1959

Development of a Recording Volumetric Transducer for Studying Effects of Soil Parameters on Compaction

J. D. Hovanesian, *Michigan State University*
Wesley F. Buchele, *Michigan State University*

Development of a Recording Volumetric Transducer for Studying Effects of Soil Parameters on Compaction

J. D. Hovanesian and W. F. Buchele
Assoc. Member ASAE Member ASAE

INCREASED research effort has been directed toward the solution of soil compaction problems in the past few years. Research workers dealing with these problems have been handicapped for at least the following four reasons:

1 Lack of definition and understanding of soil stress and soil compaction; a need for a suitable mathematical model

2 Lack of suitable recording equipment for measuring changes in soil physical phenomena, particularly soil compaction

3 Lack of basic engineering information about the mechanics of agricultural soils

4 Lack of adequate information showing the effect of certain parameters (soil moisture, soil type, and others) upon soil properties.

The investigation reported in this paper has made contributions under points 2, 3, and 4 above. Another research worker (5)* has made contributions under point 1.

VandenBerg (5) suggests the use of the model of a continuous media for the description of soil physical phenomena. With this model the description of soil stress requires a set of quantities in the form of a stress tensor, rather than a single value. He showed that changes in the scaler component of the stress tensor (change in mean stress) is simply related to changes in bulk density. Changes in bulk density is a measure of soil volumetric strain often called "soil compaction."

Cooper and VandenBerg (1) have developed a transducer which is capable of measuring a single vectorial component of soil stress. In this work, an instrument was developed which is capable of continuously measuring and recording volume changes in soil. This instrument, known as the recording volumetric transducer can measure volume changes as small as 0.01 ml (milliliter) at a point in soil media.

Paper prepared expressly for publication by the American Society of Agricultural Engineers. Publication authorized as Journal Article No. 2367 of the Michigan Agricultural Experiment Station, East Lansing.

The authors—J. D. HOVANESIAN and W. F. BUCHELE—are, respectively, assistant professor of agricultural engineering, Pennsylvania State University, and associate professor of agricultural engineering, Michigan State University.

Acknowledgment: The authors express their appreciation of the helpful suggestions of K. V. Lask, G. E. VandenBerg, and H. F. McColly in connection with the research project reported in this paper.

* Numbers in parentheses refer to appended references.

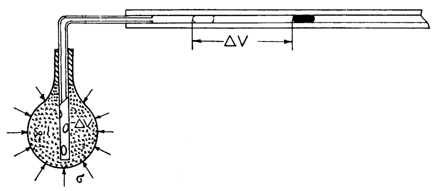

FIG. 1 Indicating volumeter. (A volume change ΔV resulting in a soil sample due to an applied load σ is indicated by displacement of a mercury droplet in a calibrated capillary tube).

Development of the Recording Volumetric Transducer and Indicating Volumeter

The indicating volumeter (Fig. 1) consists of a capillary tube connected to a spherical-shaped rubber balloon (approximately 3 cm in diameter). A non-collapsible plastic tubing is located between and connects the capillary tube and balloon. A droplet of mercury is placed in the capillary tube and acts as a movable piston in the tube. When the balloon, filled with soil, is surrounded with a larger quantity of the same type soil, the encompassed minute volume of soil in the balloon represents a point. When the soil medium is subjected to stresses, the change in volume of the point mass is easily determined by displacement of the mercury droplet in the capillary tube since the volume/linear unit of displacement of the capillary tube is known.

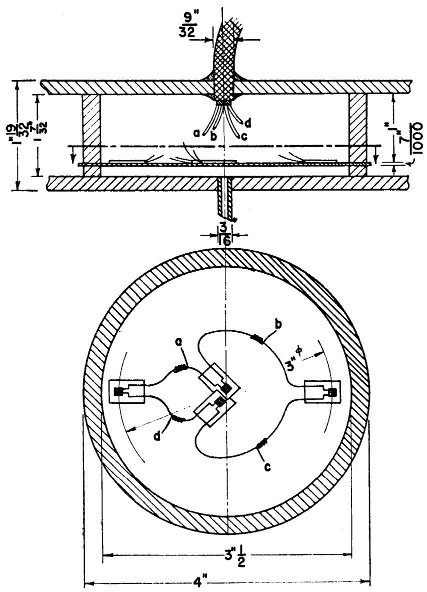

FIG. 2 Volumetric transducer. (Strain gages measure deflection of diaphragm which results from volume changes in soil).

While tests indicated that the indicating volumeter would accurately measure changes in bulk density, its use was limited to static loadings.

The recording volumetric transducer (Fig. 2) can measure and record dynamic changes and is more accurate and sensitive than the volumeter. It is used with conventional strain-gage equipment. The instrument was constructed in two models, the first being built with a 0.04-in. thick plexiglas diaphragm and the second with a 0.007-in. thick stainless steel diaphragm. Other portions of the transducer were constructed from plexiglas material.

Basic Principles of Transducer

Fig. 3 is a skeleton sketch of the volumetric transducer, soil sample and calibration means. The basis of operation may be analyzed as follows:

$$\Delta P_i = \frac{dP_i}{dV_i}\Delta V_i + \frac{dP_i}{dT_i}\Delta T_i \quad \ldots\ldots i = 1,2 \ldots\ldots \quad [1]$$

Note: d symbolizes partial derivatives.

and if $\Delta T_1 = \Delta T_2 = 0$, and $\Delta V_2 = 0$,

$$\text{then } \Delta P_{net} = \Delta P_1 - \Delta P_2 = \frac{P_1}{V_1}\Delta V_1 \quad \ldots\ldots\ldots\ldots \quad [2a]$$

When

$\Delta V_2 \cong 0$, and $\Delta T_1 = \Delta T_2 = \neq 0$

$$\text{then } \Delta P_{net} = \frac{P_1}{V_1}\Delta V_1 - R\left(\frac{N_2}{V_2} - \frac{N_1}{V_1}\right)\Delta T \quad \ldots\ldots\ldots\ldots \quad [3]$$

But if $P_1 = P_2$, then $\frac{N_2}{V_2} = \frac{N_1}{V_1}$

since $T_1 = T_2$

$$\text{Thus } \Delta P_{net} = \frac{P_1}{V_1}\Delta V_1, \text{ or } \Delta V_1 = \frac{V_1}{P_1}\Delta P_{net} \quad \ldots \quad [2b]$$

where

ΔP_1, ΔP_2, (See Fig. 3) are partial pressures on each side of diaphragm

ΔT_1, ΔT_2 are temperature changes on either side of diaphragm

ΔV_2, ΔV_1 are volume changes on either side of diaphragm

R is the universal gas constant

N_1, N_2 are the number of gas molecules on either side of diaphragm.

The above derivations show a simple linear relationship between ΔV_1 and

ΔP_{net}. Since ΔP_{net} is easily and accurately measurable with the transducer of Fig. 3, ΔV_1, the unknown quantity, is easily determined. Calibration is obtained directly in the following manner:

1 Soil sample (encompassed in balloon) is subjected to preload value, σ_o.

2 A calibration volume ΔV_c is imposed into the system by means of an accurately calibrated pipette.

3 The gain on the amplifier is adjusted so that the recorder pen deflection is some convenient multiple of ΔV_c of step 2 above.

In practice ΔV_c was equal to 1 ml and the corresponding pen deflection was adjusted to 10 lines; thus each line represented 0.1 ml volume change. As straightforward as the above calibration means seems, a still easier and simpler method can be used if a series of consecutive tests are to be run. The steps of the simpler calibration for a series of tests are as follows:

1 Make the first calibration as outlined in the above steps.

2 Depress the calibration switch on the strain-gage amplifier. (This places a shunt resistance into the circuit with one of the strain gages.)

3 Observe what pen deflection occurs as a result of step 2.

4 For subsequent calibrations, adjust the amplifier gain such that the same pen deflection (observed in step 3) occurs as a result of depressing the calibration switch.

In order to minimize calibration errors in using the latter simplified method, it is necessary to add a volume enlargement as shown in Fig. 3 to the system to make V_1 sufficiently large.

If δV_1†, the air volume variation expected between soil samples is known (usually much less than 4 ml), and it is desirable to keep the error in ΔP, i.e., $\delta(\Delta P_1)/\Delta P_1$ less than 1 percent; then the required volume of V_1 can be determined by the following error analysis: To find the error in ΔP, $\delta(\Delta P_1)/\Delta P_1$ resulting from variations in ΔV_1, (δV_1), one must take the partial derivative of equation [2b] considering ΔP as the dependent variable and V_1 the independent variable. To put the result in proper form, it is convenient to take first the logarithm of both sides of equation [2b] to get:

† ΔV_1 is a change in V_1 resulting from an applied load. δV_1 is variations in V_1, existing between different soil samples.

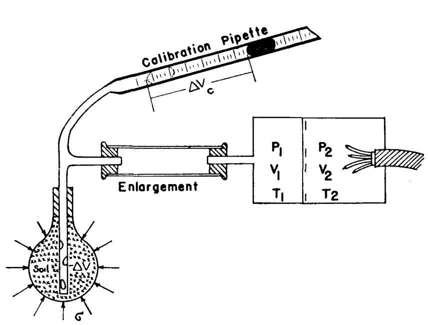

FIG. 3 Skeleton sketch of volumetric transducer, soil sample and calibration means.

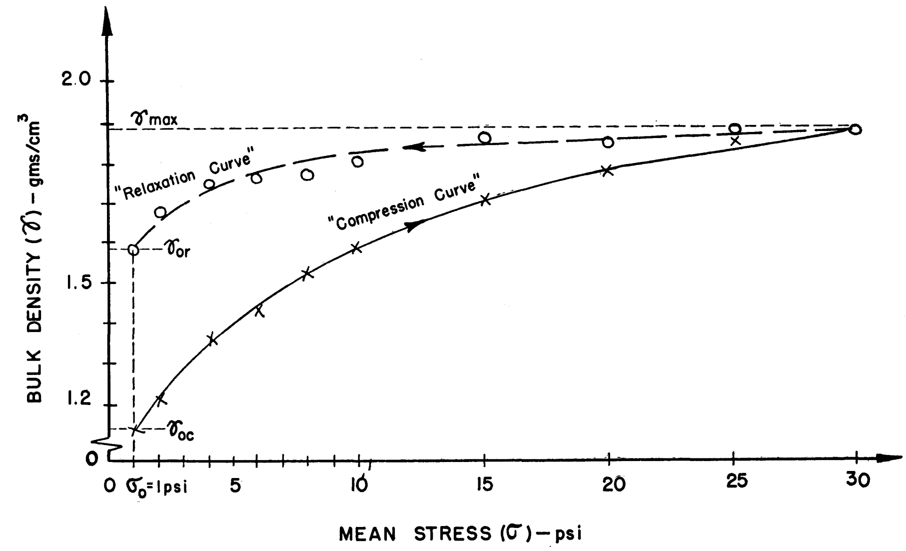

FIG. 4 A typical bulk density γ versus mean-stress σ relationship for agricultural soils. NOTE: The function changes when the load is relieved, thus resulting in a permanent strain.

$$\ln \Delta P = -\ln V_1 + \ln P_1 + \ln \Delta V_1 \quad \ldots\ldots\ldots\ldots [2c]$$

By differentiating both sides of equation [2c] and considering variations in ΔP and V_1, the following equations are obtained:

$$\left| \pm \frac{\delta(\Delta P)}{\Delta P} \right| = \left| - \frac{\delta V_1}{V_1} \right| \leqq 0.01 \quad \ldots\ldots\ldots\ldots [2e]$$

$$\text{or } V_1 \geqq \frac{\delta(V_1)}{0.01} = \frac{4}{0.01} = 400 \text{ ml.} \quad [2f]$$

If changes in V_1 (δV_1) are expected to exceed 4 ml, a larger V_1 can be selected by the above means.

Laboratory Technique for Studying Effects of Parameters upon Soil Compaction

Data were obtained with the recording volumetric transducer described previously. Various loads (mean-stress states, σ) were applied to small samples of soil (approximately 15 grams) by means of hydrostatic pressure. Mean-stress values were varied from 1 to 30 psi by using a pressure-regulator valve. A calibrated Bourdon tube and a mercury manometer were used to indicate hydrostatic pressures (mean - stress states).

The loads on a soil sample were repeated several times during one phase of the experiment. The mean-stress values were gradually applied and varied from 1 to 30 psi, and then back to 1 psi. These repeated cycles permitted examination of the effects of repeated loadings on soil compaction. The repeated loading tests presented a minimum of difficulty since the resulting volume changes due to load variations were permanently recorded for every cycle.

Another phase of tests dealt with repeated impact loads. Application of repeated impact loads (mean stress) presented more difficulty and it was accomplished in the following manner:

A constant preload, σ_o, was subjected to the soil sample. The preload was applied by means of a water head equivalent to (1 psi). The water column was held in place by a vertical tube. The top of the tube was subjected to a rapidly applied and released load in the form of air pressure. The preload σ_o of 1 psi was maintained before and after the impact application of compressed air. The magnitude of the maximum mean stress applied in this way was equal to 27 psi. The time period of impact-load application was approximately one second. This one second period closely approximated the load-application period of field implements.

Analysis of Data

Fig. 4 shows a typical bulk density, γ, versus mean stress, σ, function for agricultural soils. All soils, except those containing a considerable amount of silt, plot as a straight line on semilog paper ($\ln \sigma$ versus γ). With silty soils, a straight line is attained on semilog paper when σ is adjusted as follows: $(\sigma/\sigma_o + K)/(1 + K)$, where K is a constant and σ_o is the initial mean-stress state (preloaded to 1 psi in these tests).

The γ versus σ function complied with a different relationship for increasing σ values from σ_o to σ_{max}, than if the values of σ were decreased from σ_{max} to σ_o. This can be expected because of the change of γ with an initial application of σ from σ_o to σ_{max} which results in a permanent soil strain, thus modifying the σ versus γ relationship. Fig. 4 shows two functions of σ versus γ and demonstrates the expected phenomenon. It will be noted that the two functions are of the "same type" because both plot straight lines on semilog paper. The two functions can be expressed by the same type equation, but with different constants.

The following mathematical equation was empirically developed and relates bulk density, γ, and mean stress, σ, if initial conditions and "soil parameters" are known:

$$\gamma = \gamma_o + \mathbf{B} \ln\left[\frac{(\sigma/\sigma_o + \mathbf{K})}{(1 + \mathbf{K})}\right] \ldots [4]$$

in which σ_o is the initial mean-stress state, γ_o is the initial bulk density (before loading) and **K**, a parametric constant depending on soil parameters, and **B** is a second parametric constant.

As previously discussed, this relationship in equation [4] is valid for both soil "compression" (σ increasing) or for soil "relaxation" (σ decreasing). The authors reason that the constants in equation [4] take different values because permanent strain occurs as a result of a load application to the soil. The same general equation holds because the physical phenomena prescribing the γ versus σ relationship in compression also prescribes the relationship in relaxation. This relationship usually can be written in the form:

$$\gamma = \gamma_o + \mathbf{B} \ln (\sigma/\sigma_o) \;.\;.\;.\;.\;.\; [4a]$$

This simplification is possible since the parametric constant **K** is most often, except in the case of some silt soils as discussed above, equal to zero. Equations [4] and [4a] are in conformity with laboratory tests performed with three extreme textural types of soil at various moisture contents. Moisture content varied from an air-dried condition up to the lower plastic limit. The initial bulk densities were typical of actual field values.

When considering only gradually applied loads, and if mean stress values are subjected to the soil sample in several cycles, the following result was noted: the initial application of σ from σ_o to σ_{max} resulted in a σ versus γ relationship in compression as described above. Upon relieving of σ from σ_{max} to σ_o, the γ versus σ function varied along the relaxation curve as shown in Fig. 4. Subsequent applications and relief of a gradually applied load caused the γ versus σ function to vary along the relaxation curve, seemingly independent of the number of times that the cycle was repeated.

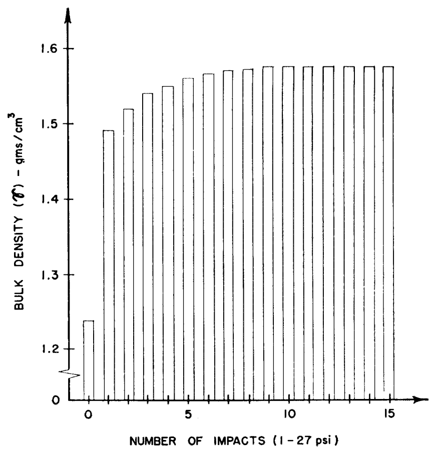

FIG. 5 Repeated impact applications of mean stress σ versus bulk density γ for 20 percent moisture content silty loam soil. NOTE: About 70 percent of the final bulk density obtained with twelve or more impacts, was attained with the first cycle.

Repeated impact test data was secured to determine qualitatively how soils behave under impact loading conditions. Figs. 5 and 6 show how bulk density γ varied as impact loads (approximately from 1 to 27 psi) were repeated. It can be seen from these bar graphs that from 70 to 90 percent of the final bulk density occurred during the first impact application. Subsequent changes in bulk density diminished rapidly as the number of repetitions were increased.

With the two soil samples tested, no additional increase in bulk density was noted when the number of repetitions exceeded fifteen. Further, the final bulk density attained as a result of fifteen or more impacts approached that which would result from a gradually applied load from 1 to 27 psi and relieved back to 1 psi. Thus, an impact load created only 70 to 90 per cent of the volumetric strain expected from a gradually applied and relieved load. It should be further emphasized that when repeated loads are gradually applied, the final bulk density occurred with the first load application. Further repetition of a gradually applied load caused no further changes in bulk density. Stated another way, beyond the first load application, the σ versus γ function varies along the "relaxation" curve as previously described.

Adaptation of Equations Relating σ and γ to Field Problems and Special Uses

The foregoing discussion shows that σ and γ can be related by means of a mathematical equation if initial conditions and parametric constants of a particular soil are known. More specifically, it can be said that the same general mathematical equation, equation [4], will be applicable to virtually all tillable soils. Other investigations (3, 4) reported a similar relationship with soils in both civil engineering and agricultural engineering work. The general mathematical equation was also satisfied by other data collected by VandenBerg (5) and Hovanesian (2).

It is known that

$$\gamma_c = \gamma_{oc} + \mathbf{B}_c \ln [(\sigma/\sigma_o + \mathbf{K}_c)/(1 + \mathbf{K}_c)] \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; [5a]$$

and

$$\gamma_r = \gamma_{or} + \mathbf{B}_r \ln [(\sigma/\sigma_o + \mathbf{K}_r)/ (1 + \mathbf{K}_r)] \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; [5b]$$

where $\mathbf{B}_c$, $\mathbf{B}_r$ are parametric constants during compression **c** or relaxation **r**.

γ_{oc} is bulk density at σ_o on the compression curve

γ_{or} is bulk density at σ_o on the relaxation curve

σ_{max} is final (max) mean-stress application

γ_{max} is final (max) bulk density resulting from σ_{max}

$\mathbf{K}_c$, $\mathbf{K}_r$ are parametric constants during compression **c** or relaxation **r**.

Equations [5a] and [5b] of the above have been adapted for the following cases:

Case 1: To find final bulk-density change $\Delta\gamma$ occurring in a soil due to a load application σ_o to σ_{max} then the load relieved back to σ_o:

$$\Delta\gamma = (\gamma_{or} - \gamma_{oc}) = \mathbf{B}_c \ln\left[\frac{(\sigma_{max}/\sigma_o + \mathbf{K}_c)}{(1 + \mathbf{K}_c)} \left(\frac{1 + \mathbf{K}_r}{\sigma_{max}/\sigma_o + \mathbf{K}_r}\right)^{\mathbf{B}_r/\mathbf{B}_c}\right] \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; [6]$$

If $\mathbf{K}_c = \mathbf{K}_r = 0$,
then $\Delta\gamma = (\mathbf{B}_r - \mathbf{B}_c) \ln (\sigma_{max}/\sigma_o)$ . . . . [7]

Case 2: γ_r in terms of σ when γ_{or} is not known

$$\gamma_r = \gamma_{oc} - \mathbf{B}_c \ln [(\sigma_{max}/\sigma_o + \mathbf{K}_c)/(1 + \mathbf{K}_c)] + \mathbf{B}_r \ln [(\sigma/\sigma_o + \mathbf{K}_r)/(\sigma_{max}/\sigma_o + \mathbf{K}_r)] \;.\;.\;.\; [8]$$

Case 3: Determination of σ_{max} for a prescribed $\Delta\gamma/\gamma_{oc} = \mathbf{j}$ and when $\mathbf{K}_r = \mathbf{K}_c = 0$

$$\sigma_{max} = \sigma_o \, \mathbf{e}^{\mathbf{j}\,\gamma_{oc}/(\mathbf{B}_c - \mathbf{B}_r)} \;.\;.\;.\; [9]$$

Problems to which the revised equations of cases 1 through 3 can be applied will be at once realized. For example, equations [6] and [7] of Case 1 can be used to predict a bulk-density change resulting from a change in mean-stress $\Delta\sigma = (\sigma_{max} - \sigma_o)$ that may be caused by implement traffic (See example 1).

Equation [8] of case 2 will enable one to predict the new bulk density of the soil resulting from an applied load not totally relieved to its initial value σ; thus equation [8] is particularly useful under this circumstance.

Equation [9] is a useful relationship which can be used to prescribe σ_{max} for certain permissible values of $\Delta\gamma/\gamma_{oc}$. For example, if one knows the critical value $\Delta\gamma/\gamma_{oc}$ for a given soil, the resulting critical value of σ_{max} may be determined for that particular soil. This critical value of σ_{max} can consequently be used as a guide for implement selection and/or design, or both (See example 2).

Examples of Adapted Quations

Example 1

Known: $\sigma_o = 1$ psi, $\sigma_{max} = 20$ psi (then relieved to 1 psi)
$\mathbf{B}_c = 0.098$, $K_c = 2$, $\mathbf{K}_r = -0.6$, $\gamma_{oc} = 1.17$, $\mathbf{B}_r = 0.023$

Required: $\Delta\gamma$ resulting from a load application from 1 to 20 psi, then relieved to 1 psi

Solution: From equation [6]

$$\Delta\gamma = 0.098 \ln \left[\left(\frac{20+2}{3}\right) \left(\frac{1-0.6}{20-0.6}\right)^{0.023/0.098} \right]$$

= 0.11 grams per cubic centimeter

Example 2

Known: $\sigma_o = 1$ psi, $\mathbf{B}_c = 0.138$, $\mathbf{B}_r = 0.009$, $\mathbf{K}_c = \mathbf{K}_r = 0$
$\gamma_{oc} = 1.48$

Required: σ_{max} such that $\Delta\gamma/\gamma_{oc} \leqq 0.2$

Solution: From equation [9]

$$\sigma_{max} \leqq \mathbf{e}^{\ 0.2(1.48)/(0.138-0.009)}$$

$\leqq$ 10 pounds per square inch

Summary

In this study, the relationship between soil bulk density and mean stress was investigated. Since mean stress is a measure of soil strain and it is related to bulk-density change in soil, the relationship between bulk density γ and mean stress σ was carefully studied. Several tests were run with tillable soils by applying known mean stress values (by means of hydrostatic pressure) to small samples of soil and observing the accompanying bulk-density changes. General mathematical equations [4] and [4a] were developed empirically. These equations depend on knowledge of initial conditions and parametric constants. When these constants and initial conditions are known, this relationship can be used to accurately predict bulk-density changes resulting from certain soil load applications.

The instrumentation problem of accurate specific-volume measurement at a point in the soil was solved by development of the volumetric transducer.

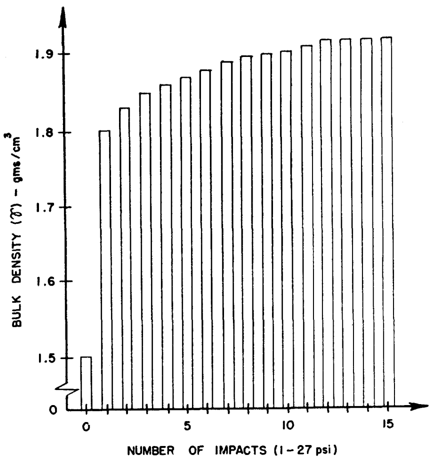

FIG. 6 Repeated impact applications of mean-stress σ versus bulk density γ for 8 percent moisture content sandy loam soil. NOTE: About 90 percent of the final bulk density obtained with 12 to 15 impacts or more was attained with the first impact.

Conclusions

1 The volumetric transducer successfully measures specific volume changes in soil at a point.

2 Temperature variations in the transducer system will not cause errors in readings provided that initial pressures are equal on both sides of the sensing diaphragm.

3 The calibration of the transducer is independent of soil-sample size, provided that a sufficiently large enlargement is added in series with the balloon.

4 With tillable soils, bulk-density γ is related to mean stress σ by the following general formula:

$$\gamma = \gamma_o + \mathbf{B} \ln\left(\frac{(\sigma/\sigma_o + \mathbf{K})}{(1+\mathbf{K})}\right)$$

5 Soil will become permanently strained; that is, the bulk-density, γ, will increase as a result of a load application and load release. With knowledge of soil parametric constants and initial conditions, this permanent strain can be predicted for a given mean-stress application and release.

6 For a given mean-stress value, impact loads will cause less soil strain than that created by a gradually applied and released load.

7 Fifteen or more repeated impact loads of a given mean-stress value will cause the same soil strain attained with a gradually applied and released load.

8 The bulk density versus mean-stress function will obey the relaxation formulas if the load is repeated after permanent deformation has occurred.

References

1 Cooper, Arthur. Investigation of and instrumentation for measuring pressure distribution in soil. Unpublished Ph.D. thesis, department of agricultural engineering, Michigan State University, 1956.

2 Hovanesian, J. D. Unpublished research data, department of agricultural engineering, Michigan State University, 1957.

3 Soehne, Walter. Fundamentals of pressure distribution and soil compaction under tractor tires, **Agricultural Engineering,** May, 1958.

4 Spangler, M. G. Soil engineering, International Textbook Co., 18:265-279, 1951.

5 VandenBerg, Glen. Application of continuous mechanics to compaction in tillable soils. Unpublished Ph.D. thesis, department of agricultural engineering, Michigan State University, 1958.