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Dynamic Indices of Building Thermal Performance

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ABSTRACT

Frequency transform and finite difference techniques are applied to a simple network developed using the equivalent thermal parameter (ETP) methodology. Subsequently a set of normalized parameter groups derived from the systems equations and solutions are discussed as indices of building thermal performance.

1. FORMULATION OF THE MODEL

The linear circuit analog used in the analysis is shown in Fig. 1. A simple model was sought which would facilitate analytic solutions and illustrate the thermally important relationships among parameters without great loss of accuracy. This particular network was adapted from those of the ETP methodology used by Kusuda on the NBS masonry building (1) and by Sonderegger on Princeton's Twin Rivers Project (2).

The building lossiness (HO), the mass-air coupling (HS), and the clamp coupling (HC) are the total conductances (BTU/HR/F) from the indoor air (TA) to the outside (TO), to the buildings equivalent thermal mass (TS), and to the isothermal clamps (TC), such as the ground or a party wall. The equivalent thermal mass (MC) is that building and/or storage mass which directly participates in the heat flows of the space. Sonderegger (3) and Goldstein (4) both discuss detailed methods for determining this quantity. However the thermal admittance method presented by Balcomb (5) is more amenable to hand calculations and was applied to the buildings examined in this paper. The other parameters, the quantities of sun striking the mass (QS) and heating the air (QA) are products of the buildings clear window area (AG) and the geometric distribution of transmitted radiation.

Writing energy balances for this network yields the systems equations of state.

\[
\begin{align*}
\frac{dTA}{dt} &= HO(TO-TA) + HS(TS-TA) \\
&\quad + HC(TC-TA) + QA \quad (1) \\
\frac{dTS}{dt} &= HS(TA-TS) + QS \\
\frac{dMC}{dt} &= MC(1 + (HO + HC)/HS) \\
&\quad \text{reduces the expression to a compact form.}
\end{align*}
\]

Figure 1. ETP Network

2. Solutions & Applications

Assuming the mass of the air, MCA, is negligible we may combine Eqs. 1 & 2. Expanding the solar gain terms, QA & QS, and defining an effective damping term, MC*=MC(1+(HO+HC)/HS), reduces the expression to a compact form.

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This expression directly relates the air temperature to the ambient driving forces in an easily understood form. It can be solved and implemented in a variety of ways.

A. Numerical Simulation
B. Regression Analysis
C. Frequency Transform Analysis
D. Derivation of Performance Indices

Since this network is roughly equivalent to PEGFIX/FLOAT (6), numerical simulation will not be discussed in this paper. The other three applications are treated individually.

2.1 Regression Analysis

Transformation of Eq. 3 to a difference expression produces an explicit expression for calculating the air temperature at discrete time intervals.

\[
TA_{t+1} = \frac{2 + \alpha}{2 + \alpha} TA_t + \frac{2 + \alpha}{2 + \alpha} \left[ \frac{HO + HC + HO + FS}{HT} \Delta T + \frac{AG + FA + MC^*}{HT} \Delta MC^* \right]
\]

\[
\alpha = \frac{(HO + HC) \Delta T}{MC^*}, \quad \Delta T = (T_{t+1} - T_t) / 2
\]

This is the form used for time domain simulation and regression analysis. To study the validity of the network model time series of performance and weather data from a monitored building are statistically regressed to obtain estimates for the coefficients of Eq. 4. If one ETP is known or an additional known parameter is introduced then the ETPs can be explicitly determined. In any case the statistical significance of the regressed coefficients and the tracking ability of the model using the regressed parameters are used to gauge the validity and appropriateness of the model. Kusuda and Sonderegger both had excellent results with this process, which confirms the accuracy of their simple models.

A validity study was conducted for this model on a freestanding greenhouse in Princeton NJ (7). Figs. 2 & 3 show the tracking ability of the simulation with regressed parameters however the significance of two coefficients was not as good. Subtle problems with monitoring equipment and regular condensation cycles inside the greenhouse introduced nonlinearities. Nonetheless the tracking is quite good and the expressiveness of the model compensates any loss of subtlety.

**Figure 2. Validation Plot 1**

**Figure 3. Validation Plot 2**

2.1 Frequency Transform Analysis

The transformation of Eq. 3 into the frequency domain allows us examine the response of the building to periodic driving functions.

\[
\simeq TA + HO + HC + HO + HT \Delta T + \frac{AG + FA}{HT} \Delta T + \frac{MC^*}{MC^*} \Delta MC^* \Delta S
\]

\[
\simeq \frac{AG}{MC^*} + \frac{HC}{MC^*}
\]

\(s\) is the Laplace Frequency variable and indicates a transformed variable.
Through some manipulation we can relate the transformed air temperature directly to the driving forces with three linear response functions.

\[
\frac{\Delta T}{\Delta \theta} = \frac{\text{AG} \cdot \text{FA} + \text{FS}}{\text{MC}^2 + \text{HC}} + \frac{\text{AG} \cdot \text{FA} + \text{HC}}{\text{HC}^2 + \text{HC}} + \frac{\text{AG} \cdot \text{FA}}{\text{MC}^2 + \text{MC}^2 + \text{HC}} \quad (6)
\]

Each response function produces a corresponding phase angle which describes the phase lag of the air temperature response.

\[
\Delta \theta_{\text{R-TO}} = \tan^{-1} \left( \frac{-\text{HS} \cdot \text{MC}}{\text{HT} \cdot \text{MC}^2 + \text{HC}} \right)
\]

\[
\Delta \theta_{\text{R-S}} = \tan^{-1} \left( \frac{\text{AG} \cdot \text{FA} \cdot \text{MC}^2}{\text{AG} \cdot \text{FA} \cdot \text{MC}^2 + \text{HC}} \right)
\]

In Table 1 are listed parameters for a variety of buildings. With the exception of those obtained by regression from the Princeton Greenhouse and Twin Rivers Project they were calculated from information available in the literature. The magnitude and phase angle of R-TO and R-S for each building are plotted below in Figs. 4 & 5.

We are primarily interested in oscillation periods of a few hours to a few weeks since this model does not address seasonal adaptations. There are three parameters which determine the shape of the plots in this range. The first are the high frequency plateaus defining the minimum response of the building to that variable. Taking the limit of the response functions we find that they are the coefficients of the differentiated temperature and solar flux.

\[
R_{\text{TO}} = \frac{\text{HO} \cdot \text{HS} + \text{HC}}{\text{MC}^2 + \text{HC}} \quad R_{\text{S}} = \frac{\text{AG} \cdot \text{FA}}{\text{MC}^2 + \text{MC}^2 + \text{HC}} \quad (9)
\]

The 'corner' where the magnitude of the response begins to increase is defined by the pole of the system, the root of the denominator function, \((\text{HO} + \text{HC}) / \text{MC}^2\). This is the system's natural frequency (indicated in Fig. 4 with black dots) below which the building response rapidly increases. Its inverse is the thermal time constant and a measure of the rate at which the building changes temperature.

Inspection of Figs. 4 & 5 illustrates the thermal character of the various building types. Hamil, SCAP, and Prince T & R (Theory and Regression) are all greenhouses with large areas of glazing, high responses to TO and S, and short time lags (20 min to 2 hour) to diurnal oscillations. Hamil is the extreme of this because it is an attached greenhouse designed to provide heat for an old farmhouse.

The three residences, Parsons, LO-CAL, and the Twin Rivers Townhouse all have lower responses and longer diurnal time lags (4-5 hours). It is striking to note the

<table>
<thead>
<tr>
<th>Table 1. Building Parameters</th>
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<tbody>
<tr>
<td>ETP</td>
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<tr>
<td>HO+HC</td>
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<tr>
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</table>

NOTE: Parameters tabulated with inverse beneath
similarity of their responses to TO in the high frequency region, though each building was the product of a vastly different design philosophy. In the Parsons residence the glazing area is 61% of the floor area while in the LO-CAL design only 8% is south glass. Their responses to S are also similar because this is a function of the distribution of sun and the building mass not just the quantity of glazing. These preliminary results indicate that comfortable buildings from all design philosophies share similar characteristics.

2.3 Performance Indices

Even without frequency analysis or time domain studies Eq. 4 contains a great deal of information about the buildings thermal character. The ETPs are assembled in six normalized groups succinctly describing their inter-relationship and their combined effect in building performance. By their normalization they are freed from the dimensions of a particular building and lend themselves to comparisons (eg SCAP is 22 times larger than Hamil). By their derivation they have physical significance and can be used as indices to dynamic performance or as sources for rules of thumb.

The network's natural frequency together with the coefficients \( H_0/(H_0+H_S+H_C) \) & \( AG/(A+H_S+H_C) \) were seen to comprise the building response to the most common weather frequencies. \( AG/(A+P_B)/MC* \) gauges the buildings overheating potential by relating solar admittance to available damping. \( H_0/MC* \) and \( HC/MC* \) are components of the natural frequency, expressing the participation of the ambient and clamp temperatures respectively.

As performance indices they are inextricably inter-connected. For example Hamil has the longest thermal time constant yet the highest response magnitude and shortest time lag because the mass-air coupling is relatively small, isolating the mass. Each of the seven ETPs is bound by its role in the network. No one parameter group solely determines the building behavior. With an awareness of these relationships and the indices to gauge them a designer can obtain a great deal of information before involving any simulations. The values in Table 1 are an indication
3. CONCLUSION

The real performance of buildings is complicated and nonlinear. Furnaces, internal gains, night insulation, and thermostatic setbacks all act to invalidate the direct application of this simple linear analysis. However, at the heart of any building there is a fundamental thermal nature which underlies all else. This dynamic character determines the heating & cooling response rates, the temperature swings, and ultimately the energy consumption of the building. Dynamic performance indices can be used as gauges of this fundamental response and as guidelines for appropriate design.

4. NOMENCLATURE

- AG: Clear area of glazing (SQ FT)
- FA: Fraction of sun heating air
- FS: Fraction of sun striking mass
- HC: Clamp-Air coupling (BTU/F/HR)
- HO: Building Lossiness (BTU/F/HR)
- HS: Mass-Air coupling (BTU/F/HR)
- HT: HO+HS+HC (BTU/F/HR)
- MC: Equivalent thermal mass (BTU/F)
- MC*: Effective thermal mass (BTU/F)
- QA: Solar gain to air (BTU/HR)
- QS: Solar gain to mass (BTU/HR)
- R-TO: Response function to TO
- R-S: Response function to S
- s: Laplace variable (Radians/HR)
- S: Solar flux (BTU/SQ FT/HR)
- TA: Indoor air temperature (F)
- TC: Clamp temperature (F)
- TO: Outdoor air temperature (F)
- TS: Mass temperature (F)

5. REFERENCES

(2) R. Sonderegger, "Dynamic Models of House Heating Based on Equivalent Thermal Parameters", Princeton Univ.: Center for Environmental Studies, (Sept. 1977)
(3) Sonderegger, Chapter IV.