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Consumption Habit Formation and Income Variability

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By Xiaoying Liu*

This paper develops a new way to estimate the consumption habit persistence coefficient using household panel data. In contrast to the approach of estimating an approximate Euler equation, the identification comes from comparing the coevolution of income and consumption. Using Spanish data from 1985 to 1995, we estimate a joint model of habit formation and income variability. We find habit persistence smooths the impact of permanent income shocks on consumption. With our estimated parameters, we find habit persistence reduces the variance of consumption by 40% over the life-cycle. Consumption habits offers an explanation for the “excess smoothness of consumption” puzzle.

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Árbol que nace torcido, jamás su tronco endereza (A tree that is born twisted never grows straight) — Spanish proverb.

The past three decades have seen efforts to move away from the intertemporally separable preference model. Common ways to introduce intertemporal nonseparability in life cycle consumption models include habit formation (Constantinides, 1990) and the durability of consumption (Hayashi, 1985). These mechanisms, in addition to the usual income and prices channels, can influence one’s consumption in a fundamental way. In particular, with habit formation, current utility does not only depend on current consumption, but on past consumption in a form of habit stock. If habit plays an important role, then there can be much slower adjustments in consumption following shocks to income. Because of this important feature, habits have been proposed as an explanation for several apparent “puzzles” under the standard separable utility model. Examples include the excess sensitivity or excess smoothness of aggregate consumption (Deaton, 1992), the equity premium puzzle (Abel (1990); Campbell and Cochrane (1999); Constantinides (1990)), and the causal relationship between savings and economic growth (Carroll, Overland and Weil, 2000). Notwithstanding the great benefits of introducing habits into the utility function as well as the straightforward intuition it provides, the identification and estimation of the habit persistence has remained a very challenging problem. The objective of this paper is to develop a model that allows us to identify habit persistence in consumption, and then to empirically investigate the quantitative importance using Spanish Household Consumption and Income Panel (ECPF) data. By examining the coevolution of consumption and income inequality, we find that households’ quarterly consumption displays a strong degree of habit persistence, with this persistence smoothing the impact of income shocks on consumption.

Most previous attempts to identify habit persistence estimate Euler equation, i.e. the first order condition of the intertemporal maximization problem, exploring cross-sectional

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1 The importance of habits in consumption has long been recognised. See Constantinides (1990) for a review on history of habit research.
variations in consumption. Examples of this approach with additive habits\(^2\) include Alessie and Lusardi (1997) and Dynan (2000). These models are then estimated using an GMM procedure with variables at lagged periods as instruments. These papers rely upon the use of a linearized Euler equation based on a Taylor series expansion, whose validity is the source of much disagreement (see the discussion between Carroll (2001) and Attanasio and Low (2004)). Carroll (2001), as well as Ludvigson and Paxson (2001), argues in estimating linearized Euler equation, instruments (usually interest rates at lagged periods) are very likely to be correlated with the omitted higher-order approximation error. However, this argument has been refuted by Attanasio and Low (2004) who argue that it is the lack of cross-sectional variability in interest rates and lack of information on individual discount rates that contributes to the failure of sensible estimates of system parameters such as intertemporal elasticity of substitution. Instead, they show that using "large-\(T\)" asymptotics instead of cross-sectional variation can produce unbiased estimates in simulation exercises unless discount rate is exceedingly and implausibly high, and hence log-linear approximation shouldn’t be taken to account for the unreasonable estimates. This argument is joined by Yogo (2004) who particularly show weak instruments can explain the puzzle in estimating log-linearized Euler equation. Besides GMM-Euler equation approach, there exist other methods in literature to examine consumption habit. For example, Fuhrer (2000) considers a consumer model with multiplicative habits. He linearizes the Euler equation around steady state and estimates the model using maximum likelihood estimation. In contrast to Dynan (2000), which doesn’t find evidence of habit persistence using household food consumption data from the Panel Study on Income Dynamics (PSID), Fuhrer (2000) finds habit formation is economically important using US quarterly aggregate consumption and income data. An alternative empirical approach is provided in Browning and Collado (2007). Here, the difference in additivity between different consumption goods is recognised, and Engel curve relationships are then estimated to test whether last period’s budget share for a given good is correlated with this period’s share. Meghir and Weber (1996) develop a structural model which is the first to disentangle dynamics in preferences (i.e. habit formation) from dynamics due to borrowing constraints – both features introduce dependence on variables in the information set of the consumer which invalidates the standard Euler equation. Although they reject intertemporal nonseparable preference over three types of nondurable goods using US Consumer Expenditure Survey, Carrasco et al. (2005) do find habit persistence by applying the same model to ECPF data and taking account of household fixed effects\(^3\). More recently, Crawford (2010) implements a nonparametric revealed preference test for habits using the same data set as we use in this study, and finds evidence for habits in consumption. However, constrained by the methodologies used, most of these papers can’t produce point estimation of structural parameters such as habit persistence coefficients.

This paper provides a new method to identify and estimate habits in consumption. It differs from much of the existing literature in that it examines the co-evolution of both income and consumption, and the identification comes from the comparison of variances and covariance between consumption and income. When the interest rate and the rate of time preference are similar, we demonstrate that intrinsic consumption (defined as the part of consumption that directly contributes to current utility), rather than consumption itself as under the permanent income hypothesis, should be a martingale. Moreover, the cross-sectional variance of intrinsic consumption should only reflect the variability of unexpected permanent income shocks. Faced with both income and consumption panel data at household level, the model allows us to identify the coefficient of habit persistence, together with the stochastic permanent and transitory income processes. By applying the model

\(^2\)In additive habit model, habit stock enters utility function in a additive form to the current consumption; while in multiplicative habit model, habit stock enters utility function in a multiplicative form to the current consumption.

\(^3\)They can’t reject the absence of habit in consumption if they apply the exactly same Meghir and Weber (1996) method without household fixed effects to ECPF data.
to the extensive Spanish Household Consumption and Income Panel data (covering the period 1985–1995), we find evidence for the presence of consumption habits and show that it plays an important role in smoothing out income shocks, especially the persistent permanent shocks, for Spanish Households. This finding is consistent with other papers that have tested habit persistence using the same data set (see Carrasco et al., 2005; Browning and Collado, 2007; Crawford, 2010), and more importantly, it identifies the quantitative scale of importance.

This paper not only contributes to the literature on consumption habit estimation, but also advances the research on the relationship between changes in income inequality and changes in consumption inequality. Blundell and Preston (1998) document the empirical divergence between consumption inequality and income inequality in the UK over 1980s, and relate this growth in income inequality primarily to growth in transitory income shocks, with permanent income shocks fully transmitted to changes in consumption. In a more recent paper Blundell, Pistaferri and Preston (2008) provide a more detailed examination of the transmission of income shocks to consumption and develop a model which incorporates partial insurance (in terms of the degree of transmission from income shocks to consumption inequality). They provide the evidence to support partial insurance and emphasize the role of public transfers as well as heterogeneity in the ability of agents to self-insure. However, as they themselves say, they only provide “the ‘structured facts’ rather than a specific structural interpretation” about this insurance mechanism. This paper proceeds in a similar vein, and argues that besides precautionary saving, consumption habit persistence is also one of the mechanisms that can be thought of as self-insuring consumption against income shocks. This has important implication on welfare policy. As more and more researchers switch to consumption inequality from income inequality for welfare measure (for example, Cutler and Katz (1992), Deaton and Paxson (1994), Krueger and Perri (2006)), our findings suggest consumption inequality alone may not be an ample indicator if much smaller consumption inequality is attributed to habit persistence and combination of income and consumption inequality may be a more desirable indicator. Moreover, this paper also contributes to the research in explaining the “excess smoothness” of consumption to permanent income shocks (Campbell and Deaton (1989)) from the specification of utility function, following Fuhrer (2000). Although Quah (1990) shows that permanent income hypothesis can still predict excess smoothness of consumption when agents distinguish permanent and transitory movements in labour income, this paper shows that consumption can be further smoothed in the case of habit formation.

An understanding of consumers’ habit persistence also helps us to shed light on a number of important policy questions. In particular, it helps us to understand why some countries have bigger saving rates than others. It is widely believed that being frugal is a cultural tradition in East Asian countries, and this argument has been used to explain the high saving rate in those countries despite prolonged periods of consistently high economic growth. However, no empirical studies have so far provided convincing evidence for this hypothesis. Carroll, Overland and Weil (2000) provided theoretical foundations for this argument and present simulation exercises. Carroll’s argument starts from the observation that fast growing countries in East Asia such as Japan, Korea, Hong Kong and Singapore all displayed a similar trend during their growth period – that is, the increases in growth preceded the rise in saving rate. The main mechanism, as he argues, is that habit formation prevents consumption from growing as fast as income in fast growing economies, so that savings accumulate. These arguments would be strengthened significantly if backed by robust empirical evidence. While looking at a somewhat different country from those considered by Carroll, one contribution of this paper is to document evidence of habits in consumption.

This chapter proceeds as follows. In Section I, the baseline theoretical model is set up with an intertemporally nonseparable quadratic utility function, and the identification strategy for recovering the coefficient of habit persistence is discussed. Section II describes the data
set that we use and presents the empirical results. Section III then extends the model to allow for a more general CRRA utility specification, and presents estimation results based upon this alternative specification. Section IV shows some simulation exercises based on estimated parameters from the second model, before we conclude in section V.

I. Model

The model in this paper builds on that developed in Blundell and Preston (1998), which under the hypothesis of the life cycle behaviour, uses the dynamics of consumption and income variances to identify the variances of permanent and transitory income shocks. In this section we extend their model to include habits (i.e. intertemporal non-separable utility), to examine how income shocks can be decomposed if consumption shows habit persistence, and more importantly, how we may identify the habit persistence of consumption.

We begin with consumers’ intertemporal optimization problem by assuming a time-nonseparable utility function, with both current consumption and habit stock (defined below) as arguments. Each consumer chooses consumption at each period $c_{it}$ to maximize the life-time utility function:

$$\max_{\{c_{it}\}} \mathbb{E}_t \left[ \sum_{s=t}^T \left( \frac{1}{1+\delta} \right)^{s-t} u(c_{is}, h_{is}) \right],$$

where $\delta$ is discount rate and $h_{it}$ is household $i$’s habit stock at time $t$. The habit stock evolves according to:

$$h_{it} = h_{it-1} + \lambda(c_{it-1} - h_{it-1}) = (1-\lambda)h_{it-1} + \lambda c_{it-1}. \quad \text{(1)}$$

For simplicity, we assume that $\lambda = 1$, so that the habit stock is simply equal to last period’s consumption: $h_{it} = c_{it-1}$.

Consumers maximize the above life-time utility function subject to the life time budget constraint:

$$\sum_{k=0}^{T-t-1} (1+r)^{-k} c_{it+k} = A_{it} + \sum_{k=0}^{R-t} (1+r)^{-k} y_{it+k},$$

where $R$ is the retirement age, $A_{it}$ are assets holding at time period $t$, and $r$ is the risk-free interest rate. Note that the only source of uncertainty is from stochastic labour income process $y_{it+k}, k \geq 0$.

As demonstrated by Carroll (2000), the Euler equation for the simple case with $\lambda = 1$ can be written as:

$$\mathbb{E}_t \left[ u_{c,t} + \frac{1}{1+\delta} u_{h,t+1} \right] = \frac{1+r}{1+\delta} \mathbb{E}_t \left[ u_{c,t+1} + \frac{1}{1+\delta} u_{h,t+2} \right], \quad \text{(1)}$$

where $u_{c,t}$ and $u_{h,t}$ are the marginal utilities with respect to $c_t$ and $h_t$ respectively, and whose arguments have been suppressed for notational simplicity. The above Euler equation indicates the expected marginal utility by saving 1 unit of consumption today (LHS) should equal to the expected discounted marginal utility of $(1+r)$ units of consumption tomorrow (RHS). The second term on each side of the Euler equation is the marginal effect of current consumption on future utility that is carried over to the next period by habit persistence.

A. Consumption Process

We begin our analysis by assuming that habits enter the utility function additively, with one period lagged consumption as the habit stock ($h_{t} = c_{t-1}$) unchanged, so that $u(c_t, h_t) = u(c_t - \gamma c_{t-1})$, and where $\gamma$ is habit persistence coefficient. The parameter $\gamma$ measures the strength of habit formation: when larger, the consumer receives less utility from a given amount of expenditure (Dynan, 2000). If there were positive habit persistence,

\footnote{Using quarterly macro economic data, Fuhrer (2000) estimate a value of $\lambda$ that is close to 1.}
should be in the range of $[0,1]$. If $\gamma = 0$, then the model collapses to the standard intertemporally separable model, while $\gamma = 1$ means that only consumption growth matters for current utility. Alternatively, in the other case of intertemporal nonseparability – durability – $\gamma$ could be less than zero, as past consumption brings satisfaction to the current time. To simplify the subsequent notation we denote $c_{it}^* \equiv c_{it} - \gamma c_{it-1}$.

For simplicity, we assume that the within period utility function is quadratic in $c_{it}^*$, so that marginal utility is simply linear in this term. Thus, the Euler equation in the presence of habits becomes:

\[
\mathbb{E}_t \left[ c_{it}^* - \frac{\gamma}{1 + \delta} c_{it+1}^* \right] = \frac{1 + r}{1 + \delta} \mathbb{E}_t \left[ c_{it+1}^* - \frac{\gamma}{1 + \delta} c_{it+2}^* \right]
\]

This Euler equation schedules the evolution of expected consumption from period $t$ on. While as income provides the only uncertainty in this model, actual consumption will respond to the new income shocks correspondingly. In order to track how consumption evolves under uncertainty, we need to connect consumption to income innovations. This is done through the budget constraint, which by simple construction, may be written as:

\[
\sum_{k=0}^{T-t-1} (1 + r)^{-k} c_{t+k}^* = -\gamma c_{t-1} + (1 + r)^{-(T-t)} c_{T-1}
\]

By simple rearrangement we also have:

\[
\sum_{k=0}^{T-t-1} (1 + r)^{-k} c_{t+k+1}^* = -(1 + r) c_t + (1 + r)^{-(T-t-1)} c_T
\]

Subtracting equation (4) (multiplied by $\frac{\gamma}{1 + \delta}$) from equation (3) gives us:

\[
\sum_{k=0}^{T-t-1} (1 + r)^{-k} \left( c_{t+k}^* - \frac{\gamma}{1 + \delta} c_{t+k+1}^* \right)
= \left( 1 - \gamma \frac{1 + r}{1 + \delta} \right) \left( 1 - \frac{\gamma}{1 + r} \right) \left[ A_t + \sum_{k=0}^{R-t} (1 + r)^{-k} y_{t+k} \right]
- \gamma c_{t-1} + (1 + r)^{-(T-t)} c_{T-1} + \gamma \frac{1 + r}{1 + \delta} c_t - \gamma \frac{1 + r}{1 + \delta} (1 + r)^{-(T-t-1)} c_T.
\]

Since the stock of assets evolving according to $A_t = (1 + r)(A_{t-1} + y_{t-1} - c_{t-1})$ we may

\[^5\text{To simplify the notation, we have omitted the household specific subscript } i.\]
substitute this into equation (5) so that:

\[
\sum_{k=0}^{T-t-1} (1 + r)^{-k} \left( c_{t+k}^* - \frac{\gamma}{1 + \delta} c_{t+k+1}^* \right) = \\
\left( 1 - \frac{1 + r}{1 + \delta} \right) \left( 1 - \frac{\gamma}{1 + r} \right) \left[ (1 + r) A_{t-1} - (1 + r)c_{t-1} + (1 + r) \sum_{k=0}^{R-t+1} (1 + r)^{-k} y_{t+k-1} \right] \\
- \gamma c_{t-1} + \gamma (1 + r)^{-(T-t)} c_{T-1} + \gamma \frac{(1 + r)}{1 + \delta} c_t - \frac{\gamma}{1 + \delta} (1 + r)^{-(T-t-1)} c_T.
\]

By applying the period \( t \) expectations operator to both sides of equation (6), and subtracting the lagged version of equation (5) (with period \( t - 1 \) expectations operator applied) we may cancel out the \( A_{t-1} \) term. Moreover, once we impose equality of the discount and interest rate, we obtain an expression for the evolution of the differenced term \( c_{t+k}^* - \frac{\gamma}{1 + \delta} c_{t+k+1}^* \):

\[
\rho_t \left[ E_t \left( c_t^* - \frac{\gamma}{1 + \delta} c_{t+1}^* \right) - E_{t-1} \left( c_{t-1}^* - \frac{\gamma}{1 + \delta} c_t^* \right) \right] = (1 - \gamma)(1 + r - \gamma) \eta_t
\]

where \( \rho_t = 1 - (1 + r)^{-(T-t)} \) is an annuitization factor, and \( \eta_t = r \times (1 + r)^{-1} \sum_{k=0}^{R-t}(1 + r)^{-k}(E_t - E_{t-1})y_{t+k} \) is an income innovation term, or the annualized value of the expected difference of future income flows between the current and previous period.

The intuition for equation (7) is as follows: by taking into account the negative effect of current consumption on future utility, via habit persistence, the expected consumption change should only reflect a part of innovation to income at the current period. In other words, in the presence of habits, consumption should respond to income innovations more slowly. For a sufficiently small interest rate \( r \), the multiplier on \( \eta_t \) is between 0 and 1. Stronger habit persistence (larger \( \gamma \)), reduces the multiplier, and therefore lowers the response of consumption to income shocks. Thus, habits introduce a form of inertia that prevent consumption from growing as fast when there is a positive income shock. Similarly, when there is a negative income shock it prevents consumption from decreasing as quickly.

**B. From Income Change to Consumption Change**

As in Blundell and Preston (1998) and Meghir and Pistaferri (2004), we assume a standard stochastic process determining the evolution of incomes \( y_{it} \):

\[
y_{it} = y_{it}^P + u_{it} \\
y_{it}^P = y_{it-1}^P + v_{it}
\]

where \( y_{it}^P \) is permanent income, and with period \( t \) changes in permanent income given by innovation term \( v_{it} \). Similarly, transitory income shocks are given by \( u_{it} \) so that overall income evolves according to \( \Delta y_{it} = \Delta u_{it} + v_{it} \). For now, we assume that permanent and transitory income shocks are independent from each other at all leads and lags, i.e. \( u_{it} \perp v_{is} \), \( \forall t, s \), although in the much of the later analysis we will assume that transitory shocks follow a MA(1) process. For now, we do not consider measurement error in either consumption or income.

Given the above income process, the income innovation term \( \eta_{it} \) can be decomposed into transitory and permanent innovations to income:

\[
\eta_{it} = \rho_t v_{it} + \frac{r}{1 + r} u_{it}.
\]
Substitute (9) into (7) and remove the expectation operator from the LHS of equation (7), we can rewrite this expression as:

\[
\rho_t \left[ (c_t^* - \frac{\gamma}{1+r} c_{t+1}^*) - (c_{t-1}^* - \frac{\gamma}{1+r} c_t^*) \right] = (1 - \gamma)(1 + r - \gamma) \eta_{it} + \rho_t \xi_{t+1} - \rho_t \xi_t
\]

where \( \xi_{t+1} = -\frac{\gamma}{1+r} (c_{t+1} - E_t c_{t+1}) \) is the expectation error of consumption at period \( t+1 \), which includes information that only arrives at \( t+1 \), and \( \xi_t = -\frac{\gamma}{1+r} (c_t - E_{t-1} c_t) \) is similarly defined. Theoretically, the expectation error at time \( t \) will be a function of the income innovations \( u_{it+1} \) and \( v_{it+1} \) as it includes consumption shock in response to newly arrived information on income at \( t+1 \). For now, we treat these consumption expectation errors as unknown variables. Equation (10) implies that the growth in intrinsic consumption at time \( t \), after removing the part which is carried over to the future period by habit persistence, can be decomposed into two parts: the corresponding annuitized income innovation at time \( t \), and the difference between the expectation error of one-period forward consumption in the current period and that of the previous period.

For notational simplicity we define composite consumption \( Q_t = c_t^* - \frac{\gamma}{1+r} c_{t+1}^* \) and \( Q_{t-1} = c_{t-1}^* - \frac{\gamma}{1+r} c_t^* \). We refer to \( Q_t \) as intrinsic consumption, as it is the marginal current utility of current consumption with the effect on future utility (carried over by habit persistence) removed. So Equation (10) actually describes the martingale property of the intrinsic consumption. If we divide by the annuitization factor and take variances on both sides of equation (10), we can obtain the following equation:

\[
\Delta \text{var}(Q_{it}) = (1 - \gamma)^2(1 + r - \gamma)^2 \left( \text{var}(v_{it}) + \frac{r^2}{(1+r)^2 \rho_t^2} \text{var}(u_{it}) \right) + \Delta \text{var}(\xi_{it+1})
\]

For small enough interest rate \( r \) and sufficient long time horizon \( T \), we obtain \( \rho_t \to 1 \) which allows us to derive an approximate version of the above equation:

\[
\Delta \text{var}(Q_{it}) \simeq (1 - \gamma)^2(1 + r - \gamma)^2 \text{var}(v_{it}) + \Delta \text{var}(\xi_{it+1}).
\]

To better understand the equation (11), we are comparing it with its counterpart in Blundell and Preston (1998):

\[
\Delta \text{var}(c_{it}) \simeq \text{var}(v_{it})
\]

As Blundell and Preston (1998) assume a life cycle model with a quadratic intertemporal separable utility function, equation (12) says the growth in consumption variance for each cohort should reflect the variance in permanent income shocks only at one to one ratio, with the effect of transitory income shocks being smoothed away. Similarly but differently, the moment condition Equation (11) in the case of habit persistence (\( \gamma > 0 \)) states that, after being adjusted for the change in the variance of consumers’ expectation error, the change in the variance of intrinsic consumption which contributes to current utility responds only to the permanent income innovation variance at a discounted rate. That is to say, consumers’ habits act to passively smooth out income shocks. The permanent income hypothesis tells us how individuals use consumption to smooth out the impact of transitory income shocks on utility. While passively, habits provide some form of self-insurance against more persistent shocks, so that consumption responds even more gradually to income changes. As we now discuss, understanding this co-evolution of income and consumption is essential for empirically identifying the presence of habits.
C. Identification

Equation (11) suggests a method for estimating the coefficient of habit persistence $\gamma$. As in Blundell and Preston (1998), we can derive a set of moment conditions which will then allow us to separately identify the variance of permanent and transitory income innovations, as well as $\gamma$. We obtain:

$$\Delta \text{var} \left( c_t - \frac{\gamma}{1 + r} c^*_{t+1} \right) \simeq \left[ (1 - \gamma)^2 (1 + r - \gamma)^2 \right] \text{var}(v_t) + \Delta \text{var}(\xi_{t+1})$$

$$\Delta \text{var}(y_t) - \frac{1}{(1 - \gamma)^2 (1 + r - \gamma)^2} \Delta \text{var} \left( c_t - \frac{\gamma}{1 + r} c^*_{t+1} \right) \simeq \Delta \text{var}(u_t) - \frac{1}{(1 - \gamma)^2 (1 + r - \gamma)^2} \Delta \text{var}(\xi_{t+1})$$

$$\frac{1}{(1 - \gamma)(1 + r - \gamma)} \Delta \text{cov} \left( c_t - \frac{\gamma}{1 + r} c^*_{t+1}, y_t \right) \simeq \text{var}(v_t).$$

With $\gamma = 0$, these three equations collapse to the three moment conditions used in Blundell and Preston (1998):

$$\Delta \text{var}_k(c_t) \simeq \text{var}_k(v_t)$$

$$\Delta \text{var}_k(y_t) - \Delta \text{var}_k(c_t) \simeq \Delta \text{var}_k(u_t)$$

$$\Delta \text{var}_k(c_t, y_t) \simeq \text{var}_k(v_t)$$

Equation (13) is the first moment condition we derive in the last section. Equation (14) is analogous to (17) in which permanent shocks that is identified from (13) can be removed from change in variance of income to obtain change in transitory income shocks. The covariance between composite consumption and income also provides information on permanent income shocks (equation (15)). So far, for each period there are three parameters to be identified: $\text{var}(v_t)$, $\Delta \text{var}(u_t)$, $\Delta \text{var}(\xi_{t+1})$, plus a time-invariant $\gamma$, while we have only three moment conditions for each period as listed above. To obtain full identification of the whole model we proceed to use the income covariance conditions as presented in Meghir and Pistaferri (2004), which are sufficient to identify the variances of permanent income shocks:

$$\mathbb{E} \left[ g_{it} \left( \sum_{j=-(1+q)}^{1+q} g_{it+j} \right) \right] = \mathbb{E}(v^2_{it}) = \text{var}(v_{it}),$$

where $g_{it} = \Delta y_{it} = \Delta u_{it} + v_{it}$ is stochastic income growth. $q$ denotes the degree of moving average process of transitory shock $u_{it}$. As we assume $u_{it}$ is independently distributed for the moment, $q = 0$.

The model moment conditions (equations (13)–(15) and (19)) use information on the relationship between income and consumption. While consumption habits can partly smooth
out permanent shock of income, the covariance of the income series itself can reveal information about permanent income shocks. Furthermore, equation (15) and (19) allow us to identify habit persistence \( \gamma \) and variances of permanent shocks together, which when used with the other two conditions ((13) and (14)) allow us to identify the growth in variances of transitory shocks and prediction errors.

In the above model, we assumed that both \( u_{it} \) and \( v_{it} \) are independent from each other at all leads and lags. However, in reality, we may expect some form of correlation over time. Indeed, should transitory shocks follow MA(1) process, we set \( q = 1 \) in equation (19) and equation (15) is replaced by the following modified condition:

\[
(20) \quad \Delta \text{cov} \left( c_{t+1}^\ast - \frac{\gamma}{1 + r} c_{t+2}^\ast, y_t \right) = (1 - \gamma)(1 + r - \gamma) \text{var}(v_t)
\]

It is important to note that, different from Blundell and Preston (1998) using repeated cross-sectional data in estimation, identification of this model requires at least 4 waves of panel data with both consumption and income in the presence of MA(0) transitory shocks as variance and covariance are not linear functions. And for the model with MA(1) transitory shocks, at least 6 waves then becomes necessary. The minimum number of waves required for identification in general is \( 4 + 2q \).

II. Data and initial results

We estimate the model using a detailed Spanish household panel data set (Encuesta Continua de Presupuestos Familiares, ECPF), 1985–1995. The ECPF has been used in several papers to test consumption intertemporal non-separability owing to the long panel that it covers (Carrasco et al., 2005; Browning and Collado, 2007; Crawford, 2010). Each household is surveyed for at most eight consecutive quarters, with one-eighth of the sample being rotated each wave. The sample size for each wave is around 3200 households. It provides detailed information on expenditure, income and demographics at the household level (see Browning and Collado, 2001, for a detailed description of the data set), with food and other non-durable goods consumption taken at weekly base, and durable goods consumption each quarter.

A. Sample selection and variable definition

We only consider families reporting complete information for at least 4 waves, and of couples with or without dependent children (34253 observations excluded from the original data). We also drop households if their household heads have changed over time (23 observations), those with at least one permanent guest (658 observations), or having household size changed by more than one person (2771 observations). We further drop those with age inconsistencies (1931 observations), or spouse’s employment status changed (11,667 observations) to avoid spousal labour supply insurance which may confound our results. Finally, we dropped household observations with zero reported food consumption (204 observations). The final sample consists of 7,717 households (45,983 observations), with heads’ age between 25 and 60 in the sample. For the detailed composition of sample, please see Table B1 in Appendix 1.A.

We define three categories of consumption and estimate our model with these alternative consumption definitions. Non-durable consumption and services is defined as the total household expenditure on food at home (including alcohol and tobacco), all kinds of services (including utility bills) and all the non-durable goods. We add semi-durable consumption (semi-durable household goods and clothes etc.) to non-durable consumption and services to form the second consumption category. The last category includes nearly all consumption items (including car purchasing and education payment for example), by adding durable goods consumption on top of the second category. Following Pijoan-Mas and Sánchez-Marcos (2010), we also try four measurements of income in estimation: net labour earnings
(wages + 2/3 self employment earnings); net labour earnings plus private transfers; net income (all types of income, including wages, self-employed earning, capital income, and all private transfer); total net disposable income (net income plus public transfers such as pensions and unemployment benefit). We trim off the bottom and top 1% observations in consumption (the third definition) and income (the fourth definition) distribution in our estimation. Table C1 lists the mean and the variances of income and consumption under each definition.

To understand the movement of income and consumption shocks, the predictable part of real income and consumption (conditional on demographic characteristics) must first be removed. Following Blundell, Pistaferri and Preston (2008), we run a simple linear regression:

$$\log Y_{it} = Z_{it}' \varphi_t + P_{it} + u_{it},$$

where $P_{it}$ is $i$’s permanent income at time $t$, and $u_{it}$ is transitory income shock as before. $Z_{it}$ is a set of household’s (and head’s) characteristics observable and known by consumers at time $t$. These include household demographics, head’s education, head’s job type, and seasonal effects. We allow these effects to interact with time or cohort. We also allow for general time trends$^6$ and cohort effects$^7$ in the intercept term. Both consumption and income are deflated by a Stone Price Index (which is the weighted price index with the household specific weights determined by each period’s expenditure shares. Linear predictions are taken following these regressions; the distribution of residuals is presented in Figure B1 in the Appendix A.

B. The autocovariance of consumption and income

Figure 1 plots the raw estimates of unexplained consumption and income variance, which are obtained after removing the predictable parts of income and consumption. The upper left panel is calculated using the entire estimation sample. It shows that income variance falls rapidly in the mid 1980s and then fluctuates around a steady level thereafter before rising again slowly in the last few quarters. Furthermore, the variance of consumption is much less variable and is around 0.15 throughout the sample period. Not surprisingly, the variance of total net disposable income is much lower than that of net labour income. Public and private transfers apparently work to lower the degree of inequality across population. The other three panels consider different (heads’) education groups and show the trend of within-group variances over the same sample period. Households where the head has a university degree group shows a lower-than-average variance in incomes, while the lowest education group displays a much higher inequality, which drops by a large amount after taking account of transfers. These patterns are consistent with the conclusion of Pijoan-Mas and Sánchez-Marcos (2010) that “public transfers have played a crucial role in smoothing out the inequality arising in the labour market” during that period.

For the auto-covariance between consumption and income, we calculate several moments of the income and consumption processes, as shown in Table $(D1)$. Estimates are reported for each wave. The table shows that there is a decrease in the variance of income growth in the second half of 1980s, while it increases slightly in the first part of 1990s. The first-order auto-covariances of income are of the expected sign ($-$) and show a slight increase over time in absolute value. Second- and higher- order auto-covariances are informative about the presence of serial correlation in the transitory income components (Blundell, Pistaferri and Preston, 2008). Second order auto-covariances are small economically and

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$^6$We have experimented with two alternative types of time trend: an annual trend (one dummy for each year) and wave trend (one dummy for each wave). The main the empirical results are qualitatively similar for both types, and we present only those obtained using the annual trend.

$^7$We adopt the same cohort definition as in Albarran, Carrasco and Martinez-Granado (2009): Cohort 1 – born between 1920 and 1934; Cohort 2 – born between 1935 and 1944; Cohort 3 – born between 1945 and 1954; Cohort 4 – born between 1955 and 1964.
only statistically significant from zero in some time periods of the whole sample\footnote{However, the third-order auto-covariances of income are not small. In practice this data shows higher order auto-covariances are not necessarily small in size because of the quarterly data structure. To see how it works, look at $\text{cov}({\Delta y}_{it},{\Delta y}_{it+3}) = \text{cov}({y}_{it} - {y}_{it-1} ,{y}_{it+3} - {y}_{it+2})$, where $y_{it+3}$ is the income in the same quarter exactly one year after when $y_{it-1}$ is surveyed, which can be highly correlated, especially to seasonal job takers. This pattern exists even after we condition our incomes and consumptions on seasonal dummies. It’s also important to notice the estimated standard errors don’t take account of the fact that income data are the estimated residuals. Estimating the standard errors by bootstrapping may mitigate this problem. However, this is a data issue that seems hard to be completely resolved within the framework of this paper.}. In the following analysis we assume the transitory income process as MA(1).

\section*{C. Results}

\subsection*{Habit persistence}

The above model has been estimated by minimum distance estimation. In estimation we allow the habit persistence coefficient $\gamma$ to be negative. This therefore allows for both habit persistence ($\gamma > 0$) and durability of consumption ($\gamma < 0$) across time. Besides positive constraints on permanent income shock variances, no other linear nor non-linear constraints are imposed \footnote{We do not constrain $c_{it} - \gamma c_{it-1} \geq 0$ during estimation. However, ex-post we can easily verify whether this condition is violated in any $t$ given our estimate of $\gamma$. We do not find any such violations.}. To obtain greater efficiency, we weight the moment conditions using the diagonal of their inverse variance matrix \footnote{That is, the off diagonal elements are neglected. This treatment is to avoid the small sample bias of optimal minimum distance estimator (Altonji and Segal (1996)).}. We also assume MA(1) structure of transitory income shocks. The results of this diagonally optimally weighted minimum distance estimation (DWMD) are presented in Table (1).

The table shows that the estimated habit persistence coefficients for non-durable and semi-durable goods consumption is positive and both economically and statistically significant (around 0.4–0.5). This means nearly half of the past consumption serves as baseline to which only the additional part of current consumption can be brought to current satisfaction. The habit persistence coefficient decreases when we move from non-durable goods consumption to durable goods consumption, which reflects the fact that the identified coefficient captures both habit persistence and durability. When the degree of habit persistence is stronger than durability (for example in the case of non-durable goods consumption), the
estimated coefficient tends to be positive and large. However, when the negative durability
effect dominates the positive habit persistence (in the case of durable goods consumption)
it is possible to obtain small or even be negative coefficients. Consistent with this intuition,
when estimating the model with durable goods we obtain an estimate of $\gamma$ that is signifi-
cantly smaller than we obtained with non-durable goods (0.41–0.45), although economically
this difference is not especially large. We also note that we find that the estimated habit
persistence coefficient is much smaller when using total net disposable income, which is
much less volatile than the other incomes without public transfers. This is consistent with
our identification idea: given the same consumption variance, larger income variance implys
larger habit persistence as habit works to smooth consumption in response to unexpected
income shocks.

### Income shocks

In Figure 2(a) we plot the variances of permanent income shocks (the solid line) and the
change in variances of transitory shocks (the dashed line) over time to see how the income
process in Spain has changed. These variances are decomposed from total net disposable
incomes across the whole sample. The figure shows that there is much larger variation
in transitory income shocks than in permanent income shocks, with a noticeable decline
in these transitory income shock variances in the most sample periods (as the growth in
this variable is negative in most periods), especially late 1980s. This decline suggests that
it was a fall in variance of transitory income shocks that was responsible for decrease in
overall income inequality seen in late 1980s. The variances of permanent income shocks
are on the contrary quite stable and relatively small, at about 0.02 on average per quarter.

### III. Incorporating precautionary saving

#### A. Habit persistence under CRRA utility

The model presented in Section I was based upon a life-cycle consumption model with
a within-period quadratic utility function. Such a specification does not incorporate any
precautionary savings motive. Precautionary saving provides another mechanism through
which consumers may only partially adjust their consumption following a change in their
permanent income. Ignoring it may confound our estimate of habit persistence coefficient.
The main distinction between habit and precautionary saving lies in the fact that habit is
formed before consumption (for example, it may be inherited from family) and therefore
consumers are more unconscious of its effect; while precautionary saving is a conscious
decision made by consumers, according to the predicted future income risk. This section
extends the baseline model to allow for this property. Although we are not trying to disen-
tangle the effects on consumption of habit persistence and precautionary saving, the model
presented in the following does allow us to identify the coefficient of habit persistence in a
framework in which precautionary saving is possible.

To incorporate precautionary saving, a utility function with convex marginal utility curve
is necessary. We follow many papers that have incorporated habits, by using a constant
relative risk aversion (CRRA) utility function. The setting is otherwise the same as in the quadratic utility case, and we continue to use last periods consumption as the habit stock. But in contrast to the quadratic case the stock of habits now enter the utility in multiplicative form, as in Carroll (2000). We therefore have the following within period utility function:

$$u(c_{it}, h_{it}) = u \left( \frac{c_{it}}{h_{it}^{\gamma}} \right) = \frac{1}{1 - \sigma} \left( \frac{c_{it}}{h_{it}^{\gamma}} \right)^{1 - \sigma},$$

with $\gamma$ again used to denote the coefficient of habit persistence (although its scale is not comparable to the quadratic case). This coefficient should always be in the interval $[0, 1]$. A value $\gamma > 1$ would imply that steady-state utility is falling in consumption (Fuhrer, 2000). In contrast to the additive habit model, it is not the growth of consumption, but rather the relative ratio between consumption and the stock of habits that now matters for current utility. If $\gamma = 0$, this is simply the model without habits; while if $\gamma = 1$, only the ratio between current consumption and past consumption matters to the current utility; if it is between these values then both the consumption level and the ratio contribute to current utility, with $\gamma$ determining the relative importance of each.

As before, we still assume expected utility framework, i.e. households choose consumption to maximize the sum of expected utility over time. If we substitute expressions for marginal utility, $u_{c,t} = (\frac{c_{it}}{h_{it}^{\gamma}})^{-\sigma} \frac{1}{h_{it}^{\gamma}}$ and $u_{h,t} = -\gamma \frac{c_{it}}{h_{it}^{\gamma}} u_{c,t}$ into the generic Euler equation expression (Equation (1)) then we obtain:

$$E_t \left[ \frac{c_{it}^{\sigma}}{h_{it}^{\gamma \sigma + \gamma}} - \frac{\gamma}{1 + \delta} \frac{c_{it+1}^{\sigma}}{h_{it+1}^{\gamma \sigma + \gamma + 1}} \right] = \frac{1 + r}{1 + \delta} E_t \left[ \frac{c_{it+1}^{\sigma}}{h_{it+1}^{\gamma \sigma + \gamma}} - \frac{\gamma}{1 + \delta} \frac{c_{it+2}^{\sigma}}{h_{it+2}^{\gamma \sigma + \gamma + 1}} \right].$$

This Euler equation again shows how consumption in the future should evolve over time. Theoretically it could be estimated by the usual Euler-equation GMM estimation.
method, with lagged variables as instruments. However, it is well known that estimating this non-linear Euler equation in the presence of measurement error with GMM generates inconsistent estimates (Amemiya (1985)). Therefore, log-linearization of the Euler equation is still necessary. Specifically we can show in the following how to remove the effect of measurement errors by log-linearization under standard assumptions of multiplicative measurement error. Following Carroll (2000), we write the above Euler equation as a function of $c_{it}/h_{it}$ and linearize it around the steady state ratio $c_0/h_0$. As demonstrated in it, in the steady state of the model without stochastic shocks, the ratio between consumption and habit stock is equal to a constant: $c_0/h_0 = \chi$. Moreover, in the special case that $\lambda = 1$, as we examine here, the value of $\chi$ equals $1^{11}$.

B. Linearization of Euler equation under CRRA utility

In a situation without uncertainty, the system reaches its steady state, in which the ratio between consumption and habit stock is a constant. As proved by Carroll (2000), this constant equals 1 if interest rate is the same as discount rate. We linearize the Euler equation around the log of this steady state ratio $c_0/h_0$. After removing the expectation terms, we obtain the following linearized Euler equation (see the Appendix 1.B for a proof):

$$\left(-\gamma - \frac{\sigma}{\sigma-1}(\gamma - 1 - r)\right) \Delta \ln c_{i,t+1} + \gamma(1+r)\Delta \ln c_{i,t} + \gamma(\Delta \ln c_{i,t+2} - \gamma \Delta \ln c_{i,t+1})$$

$$\simeq \left(-\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r)\right) \epsilon_{i,t+1} + \gamma(\xi_{i,t+2} - \epsilon_{i,t+1} - \gamma \epsilon_{i,t+1}) + O(\epsilon^2_{i,t+1}) + O(E \epsilon^2_{i,t+2})$$

(22)

$$= \left(-\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r) - \gamma^2\right) \epsilon_{i,t+1} + \gamma \xi_{i,t+2} + O(\epsilon^2_{i,t+1}) + O(E \epsilon^2_{i,t+2})$$

where $\epsilon_{i,t+1} = \ln c_{i,t+1} - E_t \ln c_{i,t+1}$ and $\xi_{i,t+2} = \ln c_{i,t+2} - E_t \ln c_{i,t+2}$ are the respective (one period and two periods) log consumption innovation. As in Blundell, Low and Preston (2008), $O(x)$ denotes a term with the property that there exists a $K < \infty$ such that

$$|O(x)| < K |x|$$

Based on the loglinearized budget constraint first used by Campbell (1993), Blundell, Low and Preston (2008) has provided a way to connect this log consumption innovation with log income innovation. They show that $\epsilon_t = \pi_t(v_t + \alpha t u_t + \Omega_t)$, where $\pi_t$ is (roughly) the share of expected future labour income in current human and financial wealth, an idiosyncratic insurance coefficient against permanent income shocks, by for example precautionary saving; $v_t$ and $u_t$ are idiosyncratic permanent and transitory log income shocks at time $t$; $\Omega_t$ is the common income trend and doesn’t vary across cohorts/population, and $\alpha_t$ can be seen as an annuitisation factor for income that is close to zero under a long time horizon. Similarly, it is easy to demonstrate that $\xi_{i,t+2} = \pi_{i,t+2}(v_{i,t+2} + \alpha_{i,t+2}u_{i,t+2} + (1 - \alpha_{i,t+1})v_{i,t+1} + \alpha_{i,t+1}\alpha_{i,t+2}u_{i,t+1} + \Omega_{i,t+1} + \Omega_{i,t+2})$. As $\alpha_t$ is close to 0 for long $T$, the evolution of consumption can then be written as the following function of the income innovation:

$$\left(-\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r)\right) \Delta \ln c_{i,t+1} + \gamma(1+r)\Delta \ln c_{i,t} + \gamma(\Delta \ln c_{i,t+2} - \gamma \Delta \ln c_{i,t+1})$$

$$\simeq \left(-\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r)\right) \pi_t(v_{i,t+1} + \Omega_{i,t+1}) + \gamma \pi_{i,t+2}(v_{i,t+2} + \Omega_{i,t+2}) + O_p(E_t|\nu_{i,t+1}^R|)$$

where $O_p(x)$ denotes a term with the property that for each $\kappa > 0$ there exists a $K < \infty$ such that

$^{11}$Please refer to Carroll (2000) for derivation
For the purposes of estimation, we develop a similar set of moment conditions in the presence of MA(1) transitory shocks. We assume $\pi_{it}$ is a constant across individuals and over time series. Defining $X_{it} \equiv (-\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r)) \ln c_{it} + \gamma(1+r) \ln c_{it-1} + \gamma(\ln c_{it+1} - \gamma \ln c_{it})$, these may be written as:

\begin{align*}
(23) \quad \Delta \text{var}(X_{it+1}) &= \left[ -\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r) - \gamma^2 + \gamma \right]^2 \pi^2 \text{var}(v_{it+1}) \\
&\quad + \gamma^2 \pi^2 \Delta \text{var}(v_{it+2}) + O(\mathbb{E}_t \|v_{it+1}\|^3) \\
(24) \quad \Delta \text{var}(\ln y_{it}) &= \text{var}(v_{it}) + \Delta \text{var}(u_{it}) \\
(25) \quad \Delta \text{cov}(X_{it+2}, \ln y_{it+1}) &= \left[ -\gamma + \frac{\sigma}{\sigma-1}(\gamma - 1 - r) - \gamma^2 + \gamma \right] \pi \text{var}(v_{it+1}) \\
&\quad + O(\mathbb{E}_t \|v_{it+1}\|^3) \\
(26) \quad \mathbb{E}_t \left[ g_{it} \left( \sum_{j=-2}^{2} g_{it+j} \right) \right] &= \text{var}(v_{it})
\end{align*}

These moment conditions are analogous to Equations (13)–(15) and (19). Up to a term which is $O(\mathbb{E}_t \|v_{it+1}\|^3)$, the growth of the intrinsic consumption variance responds to variances of unexpected permanent income shocks (Eq. (23))\textsuperscript{12}. Although in reality we estimate the parameters all in one step, identification however can be considered as consisting of two steps: (i) decomposing variances of income shocks into permanent and transitory parts using (24) and (26); (ii) substituting the permanent income shock variances into (23) and (25) to back up the habit persistence coefficient $\gamma$ and the coefficient of risk aversion $\sigma$. As 6 waves are required to identify one permanent income shock variance according to Equation (26), the variances of permanent income shocks are only identified starting from the fourth period till the last third period, i.e., $\text{var}(v_{i4}), ..., \text{var}(v_{iT-2})$, and likewise for the change in variance of transitory income shocks.

Measurement errors

So far we have not discussed the possible measurement errors in consumption and income. In the case of multiplicative measurement errors (i.e., $\tilde{q}_t = q_t \varepsilon_t^q$, where $q_t$ denotes the true value of consumption or income, $\tilde{q}_t$ for the observed value, and $\varepsilon_t^q$ as measurement error on $q_t$), if we make the following assumption:

**Assumption** Measurement errors in consumption and income are stationary and independent of each other and other variables in the model, and also serially uncorrelated.

then the estimation of the parameters in interest ($\gamma, \sigma$) is robust to the presence of measurement errors. We can see this point clearly from the first moment condition Eq. (23), as we take the growth in variances of intrinsic consumption which is a linear function of log consumption, as long as measurement errors in consumption are stationary and independent from all consumptions and other parameters, the variance of measurement error in consumption would disappear by taking the first difference. Estimation of permanent income shocks is also robust to the presence of measurement error in income. In the last moment condition Eq. (26), if we use the observed income data with measurement errors we can get LHS as:

\textsuperscript{12}Blundell, Low and Preston (2008) shows by Monte-Carlo experiment approximation omitting $O(\mathbb{E}_t \|v_{it+1}\|^3)$ doesn’t bias estimates.
\[
\mathbb{E}[\Delta \ln \tilde{y}_{it} \cdot (\Delta \ln \tilde{y}_{it+2} + \Delta \ln \tilde{y}_{it+1} + \Delta \ln \tilde{y}_{it} + \Delta \ln \tilde{y}_{it-1} + \Delta \ln \tilde{y}_{it-2})] = \mathbb{E}[(\Delta \ln y_{it} + \Delta \varepsilon_{it}) (\ln y_{it+2} + \varepsilon_{it+2} - \ln y_{it-3} - \varepsilon_{it-3})] = \mathbb{E}[(\Delta \ln y_{it}) (\ln y_{it+2} - \ln y_{it-3})]
\]

i.e., the result is the same as using the true value. Again, here we just use the assumption that measurement error is multiplicative and serially uncorrelated. It’s also easy to see that this robustness maintains in the other two moment conditions as long as the above assumption holds. This feature of the model makes it superior to the Euler equation approach as estimating nonlinear Euler equation (especially in the habit model) using GMM yields inconsistent estimates in the presence of measurement error (Amemiya (1985)).

As consumption and income data are in log form, this model actually identifies growth in variances of proportionate transitory and permanent shocks, and habit persistence as the relative importance of consumption change to current consumption level in the current satisfaction of consumption. Again when \(\gamma = 0\), the model collapses to the basic model (with \(c_t\) replaced by \(\ln c_t\)). In this case, the coefficient of risk aversion \(\sigma\) is not identified as it would be cancelled out in log-linearization. The income transmission term \(\omega^2 = \left[\frac{\sigma}{\sigma - 1}(\gamma - 1 - r) - \gamma^2\right]^2\) contains information of both risk aversion (and hence precautionary saving) as well as habit persistence. If it is less than 1 in value, then the influence of current permanent income shocks on consumption is dulled by a combination of these effects.

All \((2T - 8)\) parameters \((\text{var}(\varepsilon_4)\ldots\text{var}(\varepsilon_{T-2}), \Delta\text{var}(u_4)\ldots\Delta\text{var}(u_{T-2}), \gamma, \sigma)\) are estimated jointly using DWMD with the inverse of the variance of the empirical moments as weights. \(\pi\) is set to be fixed at 0.85. The point estimate results of \(\gamma\) and \(\sigma\) are presented in Table 2. The habit persistence coefficient \(\gamma\) is around 0.75–0.85 for non-durable or semi-durable consumption, while it is generally lower for the broader consumption category. This is in line with our earlier empirical findings, and is consistent with the hypothesis that the durability of consumption goods counteracts the influence of habit persistence, resulting in a smaller estimated \(\gamma\). As the income and consumption are in log forms, the habit persistence in this model shows the relative importance of the consumption growth ratio in utility function. Higher this coefficient, more important the consumption growth relative to the absolute size of consumption in current satisfaction, and therefore smoother consumption change. Although the estimated coefficients of risk aversion are smaller than the usual range used in the literature (around 1.5), they are not significantly different from 1, which is in line with some studies (Chetty (2006)). The size of the coefficient of risk aversion is in general smaller for consumption measures which include durable goods than those without, consistent with the finding that households display a high degree of risk aversion with respect to consumption of basic goods compared to luxury goods (AIT-SAHALIA, PARKER and YOGO (2004)).

We can calculate the income transmission coefficient \(\omega\) using the estimated values of \(\gamma\) and \(\sigma\). The results are presented in the bottom panel in Table 2, and show that this coefficient is well below 1, or very close to 0 in value. This suggests that the combination of both habit persistence and precautionary saving leads to intrinsic consumption adjusting mainly to the growth in variance of permanent income shocks in the next period (with coefficient \(\gamma^2 \approx 0.6\) for nondurable consumption), which makes consumption adjustment a much smoother process. The identified income shocks are depicted in panel (b) in Figure 2. The variance of permanent income shocks display a slight increasing trend, while the variance of transitory shocks show a continual decline over the same period.

Table 3 presents the estimation results with a smaller sample by removing the seasonal workers and employers (mainly in the agricultural sector) from the original sample to mitigate the seasonal effect in income process. Both estimated coefficients of habit persistence and relative risk aversion are slightly bigger than the ones in Table 2, and the general
Table 2—Estimated Habit Persistence (CRRA preference)

<table>
<thead>
<tr>
<th>Income</th>
<th>consumption</th>
<th>net labour earnings</th>
<th>net labour earnings + private transfer</th>
<th>net income</th>
<th>total net disposable income</th>
</tr>
</thead>
<tbody>
<tr>
<td>habit</td>
<td>non-durable</td>
<td>0.761 (0.395)</td>
<td>0.742 (0.338)</td>
<td>0.752 (0.377)</td>
<td>0.647 (0.398)</td>
</tr>
<tr>
<td>persistence</td>
<td>non+semi</td>
<td>0.835 (0.328)</td>
<td>0.792 (0.306)</td>
<td>0.832 (0.233)</td>
<td>0.853 (0.290)</td>
</tr>
<tr>
<td>γ</td>
<td>all</td>
<td>0.708 (0.320)</td>
<td>0.497 (0.281)</td>
<td>0.656 (0.292)</td>
<td>0.801 (0.347)</td>
</tr>
<tr>
<td>coef risk</td>
<td>non-durable</td>
<td>0.696 (0.417)</td>
<td>0.665 (0.381)</td>
<td>0.683 (0.395)</td>
<td>0.541 (0.408)</td>
</tr>
<tr>
<td>aversion</td>
<td>non+semi</td>
<td>0.812 (0.415)</td>
<td>0.743 (0.419)</td>
<td>0.801 (0.404)</td>
<td>0.856 (0.424)</td>
</tr>
<tr>
<td>σ</td>
<td>all</td>
<td>0.656 (0.380)</td>
<td>0.349 (0.357)</td>
<td>0.561 (0.385)</td>
<td>0.770 (0.420)</td>
</tr>
<tr>
<td>income</td>
<td>non-durable</td>
<td>1.40E-003</td>
<td>3.67E-004</td>
<td>9.60E-004</td>
<td>1.08E-003</td>
</tr>
<tr>
<td>transmission</td>
<td>non+semi</td>
<td>2.10E-002</td>
<td>3.74E-003</td>
<td>1.03E-002</td>
<td>3.03E-002</td>
</tr>
<tr>
<td>ω²</td>
<td>all</td>
<td>1.28E-002</td>
<td>1.59E-003</td>
<td>2.28E-003</td>
<td>1.52E-002</td>
</tr>
<tr>
<td>Sample size</td>
<td></td>
<td></td>
<td></td>
<td>4649</td>
<td></td>
</tr>
</tbody>
</table>

Note: Bootstrapped standard errors are in parentheses.

patterns described above still maintain. Again, the estimated coefficients of relative risk aversion are smaller than, but not significantly different from 1. These results are improved version of the model estimates because of the following reasons: 1) the new sample is more homogeneous than the original one, and therefore households are more likely to face the same income shocks; 2) seasonal pattern is less likely to be present in the new sample data, especially the income data.

Table 3—Estimated Habit Persistence (CRRA preference w/o seasonal inc earners)

<table>
<thead>
<tr>
<th>Income</th>
<th>consumption</th>
<th>net labour earnings</th>
<th>net labour earnings + private transfer</th>
<th>net income</th>
<th>total net disposable income</th>
</tr>
</thead>
<tbody>
<tr>
<td>habit</td>
<td>non-durable</td>
<td>0.897 (0.396)</td>
<td>0.896 (0.373)</td>
<td>0.903 (0.407)</td>
<td>0.902 (0.452)</td>
</tr>
<tr>
<td>persistence</td>
<td>non+semi</td>
<td>0.909 (0.308)</td>
<td>0.915 (0.260)</td>
<td>0.920 (0.245)</td>
<td>0.919 (0.200)</td>
</tr>
<tr>
<td>γ</td>
<td>all</td>
<td>0.861 (0.214)</td>
<td>0.861 (0.192)</td>
<td>0.882 (0.227)</td>
<td>0.885 (0.232)</td>
</tr>
<tr>
<td>coef risk</td>
<td>non-durable</td>
<td>0.885 (0.447)</td>
<td>0.881 (0.447)</td>
<td>0.893 (0.444)</td>
<td>0.892 (0.444)</td>
</tr>
<tr>
<td>aversion</td>
<td>non+semi</td>
<td>0.909 (0.432)</td>
<td>0.916 (0.433)</td>
<td>0.922 (0.422)</td>
<td>0.922 (0.427)</td>
</tr>
<tr>
<td>σ</td>
<td>all</td>
<td>0.847 (0.425)</td>
<td>0.844 (0.419)</td>
<td>0.871 (0.421)</td>
<td>0.875 (0.419)</td>
</tr>
<tr>
<td>transmission</td>
<td>non+semi</td>
<td>1.40E-001</td>
<td>1.74E-001</td>
<td>2.14E-001</td>
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Note: Bootstrapped standard errors are in parentheses.

C. Further into the past?

In the last section, habit stock is assumed being equal to last period’s consumption, i.e. \( \lambda = 1 \). A natural question follows: what if habit has longer memory? After all, habit is not built in a day. It could be resourced back to early life, or even past generations. For this purpose, we extend the above model to the case with more two lags in habit stock equation\(^{13}\).

Habit stock evolves according to \( h_{it} = (1-\lambda)h_{it-1} + \lambda c_{it-1} \), where \( \lambda \in [0, 1] \) describing the speed of habit catching up with consumption. As \( \lambda \) approaches to 1, recent consumptions become more important in habit stock. In the extreme case with \( \lambda = 1 \), only last period’s

\(^{13}\)Theoretically, we could have extended analysis to more lags. While the empirical extension would be constrained by the data set available as long panel data would be required for long memory.
consumption is enough to represent one’s habit stock. While when \(0 < \lambda < 1\), we can repeatedly iterate substitution and get the evolution of habit as:

\[
h_{it} = (1 - \lambda)s h_{it-s} + \lambda \sum_{k=1}^{s} (1 - \lambda)^{k-1} c_{it-k}
\]

If \(\lambda \to 1\), \((1 - \lambda)^n \to 0\), for \(n > 1\). Therefore we can truncate the higher orders and approximate the habit stock as:

\[
h_{it} \approx (1 - \lambda)\lambda c_{it-2} + \lambda c_{it-1}
\]

With two lags of consumption in habit stock, we log-linearize Euler equation (21) around steady state as follows (See Appendix 1.B for proof). Keep in mind that with assumption that interest rate equal to the discount rate, the steady state ratio \(\frac{c_{t}^{s}}{h_{t}^{s}} = 1\) and \(\frac{c_{t-1}^{s}}{c_{t-1}^{s}} = 1\) still hold.

\[
E_t \left[ -\sigma \Delta \ln c_{it+1}^s + (\gamma \sigma - \gamma) \Delta \ln h_{it+1}^s - \frac{\gamma}{1+\delta} \Delta \ln c_{it+2}^s + \frac{\gamma}{1+\delta} \Delta \ln h_{it+2}^s \right] = 0
\]

where, \(\ln c_{it}^s = \ln c_t - \frac{\gamma}{1+\delta} \ln c_{it+1}^s, \ln h_{it}^s = \ln h_t - \frac{\gamma}{1+\delta} \ln h_{it+1}^s\).

If we define \(\Xi_{it+1} = -\sigma \ln c_{it+1}^s + (\gamma \sigma - \gamma) \ln h_{it+1}^s - \frac{\gamma}{1+\delta} \ln c_{it+2}^s + \frac{\gamma}{1+\delta} \ln h_{it+2}^s\). Euler equation can be written as the following martingale form:

\[
E_t \Xi_{it+1} = E_t \Xi_{it}
\]

Remove expectation, we can again write the consumption composite growth as the function of consumption innovations:

\[
\Delta \Xi_{it+1} = -\sigma (\epsilon_{it+1} - \frac{\gamma}{1+\delta} \xi_{it+2}) + \frac{\gamma}{1+\delta} \epsilon_{it+1} - (\gamma \sigma - \gamma) \frac{\gamma}{1+\delta} \epsilon_{it+1} - \frac{\gamma}{1+\delta} (\xi_{it+2} - \epsilon_{it+1}) + \frac{\gamma}{1+\delta} \epsilon_{it+1} = (-\sigma - \frac{\gamma}{1+\delta} + \frac{\gamma^2 (\sigma - 1)}{1+\delta} + \frac{2\gamma}{1+\delta}) \epsilon_{it+1} + (\gamma \sigma - \frac{1}{1+\delta}) \xi_{it+2}
\]

Call \(A = -\sigma - \frac{\gamma}{1+\delta} + \frac{\gamma^2 (\sigma - 1)}{1+\delta} + \frac{2\gamma}{1+\delta}\), \(B = \gamma \sigma - \frac{1}{1+\delta}\), the above function can be simplified as:

\[
\Delta \Xi_{it+1} = A \epsilon_{it+1} + B \xi_{it+2} = A \pi_{it+1} \epsilon_{it+1} + B \pi_{it+2} \xi_{it+2} = A \pi_{it+1} v_{it+1} + B \pi_{it+2} (v_{it+2} + v_{it+1})
\]

Substitute the consumption innovation with income innovations, we can again relate consumption change with income innovations, and therefore form the moment conditions as follows:

\[
\Delta \text{var}(\Xi_{it+1}) = (A + 2B)^2 \pi^2 \text{var}(v_{it+1}) + B^2 \pi^2 \Delta \text{var}(v_{it+2})
\]

\[
\Delta \text{var}(\ln y_{it}) = \text{var}(v_{it}) + \Delta \text{var}(u_{it})
\]

\[
\Delta \text{cov}(\Xi_{it+2}, \ln y_{it+1}) = (A + 2B) \pi \text{var}(v_{it+1})
\]

\[
\Delta \text{cov}(\Xi_{it+2}, \ln y_{it+2}) = (A + 2B) \pi \text{var}(v_{it+2})
\]

\[
E_t \left[ g_{it} \left( \sum_{j=-2}^{2} g_{it+j} \right) \right] = \text{var}(v_{it})
\]
As the moment conditions involve \( \{C_{it-2}, C_{it-1}, C_{it}, C_{it+1}, C_{it+2}, C_{it+3}\} \) for each \( 3 \leq t \leq T-3 \), 6 waves panel is again necessary and sufficient to identify \( 2T-7 \) parameters, including all the income shocks as well as \( \gamma, \sigma \) and \( \lambda \).

This model is brought to nondurable goods consumption and all income categories. The results are shown in Table 4. Estimated \( \lambda \) is not significantly different from 1, implicating a short memory of habit.

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<th>( \gamma )</th>
<th>( \sigma )</th>
<th>( \lambda )</th>
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<td>0.792 (0.072)</td>
<td>0.923 (0.053)</td>
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Note: Consumption variable is nondurable goods consumption. Bootstrapped standard errors are in parentheses.

IV. Simulation

In order to give a more intuitive demonstration of the effect of habit persistence on consumption, I have done two types of simulations. The first one examines the life cycle evolution of consumption inequality, and the second one shows whether the prediction from our model can replicate the original data set. As variances of income shocks can be identified using income information alone, we generate a series of normal distributed idiosyncratic permanent income shocks using the multi-year average of the identified variance of income shock and the assumption of mean zero of the shocks. For the first simulation, we focus on young cohorts as it is generally believed that consumption inequality increases over life time (Deaton and Paxson (1994)) and therefore by conditioning on cohorts we can exclude cohort effects from the age effect. We assume all the households start with a mean nondurable consumption (i.e. variance is zero) in the initial period(s)\(^{14}\), and simulate their consumption series in the future periods given the randomly assigned idiosyncratic permanent income shocks, under two scenarios: without habit and with habit identified in the previous section. We then plot the variances of consumption series in Figure 3. As permanent income shocks have strong persistence, it is not surprising to find that inequality in consumption increases over time given a series of persistent shocks. Moreover, although the variances of consumption increase under both scenarios, the one with habit persistence increases however much more slowly than the one without habit persistence. More specifically, given the income shocks identified from our model, the variance of nondurable consumption increases from 0 to 0.65 during the period 1987-1995, by on average 0.08 each year, or 0.02 each quarter, which is exactly the variance of quarterly permanent shock. As in CRRA model this variance is in proportional term, in other words, the variance of consumption would be increasing by 8% every year in the absence of habit persistence. However, this level is nearly halved by the presence of consumption habit, i.e. by increasing around 5% each year. That is to say, consumption habit can help smooth consumption even further compared to the one predicted by permanent income hypothesis in response to persistence income shocks, which confirms our argument that habit is an insurance mechanism for consumers to smooth their consumption.

\(^{14}\)For non-habit case, only one initial period is necessary to generate future consumption series by random walk; while multi-periods are required in the habit model.
The second simulation shows whether our model can replicate the original data trend. We again focus on youngest cohort. Since the data set is an unbalanced panel data, we use the actual data as initial values and only predict the future value under each scenario if there has been an observation in the corresponding period for the household. The permanent shocks are generated according to the actual variances identified from the previous section (instead of the multi-year average as in the first simulation). We then calculate the variances of consumption for each period and plot them along with the variance of consumption from the real data in Figure 4. It is not difficult to see that the predicted trend by the habit model match the real data more closely, with variance of consumption lower than the result predicted by the model without habit, which leads to less volatile consumption.

In summary, the above two simulations show that habit model in general predict the data much better than its intertemporal separable alternative, and reconciles the “excess smoothness” puzzle of consumption.

V. Conclusion

This paper has provided a new way to estimate the habit persistence coefficient in a life-cycle consumption model. This is achieved by examining the evolution of cross-sectional variance...
variances and covariances of consumption and income, so that we can back up the scale of habit persistence coefficient. The identification strategy comes from the fact that with habit formation, consumers not only smooth consumption, but also the rate of consumption change, and therefore it can introduce sluggish response of consumption to unexpected income shocks, especially the permanent ones. This information is correspondingly reflected in variances of consumption and income. Using an extensive Spanish household panel data set, we find evidence to support the existence of habits in consumption, with our results both economically and statistically significant. In our baseline model with quadratic utility and additive habits we obtain habit coefficients in the range 0.4–0.5, while in our extended model with CRRA utility with multiplicative habits we obtain consumption habit coefficient is about 0.75-0.85, although the scale of coefficients are not comparable across specifications. The scale of habit persistence coefficient in CRRA framework fits the size which can explain most of empirical puzzles (Deaton (1988); Carroll, Overland and Weil (2000); Constantinides (1990)). We also estimate the coefficient of relative risk aversion (RRA) in the second model, as about 0.8-0.9, and not significantly different from 1. We show in simulation exercises that habit model can reconcile the “excess smoothness” of consumption.

By combining two important strands of literature, this paper has shown how habit persistence can be identified by exploring the co-evolution of income and consumption. In doing so it avoids estimating (log) linearized approximate Euler equation whose validity hinges upon the ability to find suitable instruments (mainly lagged variables) which has enough variation and does not correlate to the approximation error. Within our framework, the presence of habits means that the consumption which contributes to the current utility is, to some extent, insulated from permanent income shocks. Therefore, while income information alone can be used to identify income shocks, its combination with consumption data can provide a way to identify habit persistence as well as other parameters. As predicted by theory, our results also suggest that consumers have stronger habit persistence when considering non-durable goods rather than durable goods.

Our analysis is not without its limitations. First, quarterly income data displays different patterns of auto-covariation from annual data due to seasonal correlation. This may invalidate the assumption of an MA(1) structure of transitory shocks and therefore the identification of permanent income shocks. This is however, a data rather than a conceptual challenge. Annual panel data (at least 6 years panel for income and 3 years panel for consumption), or quarterly consumption excluding seasonal goods in this sense would be more desirable. Second, in the current paper we assume the whole population face the same uncertainty. However, estimation distinguishing cohorts and education group should be implemented given a larger data set. Different cohorts and education groups are supposed to face income shocks at different level and persistence. Thirdly, credit constraints are not considered in the current model while they may also have the effect of delaying consumption to later periods. Meghir and Weber (1996) disaggregate intertemporal nonseparability from borrowing constraints by comparing the intertemporal substitution of consumer goods with the intratemporal one, as the latter should be immune to the borrowing constraints but still affected by intertemporal nonseparability. Although Carrasco et al. (2005) apply this method to ECPF data and do find the existence of habit persistence with consideration of borrowing constraints, this paper however can’t disentangle two effects and therefore the quantitative scale of the habit persistence coefficient.

Nonetheless, our results have important implications for both public and macro economic policy. In particular, understanding the quantitative importance of habits can help researchers better understand how households smooth consumption in different economic environments, and how they respond to redistributive policies. More broadly, the results also shed important insight into how fast growing countries accumulate saving and design the optimal fiscal policy.
REFERENCES


* Appendix
In Figure B1 we show the distribution of regression residuals of consumption and income using data from 1985Q1, 1990Q1 and 1995Q1. In Table B1 we show the composition of our sample; summary statistics are presented in Table C1. Finally, in Table D1 we show the autocovariance matrix of total net disposable income growth. The reader should refer to the main text for further details and discussion of these.

**Figure B1. Residuals of (log) Consumption and Income.**
Table B1—Sample Definition

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<td>-0.015 (0.004)</td>
</tr>
<tr>
<td>1990</td>
<td>4</td>
<td>0.072 (0.007)</td>
<td>-0.044 (0.007)</td>
<td>0.016 (0.003)</td>
<td>-0.015 (0.005)</td>
</tr>
</tbody>
</table>
Table D1—The Autocovariance Matrix of Total net Disposable Income Growth

<table>
<thead>
<tr>
<th>Year Quarter</th>
<th>var (△y&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>cov (△y&lt;sub&gt;t&lt;/sub&gt;, △y&lt;sub&gt;t-1&lt;/sub&gt;)</th>
<th>cov (△y&lt;sub&gt;t&lt;/sub&gt;, △y&lt;sub&gt;t-2&lt;/sub&gt;)</th>
<th>cov (△y&lt;sub&gt;t&lt;/sub&gt;, △y&lt;sub&gt;t-3&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991 1</td>
<td>0.062 (0.005)</td>
<td>-0.034 (0.005)</td>
<td>0.009 (0.003)</td>
<td>-0.015 (0.003)</td>
</tr>
<tr>
<td>1991 2</td>
<td>0.055 (0.004)</td>
<td>-0.026 (0.003)</td>
<td>0.008 (0.003)</td>
<td>-0.012 (0.003)</td>
</tr>
<tr>
<td>1991 3</td>
<td>0.060 (0.004)</td>
<td>-0.030 (0.004)</td>
<td>0.004 (0.003)</td>
<td>-0.014 (0.003)</td>
</tr>
<tr>
<td>1991 4</td>
<td>0.074 (0.006)</td>
<td>-0.030 (0.003)</td>
<td>0.007 (0.003)</td>
<td>-0.011 (0.003)</td>
</tr>
<tr>
<td>1992 1</td>
<td>0.073 (0.006)</td>
<td>-0.040 (0.005)</td>
<td>0.010 (0.002)</td>
<td>-0.011 (0.003)</td>
</tr>
<tr>
<td>1992 2</td>
<td>0.066 (0.006)</td>
<td>-0.033 (0.004)</td>
<td>0.016 (0.004)</td>
<td>-0.005 (0.005)</td>
</tr>
<tr>
<td>1992 3</td>
<td>0.065 (0.005)</td>
<td>-0.033 (0.004)</td>
<td>0.017 (0.004)</td>
<td>-0.023 (0.005)</td>
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<tr>
<td>1992 4</td>
<td>0.070 (0.006)</td>
<td>-0.034 (0.004)</td>
<td>0.015 (0.003)</td>
<td>-0.020 (0.006)</td>
</tr>
<tr>
<td>1993 1</td>
<td>0.069 (0.006)</td>
<td>-0.034 (0.004)</td>
<td>0.014 (0.003)</td>
<td>-0.018 (0.004)</td>
</tr>
<tr>
<td>1993 2</td>
<td>0.081 (0.008)</td>
<td>-0.039 (0.005)</td>
<td>0.012 (0.003)</td>
<td>-0.021 (0.004)</td>
</tr>
<tr>
<td>1993 3</td>
<td>0.082 (0.008)</td>
<td>-0.041 (0.005)</td>
<td>0.014 (0.004)</td>
<td>-0.013 (0.004)</td>
</tr>
<tr>
<td>1993 4</td>
<td>0.089 (0.008)</td>
<td>-0.047 (0.007)</td>
<td>0.015 (0.005)</td>
<td>-0.013 (0.004)</td>
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<tr>
<td>1994 1</td>
<td>0.063 (0.004)</td>
<td>-0.039 (0.004)</td>
<td>0.009 (0.004)</td>
<td>-0.013 (0.006)</td>
</tr>
<tr>
<td>1994 2</td>
<td>0.070 (0.006)</td>
<td>-0.037 (0.005)</td>
<td>0.023 (0.005)</td>
<td>-0.018 (0.005)</td>
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<tr>
<td>1994 3</td>
<td>0.073 (0.005)</td>
<td>-0.032 (0.003)</td>
<td>0.013 (0.003)</td>
<td>-0.028 (0.006)</td>
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<tr>
<td>1994 4</td>
<td>0.071 (0.006)</td>
<td>-0.038 (0.004)</td>
<td>0.008 (0.003)</td>
<td>-0.010 (0.004)</td>
</tr>
<tr>
<td>1995 1</td>
<td>0.081 (0.006)</td>
<td>-0.035 (0.004)</td>
<td>0.010 (0.004)</td>
<td>-0.012 (0.004)</td>
</tr>
<tr>
<td>1995 2</td>
<td>0.069 (0.005)</td>
<td>-0.041 (0.004)</td>
<td>0.015 (0.004)</td>
<td>-0.017 (0.005)</td>
</tr>
<tr>
<td>1995 3</td>
<td>0.071 (0.006)</td>
<td>-0.031 (0.004)</td>
<td>0.009 (0.003)</td>
<td>-0.019 (0.007)</td>
</tr>
<tr>
<td>1995 4</td>
<td>0.078 (0.005)</td>
<td>-0.040 (0.004)</td>
<td>0.013 (0.003)</td>
<td>-0.017 (0.004)</td>
</tr>
</tbody>
</table>
Mathematical Appendix

**Proof of equation (11) in quadratic utility model**

The LHS of equation 10 after taking variance:

\[
\var(Q_t - Q_{t-1}) = \var(Q_t) + \var(Q_{t-1}) - 2\cov(Q_{t-1}, Q_t) \\
= \var(Q_t) + \var(Q_{t-1}) - 2\var\left(\frac{(1 - \gamma)(1 + r - \gamma)}{\rho_t} \eta_{it} + \xi_{t+1} - \xi_t, Q_{t-1}\right) \\
= \Delta\var(Q_t) - 2\cov\left(\frac{(1 - \gamma)(1 + r - \gamma)}{\rho_t} \eta_{it} + \xi_{t+1} - \xi_t, c_{t-1} - \frac{\gamma}{1 + r} c_t^2\right) \\
= \Delta\var(Q_t) - 2\cov\left(\frac{(1 - \gamma)(1 + r - \gamma)}{\rho_t} \eta_{it} + \xi_{t+1} - \xi_t, -\frac{\gamma}{1 + r} (c_t - c_{t-1})\right) \\
= \Delta\var(Q_t) - 2\cov\left(\frac{(1 - \gamma)(1 + r - \gamma)}{\rho_t} \eta_{it} + \xi_{t+1} - \xi_t, -\frac{\gamma}{1 + r} \xi_{t-1} c_t + \xi_t\right) \\
= \Delta\var(Q_t) - 2\cov\left(1 - \gamma)(1 + r - \gamma) \left(v_{it} + \frac{r}{(1 + r)\rho_t} u_{it}\right), \xi_t\right) + 2\var(\xi_t)
\]

Where the second equation is achieved by applying equation (10). While on the RHS, we have

\[
\var\left(1 - \gamma)(1 + r - \gamma) \left(\frac{\eta_{it}}{\rho_t} + \xi_{t+1} - \xi_t\right) \\
= \var\left(1 - \gamma)(1 + r - \gamma) \left(\frac{\eta_{it}}{\rho_t}\right) + \var(\xi_{t+1} - \xi_t) + 2\cov\left(\frac{(1 - \gamma)(1 + r - \gamma)}{\rho_t} \eta_{it}, \xi_{t+1} - \xi_t\right) \\
= (1 - \gamma)^2 (1 + r - \gamma)^2 \left(\var(v_{it}) + \frac{r^2}{(1 + r)^2\rho_t^2} \var(u_{it})\right) \\
+ \var(\xi_{t+1}) + \var(\xi_t) \\
- 2\cov\left(1 - \gamma)(1 + r - \gamma) \left(v_{it} + \frac{r}{(1 + r)\rho_t} u_{it}\right), \xi_t\right)
\]

For small enough \(r\) and sufficiently long \(T\), the equation may be simplified as follows:

\[
\Delta\var(Q_t) = (1 - \gamma)^2 (1 + r - \gamma)^2 \left(\var(v_{it}) + \frac{r^2}{(1 + r)^2\rho_t^2} \var(u_{it})\right) + \var(\xi_{t+1}) + \var(\xi_t) \\
- \var(\xi_t) - 2\var(\xi_t) \\
= (1 - \gamma)^2 (1 + r - \gamma)^2 \left(\var(v_{it}) + \frac{r^2}{(1 + r)^2\rho_t^2} \var(u_{it})\right) + \Delta\var(\xi_{t+1}) \\
\approx (1 - \gamma)^2 (1 + r - \gamma)^2 \var(v_{it}) + \Delta\var(\xi_{t+1})
\]

**Proof of equation (15) in quadratic utility model**

\[
\Delta\cov(Q_t, y_t) = \cov(Q_t, y_t) - \cov(Q_{t-1}, y_{t-1}) \\
= \cov(Q_{t-1} + \Delta Q_t, y_{t-1} + \Delta y_t) - \cov(Q_{t-1}, y_{t-1}) \\
= \cov(\Delta Q_t, \Delta y_t) + \cov(Q_{t-1}, \Delta y_t) \\
= \cov(\Delta Q_t, \Delta y_t) + \cov(Q_{t-2} + \Delta Q_{t-1}, \Delta y_t) \\
= \cov(\Delta Q_t + \Delta Q_{t-1}, \Delta y_t) + \cov(Q_{t-3} + \Delta Q_{t-2}, \Delta y_t) \\
= \cov(\Delta Q_t + \Delta Q_{t-1} + \Delta Q_{t-2}, \Delta y_t) + \cov(Q_{t-3}, \Delta y_t)
\]
where \( \text{cov}(Q_{t-3}, \Delta y_t) = 0 \) if the transitory shock \( u_{it} \) is i.i.d. Since we also have:

\[
\Delta Q_t = R v_{it} + R \frac{r}{(1 + r) \rho_t} u_{it} + \xi_{t+1} - \xi_t,
\]

with \( R = (1 - \gamma)(1 + r - \gamma) \), and \( \Delta y_{it} = \Delta u_{it} + v_{it} \), the difference of the covariance then becomes:

\[
\Delta \text{cov}(Q_t, y_t) = \text{cov} \left( R v_{it} + R \frac{r}{(1 + r) \rho_t} u_{it} + \xi_{t+1} - \xi_t \right.
\]
\[
+ R v_{it-1} + R \frac{r}{(1 + r) \rho_{t-1}} u_{it-1} + \xi_t - \xi_{t-1}
\]
\[
\left. + R v_{it-2} + R \frac{r}{(1 + r) \rho_{t-2}} u_{it-2} + \xi_{t-1} - \xi_{t-2} \Delta u_{it} + v_{it} \right) = R \cdot \text{var}(v_{it}) + R \frac{r}{1 + r} \Delta \frac{1}{\rho_t} \text{var}(u_{it})
\]

with the approximation holding for small \( r \) and large \( t \).

If we assume instead assume an MA(1) structure for transitory shocks, then we can derive the following condition:

\[
\Delta \text{cov}(Q_{t+1}, y_t) = \text{cov}(\Delta Q_{t+1} + \Delta Q_t + \Delta Q_{t-1} + \Delta Q_{t-2} + \Delta Q_{t-3}, \Delta y_t) + \text{cov}(Q_{t-4}, \Delta y_t)
\]

\[
= \text{cov}(\Delta Q_{t+1} + \Delta Q_t + \Delta Q_{t-1} + \Delta Q_{t-2} + \Delta Q_{t-3}, \Delta y_t)
\]

\[
= R \cdot \text{var}(v_{it}) + R \frac{r}{1 + r} \text{var}(\epsilon) \left( \frac{\theta}{\rho_{t+1}} + \frac{\theta^2 - \theta + 1}{\rho_t} + \frac{\theta - 1 - \theta^2}{\rho_{t-1}} - \frac{\theta}{\rho_{t-2}} \right)
\]

where, we assume \( u_{it} = \epsilon_t - \theta \epsilon_{t-1} \), and \( \epsilon_t \) is i.i.d with mean 0 and constant \( \text{var}(\epsilon) \)

The same method applies for the last equation in model 3.

\[
\Delta \text{cov}(c_{t+1}^*, y_t) = \text{cov}(c_{t+1}^*, y_{t+1}) - \text{cov}(c_t^*, y_{t-1})
\]

\[
= \text{cov}(c_t^* + (1 - \gamma \frac{r}{1 + r}) v_{t+1} + (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t+1}} u_{t+1}, y_{t+1} - \Delta u_t + v_t)
\]

\[
- \text{cov}(c_t^*, y_{t+1})
\]

\[
= \text{cov}(c_t^*, \Delta u_t + v_t) + (1 - \gamma \frac{r}{1 + r}) \text{cov}(v_{t+1} + \frac{r}{(1 + r) \rho_{t+1}} u_{t+1}, \Delta u_t + v_t)
\]

\[
+ (1 - \gamma \frac{r}{1 + r}) \text{cov}(v_{t+1} + \frac{r}{(1 + r) \rho_{t+1}} u_{t+1}, y_{t+1} - \Delta u_t + v_t)
\]

\[
= \text{cov}(c_t^* + (1 - \gamma \frac{r}{1 + r}) v_{t+2} + (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t-2}} u_{t+2} + (1 - \gamma \frac{r}{1 + r}) v_{t-1}
\]

\[
+ (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t-2}} u_{t+2} + (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t-2}} u_{t+2} + \Delta u_t + v_t)
\]

\[
+ (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t-2}} \text{cov}(u_{t+1}, \Delta u_t)
\]

\[
= (1 - \gamma \frac{r}{1 + r}) \text{var}(v_t)
\]

\[
+ (1 - \gamma \frac{r}{1 + r}) \frac{r}{(1 + r) \rho_{t-2}} \text{cov}(\frac{1}{\rho_{t-2}} u_{t+2} + \frac{1}{\rho_{t-2}} u_{t+2} + \frac{1}{\rho_t} u_t + \frac{1}{\rho_{t+1}} u_{t+1}, \Delta u_t)
\]

Again, if we assume MA(1) structure for transitory shock, i.e. \( u_{it} = \epsilon_t - \theta \epsilon_{t-1} \), and \( \epsilon_t \) is i.i.d with mean 0 and constant \( \text{var}(\epsilon) \), we can write the above equation into
\[ \Delta \text{cov}(c_{t+1}^*, y_t) = (1 - \frac{\gamma}{1 + r}) \text{var}(v_t) \]
\[ \approx (1 - \frac{\gamma}{1 + r}) \text{var}(v_t) \]

approximation holds for small \( r \) and big \( T - t \).

**Proof of linearization equation in CRRA model**

This proof borrows heavily from Carroll (2000).

Linearize the Euler equation (21) around steady state ratio \((c_0/h_0)\):

\[ \text{Et} \left[ \frac{c_{it}^{\sigma}}{h_{it}^{\sigma + \gamma}} - \frac{\gamma}{1 + \delta} \frac{c_{it+1}^{\sigma+1}}{h_{it+1}^{\gamma+\sigma+1}} \right] = \frac{1 + r}{1 + \delta} \left[ \frac{c_{it+1}^{\sigma}}{h_{it+1}^{\gamma+\sigma+1}} - \frac{\gamma}{1 + \delta} \frac{c_{it+2}^{\sigma+1}}{h_{it+2}^{\gamma+\sigma+1}} \right] \]

Divide both sides of the Euler equation by the first ratio on LHS:

\[ \text{Et} \left[ 1 - \frac{\gamma}{1 + \delta} \frac{c_{it+1}^{\sigma+1} h_{it+1}^{\gamma+\sigma+1} c_{it+1}^{\sigma-1}}{c_{it}^{\sigma} h_{it}^{\gamma+\sigma+1} h_{it+1}} \right] = \frac{1 + r}{1 + \delta} \text{Et} \left[ \frac{c_{it+1}^{\sigma}}{h_{it+1}^{\gamma+\sigma+1}} - \frac{\gamma}{1 + \delta} \frac{c_{it+2}^{\sigma+1}}{h_{it+2}^{\gamma+\sigma+1}} \right] \]

Define \( \psi_t = c_t/c_{t-1}, \chi_t = c_t/h_t, \) and remember \( h_{t+1} = c_{t+1} \), the above equation can be written as:

\[ \text{Et} \left[ 1 - \frac{\gamma}{1 + \delta} \psi_{it+1}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+1} \right] = \frac{1 + r}{1 + \delta} \text{Et} \left[ \psi_{it+1}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+1} - \frac{\gamma}{1 + \delta} \psi_{it+2}^{\sigma-1} \psi_{it+1}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+2} \right] \]

Multiply both sides by \( \psi_{it+1}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+1} \)

\[ 1 = \text{Et} \left[ 1 + \frac{\gamma}{1 + \delta} \left( \psi_{it+1}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+1} - \frac{\gamma}{1 + \delta} \right) + \frac{\gamma}{1 + \delta} \psi_{it+2}^{\sigma-1} \chi_{it+1}^{\gamma-1} \chi_{it+2} \right] \]

Linearize the above equation around the steady state ratio \( \ln \psi_0 \) and \( \ln \chi_0 \):

\[ 1 = \text{Et} \left[ 1 + \frac{\gamma}{1 + \delta} \left( \psi_0^{\sigma} \chi_0^{\gamma-1} + \sigma \psi_0^{\sigma-1} \chi_0^{\gamma-1} \ln \psi_0 - \gamma(1 - \sigma) \psi_0^{\sigma-1} \chi_0^{\gamma-1} \ln \psi_0 + \gamma(1 - \sigma) \psi_0^{\sigma-1} \chi_0^{\gamma-1} \ln \psi_0 + O(n) \right) \right] \]

Use the fact that \( \chi_t = \psi_t \) as we assume \( h_t = c_{t-1} \) for simplicity and assume \( r \simeq \delta \), therefore \( \psi_0 \simeq 1 \), we can simplify above linear equation into:

\[ 0 = \text{Et} \left[ \sigma \psi_0^{\sigma} \chi_0^{\gamma-1} \ln \chi_0 - \frac{\gamma}{1 + \delta} \ln \chi_0 + \frac{\gamma}{1 + \delta} \psi_0^{\sigma-1} \chi_0^{\gamma-1} \ln \psi_0 \right] \]

\[ + \frac{\gamma}{1 + \delta} \left( \ln \chi_0 + \ln \psi_0 \right) + O(n) \]
\[ 0 = \mathbb{E}_t \left[ \sigma \ln \psi_{t+1} + \gamma(1 - \sigma) \ln \psi_t - \frac{\gamma}{1 + r} \ln \psi_{t+1} - \frac{\gamma \sigma}{1 + r} \ln \psi_{t+2} + \frac{\gamma}{1 + r} (\gamma(\sigma - 1) \ln \psi_{t+1} + \ln \psi_{t+2} + \mathcal{O}(n)) \right] \]

Substitute \( \ln \psi_t = \ln c_t - \ln c_{t-1} \) and rearrange,

\[ \mathbb{E}_t \left[ (\sigma - \frac{\gamma}{1 + r}) \Delta \ln c_{t+1} - \gamma(\sigma - 1) \Delta \ln c_t - \frac{\gamma(\sigma - 1)}{1 + r} (\Delta \ln c_{t+2} - \sigma \Delta \ln c_{t+1}) + \mathcal{O}(n) \right] = 0 \]

where, \( \mathcal{O}(n) \) includes the second and higher order term of linearization approximation error, which is approaching to zero as \( (\ln \psi_{t+1} - \ln \psi_0) \) approaches to zero.

Multiply both sides by \( (1 + r) \) and dividing by \( (\sigma - 1) \), it becomes:

\[ \mathbb{E}_t \left[ \left( \frac{\sigma}{(\sigma - 1)} \right) (1 + r) \Delta \ln c_{t+1} - \gamma(1 + r) \Delta \ln c_t - \gamma \Delta \ln c_{t+2} - \gamma \Delta \ln c_{t+1} + \mathcal{O}(n) \right] = 0 \]

Remove expectation and define \( \epsilon_{t+1} = \ln c_{t+1} - \mathbb{E}_t \ln c_{t+1} \) and \( \xi_{t+2} = \ln c_{t+2} - \mathbb{E}_t \ln c_{t+2} \), we can get equation (22) after simple rearrangement.

**Proof of linearization equation in CRRA model with two lags:**

The Euler Equation with habit is:

\[ \mathbb{E}_t \left[ 1 - \frac{\gamma}{1 + \delta} \frac{c_{t+1}}{c_t} \frac{h_t}{h_{t+1}} + \sigma \frac{\gamma^{\sigma + \gamma} c_{t+1}}{c_t} \frac{\gamma^{\sigma + \gamma} h_t}{h_{t+1}} + \sigma^{\gamma + \gamma} c_{t+1} \frac{\gamma^{\sigma + \gamma} h_t}{h_{t+1}} \right] = 1 + \delta \mathbb{E}_t \left[ \frac{c_{t+1}}{c_t} \frac{h_t}{h_{t+1}} \frac{\gamma^{\sigma + \gamma} c_{t+1}}{c_t} \frac{\gamma^{\sigma + \gamma} h_t}{h_{t+1}} \right] \]

In order to linearize around the steady state ratio, we need to write it in terms of the rate of consumption over habit:

\[ \mathbb{E}_t \left[ 1 - \frac{\gamma}{1 + \delta} \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] = 1 + \delta \mathbb{E}_t \left[ \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] \]

Define \( \psi_{it} \equiv \frac{c_{it}}{c_{it-1}} \) and \( \chi_{it} \equiv \frac{c_{it}}{h_{it}} \), we then have:

\[ \mathbb{E}_t \left[ 1 - \frac{\gamma}{1 + \delta} \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] = 1 + \delta \mathbb{E}_t \left[ \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] \]

Or

\[ \mathbb{E}_t \left[ 1 - \frac{\gamma}{1 + \delta} \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] = 1 + \delta \mathbb{E}_t \left[ \psi_{t+1}^{\sigma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \psi_{t+1}^{\gamma - \gamma} \right] \]

Move the second term on LHS to RHS and linearize around steady state \( \ln \chi_0, \ln \psi_0 \):
1 = E_t \left[ \frac{\gamma}{1 + \delta} \psi_0^{\sigma - \gamma - \sigma} \xi_{0t} + \frac{\gamma}{1 + \delta} \left( \gamma \sigma - \gamma - \sigma \right) \psi_0^{\sigma - \gamma - \sigma} (\ln \psi_{it+1} - \ln \psi_0) \right] \\
+ \frac{\gamma}{1 + \delta} \psi_0^{\sigma - \gamma - \sigma} \xi_{0t} \left( (\gamma \sigma - \gamma) (\ln \chi_{it} - \ln \chi_0) + (-\gamma \sigma + \gamma + 1)(\ln \chi_{it+1} - \ln \chi_0) \right) \\
+ \frac{1 + r}{1 + \delta} E_t \left[ \psi_0^{\sigma - \gamma - \sigma} (\gamma \sigma - \gamma - \sigma) \psi_0^{\sigma - \gamma - \sigma} (\ln \psi_{it+1} - \ln \psi_0) \right] \\
+ \psi_0^{\sigma - \gamma - \sigma} ((\gamma \sigma - \gamma)(\ln \chi_{it} - \ln \chi_0) + (-\gamma \sigma + \gamma)(\ln \chi_{it+1} - \ln \chi_0)) \\
- \frac{\gamma}{1 + \delta} \psi_0^{2\sigma - 2\gamma - 2\sigma} \xi_{0t} (-\gamma \sigma - \gamma - \sigma) \psi_0^{2\sigma - 2\gamma - 2\sigma} \xi_{0t} (\ln \psi_{it+1} - \ln \psi_0) + (\ln \psi_{it+2} - \ln \psi_0)) \\
- \frac{\gamma}{1 + \delta} \psi_0^{2\sigma - 2\gamma - 2\sigma} \xi_{0t} ((\gamma \sigma - \gamma)(\ln \chi_{it} - \ln \chi_0) + (-\gamma \sigma + \gamma + 1)(\ln \chi_{it+2} - \ln \chi_0)) \right]

As proved by Carroll (2000), \( \xi_{0t} = 1, \psi_0 = 1 \) if \( r = \delta \). Substitute into the above equation:

\[
1 = E_t \left[ \frac{\gamma}{1 + \delta} \left( 1 + (\gamma \sigma - \gamma - \sigma) \ln \psi_{it+1} + (\gamma \sigma - \gamma) \ln \chi_{it} + (-\gamma \sigma + \gamma + 1)(\ln \chi_{it+1}) \right) \right] \\
+ E_t \left[ 1 + (\gamma \sigma - \gamma - \sigma) \ln \psi_{it+1} + (\gamma \sigma - \gamma) \ln \chi_{it} + (-\gamma \sigma + \gamma) \ln \chi_{it+1} \right] \\
- \frac{\gamma}{1 + \delta} \left( 1 + (\gamma \sigma - \gamma - \sigma) \ln \psi_{it+1} + (\gamma \sigma - \gamma) \ln \chi_{it} + (-\gamma \sigma + \gamma + 1)(\ln \chi_{it+2}) \right) \\
+ \left( \gamma \sigma - \gamma - \sigma \right) \ln \psi_{it} + \left( \gamma \sigma - \gamma - \sigma \right) \ln \chi_{it} + (-\gamma \sigma + \gamma + 1)(\ln \chi_{it+2}) \right]

by rearrangement,

\[
0 = E_t \left[ \frac{\gamma}{1 + \delta} \left( \gamma \sigma - \gamma - 1 \right)(\ln \chi_{it+1} + (\gamma \sigma - \gamma - \sigma) \ln \psi_{it+1} + (\gamma \sigma - \gamma) \ln \chi_{it} + (-\gamma \sigma + \gamma) \ln \chi_{it+1} \right) \right] \\
- E_t \left[ \frac{\gamma}{1 + \delta} \left( \gamma \sigma - \gamma - \sigma \right) \ln \psi_{it+2} + \gamma \left( \gamma \sigma - \gamma + 1 \right)(\ln \chi_{it+2}) \right]

Replace back the definition of \( \chi_{it} = \frac{c_{it}}{h_{it}} \) and \( \psi_{it} = \frac{c_{it}}{c_{it}} \),

\[
E_t \left[ -\frac{\gamma}{1 + \delta} \left( \gamma \sigma - \gamma - 1 \right)(\ln c_{it+1} - \ln h_{it+1}) + (\gamma \sigma - \gamma - \sigma)(\ln c_{it+1} - \ln c_{it}) \right] \\
+ (\gamma \sigma - \gamma)(\ln c_{it} - \ln h_{it}) + (\gamma \sigma - \gamma)(\ln c_{it+1} - \ln h_{it+1}) \right] \\
= E_t \left[ \frac{\gamma}{1 + \delta} \left( \gamma \sigma - \gamma - \sigma \right) (\ln c_{it+2} - \ln c_{it+1}) - \gamma \left( \gamma \sigma - \gamma - 1 \right)(\ln h_{it+2} - \ln h_{it+1}) \right]

or it can be written as,

\[
E_t \left[ \left( \gamma \sigma - \gamma - \sigma \right) \left( (\ln c_{it} - \frac{\gamma}{1 + \delta} \ln c_{it+1}) - (\ln c_{it} - \frac{\gamma}{1 + \delta} \ln c_{it+1}) \right) \right] \\
+ (\gamma \sigma - \gamma) \left( (\ln c_{it} - \frac{\gamma}{1 + \delta} \ln c_{it+1}) - (\ln h_{it} - \frac{\gamma}{1 + \delta} \ln h_{it+1}) \right) \\
- (\gamma \sigma - \gamma) \left( (\ln c_{it} - \frac{\gamma}{1 + \delta} \ln c_{it+1}) - (\ln h_{it-1} - \frac{\gamma}{1 + \delta} \ln h_{it+2}) \right) \\
+ \frac{\gamma}{1 + \delta} (\ln c_{it+1} - \ln h_{it+1}) - \frac{\gamma}{1 + \delta} (\ln c_{it+1} - \ln h_{it+2}) \right]
\]

Define \( \ln c_{it} = \ln c_{it} - \frac{\gamma}{1 + \delta} \ln c_{it+1}, \ln h_{it} = \ln h_{it} - \frac{\gamma}{1 + \delta} \ln h_{it+1}, \).

\[
E_t \left[ -\sigma \Delta \ln c_{it+1} + (\gamma \sigma - \gamma \Delta \ln h_{it+1} - \frac{\gamma}{1 + \delta} \Delta \ln c_{it+1} + \frac{\gamma}{1 + \delta} \Delta \ln h_{it+2} \right] = 0
\]

Or if we define \( \Xi_{it+1} = -\sigma \ln c_{it+1} + (\gamma \sigma - \gamma \ln h_{it+1} - \frac{\gamma}{1 + \delta} \ln c_{it+2} + \frac{\gamma}{1 + \delta} \ln h_{it+2} \), we can simply write the above as:

\[
E_t \Xi_{it+1} = E_t \Xi_{it}
\]