Infiltration Function from Furrow Stream Advance

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By William R. DeTar

ABSTRACT: Average infiltration rates for large land areas can be quickly determined using a new procedure for obtaining the coefficients of the Kostiakov equation. By introducing the concept of the average opportunity time it is possible to plot the infiltration function directly from tabulated data. This modification of the volume-balance method is a straightforward one, which is easy to understand and learn.

INTRODUCTION

Being able to determine infiltration rates in the field is not only important for the proper design of irrigation systems, but it is also an important research tool for characterizing the results of various soil treatments, such as compaction, erosion control, and watershed runoff studies. A procedure, which determines average infiltration rates over large areas, was presented by Christiansen et al. (1966). It is a volume-balance method in which the total volume of water entering a furrow is accounted for by the sum of that which infiltrates and that which is stored in the furrow. Christiansen et al. (1959) point out the obvious, that the higher the infiltration rate, the slower water will move down the furrow (all other factors being equal).

The procedure presented here is a modification of the Christiansen method; it produces essentially the same results with a more direct approach, and a computer is not required.

THEORETICAL DEVELOPMENT

The Kostiakov (1932) equation, one of several equations used to describe infiltration, is as follows:

\[ I_i = a t_i^m \]  

(1)

where \( I_i \) = the cumulative depth of water infiltrated, in \( m \), at any point, \( i \), along the furrow; \( t_o \) = is the opportunity time, i.e., the time, in minutes, any particular point along the furrow is exposed to the water; and \( a \) and \( m \) are experimentially determined parameters. Eq. 1 has been shown by many researchers (Christiansen et al. 1966; Criddle et al. 1956; Smerdon and Hohn 1961; Wilke and Smerden 1965; Fangmeier and Ramsey 1978; Norum and Gray 1970; Ohmes and Manges 1977; and Wu 1971) to fit data for furrows, especially during the short-term, early advance stage. It is said to be the least complex and most widely used equation for surface irrigation (Elliott and Eisenhauer 1983). Philip (1957) states that the Kostiakov function pro-

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vides an adequate description of infiltration, provided that the time frame is
not extremely large.

It is assumed by analogy that the equation
\[ \dot{I} = at^\nu \]  
holds true, where \( \dot{I} \) = the average cumulative depth of water infiltrated, in
\( m \), over any given length of furrow; and \( t_o \) = the average opportunity time,
in minutes, for any given length of furrow.

The primary goal is to find the parameters \( a \) and \( m \) in Eq. 2. They can
be obtained either graphically or by regression. The procedure that follows
concentrates on getting various values of \( \dot{I} \) and \( t_o \) for each furrow, so they
can be plotted. The first to be considered is \( \dot{I} \).

The volume balance method uses a continuity equation to account for all
water.
\[ V = QT - S \]  
where \( Q \) = the flow rate into the upper end of the furrow in \( m^3/\text{min} \) (as-
sumed constant); \( T \) = the length of time of flow, in minutes; \( V \) = the volume
of water infiltrated, in \( m^3 \); \( S \) = the volume of water in the furrow in \( m^3 \).

Dividing both sides of Eq. 3 by \( L \), the stream length, in m, at time \( T \),
and also by \( w \), a characteristic width of infiltration, in m, one gets
\[ \dot{I} = \frac{V}{Lw} = \frac{QT}{Lw} - \frac{S}{Lw} \]  
The terms on the right hand side of Eq. 4 can all be determined in the field,
at several different stages of stream advance, and thus \( \dot{I} \) is available.

The distance that the stream advances can be related to the advance time
by
\[ T = bL^n \]  
where \( b \) and \( n \) are experimentally determined parameters; \( T \) and \( L \) are defined
above. Forms of Eq. 5 have been used by many investigators, including

At any point along the stream, \( i \), a distance of \( x \), from the source, the
opportunity time is
\[ t_o = T - t_i \]  
where \( t_i \) = the time, in minutes, it took for the lead point of the stream to
reach point \( i \); and \( T \) = the total time the stream has been flowing, i.e., the
opportunity time is the length of time that water has been flowing past point
\( i \). The volume of water that has infiltrated over the full length of the stream is
\[ V = w \int_0^L I_i dx \]  
which becomes
\[ V = w \int_0^L at_o^n dx \]
when Eq. 1 is substituted into Eq. 7. Assuming $a$ is independent of $x$ and substituting Eq. 6 into Eq. 8, produces

$$V = wa \int_0^L (T - t_i) dx \hspace{1cm} (9)$$

Substituting Eq. 5 and its analogy,

$$t_i = bx_i^n \hspace{1cm} (10)$$

into Eq. 9, one gets

$$V = wab^m \int (L^m - x_i^n) dx \hspace{1cm} (11)$$

Up to this point, the analysis is similar to that used by Christiansen et al. (1966), and many others. The integral in Eq. 11 does not have an easy, direct solution. The exact solution can be obtained by numerical integration, or by using a Beta function (Christiansen et al. 1966). Approximations can be made by using the first two terms of a binomial expansion (Wu 1971), or the first four terms (Shull 1964), or by approximating a Beta function (Christiansen et al. 1966; Kiefer 1965). The main objective of this paper is to present a new, easy-to-use, procedure based on a simple, accurate approximation of the integral in Eq. 11 with the analysis that follows.

The average opportunity time can be determined by

$$\hat{t}_o = \frac{1}{L} \int_0^L t_o dx \hspace{1cm} (12)$$

which, when combined with Eq. 6, becomes

$$\hat{t}_o = \frac{1}{L} \int_0^L (T - t_i) dx \hspace{1cm} (13)$$

As before, one can substitute Eqs. 5 and 10 into Eq. 13, yielding

$$\hat{t}_o = \frac{1}{L} \int_0^L (bL^n - bx_i^n) dx \hspace{1cm} (14)$$

which, upon integrating and combining with Eq. 5, becomes

$$\hat{t}_o = \left( \frac{n}{n+1} \right) T \hspace{1cm} (15)$$

By definition

$$V = w \int_0^L I_o dx = wL\hat{t} \hspace{1cm} (16)$$

and combining Eqs. 2, 5, 15, and 16 yields

$$V = wab^m \left[ L^{mn+1} \left( \frac{n}{n+1} \right)^m \right] \hspace{1cm} (17)$$

To find the error in the assumptions for Eq. 2, the value of the term in the brackets in Eq. 17 can be compared to the numerical integration of the in-
integral in the Eq. 11. This was done for the normal range of \( n \) and \( m \) values, \((1.5 \leq n \leq 3.0, 0.3 \leq m \leq 0.7)\) and it was found that the value of Eq. 17 averaged 3\% more than that of Eq. 11. To compensate for this error, it was found that using

\[
i_o = 0.94 \left( \frac{n}{n+1} \right) T
\]

in place of Eq. 15 would cause the average error to be zero, and the maximum error is \( \pm 2\% \) at extreme values of the normal range of \( n \) and \( m \) values.

By comparison, Christiansen’s method (Christiansen et al. 1966), using Kiefer’s approximation (Kiefer 1965), produces cumulative depths of infiltration that are 0.75\% to 1.36\% higher than the actual value, and the average error is 0.9\%.

**EXPERIMENTAL APPLICATION**

Table 1 shows a complete example of the procedure. The first four columns of data used in this example are the same as that used in Christiansen et al. (1966) for Criddle’s (1956) furrow No. 3. The last column, \( i_o \), is new.

A log-log plot of \( \tilde{I} \) versus either \( T \) or \( i_o \) for many field tests reveals that the initial values are not consistent with the rest of the data. At \( T = 0 \), obviously \( \tilde{I} \) cannot possibly be less than zero, as shown in Table 1. There is a certain initial time period or distance required before a relatively stable stream size is established. From experience, it appears that the data for about the first one-fourth of the length of the furrow should not be used in any part of the analysis. The exact amount of initial data to discard is best determined from log-log plots of both \( T \) versus \( L \), and \( \tilde{I} \) versus \( i_o \); the incon-

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**TABLE 1. Example of Procedure Using Data from Christiansen et al. (1966) and Criddle et al. (1956), Furrow No. 3**

<table>
<thead>
<tr>
<th>( L ) (m)</th>
<th>( T ) (min)</th>
<th>( QT/Lw ) (m)</th>
<th>( \tilde{I} = (QT/Lw) - D_s ) (m)</th>
<th>( i_o = 0.94(n/(n+1))T ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.0031</td>
<td>0</td>
</tr>
<tr>
<td>30.5</td>
<td>13</td>
<td>0.091</td>
<td>-0.0060</td>
<td>8.00</td>
</tr>
<tr>
<td>61.0</td>
<td>42</td>
<td>0.0147</td>
<td>0.0116</td>
<td>25.86</td>
</tr>
<tr>
<td>91.4</td>
<td>90</td>
<td>0.0210</td>
<td>0.0179</td>
<td>55.42</td>
</tr>
<tr>
<td>121.9</td>
<td>158</td>
<td>0.0276</td>
<td>0.0245</td>
<td>97.29</td>
</tr>
<tr>
<td>152.4</td>
<td>239</td>
<td>0.0334</td>
<td>0.0303</td>
<td>147.2</td>
</tr>
<tr>
<td>182.9</td>
<td>333</td>
<td>0.0388</td>
<td>0.0357</td>
<td>205.0</td>
</tr>
<tr>
<td>213.4</td>
<td>457</td>
<td>0.0456</td>
<td>0.0425</td>
<td>281.4</td>
</tr>
</tbody>
</table>

Note: \( L \) = distance lead point of stream advances from beginning of furrow (m); \( T \) = time that advance was measured (min); \( Q \) = flow rate into furrow (m³/min) (in this example \( Q = 0.00227 \) m³/min); \( w \) = furrow spacing (m) (in this example \( w = 1.067 \) m); \( D_s \) = average depth of storage in furrow (m) = \( A/w \) where \( A \) is the average cross-sectional area of the stream (m²) (\( D_s = 0.00305 \) m in this example); \( \tilde{I} \) = average depth of water infiltrated over entire length of stream (m); \( n \) = the slope of the plot of \( i_o \) versus \( L \) on log-log paper (in this case \( n = 1.899 \)); and \( i_o \) = the average length of time that the soil is exposed to water (min).
sistent part of the initial data generally causes a curve, rather than a straight line. Hart et al. (1968) contains more detail on this early phase discrepancy.

The \( n \) required to calculate \( \hat{t}_o \) is obtained by regressing \( \ln T \) versus \( \ln L \) (omitting \( T = 13 \) data) with the result in this case being \( T = 0.0166 L^{1.899} \), so that \( n = 1.899 \), as seen in Fig. 1, \( (T \text{ is in minutes, } L \text{ in meters}) \). It is sufficiently accurate to plot \( T \) versus \( L \) on log-log paper, and determine the slope of the line graphically; the slope equals \( n \). Elliott and Walker (1982) show more details on fitting the advance function.

The final result is obtained by regression \( \ln \hat{I} \) versus \( \ln \hat{t}_o \) (or simply plotting \( \hat{I} \) versus \( \hat{t}_o \) on log-log paper as shown in Fig. 2). In this case the result is

\[ \hat{I} = 0.00203 \hat{t}_o^{0.541} \]  \hspace{1cm} (19)

with \( \hat{I} \) in meters and \( \hat{t}_o \) in minutes.

By Christiansen’s method the result is

\[ \hat{I} = 0.00206 \hat{t}_o^{0.541} \]  \hspace{1cm} (20)

Note that with Christiansen’s method, only the exponent of the Kostiakov equation is obtained from the plotted data. The coefficient must be determined by a somewhat complex calculation. With this new method, the final infiltration function is obtained directly from the plotting of \( \hat{I} \) versus \( \hat{t}_o \) on log-log paper.

Further verification of the stream advance method is found in Shull (1958), who measured infiltration in several furrows using a furrow bypass infiltro-
FIG. 2. Infiltration Function

meter; he ran stream advance tests in the same furrows, in a Holtville silty clay-loam soil. The cumulative depth of infiltration for 60, 120, and 240 min for each method in each furrow are shown in Table 2. Analysis of variance show that there is no significant difference in the two procedures for measuring infiltration.

Field data is also available in Merriam and Keller (1978) for a sandy loam soil. The stream advance method was applied to the data given for their medium-flow furrow, using an average stream cross section of 0.0056 m². The result is the infiltration function.

\[
\hat{I} = 0.00066 \, t_o^{0.729}
\]  

(21)

| Table 2. Comparison of Shull's (1958) Bypass Infiltrometer (BP) to Stream-Advance Method (SA): Cumulative Depth of Infiltration (m) |
|---|---|---|---|---|---|---|
| Furrow number | 60 min<sup>a</sup> | 120 min<sup>b</sup> | 240 min<sup>c</sup> |
| (1) | BP (2) | SA (3) | BP (4) | SA (5) | BP (6) | SA (7) |
| 3 | 0.0544 | 0.0496 | 0.0818 | 0.0758 | 0.1162 | 0.1155 |
| 4 | 0.0563 | 0.0469 | 0.0807 | 0.0685 | 0.1102 | 0.1000 |
| 5 | 0.0858 | 0.0646 | 0.1120 | 0.0988 | 0.1409 | 0.1511 |
| 6 | 0.0748 | 0.0734 | 0.0920 | 0.0952 | 0.1120 | 0.1228 |
| 7 | 0.0551 | 0.0455 | 0.0730 | 0.0677 | 0.0932 | 0.1008 |
| 8 | 0.0566 | 0.0582 | 0.0749 | 0.0827 | 0.0956 | 0.1175 |
| Average | 0.0638 | 0.0564 | 0.0857 | 0.0815 | 0.1114 | 0.1180 |

<sup>a</sup>LSD05 = 0.0085.
<sup>b</sup>LSD05 = 0.00870.
<sup>c</sup>LSD05 = 0.0115.
Direct field measurement of infiltration using the inflow-outflow technique along several reaches of the furrow produced

\[ \dot{i} = 0.000062 \ i_o^{0.72} \]  \hspace{2cm} (22)

**Analysis**

Inconsistencies can occur in the stream advance method, observed most prominently in the value of the exponent \( m \), which will occasionally fall outside the normal range of \( 0.25 < m < 1.0 \). Some of the causes are: (1) Nonuniformity of furrow; (2) soil crusting with large cracks; and (3) extremely small or large inflow. Unnecessarily precise measurements must be made of the stream cross section whenever the volume of water infiltrated is very small relative to the storage volume in the furrow. This problem is most easily observed whenever the exponent, \( n \), on the advance function becomes smaller than about 1.20. Most of the inconsistencies can be resolved by making slight adjustments in the storage volume. The proper amount of this adjustment can be determined methodically by using the infiltration function from the first go-round as the starting point for one of the “zero inertia” procedures for predicting stream advance, such as Hall (1956). By trial and error, the storage volume is adjusted until the predicted advance matches, as closely as possible, the measured advance.

In the field, many furrows can be handled at the same time with a small team. While two or three people mark the lead point of the streams, one person can measure the inflows and the stream cross sections.

On many soils the Kostiakov equation is only applicable during the initial stages of infiltration. To make the stream-advance procedure valid for long-term infiltration on most soils, a basic-rate term needs to be added to form a modified Kostiakov equation. The only change in the procedure is to subtract this basic-rate term from the \( \dot{i} \) column in Table 1, before plotting or regressing.

This latter method was verified using data from Bali and Wallander (1987), which gives both advance functions (predicted and active) and infiltration functions (from blocked-furrow tests) for a clay-loam soil. With an average stream cross section assumed to be 0.076 \( m^2 \), and a basis rate given as 0.000172 m/min, the resulting infiltration functions by the stream-advance method, is

\[ \dot{i} = 0.0102 \ i_o^{0.18} + 0.000172 \ i_o \]  \hspace{2cm} (23)

which, for \( 30 < i_o < 90 \) min, lies within 5% of the actual infiltration function.

**Conclusions**

A new, straightforward and accurate method is presented for obtaining the infiltration function from the stream-advance function in surface-irrigated plots. It is basically a modification of the Christiansen et al. (1966) volume-balance procedure that, with the introduction of the average opportunity time concept, provides a direct plot of the infiltration function.

**Appendix I. References**


**APPENDIX II. NOTATION**

The following symbols are used in this paper:

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\( a \) = coefficient in the infiltration functions \( \dot{I} = a \dot{t}^m \) and \( I_i = a t_i^m \);

\( b \) = coefficient in the stream-advance functions \( t_i = b x_i^n \), and \( T = b L^n \);

\( I_i \) = cumulative depth of water infiltrated at any point, \( i \), along the furrow;

\( \dot{I} \) = the average cumulative depth of water infiltrated over the length of stream;

\( L \) = the length of stream at time \( T \);

\( m \) = the exponent in the infiltration functions \( \dot{I} = a \dot{t}^m \), and \( I_i = a t_i^m \);

\( n \) = the exponent in the stream advance functions \( t_i = b x_i^n \), and \( T = b L^n \);

\( Q \) = flow rate into upper end of furrow;

\( S \) = the volume of water in the furrow at time, \( T \);

\( T \) = the time that water has been flowing into the furrow;

\( t_o \) = the opportunity time (the time any particular point along the furrow is exposed to water);

\( \dot{t}_o \) = the average opportunity time over any given length of stream;

\( t_i \) = the time it takes for the lead point of the stream to reach any point \( i \) along the length of the furrow;

\( V \) = the total volume of water infiltrated at time \( T \);

\( W \) = the characteristic width of infiltration;

\( X_i \) = the distance from the upper end of the furrow to any point of interest, \( i \), along the length of the furrow.