Entry costs and economies of scope in multiproduct firms' decisions

Xosé-Luís Varela-Irimia, Universitat Rovira i Virgili
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Xosé-Luís Varela-Irimia * †
Toulouse School of Economics
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Abstract

This paper computes the scope economies associated to the commercialization of several product varieties by multiproduct firms, in a dynamic oligopoly setting. Goods are differentiated and firms decide on firm entry and exit, product entry and exit, quality and pricing. The model is applied to the Spanish automobile market. Results show moderate entry costs and substantial cost reductions when introducing a second product as compared to the first, indicating that multiproduct firms benefit from strong economies of scope when expanding their range of products. However, those economies disappear after five products have been introduced, suggesting a U-shaped curve for entry costs.

JEL classification numbers: L11, L13

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†Toulouse School of Economics. Gremaq - Inra. 21 allée de Brienne. 31000 Toulouse, France. E-mail: varela.irimia@gmail.com . http://www.eco.uc3m.es/jvarela .
1 Introduction

This paper studies the costs of product introduction and commercialization in differentiated product markets from a dynamic point of view, with an application to the Spanish automobile industry in the 1990’s. This market displays significant rates of entry and exit during that decade and is characterized by the existence of multiproduct firms that compete segment by segment with, usually, one model per segment. In this paper, I argue that incumbent firms have advantages in commercialization that make it easier for them to introduce new products compared to entrant firms. In particular, I look at the advantages when expanding the range of products: the conjecture is that a firm finds it easier to introduce a new variety of product (say a car in a new segment) when it already has other types of cars. The results give support to that claim and serve also to explain product proliferation in the industry.

Panzar and Willig coined the term “economies of scope” to describe cost savings that arise when the production of two or more product lines is combined, instead of being produced by separate firms (Panzar and Willig (1981)). Since then, many papers have been devoted to the measurement of economies of scope in different economic sectors like banking or hospitals. These approaches are based on the estimation of cost functions for multiproduct firms.

In this paper, I look at the economies of scope of product introduction abstracting from other scope economies. The model does not intend to explain scope economies in R&D, production, or plant activity. It intends to quantify the commercial advantage that a firm gets after it enters a market for the first time as the difference between the entry costs of subsequent products. I consider only economies of scope within the firm, i.e., product entry by a firm’s competitors may have market enlargement and business stealing effects, but this is not supposed to affect a firm’s entry costs. The commercial advantage may reflect brand image, continued advertising, development of dealer networks, etc. The separation between economies of scope in production and commercialization can be made because the decision to develop new products is made at the global level but
the decision to introduce new products in Spain is made at the Spanish level. Even though the concept of region for a multinational firm may exceed the boundaries of a country, the empirical evidence shows that significant differences arise in the entry and exit of identical products across markets\(^1\). The intuition is that, apart from the R&D costs, the effective commercialization of a new product depends on market-specific factors. Demand conditions, regional or national tastes for characteristics, etc. can render a product successful in one given market while it fails in another. This separation serves also to avoid economies of scope in production being captured by the measure of economies of scope in commercialization, as each of them belong to a different stage.

In this way we can talk of scope economies in the commercialization of new products and give a measure of their importance, which is the main contribution of the paper.

Contrary to standard productive costs functions, entry costs have the special characteristic that they are paid only once, but their effects spread over future periods. Therefore, they must be treated differently from usual recurrent costs. The first structural works of entry proposed multi-stage game theory models (see Toivanen and Waterson (2000) for a review). However, static models are not able to capture the intrinsic dynamic nature of entry costs. For this purpose, following the approach of Ericson and Pakes (1995), recently revised by Doraszelski and Satterthwaite (2007), I construct a model where multiproduct firms decide whether to introduce and quit products and potential entrant firms decide on entry. Product is differentiated in quality, which is allowed to exogenously vary over time at some cost. Product characteristics are summarized in a quality index representing the utility obtained by consumers. Finally, there is price competition. Firms play a game that lasts an infinite number of periods, differing from the classical “supergames” in that it is a single game, rather than the infinite replica of a multi-stage game.

The large computational costs in solving for an equilibrium of this kind of models has limited the range of empirical applications. Recent methodological developments (Aguirregabiria and Mira (2007); Bajari, Benkard, and Levin (2007); Pakes, Ostrovsky, and

\(^1\)This evidence is described in a companion paper available from the author
Berry (2007); Pesendorfer and Schmidt-Dengler (2007)), which make it possible to estimate the structural dynamic parameters without solving for an equilibrium, have boosted the literature on applied dynamic oligopoly models. The topics covered include firm entry and exit in homogeneous good markets (Ryan (2006); Collard-Wexler (2006)), entry in geographic markets (Dunne, Klimek, Roberts, and Xu (2006)), entry and competition in local retail markets (Aguirregabiria and Mira (2007)), and horizontal location of firms (Sweeting (2007)). So far, there has been no previous attempt to estimate product introduction costs by multiproduct firms, and the applications to the automobile industry focus only on the relation between market structure and innovation (Hashmi and Van Biesebroeck (2007)). To the best of my knowledge, the concept of scope economies defined in Panzar and Willig (1981) has not been used to provide a rationale for firms’ product introduction and product proliferation strategies.

The topic of entry and exit in automobile markets has been addressed from different perspectives. For example, Geroski and Murphy (1991) examine entry patterns across three segments of the U.K. car industry. They develop a probit model of the entry decision where post-entry profits depend on post-entry advertising shares. They find evidence that prior experience in the market may have had a small effect on entry in a particular segment. Geroski and Mazzucato (2001) study the relation between entry and advertising in the US automobile market. Requena-Silvente and Walker (2005) study how model survival in the UK car market relates to competition. They find that inter-firm competition determines survival of sports and luxury models while intra-firm competition is determinant for the rest. However, these works are based on the estimation of reduced form models, a usual characteristic in the earlier papers in this literature, whereas I am proposing a structural model. Regarding the Spanish market, the focus has been on testing pricing behavior (Jaumandreu and Moral (2006)) or on the role of advertising (Barroso (2007)).

The paper is organized as follows: in the next section I present a model of firm and product entry and exit. Then I explain the estimation strategy. Section 4 describes the data. Section 5 presents the results and further details. I conclude in Section 6.
2 A Model of Firm and Product Entry and Exit

The \( i^{th} \) firm maximizes the discounted sum of expected profits from the sum of its \( N_t \) products (indexed by \( j \)); \( N_t \) is the total number of products at \( t \):

\[
\Pi_i = E \sum_{t=0}^{\infty} \sum_{j} \delta^t \pi_{ijt}, \quad \forall \ i = 1, ..., N_t
\]  

(1)

A common discount factor \( \delta \) is assumed for all firms. Variable profits are given by:

\[
\pi_{ijt}^{\text{var}} = (p_{ijt} - c_{ijt}) D_{ijt} (P_t; K_t)
\]

Individual demand depends on the vector of all competing product prices and characteristics:

\[
P_t = (p_{1t}, ..., p_{N_t})
\]

\[
K_t = (k_{1t}, ..., k_{N_t})
\]

\( k_{ijt} \) is a quality index summarizing product characteristics (excluding price). I do not consider the problem of choosing characteristics in a multidimensional framework, hence each product is just a bundle of diverse features added up using a hedonic weight, \( \gamma_q \), for each one:

\[
k_i = \gamma_1 k_{i1} + \gamma_2 k_{i2} + ... + \gamma_q k_{iq}
\]  

(2)

I assume that products are exogenously classified in segments (groups). Then following Berry (1994), demand is modeled using a standard nested logit model:

\[
D_{it} (P_t, K_t) = S_{it} (P_t, K_t) * M_t
\]
where $M_t$ is the market size and $S_{ijt} (\cdot)$ is the share of product $j$. I also assume a constant marginal cost of production, $c_{ijt}$.

There is a product specific cost\(^2\), $F(k_{ijt}, i_{ijt})$, of implementing a quality index $k_{ijt}$ with quality change $i_{ijt}$.

**State Variables and Controls**

I describe here the state space of the model. I try to represent it in a parsimonious way, although this requires making some simplifications. I comment on them as I describe each element of the state space.

In period $t$, the controls are the decisions of entry and exit $(\chi_{ijt})$ and the decision of whether to invest. Entry and exit determine the number of products at the beginning of $t + 1$. Therefore, $N$ is an endogenously evolving state variable. However, market shares do not depend directly on the number of products. The larger the number of competitors, the less likely a particular product is consumed because there are more potential options that can give more utility. Thus, $N_t$ influences shares through the pair $(K_t, P_t)\(^3\). An alternative way of considering the number of competitors would be to split $N$ into its components: number of products in the segment and number of products in other segments. The former may capture the incentives for product exit in a segment as a consequence of competition. The latter could reflect the incentives for product entry in

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\(^2\)The cost of change is just the cost of adapting the product to the new specification (and not the cost of developing it), e.g., the cost of adapting the car to embody a more powerful engine, or perhaps the cost of adapting the productive chain of that model. It is true that in order to use a more powerful engine, it must first be designed and developed. In this stage, spillovers arise within multiproduct firms, which can make it easier to develop new engines for other models. These are the scope economies in production (or R&D), but I am looking only to scope economies in commercialization.

\(^3\)In my model, we can only have a direct effect of the number of incumbents over market shares when all firms offer the same pair $(k, p)$, then: $S_{ijt} = \frac{\exp[k-op]}{1-N\exp[k-op]}$. 
new segments to relax competition.

The total number of firms is fixed and known. They may be incumbents or potential entrants. Candidate products not introduced are discarded and in the following period a new set of products will be available for entry. There is an implicit but important assumption, which is that every period the feasible set of products for a firm is somehow “given”. The decisions of entry are made on these products. This first stage where products become available is not modelled. This caveat is partially justified in the application I have in mind: product entry in a national market by a global firm, where R&D is made at the global level, so that every period a range of products becomes available worldwide but the decision of entry is made independently in each national market. Needless to say, the ideal model should account for that stage as well.

Initial quality is given by initial product characteristics, which are drawn from some distribution and then aggregated to form the index, whose initial value is thus random. Given that I am not modelling the choice of characteristics and I only need the quality index, it is in fact simpler to assume randomly drawn initial quality, understanding that it is linked to characteristics through equation (2). Firms modify quality as time goes by. The amount of change, if any, is exogenously given. The model does not deal with choice of characteristics and, as noted above, when they vary I assume they do it in exogenous amounts. However, the decision of modification is endogenous because the firm can always keep the product “as is” without modifications. Therefore, the model can explain entry and exit decisions controlling for the empirical fact that quality is changing over time. In the context of product differentiation, it is necessary to keep track of product characteristics even though we are only interested in the entry-exit process, because characteristics define products. Notice that quality changes are an alternative to product replacement, but if the cost of those changes is too high, firms may prefer to quit the product.

The dimension of $K$ becomes large as the number of products increases and this poses a problem in estimation because the state space is too large. Therefore, I consider an
alternative representation of the state space in the spirit of Weintraub, Benkard, and Van Roy (2007), where for each product the state is defined by its quality index, the average quality index $\bar{K}$, and the number of competing products. Likewise, I do not include market size or demand conditions in the state space.

**Timing**

Each firm receives a private draw from the distribution of sunk costs of entry/sell-off values (depending on whether it is an entrant or an incumbent) and decides $\chi_{ijt}$. If a firm does not quit the product, it receives an exogenous shock that determines whether the quality of the good is going to be changed or not. Entrants can immediately start to sell their product; exiting firms receive their scrap value and disappear. Given the new $(K_t, N_t)$, firms simultaneously set prices and receive variable profits. The important thing here is that firms cannot change $k_{ijt}$ or $\chi_{ijt}$ when they are about to compete in prices. Shocks happen before decisions are made and are the key to rationalizing the variability of decisions from firms with the same observed features.

**Decision Rules**

Firms make several decisions every period. An incumbent firm decides, for each segment, whether to introduce a new product or not. If it has no product in a segment, that decision is equivalent to entering or not entering that segment. If it already has other product(s) in the segment, then it can decide whether to quit any of them. Firms that are out of the market may decide to enter or not. Entry may take place in more than one segment at the same time. Firms are allowed to introduce at most one new product per segment and period, but they can quit as many products as they have. There is a maximum for the number of products a firm can commercialize in each segment. Enter/Not enter decisions are represented by the indicator $\chi'_{ijt} = \{0,1\}$ and stay/exit by $\chi_{ijt} = \{0,1\}$. After all firms have decided about entry and exit, products are modified if necessary. Finally, there is price competition.
**Entry/Stay/Exit**

Every period each incumbent (entrant) receives a shock from a known distribution on the sell-off value (sunk cost of entry), $\phi$ ($\kappa$). If the discounted sum of expected payoffs is smaller than that draw, the firm exits (does not enter), i.e.:

\[
\chi_{ijt} = 0 \iff EDV_{t}^{incumbent} (K_{t-1}, N_{t-1}) \leq \phi
\]

\[
\chi_{ijt}^e = 0 \iff EDV_{t}^{entrant} (K_{t-1}, N_{t-1}) \leq \kappa
\]

where $EDV_{t}^{incumbent} (K_{t-1}, N_{t-1})$ and $EDV_{t}^{entrant} (K_{t-1}, N_{t-1})$ represent the expected discounted sum of payoffs for an incumbent and an entrant, respectively, conditional on staying/entering at $t$ given $(K_{t-1}, N_{t-1})$. I assume that when a firm quits a product, it can never re-enter. Instead, it may introduce a new product. Also, all firms decide $\chi_{ijt}$ simultaneously. These are dynamic decisions because their effects spread over a number of periods.

**Pricing**

This is a static decision: $p_{ijt+s}$ is not a function of $p_{ijt+1}$ $\forall s \neq 1$. Pricing decisions become determined by the state variables and Bertrand competition. Therefore, prices are irrelevant in the dynamic problem and they can be substituted by their optimal expressions in the one-period payoff function.

Given $k_{ijt}$ and $N_t$, $p_{ijt}$ is chosen so as to maximize the profit stage. As I argued above, this is a “self-contained” decision and I can substitute back in the one-period payoff function to obtain a reduced form:

\[
\pi_{it} = \pi_{it} (K_t, N_t)
\]
It can be shown that the first-order condition of a multiproduct firm in a nested logit is:

\[
(p_j - c_j) S_j = \frac{1 - \sigma}{\alpha} S_j + \sigma S_{j/g} \sum_{j \in G_g} (p_j - c_j) S_j \\
+ (1 - \sigma) S_j \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j , \forall j, \forall g
\]

From this system of FOC’s, it is possible to obtain an equilibrium expression of product variable profit as a function of the state variables:

\[
\pi^\text{var}_j (K_t, N_t) = \frac{1 - \sigma}{\alpha} S_j (K_t) M + \sigma S_{j/g} (K_t) \pi^\text{var}_{i_g} (K_t, N_t) \\
+ (1 - \sigma) S_j (K_t) \pi^\text{var}_i (K_t, N_t) , \forall j, \forall g
\]

where \(\alpha\) is the marginal utility of income, \(\sigma\) is the degree of intra-group correlation and \(S_{j/g}\) is the market share of \(j\) conditioned to group \(g\). \(\pi^\text{var}_{i_g}\) is the variable profit of firm \(i\) in segment \(g\), and \(\pi^\text{var}_i\) is the total profit of firm \(i\):

\[
\pi^\text{var}_{i_g} = \frac{(1 - \sigma) \sum_{j \in G_g} S_j}{1 - \sigma \sum_{j \in G_g} S_{j/g}} \left[ \frac{M}{\alpha} + \pi^\text{var}_i \right] , \forall g
\]

\[
\pi^\text{var}_i = \frac{M \sum_{g \in G_g} \frac{1 - \sigma}{\alpha} S_j}{1 - \sigma \sum_{j \in G_g} S_{j/g}} \left[ \frac{(1 - \sigma) \sum_{j \in G} S_j}{1 - \sigma \sum_{j \in G_g} S_{j/g}} \right] , \forall g
\]

Notice that there is an implicit, non-linear, one-to-one relationship between \(p\) and \(k\). \(p\) is determined with no direct influence of the shock \(\varepsilon\), although \(p\) is affected by \(\varepsilon\) through \(k\).
Characteristics

As discussed above, firms modify product characteristics in response to exogenous changes. When this happens, firms adjust product quality to a new level such that:

\[ i_{ijt} = k_{ijt} - k_{ijt-1} \]

\( i_{ijt} \) can be interpreted as the change of value of product \( j \) in hedonic terms. This is a deterministic law of motion for \( k_{ijt} \); what is random is the decision to change product specification.

Summarizing, the one-period payoff function is given by:

\[
\pi_{it} = \sum_{j=1}^{N_t} \chi_{ijt} \left[ (p_{ijt} - c_{ijt}) S_{ijt} M_t + \rho \mathbf{1} (i_{ijt} \neq 0) (F (k_{ijt}, i_{ijt}) + \varepsilon_{ijt}) \right] \\
- \chi_i^{e} e_{ijt} + (1 - \chi_{ijt}) \phi_{ijt}
\]

(3)

where \( \mathbf{1} (inv \neq 0) \) is an indicator function whose value is zero if investment is zero, and one otherwise. Notice that \( \chi_e = 1 \implies \chi = 1 \) and \( \chi = 0 \implies \chi_e = 0 \). A model that is in the market and continues is represented by \( \chi = 1, \chi_e = 0 \). \( \chi_e = 1, \chi = 0 \) is not possible.

The profit from a product continuing in the market is:

\[
\pi_{ijt} (K_t, N_t^I) = \pi^\text{var} (K_t, N_t) + \rho \mathbf{1} (inv \neq 0) (F (k_{ijt}, i_{ijt}) + \varepsilon_{ijt})
\]

(4)

\( \pi_{ijt} \) depends on \( K_t \) in a highly non-linear way through market shares.

Bellman Equation

The Bellman equation of the problem can be written as:
For an incumbent:

\[
V_{ij} (K_{t-1}, N_{t-1}) = \max \left\{ \phi_{ijt} , \max_{\pi_{ijt}} \left\{ \pi_{ijt} + \beta \int V_{ij} (K_t, N_t) \right\} \right\} \\
\max_{ijt} \left\{ dG_K (K_t|K_{t-1}) dG_N (N_t|N_{t-1}) \right\}
\]

and \(\pi_{ijt}\) is given by (4).

For a potential entrant:

\[
V_{ij} (K_{t-1}, N_{t-1}) = \max \left\{ 0 , \max_{\pi_{ijt}} \left\{ -\kappa_{ijt} + \pi_{ijt} + \beta \int V_{ij} (K_t, N_t) \right\} \right\} \\
\max_{ijt} \left\{ dG_K (K_t|K_{t-1}) dG_N (N_t|N_{t-1}) \right\}
\]

and \(\pi_{ijt}\) is given by (4). Notice that the entrant incurs the cost \(\kappa_{ijt}\) right after entry, and it can immediately start to sell the product.

\(G_K\) and \(G_N\) are the distribution functions giving the transition probabilities of \(K\) and \(N\), respectively.

**Equilibrium Concept**

Firms make decisions with an infinite horizon and so the potential number of Nash equilibria (NE) is likely to be large, involving complex combinations of decision rules. Therefore, I consider a restricted class of NE, the pure strategy Markov perfect equilibria (MPE), by assuming firms play Markov strategies, which means that strategies depend on all payoff relevant history. Formally, a Markov strategy is a map from the state space to the action space:

\[
\sigma_{ijt} : K_t \times N_t \rightarrow A_{ijt}
\]

such that:

\[
\sigma_{ijt} (K_{t-1} N_{t-1}, K_{t-2}, N_{t-2}, K_{t-3}, N_{t-3}, ...) = \sigma_{ijt} (K_{t-1}, N_{t-1})
\]
Let’s define a profile of Markov strategies as the vector:

\[ \sigma_t = (\sigma_{1t}, \ldots, \sigma_{Nt}) \]

Following Ackerberg, Benkard, Berry, and Pakes (2005), we can say that a Markov strategy profile, \( \sigma \), is an MPE if for all \( ij \), all states, and all Markov strategies, \( \sigma'_{ij} \):

\[ V_{ij} (K, N \mid \sigma_{ij}, \sigma_{-ij}) \geq V_{ij} (K, N \mid \sigma'_{ij}, \sigma_{-ij}) \]

Doraszelski and Satterthwaite (2007) (DS) show the existence of (at least one) MPE in an Ericson and Pakes (1995) setting like mine. In particular, proposition 1 in DS shows that an MPE equilibrium exists in cutoff entry-exit and pure investment strategies under three assumptions. Assumption 1 states boundness of the model’s primitives (finite state space, bounded profits and investment, continuity and bounded support of entry costs and scrap values, and discounting). Assumption 2 is basically a continuity assumption in payoff functions. Assumption 3 requires that a firm’s investment choice is always uniquely determined. The first two are standard assumptions easy to verify. The third one is a bit more restrictive, but not a big issue in my model. I consider exogenously determined investment with one unique investment level for each state and this is optimal provided the policy function has been accurately recovered.

I assume that if there is more than one equilibrium, then the data is generated from only one of them.

Therefore, the value function can be written in recursive form:

\[
V_{ij} (K, N | \sigma) = \pi_{ij} (\sigma (K, N), K, N) \\
+ \beta \int V_{ij} (K', N' | \sigma) \, dG_K (K' | \sigma (K, N), K) \, dG_N (N' | \sigma (K, N), N)
\]
3 Estimation Strategy

In this section, I follow the two-step methodology developed in Bajari, Benkard, and Levin (2007) (BBL). In the first step, the goal is to estimate policy functions and all parameters not involved in the dynamics of the problem. These allow the simulation of alternative histories for the industry which are then used in the second step to recover the dynamic parameters and value function estimates in equilibrium.

First Stage

The target parameters here are those from market demand, variable profit function, investment, and entry/stay/exit decision rules.

Variable static profits are computed by making use of the equilibrium expression obtained from the logit specification:

\[
\pi_{ijt}^{\text{var}} = \frac{1 - \sigma}{\alpha} S_{ijt} M + \sigma S_{j/g} \pi_{ig}^{\text{var}} + (1 - \sigma) S_{ijt} \pi_i^{\text{var}}
\]  

(5)

\(\alpha\) is the marginal utility of income, obtained from demand estimation, \(M_t\) is (observed) market size, and \(\pi_{ig}^{\text{var}}\) and \(\pi_i^{\text{var}}\) are functions of market shares defined above. Therefore, the key element is the estimation of market shares, which is discussed below.

As I handle a reduced form of variable profits where marginal cost is substituted away, I do not have to be concerned about estimating variable production costs.

The policy functions for entry and exit are kept simple (as, for example, in Ryan (2006)). I model the probability of firm \(i\) introducing a new model \(j\) at time \(t\) in segment \(g\), conditional on the number of models of the firm \(N_i\), as a function of the number of models it has in other segments, \(N_{i,-g}\), and on the average quality in the segment, \(Avksg\). (Other measures of the number of competitors were considered, but their coefficients had significance problems).

The dimension of the state space would require a lot of data to be able to estimate the policy function parameters for each combination of quality for all competing models. This
is why I consider the average quality in the segment as a proxy for the vector of product qualities. For the same reason, I consider the number of models instead of the particular portfolio of the firm’s products. This is to say that, for example, the decision of entry of the third, large model having a small and a mini is equivalent to having a medium and a mini. The implicit assumption under these two simplifications is that I am still able to recover the optimal policies coming from the equilibrium of the model.

Using Bayes’ rule:

$$
\text{prob}(\text{entry}_{ijt} | N_{it}) = \frac{\text{Pr}(N_{it}|\text{entry}) \ast \text{Pr}(\text{entry})}{\text{Pr}(N_{it}|\text{entry}) \ast \text{Pr}(\text{entry}) + \text{Pr}(N_{it}|\text{no entry}) \ast \text{Pr}(\text{no entry})}
$$

where \( \text{Pr}(N_{it}|\text{entry}) \) and \( \text{Pr}(N_{it}|\text{no entry}) \) are modeled using ordered probits:

$$
\text{Pr}(N_{it} = 0 | \text{entry}) = F_n (c_1 - \beta_1 \ast N_{i-g,t-1} - \beta_2 \ast Avks g_{gt-1}) = F_n (c_1 - x \beta)
$$

$$
\text{Pr}(N_{it} = n | \text{entry}) = F_n (c_{n+1} - x \beta) - F_n (c_n - x \beta) \quad , \quad n = 1, ..., N - 1
$$

$$
\text{Pr}(N_{it} = N | \text{entry}) = 1 - F_n (c_N - x \beta)
$$

where the \( c \)'s are the cutoffs determining when each category is chosen. The same is done for \( \text{Pr}(N_{it}|\text{no entry}) \). The \( \text{Pr}(\text{entry}) \) is estimated as the sample rate of entry. I decompose the conditional probability of entry in the reverse conditional probabilities because for some \( N \) there are few observations on entry and this poses some difficulties in the estimation. The \( \text{Pr}(N_{it} = n | \text{entry}) \) and \( \text{Pr}(N_{it} = n | \text{no entry}) \) turn out to be easier to estimate and simplify the computations for the second stage. For this reason, here I have not considered other smoothing or interpolation techniques such as kernels or splines.

The probability of exit is modeled using a probit on the deviation of \( k \) with respect to its segment mean (a parsimonious way of modelling the relation of product \( j \) with its competitors), the number of models of the firm in the segment, \( N_{ig,t-1} \), and the age of
the firm’s oldest product in the segment, \( maxage_{ig} \):

\[
prob(\text{exit}_{it}) = F_n (\beta_0 + \beta_3 * N_{ig,t-1} + \beta_4 * maxage_{igt-1} + \beta_5 * DevkSg_{ijt-1})
\]  

(7)

The state space increases as the number of products becomes large. It is necessary to reduce the dimension of the state space to be able to estimate the policy functions, because it is not possible to estimate one parameter for each element of vector \( K \). I overcome this by considering the average quality, instead of the vector of qualities, in the entry and exit probits. Moreover, the large number of zeros in the entry and exit decisions can pose some identification problems in those equations unless some exclusion restriction is imposed. For this reason I add \( maxage \) as an explicative variable in the exit equation. This variable is a proxy for the degree of obsolescence of the product and it serves as a complement to \( k_j \) (which is included within \( DevkSg \)).

The different approach for entry and exit policies is because we are interested in how the entry cost changes as the number of models of the firm increases. Therefore, the entry policy must be sensitive to that fact while the exit policy can be more parsimonious.

Quality changes are modeled in the following manner. The probability of change depends on the current level of \( k \) and its deviation with respect to the mean of the segment. The firm is shocked by an exogenous cutoff such that if the probability of change is larger, then the quality of the product is adjusted. I use a probit for the probability of change:

\[
prob(\text{invest}) = F_n (\beta_{inv}^0 + \beta_6 * k_{ijt-1} + \beta_7 * DevkSg_{ijt-1})
\]  

(8)

The quality adjustment is given by a cubic B-spline policy on the deviation of \( k_j \) with respect to its segment mean:

\[
i_{it} = \beta_8 * Sp (DevkSg_{ijt-1})
\]  

(9)

This setup allows for a better fit of the observed quality changes.
I model the adjustment cost function as an exponential of the absolute value of the change in characteristics, and zero when investment is zero:

\[ F(k_{jt}, i_{jt}) = 1 (i_{jt} \neq 0) \times \exp(\text{abs}(i_{jt})) \]

I obtain the estimates for all the \( \alpha, \sigma, c, \) and \( \beta \) parameters. Then I can generate a set of simulated paths from different initial conditions and confront them with perturbed, non-optimal paths to obtain the estimates of the dynamic parameters of the model in the second stage.

**Second Stage**

The second stage deals with the estimation of dynamic parameters (investment cost, scrap value, entry costs). Given the actual and simulated paths for the industry, the estimation goes as follows: recall the equilibrium condition

\[ V_{ij}(K, N | \sigma_{ij}, \sigma_{-ij} ; \theta) \geq V_{ij}(K, N | \sigma'_{ij}, \sigma_{-ij} ; \theta) \]

Recall that from equation (3), the one-period payoff function is linear in the dynamic parameters and then \( V(\cdot) \) is linear in \( \theta \):

\[
V_{ij}(K, N; \sigma_{ij}, \sigma_{-ij}; \theta) = E \left[ \sum_{t=0}^{\infty} \delta^t \Psi_{ijt}(\sigma_{ijt}, K_t, N_t, \nu_{ijt}) | K_0 = K, N_0 = N \right] \cdot \theta \\
= W_{ij}(K, N; \sigma_{ij}, \sigma_{-ij}) \cdot \theta
\]

where \( \Psi_{ijt}(\cdot) = (\pi_{ijt}^{\text{opt}}(K), F(k_{ijt}, i_{ijt}, \phi_{ijt}) \) is the vector of basis functions for payoffs and \( \theta' = (1, \rho, \phi) \); then the equilibrium condition becomes:

\[ [W_{ij}(K, N | \sigma_{ij}, \sigma_{-ij}) - W_{ij}(K, N | \sigma'_{ij}, \sigma_{-ij})] \cdot \theta \geq 0 \]

Let \( x \in X \) be an index for the equilibrium conditions such that each \( x \) represents a
combination of product, alternative action, and states, \((ij, K, N, \sigma'_{ij})\). Then each condition \((11)\) of the set of inequalities \(X\) can be rewritten as

\[
g (x; \theta) = [W_{ij} (K, N; \sigma_i, \sigma_{-i}) - W_{ij} (K, N; \sigma'_{ij}, \sigma_{-ij})] \cdot \theta
\]

(12)

The vector of dynamic parameters \(\theta\) satisfies an equilibrium condition defined by \(x\) if \(g (x; \theta) \geq 0\). Therefore, the estimation strategy consists in taking many such conditions and finding a \(\theta\) such that profitable deviations from the optimal policies (represented by \(g (x; \theta) \leq 0\)) are minimized. For this purpose, define the function

\[
Q (\theta) = \int (\min \{g (x; \theta), 0\})^2 dH (x)
\]

where \(H (\cdot)\) is a distribution over the set of inequalities, \(X\), which the \(g\) conditions belong to. At the true parameter value, \(Q (\theta_0) = 0 = \min \limits_{\theta} Q (\theta)\), i.e., the objective function is minimized at \(\theta_0\). Its empirical counterpart can be written as:

\[
Q_n (\theta) = \frac{1}{n_I} \sum_{k=1}^{n_I} (\min \{\tilde{g}_k (x; \theta), 0\})^2
\]

(13)

where the \(\tilde{g}_1, \ldots, \tilde{g}_{n_I}\) is a set of \(n_I\) inequalities drawn from \(H (\cdot)\). \(\tilde{g}\) is the sample counterpart of \(g\) that results from replacing \(W_{ij}\) with simulated estimates \(\hat{W}_{ij}\).

Following the methodology of BBL, I randomly draw the \(n_I\) inequalities to construct (13). Then I compute \(W_i\) for the observed and alternative policies using observed and simulated industry paths. Alternative policies are generated by adding small, random perturbations to the policy functions. The \(W_{ij}\)’s are used to obtain the \(\tilde{g}_k (\cdot)\)’s. Finally, \(Q_n (\cdot)\) is minimized in \(\theta\) for the non-positive \(\tilde{g}_k (\cdot)\) conditions using standard optimization procedures. BBL show that under some regularity assumptions, \(\hat{\theta}\) is consistent for \(\theta\).
Entry Costs

Once the vector of dynamic parameters, \( \hat{\theta} \), is obtained, it is possible to estimate the distribution of sunk costs of entry in a simple manner: for each relevant state configuration, simulate the expected discounted value of entry (EDV) for an entrant at that state. Also compute the probability of entry using the entry policy. We know that firms enter only if the EDV is not smaller than the sunk cost of entry; if we match this with the predicted probability of entry, we obtain the following relationship:

\[
\text{prob}(\text{entry}|N) = \text{prob}(\kappa \leq EDV; \lambda) = F(EDV; \lambda)
\]

i.e., the observed probability of entry is the value of the cumulative density function evaluated at EDV. I assume a normal distribution for \( F \) and I minimize the squared distance between both parts of the equation:

\[
\min_{\mu, \sigma} \frac{1}{n_e} \sum_{i=1}^{n_e} \left[ \text{prob}_i (\text{entry}|N) - F(EDV_i) \right]^2
\]

(14)

where \( n_e \) is the number of states for which the EDV of entry is computed. With \( \mu \) and \( \sigma \), the distribution of sunk entry costs is characterized under the assumption of normality.

This basic procedure can be used to estimate different types of entry costs. In particular, the model allows us to compute EDV’s and entry costs for:

- Firm entry (the entrant model belongs to a firm which has no other model in any other segment).
- Segment entry (the firm is in other segments but not in this one).
- Model entry (the firm is already in the segment and introduces a new model).

Case 1:

This is the simplest case described above. We can compute the empirical probability of entry using the policy function, and simulate the EDV starting from an industry
configuration where this firm is not in the market.

Case 2:

In this case, the firm is already in and we want to estimate the cost of entering another segment. The number of possible industry configurations increases with respect to case 1 because the incumbent firm may have several models in other segments. For example, with a classification in 8 segments and a firm that has only one previous model, the number of alternatives with respect to case 1 multiplies by 7. If the firm already had 2 models, the number of alternatives is multiplied by $\binom{3}{1} = 21$, and so on.

Again, the empirical probability can be computed and the EDV’s are computed for an industry configuration restricted in the appropriate manner.

Case 3:

The number of combinations is the same as case 1 or case 2 depending on the particular restrictions we want to impose on the firm. In particular, we could think of the cost of introducing the second product in the same segment and compare it to the cost of introducing it in another segment (as in case 2).

**Estimation of Entry Cost Parameters and the Measure of Scope Economies**

I argue that the entry cost is different depending on the number of models commercialized by the firm. This implies that, conditional on the number of models, all firms receive iid shocks from the same (normal) distribution over time, but this distribution changes as the number of models changes. I introduce a parametric restriction which is that all distributions have the same variance and they differ only in the mean. This implies that those distributions are shifted to the left or to the right as the number of models increases. Taking as a reference the entry cost with no previous products (firm entry), the existence of economies of scope in commercialization follows from these distributions shifting to the left (at least for a small number of products). Recall that the basic equation for sunk
costs of entry is:

\[
\begin{align*}
\text{prob}(\text{entry}) &= F_N(\kappa \leq EDV; \lambda) \\
\text{prob}(\text{entry}) &= F_N\left(\frac{\kappa - \mu}{\sigma_e} \leq \frac{EDV - \mu}{\sigma_e}\right)
\end{align*}
\]

Consider for simplicity the case where we want to estimate the cost of firm entry and the cost of introducing a second and third product in other different segments. We can argue that they are different by factors \(d_1\) and \(d_2\) such that the mean firm entry cost is \(\mu_0\), the mean entry cost of a second product is \(\mu_1 = \mu_0 + d_1\), and for the third product \(\mu_2 = \mu_0 + d_2\). The above probability equation becomes:

\[
\text{prob}(\text{entry} \mid N = 1) = F_N\left(\frac{\kappa - d_1 - d_2 - \mu_0}{\sigma_e} \leq \frac{EDV - d_1 - d_2 - \mu_0}{\sigma_e}\right)
\]

If the firm is about to introduce a third product:

\[
\text{prob}(\text{entry} \mid N = 2) = F_N\left(\frac{\kappa - d_1 - d_2 - \mu_0}{\sigma_e} \leq \frac{EDV - d_1 - d_2 - \mu_0}{\sigma_e}\right)
\]

The firm entry cost can be identified from the variation in the observed rates of entry, the normality assumption, and the variation of present discounted values. For a given initial state, the entry policy function provides an estimate of the probability of entry. The forward simulation procedure yields the correspondent expected value of entry for that initial state. The quality adjustment cost parameter and the scrap value required in the forward simulation are also identified. These parameters are computed such that profitable deviations from optimal observed behavior, summarized in the policy functions, are minimized. Therefore, the variability in adjustment decisions, conditional on the state, and their difference with respect to the optimal ones identify the adjustment cost parameter such that the policy function is indeed optimal. A similar argument holds for the scrap value.

The identification of \(d\)'s comes from the observed variation in the number of previous
products of the firm. In practice, $d$ should be estimated as the coefficient of a dummy for the number of firm models. For example, $d_2$ is the coefficient of the indicator variable $1(\#\text{models} = 2)$. It is clear that this variable is zero in the first equation, but it is still good to include it because the joint estimation of $\mu_0$ and $d$'s is more efficient. The difference of entry costs in other scenarios can be captured by adding the corresponding dummy variables and computing the EDV for all the possible states involved.

4 Data

I apply the methodology described above to the Spanish car industry. I use a unique monthly data set of car models in Spain from 1990 to 2000. These data were initially collected by, and first used in Moral (1999)\(^4\), who also provides a thorough description of the data base. It contains information on model characteristics such as speed, size, consumption, and horse power, among others. I also have the number of registrations by model and listed prices.

A descriptive look at the evolution of main characteristics (see Table 1) shows that, on average, we observe variation in at least one characteristic in roughly 60% of the sample of yearly observations (the ratio is obviously smaller when looking at monthly observations). Table 2 shows the percentage of variation by segment on a monthly basis for the same characteristics. Overall, the average variation across segments is 4.8%. This variability is also confirmed by casual observation of specialized press reports.

Table 3 summarizes entries and exits by segment: the rate of entry is around 17% per year per 9% of exit. The persistent gap is the reason for the increasing number of models in the industry during the 1990’s.

I construct the index of characteristics of each model in the sample as a weighted sum whose weights are the estimated coefficients of each characteristic in the demand

\(^4\text{The data base here, which runs from January 1990 to December 1996, has later been extended up to December 2000.}\)
equation (in a sense, we could call that index a gross hedonic index as price is excluded and considered separately). The index can be interpreted as the average utility that a consumer could obtain from that product, without taking into account its price. Table 4 gives a summary of characteristics and prices per segment. The coefficients of the index are in Table 5.

5 Results and Further Details

5.1 First Stage Estimates

In this stage, I estimate demand and the policy functions for entry, exit, and investment.

Demand Estimation Using Nested Logit

Following Berry (1994), we can write the shares equation as follows:

\[
\ln (S_{jt}) - \ln (S_{0t}) = k_{jt} - \alpha_g p_{jt} + \sigma_g \ln (S_{jt/g}) + \eta_j
\]  

(15)

where the marginal utility of income (\( \alpha \)) and the degree of intra-group correlation (\( \sigma \)) are allowed to vary across segments. \( S_{jt/g} \) is the market share of product \( j \) in its group \( g \), \( \eta_j \) is an unobserved fixed effect, and the index of characteristics is constructed as:

\[
k_{jt} = \gamma_1 Carsize_{jt} + \gamma_2 HP_{jt} + \gamma_3 KmL_{jt} + \gamma_4 AC_{jt} + \gamma_5 ABS_{jt}
\]

The endogeneity of prices and conditional market shares is controlled for with the following instruments: following Berry, Levinsohn, and Pakes (1995) (BLP), as instruments I use product characteristics, the sum across own-firm products of each characteristic, and the sum across rival firms’ products of each characteristic. I also include the total number of models per segment (as in Brenkers and Verboven (2006)), and finally the differences of prices with respect to their individual time means, \( \tilde{p}_{jt} = p_{jt} - \frac{1}{T_j} \sum_{t=1}^{T_j} p_{jt} \), lagged 12
months (first introduced by Bhargava and Sargan (1983) and studied in Arellano and Bover (1995)). I also control for the existence of tariffs over imported cars.

Consistent estimators are obtained by first using the within transformation to remove the fixed effect and then applying two-stage least squares to the transformed model.

Table 5 summarizes estimation results. The coefficients of real price and characteristics have the expected sign and almost all of them are significant at the 1% level.

The own-price elasticities implied by the estimates of Table 5 suffer from the rigidity in substitution patterns imposed by the logit assumption. The nested logit helps in correcting the problem but the elasticities for cheaper cars are still small, a bit far from the pattern for the US automobile industry (Berry, Levinsohn, and Pakes (1995)), but at least not so far from previous estimates for European markets (Brenkers and Verboven (2006)). In facing the trade-off between accuracy and computational simplicity, the loss of precision at this stage might not be excessively harmful. Nested logit is still a common approach to demand estimation in automobile markets. The alternative would be to estimate demand following the BLP methodology.

Demand estimation yields the estimate of $\alpha$ and $\sigma$ in equation (5) and the hedonic coefficients for characteristics, used in the construction of the index $k$.

**Policy Functions**

Tables 6 and 7 summarize the probits for quality changes and the entry and exit policies. For entry, the coefficients are in general significant although their interpretation is not direct because we are interested in the reversed probability $\text{Pr}(\text{entry} \mid N)$ and also because in ordered probit models the sign of the marginal effect does not always coincide with the sign of the coefficient for the intermediate categories. For exit, the interpretation varies depending on the segment considered, but in general the parameters are significant. Regarding the probit for investment, the probability of investing increases with the level of $k$ and is decreasing in the distance to the mean. It seems that models with a large endowment of characteristics are modified more frequently and last longer than smaller
cars. At the same time, cars that are “too different” from their competitors are more likely to quit and less likely to be modified.

I choose cubic B-splines because of their flexibility and computational simplicity. Splines are interpolation methods used to make predictions of a variable based on other(s) when the functional relation between them is not known. (see Judd (1998) chapter 6 or Cheney and Kinkaid (1985) chapter 7 for a survey). B-splines are defined with reference to a set of knots. A k-degree B-spline is just a set of different k-degree polynomials, one for each of the intervals defined by the set of knots. It has the property that the derivatives from 0 to $k - 1$ at each knot are the same for contiguous polynomials. This produces smooth interpolations.

I use cubic B-splines with 20 interior knots to tabulate the investment policy function. I construct a grid for the explicative variable with precision $10^{-5}$ and then I compute the predicted value of investment for each element in the grid using the cubic B-splines. The tabulated policy stays in memory and it is called when a value for predicted investment is needed. The grid is fine enough as the explicative variable does not show significant variability further than the 4th or 5th decimal place. There is no particular economic interpretation to be given to those parameters, but they provide a good fit of the tabulated investment policy to the observed one.

Prices and market shares have no dynamic implication and are solved every period given the level of $k$. Unfortunately, there is no explicit analytical expression of $p$ as function of $k$ within the logit framework. Therefore, I again use cubic B-splines to obtain $p$ as a tabulated function of $k^5$, $p_i = p_i(k_i)$.

In generating the simulated paths, a random shock or bias is added to each of the three policies. For entry and exit, I draw from a random uniform distribution. For investment, I draw from a lognormal distribution (in fact, this is as if it were a bias over the level of $k$; that’s the reason for assuming log-normality). The cutoff values for entry, exit, and

\footnote{Alternatively, the first-order conditions could be solved numerically for $p$, at the cost of increasing the computational burden.}
investment policies are also drawn from a uniform distribution, rescaled in each case to meet sample moments.

The simulation of the alternative paths goes as follows: I draw initial values for $k$ from a lognormal distribution. With these initial values, I can use policy functions to obtain the correspondent price and exit decisions. Then I only have to recursively apply the policy functions until a whole history for all firms (incumbents and potential entrants) is filled up. I repeat the same process for the same initial state but this time adding a small random shock to the policies to simulate alternative, non-optimal paths. I do it for 132 periods (months). Given a simulated history, I can compute the market shares for each firm. Then I can compute the $W_{ij}$ vector in equation (10) as the difference between the present discounted values from the actual and the alternative histories. The discount factor is the monthly equivalent of a 10% annual interest rate. During the 1990’s, interest rates in Spain ranged from 3% in 2000 up to values close to 10% at the beginning of the decade. I stick to the conservative perspective.

5.2 Estimates of Dynamic Parameters

The equilibrium condition (11) and its empirical counterpart can be constructed with the simulated histories. The minimization of (13) yields the vector of dynamic parameters for investment cost and scrap value. Table 8 provides estimates for the scrap value and investment cost parameters for the whole Spanish market. It turns out that scrap values are moderate compared to industry profits. The implied price elasticities combined with average prices in Table 4 yield margins roughly between 5000 and 8000 euros per car. This is equivalent to $5100 \text{ – } 8200$ units. The scrap value in this context can be interpreted as what remains for a firm after it quits a model. For example, a successful car may induce consumers to go to that firm looking for the new model because of the positive image of the previous one. The sell-off value would be the value of the goodwill generated by the model quitting the market.

The parameters for investment cost reveal a small or moderate cost of changing char-
acteristics. The large value of the coefficient is just an effect of the rescaling of investment. In fact, the changes in $k$ are usually small (on average, in absolute value 0.18 with standard deviation 0.17) and magnitudes of $10^{-2}$ or much smaller are frequent. Continuing with the High-Intermediate segment, the investment cost of, for example, a change of 0.02 in $k$ is 1 million € or, equivalently, the profits from selling just $130 - 210$ units.

Before discussing the results on entry cost, a word on the way they are computed may be useful. I compute the EDV of introducing a product when the firm has no other product (thus this is firm entry) and when the firm has up to five products. In the latter case, previous products are always in different segments than the one the firm is currently entering (thus this is segment entry). I do this for each segment. For example, I compute the EDV of entering segment 1 (the same for all other 7 segments) when the firm has no previous product in segment one, and when it has one product in another segment. In this latter case, there are 7 alternative situations (the previous product being in each of the 7 remaining segments). Finally, each of the alternative situations of entry described above is computed under different industry structures, i.e., for alternative numbers of rival products in the segment of entry. For example, I compute the EDV of firm one introducing its second product in segment 1 when its first product is in segment 6 and the number of rival products in segment 1 (the segment of entry) is 5. I do this for all the combinations of: 1) segments; 2) the number of previous products in different segments (up to five); 3) the number of rival products in the segment of entry. It is easy to see that the number of alternative starting states becomes large as we allow for diversity: with 8 segments, allowing for five previous products at most and considering only 4 alternative numbers of competing products, we have $3840$ different initial states $(8 \times (1+7+21+35+35+21+4))$. The initial states described above are generated randomly. In the segment of entry and in the starting period, all firms are forced to have only one product (they may introduce new products from period two onwards). In all other segments, there is no constraint. For the entering firm, and in the case it is allowed to have 2 or more previous products, these are forced to be in different segments. This is to reduce the number of possible
alternative combinations.

Once the alternative initial states are devised, the usual forward simulation procedure is used to compute the EDV of the product that has been introduced. The EDV’s are normalized such that they have variance 1. The empirical probability of entry is computed using the policy function. The variable accounting for the number of previous models is also easily obtained. All of this allows the estimation of (14).

The distribution of entry costs has a mean of 2,439 million euros. In terms of units of product, this is roughly equivalent to 304,000 to 487,000 units. Although this may look large, it has to be taken into account that it corresponds to the cost of entry of a firm for the first time. Once the firm is established and operating in a given segment, the introduction of a second model in another segment is substantially cheaper: the cost of entering a second product in a different segment is equivalent to 271,000 – 433,000 units, 12.5% less. Entry costs remain low for the range of products between 2 and 4 and tend to rise again with 5 products. These estimates seem reasonable compared to the 10,000 units per model sold on average every year and the 50,000 units per year for the most popular models of different firms.

So far, I have not computed standard errors for the dynamic parameters. I plan to do it using non-parametric bootstrap which is robust to sampling error introduced in the first stage of estimation or induced by the simplification of the state space.

The results show that there exist economies of scope in commercialization and that these economies tend to disappear as the profile of a firm’s product goes large (Figure 1). Once a firm has entered the market, it has incentives to expand its range of products. However, when the firm has products in 4 different segments, starting to cover a fifth segment does not turn out to be so cheap. It is easy to see the implications in the automobile industry: we can see firms covering a wide range of products, but not the full range of products because, as the results suggest, it is too costly. Citroen may have a good profile of products in the low and medium-quality segments but producing in all segments would imply it is also producing high-quality cars, and it may not be prepared
for that. On the other side, Mercedes Benz can be good in luxury and sports cars but very bad in less expensive ones.

This suggests that some advantages can be obtained, among others, in the process of commercialization and distribution, and not only at the productive plant level (whose analysis is beyond the scope of this paper). It also provides an explanation for the dramatic increase in the number of models for sale in the Spanish market during the 1990’s.

6 Concluding Remarks

This paper presents a structural dynamic model of entry and exit for the Spanish car industry that allows the computation of entry costs in different scenarios. In particular, it permits the comparison between the cost of firm entry, understood as the cost of introducing the first product, and the cost of introducing a second and further models. This difference gives a measure of the scope economies in commercialization, and a quantification of the advantages of being an incumbent when a firm is about to introduce new products. The estimation strategy is based on the methodology proposed by Bajari, Benkard, and Levin (2007). The results show that entry costs are moderate and that there is a substantial reduction in the cost of introducing a second product with respect to the introduction of the first product. The advantage extends to the third, fourth, and fifth product and seems to be exhausted when the firm wants to introduce a sixth one. This gives support to the idea of firms having an optimal number of products and can also explain product proliferation in the automobile industry.

There are some issues that call for future work. Firstly, a more flexible approach to demand estimation may help to obtain better estimates of price elasticities, in line with Berry, Levinsohn, and Pakes (1995). Secondly, the paper shows results for entry costs in different segments, but the same framework can be used to compute the costs of introducing the second, third, etc. product in the same segment. This would provide a measure of the advantages of incumbent firms in product replacement by opposition
to newcomers. In third place, standard errors for the estimated parameters are needed. The most suitable technique seems to be the bootstrap, even if it makes the problem more computationally burdensome. Finally, the model should be extended to account for a previous development stage where the products are technically devised before it is decided whether they will be introduced or not.
References


Appendix: Pricing Equations and Profit Functions in Nested Logit

Consider a multiproduct firm $i$ facing nested logit demand and competing in prices. Its objective function is:

$$\pi_i = \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j M$$

where $M$ is market size and $g$ is segment from the total number of segments $G$. Also: $S_{j/g}$ is the share of product $j$ in group $g$, $S_{i/g} = \sum_{j \in G_g} S_j$, $S_{j/g}$ is the share of firm $i$ in segment $g$, and $S_g = \sum_{g \in G} \sum_{k \in G_g} S_k$ is the share of group $g$, such that $S_j = S_g * S_{j/g}$.

The FOC for the maximization problem of a multiproduct firm (several products in several segments) under Nested Logit demand is:

$$(p_j - c_j) S_j = \frac{1 - \sigma_g}{\alpha_g} S_j + \sigma_g S_{j/g} \sum_{j \in G_g} (p_j - c_j) S_j$$

$$+ (1 - \sigma_g) S_j \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j, \forall j, \forall g \quad (16)$$

Divide by $S_j$ and rearrange $S_{j/g}$:

$$(p_j - c_j) = \frac{1 - \sigma_g}{\alpha_g} + \sigma_g \sum_{j \in G_g} (p_j - c_j) S_{j/g} + (1 - \sigma_g) \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j, \forall j, \forall g$$

The three summands on the right hand side are equal for all products within the same segment, but different across segments. Therefore, we can take $(p_j - c_j)$ out of within-segment summations:

$$(p_j - c_j) = \frac{1 - \sigma_g}{\alpha_g} + \sigma_g (p_j - c_j) S_{i/g} + (1 - \sigma_g) \sum_{g \in G} (p_j - c_j) \sum_{j \in G_g} S_j, \forall j, \forall g \quad (17)$$
Now go back to (16), multiply by \( M \) and sum over \( g \) and \( j \):

\[
\sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j M = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} + M \sum_{g \in G} \sigma_g (p_j - c_j) S_{ig} S_{i/g} \\
+ \left( \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j M \right) \sum_{g \in G} (1 - \sigma_g) S_{ig}
\]

\[
\pi_i^{\text{var}} = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} + M \sum_{g \in G} \sigma_g (p_j - c_j) S_{ig} S_{i/g} + \pi_i^{\text{var}} \sum_{g \in G} (1 - \sigma_g) S_{ig} \tag{18}
\]

Take (16) and sum over \( j \):

\[
\sum_{j \in G_g} (p_j - c_j) S_j = \frac{1 - \sigma_g}{\alpha_g} S_{ig} + \sigma_g (p_j - c_j) S_{ig} S_{i/g} \\
+ \left[ (1 - \sigma_g) \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j \right] S_{ig}, \forall g
\]

\[
\sum_{j \in G_g} (p_j - c_j) S_j = \frac{1 - \sigma_g}{1 - \sigma_g S_{i/g}} \frac{1 - \sigma_g}{\alpha_g} S_{ig} \\
+ \frac{1}{1 - \sigma_g S_{i/g}} \left[ (1 - \sigma_g) \sum_{g \in G} \sum_{j \in G_g} (p_j - c_j) S_j \right] S_{ig}, \forall g \tag{19}
\]

Substitute back in (18):

\[
\pi_i^{\text{var}} = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} + \sum_{g \in G} \sigma_g S_{i/g} \left[ \frac{1 - \sigma_g}{\alpha_g} S_{ig} + \frac{1 - \sigma_g}{1 - \sigma_g S_{i/g}} M + \frac{(1 - \sigma_g) \pi_i^{\text{var}}(S_{ig})}{1 - \sigma_g S_{i/g}} \right] \\
+ \pi_i^{\text{var}} \sum_{g \in G} (1 - \sigma_g) S_{ig}
\]
\[ \pi_i^{\text{var}} = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} + \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} \frac{(1 - \sigma_g) S_{ig}}{1 - \sigma_g S_{ig}} M + \pi_i^{\text{var}} \sum_{g \in G} (1 - \sigma_g) S_{ig} \]

\[ \left[ 1 - \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} \right] \pi_i^{\text{var}} = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} \]

\[ = M \sum_{g \in G} \frac{(1 - \sigma_g) S_{ig} - \sigma_g S_{ig} (1 - \sigma_g) S_{ig}}{1 - \sigma_g S_{ig}} \]

\[ \left\{ 1 - \sum_{g \in G} \frac{(1 - \sigma_g) S_{ig}}{1 - \sigma_g S_{ig}} \right\} \pi_i^{\text{var}} = M \sum_{g \in G} \frac{1 - \sigma_g}{\alpha_g} S_{ig} \]

Then \( \pi_i^{\text{var}} = f(\sigma, \alpha, S, S_{j/g}, M) \). Rearrange (19) using the definition of \( \pi_i^{\text{var}} \):

\[ \sum_{j \in G_g} (p_j - c_j) S_{j} M = \frac{1 - \sigma_g}{\alpha_g} S_{ig} M + \frac{(1 - \sigma_g) S_{ig}}{1 - \sigma_g S_{ig}} \pi^{\text{var}}_i, \forall g \]

\[ \pi_{ig}^{\text{var}} = \frac{1 - \sigma_g}{\alpha_g} S_{ig} M + \frac{(1 - \sigma_g) S_{ig}}{1 - \sigma_g S_{ig}} \pi^{\text{var}}_i, \forall g \]

Now go back to (16):
\[(p_j - c_j) S_j M = \frac{1 - \sigma_g}{\alpha_g} S_j M + \pi_{ig} \sigma_g S_{j/g} + \pi_i (1 - \sigma_g) S_j, \ \forall j, \ \forall g\]

Then:

\[\pi_j = \frac{1 - \sigma_g}{\alpha_g} S_j M + \pi_{ig} \sigma_g S_{j/g} + \pi_i (1 - \sigma_g) S_j, \ \forall j, \ \forall g\]

where \(\pi_{ig}\) and \(\pi_i\) are the functions of market shares computed above.
### Tables

**Table 1**

<table>
<thead>
<tr>
<th>Charac.:</th>
<th>Description</th>
<th>% of variation (year)</th>
<th>% of variation (month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCKg</td>
<td>Cubic centimeters by kilo</td>
<td>50.0%</td>
<td>3.8%</td>
</tr>
<tr>
<td>CarSize</td>
<td>Length times width (m²)</td>
<td>36.9%</td>
<td>2.8%</td>
</tr>
<tr>
<td>KmL</td>
<td>Kilometers driven by litre</td>
<td>47.7%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Maxsp</td>
<td>Maximum speed in Km/H</td>
<td>43.7%</td>
<td>3.1%</td>
</tr>
<tr>
<td>AC</td>
<td>Air conditioning</td>
<td>27.3%</td>
<td>2.3%</td>
</tr>
<tr>
<td>ABS</td>
<td>ABS</td>
<td>25.6%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>59.9%</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>% of monthly variation</th>
<th>CCKG</th>
<th>CarSize</th>
<th>KmL</th>
<th>MaxSp</th>
<th>AC</th>
<th>ABS</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small-Mini</td>
<td>2.2</td>
<td>2.0</td>
<td>2.7</td>
<td>2.0</td>
<td>1.9</td>
<td>1.7</td>
<td>3.2</td>
</tr>
<tr>
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<td>3.7</td>
<td>3.0</td>
<td>3.5</td>
<td>3.1</td>
<td>2.0</td>
<td>1.6</td>
<td>5.2</td>
</tr>
<tr>
<td>Compact</td>
<td>3.7</td>
<td>2.8</td>
<td>3.7</td>
<td>3.3</td>
<td>2.4</td>
<td>2.1</td>
<td>5.0</td>
</tr>
<tr>
<td>Intermediate</td>
<td>3.7</td>
<td>3.0</td>
<td>3.9</td>
<td>3.4</td>
<td>2.7</td>
<td>2.8</td>
<td>5.1</td>
</tr>
<tr>
<td>High Internm.</td>
<td>3.8</td>
<td>2.8</td>
<td>3.3</td>
<td>3.0</td>
<td>2.5</td>
<td>2.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Luxury</td>
<td>3.7</td>
<td>2.6</td>
<td>3.4</td>
<td>3.0</td>
<td>1.6</td>
<td>1.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Sport</td>
<td>3.9</td>
<td>2.6</td>
<td>3.2</td>
<td>2.9</td>
<td>1.9</td>
<td>2.2</td>
<td>4.5</td>
</tr>
<tr>
<td>Minivan</td>
<td>4.9</td>
<td>3.5</td>
<td>3.9</td>
<td>3.4</td>
<td>3.9</td>
<td>3.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Overall</td>
<td>3.8</td>
<td>2.8</td>
<td>3.5</td>
<td>3.1</td>
<td>2.3</td>
<td>2.1</td>
<td>4.8</td>
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</table>
### Table 3

<table>
<thead>
<tr>
<th>Category</th>
<th>Entry (%)</th>
<th>Exit (%)</th>
<th>Av. #models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monthly</td>
<td>Yearly</td>
<td>Monthly</td>
</tr>
<tr>
<td>Small-Mini</td>
<td>1.2</td>
<td>13.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Small</td>
<td>1.0</td>
<td>11.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Compact</td>
<td>1.1</td>
<td>12.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Intermediate</td>
<td>1.2</td>
<td>13.3</td>
<td>1.3</td>
</tr>
<tr>
<td>High Intermediate</td>
<td>0.9</td>
<td>10.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Luxury</td>
<td>0.8</td>
<td>8.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Sport</td>
<td>1.2</td>
<td>13.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Minivan</td>
<td>2.3</td>
<td>25.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Overall</td>
<td>1.1</td>
<td>12.3</td>
<td>0.6</td>
</tr>
</tbody>
</table>

### Table 4

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>CCKG</th>
<th>CarSize</th>
<th>Kml</th>
<th>MaxSp</th>
<th>AC (%)</th>
<th>ABS (%)</th>
<th>Power</th>
<th>Price (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small-Mini</td>
<td>1.355</td>
<td>5.321</td>
<td>20.9</td>
<td>143.6</td>
<td>0.5</td>
<td>0.0</td>
<td>48.8</td>
<td>6,301</td>
</tr>
<tr>
<td>Small</td>
<td>1.480</td>
<td>5.904</td>
<td>19.9</td>
<td>158.0</td>
<td>5.5</td>
<td>0.4</td>
<td>64.2</td>
<td>7,377</td>
</tr>
<tr>
<td>Compact</td>
<td>1.526</td>
<td>6.962</td>
<td>18.0</td>
<td>181.7</td>
<td>19.6</td>
<td>15.5</td>
<td>97.9</td>
<td>11,491</td>
</tr>
<tr>
<td>Intermediate</td>
<td>1.600</td>
<td>7.515</td>
<td>17.0</td>
<td>187.8</td>
<td>42.8</td>
<td>25.9</td>
<td>109.3</td>
<td>13,894</td>
</tr>
<tr>
<td>High Intermediate</td>
<td>1.538</td>
<td>7.664</td>
<td>16.0</td>
<td>199.0</td>
<td>56.5</td>
<td>49.3</td>
<td>124.6</td>
<td>16,877</td>
</tr>
<tr>
<td>Luxury</td>
<td>1.711</td>
<td>8.497</td>
<td>14.1</td>
<td>213.6</td>
<td>86.3</td>
<td>75.5</td>
<td>165.9</td>
<td>27,272</td>
</tr>
<tr>
<td>Sport</td>
<td>1.768</td>
<td>7.592</td>
<td>14.9</td>
<td>217.6</td>
<td>88.4</td>
<td>78.0</td>
<td>170.2</td>
<td>27,519</td>
</tr>
<tr>
<td>Minivan</td>
<td>1.428</td>
<td>7.931</td>
<td>13.7</td>
<td>175.7</td>
<td>66.4</td>
<td>39.8</td>
<td>123.7</td>
<td>19,772</td>
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<tr>
<td>Overall</td>
<td>1.573</td>
<td>7.367</td>
<td>16.5</td>
<td>189.9</td>
<td>48.8</td>
<td>39.4</td>
<td>119.2</td>
<td>17,114</td>
</tr>
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</table>
Table 5: Demand estimation

<table>
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<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real price coefficients:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small-Mini</td>
<td>-0.053</td>
<td>0.057</td>
</tr>
<tr>
<td>Small</td>
<td>-0.051**</td>
<td>0.026</td>
</tr>
<tr>
<td>Compact</td>
<td>-0.136***</td>
<td>0.015</td>
</tr>
<tr>
<td>Intermediate</td>
<td>-0.124***</td>
<td>0.021</td>
</tr>
<tr>
<td>High Intermediate</td>
<td>-0.094***</td>
<td>0.012</td>
</tr>
<tr>
<td>Luxury</td>
<td>-0.064***</td>
<td>0.007</td>
</tr>
<tr>
<td>Sport</td>
<td>-0.155***</td>
<td>0.011</td>
</tr>
<tr>
<td>Minivan</td>
<td>-0.067***</td>
<td>0.016</td>
</tr>
<tr>
<td><strong>Intra-Group correlation:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small-Mini</td>
<td>0.777***</td>
<td>0.025</td>
</tr>
<tr>
<td>Small</td>
<td>0.792***</td>
<td>0.034</td>
</tr>
<tr>
<td>Compact</td>
<td>0.739***</td>
<td>0.025</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0.742***</td>
<td>0.022</td>
</tr>
<tr>
<td>High Intermediate</td>
<td>0.367***</td>
<td>0.035</td>
</tr>
<tr>
<td>Luxury</td>
<td>0.948***</td>
<td>0.028</td>
</tr>
<tr>
<td>Sport</td>
<td>0.707***</td>
<td>0.033</td>
</tr>
<tr>
<td>Minivan</td>
<td>0.099***</td>
<td>0.026</td>
</tr>
<tr>
<td><strong>Characteristics:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car Size</td>
<td>0.201***</td>
<td>0.022</td>
</tr>
<tr>
<td>HP</td>
<td>0.009***</td>
<td>0.0005</td>
</tr>
<tr>
<td>KmL</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>Air Conditioning</td>
<td>0.042***</td>
<td>0.016</td>
</tr>
<tr>
<td>ABS</td>
<td>0.178***</td>
<td>0.016</td>
</tr>
<tr>
<td><strong>Controls:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tariffs</td>
<td>0.049***</td>
<td>0.002</td>
</tr>
<tr>
<td>Constant</td>
<td>-9.879***</td>
<td>0.219</td>
</tr>
</tbody>
</table>

(*, **, ***: significant at 10%, 5%, 1%)
Table 6: Entry policy function

|                      | Pr(N | entry) | Pr(N | no entry) |
|----------------------|----------|-------------|
| Average k by seg.    | 0.175    | 0.417       |
|                      | (0.150)  | (0.015)     |
| #models other seg.   | 1.282    | 1.165       |
|                      | (0.086)  | (0.008)     |
| cutoff 1             | 0.205    | 1.143       |
|                      | (0.508)  | (0.052)     |
| cutoff 2             | 1.618    | 2.192       |
|                      | (0.477)  | (0.050)     |
| cutoff 3             | 2.960    | 3.340       |
|                      | (0.494)  | (0.051)     |
| cutoff 4             | 4.389    | 4.733       |
|                      | (0.530)  | (0.054)     |
| cutoff 5             | 5.401    | 5.699       |
|                      | (0.561)  | (0.058)     |
| cutoff 6             | 6.473    | 6.903       |
|                      | (0.591)  | (0.062)     |
| cutoff 7             | 7.710    | 7.790       |
|                      | (0.636)  | (0.065)     |
| cutoff 8             | 9.052    | 8.760       |
|                      | (0.732)  | (0.072)     |
| cutoff 9             | 9.803    | 9.241       |
|                      | (0.781)  | (0.076)     |
| cutoff 10            | 10.890   | 10.498      |
|                      | (0.858)  | (0.086)     |
| cutoff 11            | 12.256   | 10.952      |
|                      | (1.062)  | (0.090)     |
| Observed rate of entry: | 0.011 |
Table 7: Policy functions

<table>
<thead>
<tr>
<th></th>
<th>Small-Mini</th>
<th>Small</th>
<th>Compact</th>
<th>Intermediate</th>
<th>High Interm.</th>
<th>Luxury</th>
<th>Sport</th>
<th>Minivan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probit of exit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.325)</td>
<td>(0.179)</td>
<td>(0.159)</td>
<td>(0.269)</td>
<td>(0.163)</td>
<td>(0.218)</td>
<td>(0.212)</td>
<td>(0.356)</td>
</tr>
<tr>
<td># of firm models in seg</td>
<td>0.562</td>
<td>0.303</td>
<td>0.205</td>
<td>0.027</td>
<td>0.046</td>
<td>-0.254</td>
<td>-0.035</td>
<td>0.161</td>
</tr>
<tr>
<td></td>
<td>(0.387)</td>
<td>(0.157)</td>
<td>(0.077)</td>
<td>(0.122)</td>
<td>(0.139)</td>
<td>(0.122)</td>
<td>(0.286)</td>
<td>(0.236)</td>
</tr>
<tr>
<td>age of firm oldest model</td>
<td>0.002</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.010</td>
<td>-0.001</td>
<td>0.003</td>
<td>-0.0004</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.0018)</td>
<td>(0.005)</td>
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<tr>
<td>dev. of k wrt its seg mean</td>
<td>0.341</td>
<td>-1.330</td>
<td>-0.635</td>
<td>-0.656</td>
<td>-0.198</td>
<td>0.033</td>
<td>-0.119</td>
<td>-0.212</td>
</tr>
<tr>
<td></td>
<td>(1.827)</td>
<td>(0.726)</td>
<td>(0.372)</td>
<td>(0.386)</td>
<td>(0.388)</td>
<td>(0.274)</td>
<td>(0.234)</td>
<td>(0.443)</td>
</tr>
</tbody>
</table>

|                      |            |       |         |              |               |        |       |         |
| Probit for investment|            |       |         |              |               |        |       |         |
|                      | (3.889)    | (3.436)| (1.314) | (2.341)      | (1.055)       | (1.600)| (1.480)| (2.635) |
| k                    | -1.291     | 3.941 | 1.360   | 2.288        | 1.231         | 0.328 | 0.483 | 1.651   |
|                      | (2.418)    | (1.830)| (0.539) | (0.873)      | (0.363)       | (0.459)| (0.440)| (0.902) |
| dev. of k wrt its seg mean | 1.300      | -4.659| -1.183  | -2.531       | -1.511        | -0.368| -0.533| -1.956  |
|                      | (2.680)    | (1.889)| (0.572) | (0.910)      | (0.400)       | (0.465)| (0.452)| (0.914) |
Table 8: Dynamic Parameters

<table>
<thead>
<tr>
<th>(Unit: millions of Euros)</th>
<th>Coefficient</th>
<th>Economy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment cost parameter</td>
<td>$-52.751$</td>
<td></td>
</tr>
<tr>
<td>Scrap value</td>
<td>$41.149$</td>
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</tr>
<tr>
<td>Mean firm entry cost</td>
<td>$2,439.805$</td>
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</tr>
<tr>
<td>Mean entry cost with:</td>
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<td></td>
</tr>
<tr>
<td>1 product</td>
<td>$2,168.118$</td>
<td>$12.5$</td>
</tr>
<tr>
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<td>$2,218.291$</td>
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<td>3 products</td>
<td>$2,240.368$</td>
<td>$8.9$</td>
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<td>4 products</td>
<td>$2,229.832$</td>
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</tr>
<tr>
<td>5 products</td>
<td>$2,330.066$</td>
<td>$4.7$</td>
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</tbody>
</table>

Figure 1: Mean entry cost by number of products.