interest. If the initial borrowing requirement is sufficiently high, there is again a unique equilibrium but one in which default is expected in the low income state, and the interest rate is correspondingly higher. For intermediate values of the initial borrowing requirement both equilibria can co-exist. If investors believe that the borrower will be unable to roll over debt in the low income state, they will require higher interest rates as compensation for this risk, and the greater debt burden that results will cause these beliefs to be correct. On the other hand, if they expect that refinancing will be available at either state, this too will be self-fulfilling since the debt burden will be correspondingly lower.
4.2 Credit Default Swaps

We now consider the effects of allowing for naked credit default swaps in this environment. The market for these contracts never materializes in period $T = 1$, and the same is true in period $T = 0$ if investors are confident that the borrower will be able to finance raise $b_0$ by issuing $q_0 \leq \lambda$ bonds. These bonds never default, for the debt can be rolled over even in the low income state, and this is known to all agents. But, as in the case without credit derivatives, there can be another equilibrium in which investors are not confident about the borrower’s ability to roll over its debt in the low income state.

If default protection can be purchased without holding the underlying bond, then, as in the one period model considered earlier, optimistic agents will sell protection or buy bonds without protection in equilibrium, while pessimistic agents will buy naked credit default swaps. As before, the arbitrage condition requires the swap spread $r_0$ to satisfy $p_0 + r_0 = 1$. Our earlier analysis implies that agents with $\theta > \hat{\theta}$ buy bonds without protection or sell protection, while each agent with $\theta < \hat{\theta}$ purchases protection on bonds with face value $1/(1 - p_0)$. Here the threshold type $\hat{\theta} = p_0$ as in (19).

In equilibrium we must have

$$\frac{1}{p_0} \left(1 - F(\hat{\theta})\right) = q_0 + \left(\frac{1}{1 - p_0}\right) F(\hat{\theta}).$$

Collecting terms and using $\hat{\theta} = p_0$ and $p_0 q_0 = b_0$, we get

$$1 - \left(\frac{q_0}{q_0 - b_0}\right) F\left(\frac{b_0}{q_0}\right) = b_0. \quad (21)$$

One can check that the left side is increasing in $q_0$ for $q_0 > b_0$, so there is a unique value $\tilde{q}_0(b_0) > b_0$ that satisfies the equation. Since the left side of (21) is smaller than that of (20), it also follows that $\tilde{q}_0(\lambda) > \tilde{q}_0(b_0)$.

Define $\tilde{b}_0 := \tilde{q}_0^{-1}(\lambda)$. Then, $\tilde{b}_0 < \tilde{b}_0$. The following result identifies the equilibrium set when naked credit default swaps are permitted.

**Theorem 6.** If $b_0 > \lambda$, there exists a unique equilibrium in which the borrower issues $\tilde{q}_0(b_0)$ bonds with unit face value at price $\tilde{p}_0 = b_0/\tilde{q}_0(b_0) < \hat{p}_0$. Default occurs if and only if $y_1 = 0$. If $b_0 < \hat{b}_0$, there is a unique equilibrium in which the borrower issues $q_0 = b_0$ bonds with unit face value and unit price, never defaults on these bonds, and rolls over its debt if $y_1 = 0$. If $\tilde{b}_0 \leq b_0 \leq \lambda$, then both equilibria exist.

The comparison with the case without credit derivatives is instructive. If $b_0 \in (\tilde{b}_0, \hat{b}_0)$, then banning naked credit default swaps yields the no default outcome as the unique equilibrium, whereas allowing such contracts introduces an additional equilibrium in which the borrower defaults in the low income state. Furthermore, even if multiple equilibria exist under both regimes, the terms of financing are worse for the
borrower when naked credit default swaps exist at the equilibrium with the higher interest rate. The following example illustrates.

**Example 2.** Suppose that $\lambda = 0.40$ and $F(\theta) = \theta^2$. In this case $\hat{b}_0 = 0.33$ and $\tilde{b}_0 = 0.23$. The range of initial debt levels for which multiple equilibria exist with naked credit default swaps is $[0.23, 0.40]$, but when no such contracts are allowed, this range is $[0.33, 0.40]$. Furthermore, when the more pessimistic equilibrium exists under both regimes, it is more punitive when naked credit default swaps exist (see Figure 4).

![Figure 4](image_url)

**Figure 4** – Equilibrium Bond Issues with and without Naked CDS

As is clear from the figure, the presence of naked credit default swaps has two effects. First, it expands the range of initial borrowing requirements for which an equilibrium with default in the low income state exists. And second, conditional on such an equilibrium being selected, interest rates are higher when naked credit default swaps are permitted than when they are not. The latter effect is similar to that identified in the one-period version of the model. And the former effect confirms that “self-fulfilling bear raids” are indeed more likely to occur when naked credit default swaps are permitted than when they are not.
5 Conclusions

Since naked credit default swaps are contracts with both a long and a short side and payoffs that net to zero, it is not immediately apparent what (if any) effects their presence can have on economic fundamentals. We have shown that these effects can be substantial. The availability of such contracts can shift the terms of debt contracts against borrowers by inducing optimistic investors to divert their capital away from financing real investment and towards the support of collateralized speculative positions. This can influence the project choices of firms, leading not only to lower levels of investment overall but also in some cases to the selection of riskier ventures with lower expected returns. And they can result in the emergence of equilibria in which firms are unable to rollover their debt, even when such equilibria would not exist in their absence.

Tobin (1984) observed that the advantages of greater “liquidity and negotiability of financial instruments” come at the cost of facilitating speculation, and that greater market completeness under such conditions could reduce the functional efficiency of the financial system, namely its ability to facilitate “the mobilization of saving for investments in physical and human capital... and the allocation of saving to to their more socially productive uses.” Based on our analysis, one could make the case that naked credit default swaps are a case in point. This conclusion, however, is subject to the caveat that there exist conditions under which the presence of such contracts can prevent the funding of inefficient projects. Furthermore, an outright ban may be infeasible in practice due to the emergence of close substitutes through financial engineering. Even so, it is important to recognize that the proliferation of speculative side bets can have significant effects on economic fundamentals such as the terms of financing, the patterns of project selection, and the incidence of corporate and sovereign default.
References


Appendix: Proofs

Proof of Theorem 1. Deduce from (5) and (6) that

\begin{equation}
1 - F(\hat{\theta}(\bar{p}(q), q)) = \bar{p}(q)q = q\psi(q; \hat{\theta}(\bar{p}(q), q)) \leq \Psi(\hat{\theta}(\bar{p}(q), q)),
\end{equation}

where the first equality follows from (6), the second from (5) and the inequality follows from (2). Since the left most term of (22) is decreasing in \( \hat{\theta} \) and the right most term is increasing in \( \hat{\theta} \), the middle term (total revenue) is bounded above by \( \Psi(\theta_m) \). The upper bound is attained at \( q = 1 \), \( \bar{p}(q) = \Psi(\theta_m) = \psi(1, \theta_m) \) and \( \hat{\theta}(\bar{p}(1), 1) = \theta_m \). Hence the maximum revenue that can be raised in equilibrium is \( b_m = \Psi(\theta_m) \). Any funding requirement \( b \leq b_m \) can be met since total revenue varies continuously between 0 and \( b_m \) as \( q \) varies between 0 and 1. Equilibrium exists and is unique in this case since \( q^*(b) \) is unique by definition. To prove the last statement, consider \( b < b' \leq b_m \). By definition, \( q^*(b) \leq q^*(b') \) and \( p^*(b) = \bar{p}(q^*(b)) \geq \bar{p}(q^*(b')) = p^*(b') \). We must have \( q^*(b) < q^*(b') \) and \( p^*(b) > p^*(b') \), or else \( b = p^*(b)q^*(b) = p^*(b')q^*(b') = b' \), a contradiction. Q.E.D.

Proof of Theorem 3. Recall that \( \bar{p}(q) \) satisfies (6), so for any \( p' \geq \bar{p}(q) \),

\[
\frac{1}{p'} (1 - F(\hat{\theta}(p', q))) \leq q < q + \frac{1}{1 - p'} F(\hat{\theta}(p', q)),
\]

and hence \( p' \) cannot satisfy (15). This implies \( \bar{p}(q) < \bar{p}(q) \). Next, observe that

\begin{equation}
1 - \left( \frac{1}{1 - \bar{p}(q)} \right) F(\hat{\theta}(\bar{p}(q), q)) = \bar{p}(q)q = q\psi(q; \hat{\theta}(\bar{p}(q), q)) \leq \Psi(\hat{\theta}(\bar{p}(q), q)),
\end{equation}

where the first equality follows from (15), the second follows from (5) and the inequality follows from (2). The inequality becomes an equality at \( q = 1 \). Observe that

\[
\Upsilon(q) = \sup \left\{ \Psi(\theta') \mid \exists \theta' \text{ s.t. } \Psi(\theta') = 1 - \left( \frac{1}{1 - \bar{p}(q)} \right) F(\theta') \right\}
\]

is increasing in \( q \). Hence, \( \bar{p}(q)q \) is bounded above by \( \Upsilon(1) \). Since \( \bar{p}(1) > 0 \), this bound lies strictly below \( b_m \). Total revenue is maximized at \( q = 1 \) since its upper bound \( \Upsilon(q) \) is increasing in \( q \) and this bound is attained at \( q = 1 \). Hence the maximum revenue is \( \bar{p}(1) \).

For any \( q' \in (0, q^*(b)] \),

\[
b \geq \bar{p}(q')q' > \bar{p}(q')q',
\]

where the first inequality follows from the definition of \( q^*(b) \) and the strict inequality follows from the fact that \( \bar{p}(q') < \bar{p}(q') \). The inequality implies that \( q^{**}(b) > q^*(b) \). It
follows from this that

\[ p^{**}(b) = \bar{p}(q^{**}(b)) < \bar{p}(q^{*}(b)) < p(q^{*}(b)) = p^{*}(b). \]

Hence \( r^{**}(b) = 1 - p^{**}(b) > 1 - p^{*}(b) = r^{*}(b) \). The likelihood and extent of default is higher than in the case of no credit protection since a larger quantity of bonds is issued.

Q.E.D.

Proof of Lemma 1. From (1),

\[ q\psi_G(q; \theta_0) = \int_0^q ydG(y; \theta_0) + q(1 - G(q)). \]

Integrating the first term by parts, we obtain after simplification

\[ q\psi_G(q; \theta_0) = q - \int_0^q G(y; \theta_0)dy \]

Similarly,

\[ q\psi_B(q; \theta_0) = q - \int_0^q B(y; \theta_0)dy \]

and hence

\[ q\psi_G(q; \theta_0) - q\psi_B(q; \theta_0) = \int_0^q (B(y; \theta_0) - G(y; \theta_0))dy. \]

Condition (18) can therefore be rewritten as

\[ \int_0^q (B(y; \theta_0) - G(y; \theta_0))dy \leq \Psi_G(\theta_0) - \Psi_B(\theta_0). \]

By (17), the left side is increasing in \( q \) for \( q < y^* \) and decreasing in \( q \) for \( q > y^* \). Further observe that the inequality holds at \( q = 0 \), and becomes an equality at \( q = 1 \). (The latter claim can be verified by integrating the left side of the inequality by parts.) This means that there exists \( \hat{q} \in (0, y^*) \) such that (18) holds if and only if \( q \leq \hat{q} \). Q.E.D.

Proof of Lemma 2. Suppose first that there are no credit derivatives. Analogous to the case of the superior project, the equilibrium conditions (conditional on the the funding of the inferior project) are described by:

\[ 1 - F(\hat{\theta}_B^*(b)) = b = \int_0^1 \min\{y, q_B^*(b)\}dB(y|\hat{\theta}_B^*(b)). \] (24)

The first equality implies that \( \hat{\theta}_B^*(b) = \hat{\theta}^*(b) \). The second equality and the corresponding one for the superior project implies that

\[ \int_0^1 \min\{y, q_B^*(b)\}dB(y|\hat{\theta}_B^*(b)) = \int_0^1 \min\{y, q^*(b)\}dG(y|\hat{\theta}^*(b)), \]
from which it follows that $q_B^*(b) > q^*(b)$, since $\hat{\theta}_B^*(b) = \hat{\theta}^*(b)$ and $G(y|\theta) < B(y|\theta)$ for all $(y, \theta) \in (0, 1)^2$. Since $p_B^*(b)q_B^*(b) = b = p^*(b)q^*(b)$, it follows that $p_B^*(b) < p^*(b)$.

Now suppose that naked CDSs are available. Again the equilibrium conditions, assuming the funding of the inferior project, are as follows:

$1 - \left(1 - \frac{1}{p_B^*(b)}\right) F(\hat{\theta}_B^*(b)) = p_B^*(b)q_B^*(b) = b = \int_0^1 \min\{y, q_B^*(b)\} dB(y|\hat{\theta}_B^*(b)).$ (25)

For convenience, recall the corresponding conditions for the case of the superior project:

$1 - \left(1 - \frac{1}{p^*(b)}\right) F(\hat{\theta}^*(b)) = p^*(b)q^*(b) = b = \int_0^1 \min\{y, q^*(b)\} dG(y|\hat{\theta}^*(b)).$ (26)

Comparison of the first equation of (25) with that of (24) yields $\hat{\theta}_B^*(b) < \hat{\theta}_B^*(b)$. Next, suppose $\hat{\theta}_B^*(b) \leq \hat{\theta}^*(b)$. Then, comparison of the last equation of (25) with that of (26) yields $q_B^*(b) > q^*(b)$ since $\hat{\theta}_B^*(b) \leq \hat{\theta}^*(b)$ and $G(y|\theta) < B(y|\theta)$ for all $(y, \theta) \in (0, 1)^2$. Comparison of the second equations of (25) and (26) then yields $p_B^*(b) < p^*(b)$. Comparison of the first equation of (25) and that of (26) then yields a contradiction, since $p_B^*(b) < p^*(b)$ and yet $\hat{\theta}_B^*(b) \leq \hat{\theta}^*(b)$. We thus conclude that $\hat{\theta}_B^*(b) > \hat{\theta}^*(b)$, which in turn implies (via comparison of the first equations of (25) and (26) respectively) that $p_B^*(b) < p^*(b)$ and $q_B^*(b) > q^*(b)$. Also, since $\hat{\theta}_B^*(b) < \hat{\theta}^*(b)$, it follows from the last equations of (24) and (25) respectively that $q_B^*(b) > q^*(b)$ and $p_B^*(b) < p^*(b)$. Q.E.D.