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Abstract—There has been much research into the use of algebraic iterative algorithms based on total variation (TV) minimisation for use in micro-CT for materials science and non-destructive testing. However, in these applications it is often the case that the measured projection data are severely truncated, creating an extreme interior tomography problem. With iterative algorithms, it is usually necessary for the reconstruction volume to cover the entire object support, which severely limits their application to such use cases. We describe a simple dual grid approach for applying iterative algorithms to severely truncated data, and give results with simulated data showing the effectiveness of the method. We demonstrate accurate reconstruction within an ROI with linear dimensions a factor 16 less than the size of the object.

I. INTRODUCTION

Cone beam x-ray micro-CT is now a widely-used imaging technique in materials science research [1], and also in other applications such as rock core analysis for oil exploration and non-destructive testing for the semiconductor industry, for example. In such applications, there is a drive towards faster acquisition times, motivated by a variety of reasons including higher throughput scanning, dose reduction and higher temporal resolution for time-lapse imaging. To this end, the use of algebraic iterative reconstruction algorithms involving total variation (TV) minimisation (e.g. [2], [3]) has been the subject of much research. Such algorithms have particularly high utility in these applications, as objects of interest often consist of homogeneous materials with a piecewise constant structure. A typical problem may involve calculating distribution of pore sizes based on analysis of a segmented volume, for example.

Many applications of micro-CT in materials science and non-destructive testing demand extremely high spatial resolution. Due to the limited size and resolution of the detector, the field of view of the scanner becomes smaller as resolution increases; for sub-micron spatial resolution, the field of view will typically be less than 1mm. It is therefore often the case that the region of interest (ROI) that we wish to reconstruct is covered with a much finer, high resolution grid, while the ROI is covered with a much coarser grid. Therefore the reconstruction algorithm will attempt to fit reconstruction of the ROI to data representing the whole object. The data mismatch in this case can result in severe artefacts.

In the case where the ROI represents only a very small part of the object, then covering the entire object in a high resolution grid is too costly in terms of the memory required. If low resolution data covering the whole object are available, then methods such as scout view assisted tomography [4] can be used to effectively generate data representing only the ROI. A similar technique is described in [5]. However, in many applications, it may not be possible or desirable in terms of throughput to acquire such data.

Similar to the approach used in [6], this paper describes the use of a dual grid technique to solve the interior tomography problem without needing any additional data.

II. METHODS

A. Algebraic Iterative Reconstruction Algorithms

The core of any algebraic iterative reconstruction algorithm is a discrete representation of the projection process. This forms a system of equations

$$Ax = b,$$

where the matrix \( A \) represents a discrete model of the projection process, the vector \( x \) represents the discretised object function, and the vector \( b \) represents the projection data. This system of equations can then be solved in the least-squares sense by a number of common iterative algorithms.

B. Dual Grid Approach

The basic idea of the dual grid approach is to represent the discretised object function by voxels of two sizes; a coarse, low resolution grid covers the entire object support, while the ROI is covered with a much finer, high resolution grid. This is illustrated for a 2D slice in figure 1. Similar dual grid approaches for different applications are described in [7], [8], [9]. By covering the entire object support in a low resolution grid, this allows the reconstruction algorithm to take all parts of the object into account, while avoiding the need to use large amounts of memory by covering the entire object support in a high resolution grid. Representing the object in this way, the discretised projection equations can now be written as

$$[A_{\text{fine}} \ A_{\text{coarse}}] \begin{bmatrix} x_{\text{fine}} \\ x_{\text{coarse}} \end{bmatrix} = b,$$

where \( A_{\text{fine}} \) and \( A_{\text{coarse}} \) represent the discretised projections through the fine and coarse grids respectively, and \( x_{\text{fine}} \) and \( x_{\text{coarse}} \) are the corresponding fine and coarse volumes.
The difference in size between the voxels in each grid is known as the grid magnification factor. By keeping the number of voxels in each grid the same, and using a grid magnification factor that divides the number of voxels in each dimension, the fine grid overlaps a section of the coarse grid exactly. Entries in $A_{\text{coarse}}$ and $x_{\text{coarse}}$ corresponding to the overlapping region are set to zero, keeping the two grids orthogonal to each other.

We use the length of intersection model [10] for discretising the projections. In our practical 3D implementation, coefficients of $A_{\text{fine}}$ and $A_{\text{coarse}}$ are calculated on the fly by the method described in [11]. The entire coarse grid, including the ROI, is maintained and stored; the overlapping region is simply set to zero. Although this is not efficient in terms of memory usage, for the applications that the method is intended to be used for, the amount of overlap is low, since the grid magnification factor is high. In our implementation, the discretisation in $z$ is kept the same for both grids. For the intended applications, optical magnification in the scanner is generally high, resulting in a low cone angle (typically approximately 1 degree); therefore, even with high grid magnification factors, the number of additional slices needed is low.

Since the fine and coarse grid volumes are effectively orthogonal, the forward and back projection of each can be done independently. For forward projection, the results are simply added together afterwards, while for back projection, it is necessary to set the overlapping region of the coarse grid to zero afterwards.

III. RESULTS

A. Simulated Data – Grid Magnification Factor 4

Non-truncated, 2D noise-free data were generated for a “multi-sphere” phantom, whose sinogram is shown in figure 2. Line integrals were calculated analytically using a simple length-of-intersection model, and assuming a mono-energetic spectrum. The phantom is made up of 220 spheres of random radius between 0.1 and 0.3mm, randomly positioned within a disc of radius 10mm. A uniform distribution was used for both the spheres’ radii and positions. This phantom was chosen since it gives results broadly representative of objects that are often imaged in micro-CT of materials.

The full dataset consists of 720 projections of length 512 pixels. Source-rotation axis distance was 50mm, rotation axis-detector distance was 30mm, and detector width was 40mm. Truncated data were generated from this by taking the central 128 pixels from each projection, giving a grid magnification factor of 4.

The FBP reconstruction shows a typical bright ring around the edge of the ROI, caused by the filter; this is also effectively removed by the dual grid iterative method.

Figure 3 compares the results of reconstructing the truncated simulated data with filtered back projection (FBP) (using the ASTRA toolbox [12]), and single and dual grid iterative reconstructions made with 40 iterations of the conjugate gradient least squares (CGLS) algorithm. Although not commonly used for CT reconstruction, CGLS was chosen due to its fast convergence in the sense of data fit, which highlights inconsistencies in the data. Due to the small size of the problem, numerical experiments were conducted in MATLAB by calculating the projection matrices explicitly using an implementation of Siddon’s algorithm.

Figure 4 plots the 2-norm of the image error for each iteration of the CGLS cases, with lines showing error values for FBP reconstructions of both full and truncated data. Additionally, figure 5 shows the full dual grid CGLS reconstruction, on both grids, compared to the full ground truth image.

Note that the single-grid CGLS reconstruction is almost unrecognisable; severe inconsistency of the data with the system of equations in this case results in large errors. Using the dual grid approach for iterative reconstruction removes this inconsistency and results in accurate reconstruction within the ROI, achieving a minimum error value around 40 iterations. The FBP reconstruction shows a typical bright ring around the edge of the ROI, caused by the filter; this is also effectively removed by the dual grid iterative method.

B. Simulated Data – Grid Magnification Factor 16

A second non-truncated, 2D noise-free dataset was generated for another “multi-sphere” phantom. Spheres are once again positioned randomly within a disc of radius 10mm; however, the size of the spheres is reduced by a factor of 4, and the number increased by a factor 16, giving an equivalent size and density of spheres when viewed at a $4 \times$ magnification. The sinogram for this dataset is shown in figure 6. Data were calculated in exactly the same way as for the previous example, using the same scanner geometry.

The full dataset this time consists of 720 projections of length 2048 pixels. Truncated data were generated from this
by taking the central 128 pixels from each projection, giving a grid magnification factor of 16.

Figure 7 compares the results of reconstructing the truncated simulated data with FBP, and single and dual grid iterative reconstructions made with 100 iterations of the CGLS algorithm. The same algorithms were used for reconstruction as for the first example.

Figure 8 plots the 2-norm of the image error for each iteration of the CGLS cases. Again, the error values are also given for FBP reconstructions of both the full and truncated data. As before, figure 9 shows additionally the full dual grid CGLS reconstruction, on both grids, compared to the full ground truth image.

In this case, the single-grid CGLS reconstruction is completely unrecognisable; the level of data inconsistency with the system of equations is so severe that the image error quickly grows out of control, as shown in the plot of figure 8. Using the dual grid approach for iterative reconstruction again removes the inconsistency and results in reasonably accurate reconstruction within the ROI, achieving a minimum error value in this case at 100 iterations. The FBP reconstruction again shows a typical bright ring around the edge of the ROI, though this is more severe in this case; this ring is also effectively removed by the dual grid iterative method.

C. Real Data

One of the main motivations for development of this work was to apply iterative reconstruction methods to imaging of semiconductor packages. For this use case, it is not uncommon to have an ROI of typically less than 1mm in a package size of the order several 10s of millimetres. We have successfully used
IV. Conclusions

We have presented a simple but effective method for dealing with the extreme interior tomography problem created by micro-CT scanning of certain objects of interest in materials science and non-destructive testing. The method allows iterative reconstruction to be performed in these cases, and has been demonstrated to work in practical problems where the diameter of the region of interest is up to a factor of 32 less than that of the object support.

REFERENCES