Bargaining with Variable Rhythms

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BARGAINING AT VARIABLE RHYTHMS(*)

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Abstract This note presents a modification of Rubinstein's model in which players can choose whether to be fast or slow in responding to their opponent's proposals. We characterize the effects of such choice on the outcome of the bargaining and give predictions on what rhythm will players choose in equilibrium.

Resumen Esto nota presenta una modificación del modelo tradicional de Rubinstein en el que dos jugadores pueden escoger el ritmo al que responder las propuestas de sus rivales. Caracterizamos los efectos de esta elección sobre el resultado de la negociación y ofrecemos predicciones acerca del ritmo que los jugadores utilizarán en equilibrio.

1. INTRODUCTION

The standard analysis of negotiation processes viewed as non-cooperative bargaining situations is by now well developed from Rubinstein's (1982) seminal paper. Excellent reviews of this literature are found in Osborne and Rubinstein (1990) and Binmore and Dagsupta (1987). One of the characteristics of this analysis is the fact that players involved in the bargaining process are not able to choose the rhythm at which the negotiation develops, that is to say the frequency at which proposals and counterproposals are announced.

It is a common observation that negotiations are very seldom smooth processes. Sometimes, players agree to interrupt the process and restart it some time later; some other times negotiations are broken by one of the parties unilaterally. A way to look at these interruptions is as an effort by the players that provoke the interruption to affect the initial status quo. We feel that this attitude is a crucial element to obtain a proper understanding of the bargaining process. In other words, we believe that the endogenization of the rhythm at which bargaining takes place will provide insights not available in the standard Rubinstein's model.

We propose to consider these types of behavior as a change in the rhythm of bargaining. Accordingly, we will present a model in the line of Rubinstein's where players are able to choose the rhythm of offers and counter-offers. In particular, players will be able to choose whether to be fast or slow in responding to their opponent's proposals. We characterize the effects of such choice on the outcome of the bargaining and give predictions on what rhythm will players choose in equilibrium.

Similar issues are addressed by Sákovics (1993) and Perry and Reny (1993). In their models, bargaining takes place in continuous time and players can make offers at any

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time without any predetermined order, with a fixed minimum delay between offers of the same players and between offers of different players. If the players can react immediately to their opponent’s offers, then the set of equilibria approaches the Rubinstein outcome. If it takes time to react then there are multiple equilibria that yield very different partitions of the surplus, some of them involving substantial delay.

Since the order of players’ offers as well as the possible rhythms are fixed in our model, our extension is not as rich as the one of Sákovics and Perry and Reny. However, since our simple extensive form yields a unique (subgame perfect) equilibrium, we can characterize the equilibrium outcome for each configuration of the parameters. Thus, we can explicitly relate the abilities to change rhythms and the possible delays under each rhythm to the share of the surplus that each player obtains. Our most salient result is that, unlike in the Rubinstein model, the first mover advantage does not vanish as the bargaining becomes frictionless, actually, the first player to make an offer obtains all the surplus.

2. THE MODEL

Consider the following bargaining game. Two players A and B take turns to make offers on how to split a cake of size one. An outcome \((x, 1-x, t)\) assigning a portion \(x\) to player A and \(1-x\) to player B at date \(t\) is reached if proposal \(x\) is accepted at date \(t\). Players preferences over pairs \((x, t)\), a portion of the pie and a date of agreement, are represented by utility functions \(u^i(x, t) = xe^{-\eta t}, i = A, B, \eta = a, b\). We can also write \(u^i(x, t) = x\delta^m\) with \(\delta^i = e^{-\eta A}\), where \(t = m\Delta\), and \(\Delta\) is the real time interval between proposals.

There may be \(k\), \((k = 1, 2, \ldots)\) different pairs of rhythms at which proposals can be exchanged. Given a pair of rhythms \((\Delta^A_k, \Delta^B_k)\), where \(i = A (j = B)\) we denote by \(R_k\) the pair of discounts that fully characterizes it. Abusing terminology \(R_k\) will be called a rule. At \(t = 0\) nature decides who plays first (i.e. who is called player A) and an initial rule. After hearing an offer, player \(i\) can accept it, and the game ends or reject it. Should he reject an offer he can take two decisions: either make a counteroffer within the same rule or make a new offer in the context of an alternative rule accelerating or delaying next proposal (i.e. change the rhythm of the bargaining process). We assume that changes of a rule take effect immediately. We have also considered the case when the change of rules takes one extra period, i.e. players are forced to respond in the current rhythm but can change the rhythm at which they will hear their opponent’s response. The results in this case are qualitatively very similar and the analysis is analogous.

As Rubinstein (1982) showed, a game with fixed rule \(R_k = (\delta^A_k, \delta^B_k)\) has a unique Perfect Equilibrium Partition (PEP) in which player A receives \(\frac{1-\delta^A_k}{1-\delta^A_k \delta^B_k}\) and player B receives \(\frac{\delta^B_k (1-\delta^A_k)}{1-\delta^A_k \delta^B_k}\).

**Definition 1.** We say that rule \(R_m\) is effective at rule \(R_k\) iff player B prefers to delay agreement one period and obtain the PEP of the fixed rule game with \(R_m\) rather than obtaining the PEP of the fixed rule game with \(R_k\).
For convenience, we will develop our analysis in terms of the real time intervals between proposals. If there are only two possible rhythms of bargaining $\Delta$ and $\Delta'$, every player has two discount rates: a default rate $e^{-\Delta}$ and a discount rate faced by the player that performs a switch of rules, $e^{-\Delta'}$. In this set-up there are four possible sets of rules, $R_1=(e^{-a\Delta}, e^{-b\Delta})$; $R_2=(e^{-a\Delta'}, e^{-b\Delta'})$; $R_3=(e^{-a\Delta}, e^{-b\Delta'})$; $R_4=(e^{-a\Delta'}, e^{-b\Delta})$.

**Lemma 1.** If we are at a rule $R_k$ such that rule $R_{k+1}$ is not effective, the PEP of the fixed rule $R_k$ prevails.

**Proof:** First notice that no player has any interest ever to change from rule $R_k$ to rule $R_{k+1}$, to rule $R_{k+2}$, to rule $R_{k+3}$, and back to rule $R_k$. Therefore there are four possible situations:

1. The PEP of the fixed rule $R_k$ prevails.
2. Player $j$ does not want the outcome of the fixed rule $R_{k+1}$, but he uses his ability to change to $R_{k+1}$, because it can lead to rule $R_{k+2}$. This will be the case if player $i$, being at rule $R_{k+1}$ is willing to change to the fixed rule $R_{k+2}$. But if player $i$ prefers the outcome of the fixed rule $R_{k+2}$ to the outcome of the fixed rule $R_{k+1}$, then player $j$ has to prefer the outcome of the fixed rule $R_{k+1}$ to the outcome of the fixed rule $R_{k+2}$. Hence, for player $j$ either the outcome of the fixed rule $R_k$ is better than the ones associated to $R_{k+1}$ and to $R_{k+2}$, or he will not use his ability to change to rules $R_{k+1}$ in order to get to $R_{k+2}$.
3. Player $j$ does not want the outcome of the fixed rule $R_{k+1}$, neither wants to use his ability to change to $R_{k+1}$ in order to get to $R_{k+2}$, but to use it to arrive to $R_{k+3}$. To arrive to $R_{k+3}$ requires that player $i$ will want to change to $R_{k+2}$ to reach $R_{k+3}$. But this is not possible by situation 2 above.
4. Player $j$ does not want the outcome of the fixed rule $R_{k+1}$, neither wants to use his ability to change to $R_{k+1}$ in order to get to $R_{k+2}$, but to use it to arrive to $R_{k+4} = R_k$. This is clearly not possible by a similar argument as in the previous situation.

Lemma 1 tells us that given an initial rule $R_0 = R_k$, player B may change the rhythm to $R_{k+1}$ if $k$ is odd, or to $R_{k-1}$ if $k$ is even. Accordingly, we need to check

- $R_4$ will be effective at $R_3$ iff
  
  \[ \frac{e^{-b\Delta}(1-e^{-b\Delta})}{e^{-b\Delta'}(1-e^{-a\Delta'})} \geq \frac{1-e^{-a\Delta'} e^{-b\Delta}}{1-e^{-(a+b)\Delta'}} \]  

[1]

- $R_2$ will be effective at $R_1$ iff
  
  \[ \frac{e^{-b\Delta}(1-e^{-b\Delta})}{e^{-b\Delta'}(1-e^{-a\Delta'})} \geq \frac{1-e^{-a\Delta} e^{-b\Delta'}}{1-e^{-(a+b)\Delta}} \]  

[2]

Let us consider expressions [1] and [2]. Without loss of generality we can assume $b=1$, so that we normalize player B's preferences. Equation [1] is quadratic in $\Delta$, while equation [2] is quadratic in $\Delta'$. There are four possible scenarios; (A) if both equations hold, (B) if only [1] holds, (C) if only [2] holds, and (D) if neither equation holds. We can visualize the different scenarios depending on the value of $a$, by plotting the roots of [1] and [2] in the space $(\Delta, \Delta')$. These are illustrated in figure 1, where $r_j$ stands for root $j$ of equation $i$. 
Figure 1
2.1 Equilibrium

Proposition 1. Let $\Delta < \Delta'$ and $b = 1$.

- In scenario A the PEP of the fixed rule $R_2$ prevails if $a \geq 1$ or $R_4$ if $a < 1$.
- In scenario B the PEP of the fixed rule $R_1$ prevails for any value of $a$.
- In scenario C the PEP of the fixed rule $R_2$ prevails for any value of $a$.
- In scenario D the PEP of the fixed rule $R_3$ prevails for any value of $a$.

To prove the proposition it is convenient to state the following lemma:

**Lemma 2.** Let $x_i^k$ denote the portion of the cake assigned to player $i$ in the PEP of the fixed rule $R_k$.

(i) If we are at a rule $R_k$ such that rule $R_{k+1}$ is effective and rule $R_{k+2}$ is not effective at $R_{k+1}$, the unique PEP is:

$$1 - \delta_{k+1}^b x_{k+1}^b; \quad \delta_{k+1}^b x_{k+1}^b$$  \[3\]

(ii) If we are at a rule $R_k$ such that rule $R_{k+1}$ is effective and rule $R_{k+2}$ is also effective and $R_{k+3}$ is not effective at $R_{k+2}$, the unique PEP is:

$$1 - \delta_{k+1}^b (1 - \delta_{k+2}^b x_{k+2}^b); \quad \delta_{k+1}^b (1 - \delta_{k+2}^b x_{k+2}^b)$$  \[4\]

(iii) If we are at a rule $R_k$ such that rule $R_{k+1}$ is effective, rule $R_{k+2}$ is effective, $R_{k+3}$ is effective, and $R_k$ is not effective at $R_{k+3}$, the unique PEP is:

$$1 - \delta_{k+1}^b [1 - \delta_{k+2}^b (1 - \delta_{k+3}^b x_{k+3}^b)]; \quad \delta_{k+1}^b [1 - \delta_{k+2}^b (1 - \delta_{k+3}^b x_{k+3}^b)]$$  \[5\]

Proof: Statement (i) follows immediately from Lemma 1. Statement (ii) follows from Lemma 1 and statement (i) in Lemma 2. Statement (iii) follows from Lemma 1 and statements (i) and (ii) in Lemma 2.

Proof of proposition 1:

From Lemma 2, player A chooses $R_0 = R_k$ such that $R_{k+1}$ is not effective.

- Consider scenario A. Rules that have no effective alternative are $R_2$ and $R_4$. The unique PEP, in terms of the payment to player $A$ are $\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$ and $\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$ respectively. Player $A$ prefers $R_2$ to $R_4$ if

$$\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}} > \frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$$

which holds iff $a \geq 1$.

- Consider scenario B. Rules that have no effective alternative are $R_1$ and $R_4$. The unique PEP, in terms of the payment to player $A$ are $\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$ and $\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$ respectively. Player $A$ prefers $R_1$ to $R_4$ if

$$\frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}} > \frac{1 - e^{-\Delta'}}{1 - e^{-a\Delta'} e^{-\Delta}}$$

which holds for any value of $a$. 
Consider scenario C. Rules that have no effective alternative are $R_2$ and $R_3$. The unique PEP, in terms of the payment to player A are \( \frac{1-e^{-\Delta'}}{1-e^{-a\Delta} e^{-\Delta'}} \) and \( \frac{1-e^{-\Delta}}{1-e^{-(a+1)\Delta'}} \) respectively. Player A prefers $R_2$ to $R_3$ if
\[
\frac{1-e^{-\Delta'}}{1-e^{-a\Delta} e^{-\Delta'}} > \frac{1-e^{-\Delta}}{1-e^{-(a+1)\Delta'}}.
\]
This inequality holds for any value of $a$.

Consider scenario D. Rules that have no effective alternative are $R_1$ and $R_3$. The unique PEP, in terms of the payment to player A are \( \frac{1-e^{-\Delta}}{1-e^{-\Delta'} (a+1)} \) and \( \frac{1-e^{-\Delta}}{1-e^{-\Delta'} (a+1)} \) respectively. Player A prefers $R_3$ to $R_1$ if
\[
\frac{1-e^{-\Delta}}{1-e^{-\Delta'} (a+1)} > \frac{1-e^{-\Delta}}{1-e^{-\Delta'} (a+1)}.
\]
This inequality holds for any value of $a$.

Proposition 1 tells us that scenarios B and D have equilibrium rules in which both players follow the same rhythm. In these scenarios the possibility of changing rhythm has no effect and the surplus is divided as in the Rubinstein game.

Scenarios A and C have equilibrium rules where players use different rhythms. Of these the most interesting case is scenario C. A close study of it leads to the following.

**Proposition 2.** Let $\Delta < \Delta'$ and, for simplicity, $a=b=1$. For each share of the surplus $\alpha < 1$, there is a rhythm of bargaining $\Delta$ such that if $\Delta < \Delta$, then the share of the surplus assigned to player A is greater than $\alpha$.

**Proof:** If $a=b$ we are in scenario C if the rhythms of bargaining are not too large.

Hence the portion assigned to player A is,
\[
\frac{1-e^{-\Delta'}}{1-e^{-\Delta'} e^{-\Delta}}
\]

Note that this portion is larger than $\alpha$ provided that
\[
e^{-\Delta'} < \frac{\frac{1}{\alpha} - 1}{\frac{1}{\alpha} e^{-\Delta}}
\]
which holds for $\Delta$ small enough.

**Remark 1:** As bargaining becomes frictionless, the player moving first captures all the surplus.

**Remark 2:** Giving players the ability to choose their own initial rhythms, does not substantially affect the results. Player A chooses the rhythm giving him the best PEP payoff anticipating $B'$s choice of rhythm.

**Remark 3:** Considering more than two rhythms, does not change the results. Only the fastest and the slowest rhythms matter.
REFERENCES


