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Bright Lines or Blurry Labels

Walter Effross



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**BRIGHT LINES OR BLURRY LABELS?:
INTERPRETING THE "FUZZY LOGIC" OF BANKRUPTCY LAW**

WALTER A. EFFROSS*

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* © Walter A. Effross 1998. All rights reserved. Associate Professor, Washington College of Law, American University. This Article is a revised version of a paper and program materials presented by the author at the Southwest Bankruptcy Law Institute in Dallas, Texas. The Article is dedicated to the memory of Professor Barry Zaretsky, colleague and friend.

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[U]nlike mathematical symbols, the phrasing of a document, especially a complicated enactment, seldom attains more than approximate precision . . . Apart from the ambiguity inherent in its symbols, a statute suffers from dubieties. It is not an equation or a formula representing a clearly marked process. . . .

—Justice Felix Frankfurter, 1947¹

[T]he theoretical foundation of [fuzzy logic] is actually quite precise and rather mathematical in spirit. . . . In effect, the level of precision in a particular application can be adjusted to fit the needs of the task and the accuracy of the available data.

—Professor Lotfi Zadeh, 1973²

The emerging mathematical discipline of "fuzzy logic" has recently sparked dramatic efficiencies in modern industrial processes while inviting comparisons to the age-old philosophies of Taoism and Zen. As concepts of "fuzziness" have filtered from technical journals³ to popular science magazines⁴ to trade paperbacks,⁵ they have begun to appear in legal commentaries as well.⁶

Though this unique mathematical method reflects and reduces the ambiguity in definitions and rules, the few discussions of its implications for the legal process

¹ Felix Frankfurter, *Some Reflections on the Reading of Statutes*, 47 COLUM. L. REV. 527, 528 (1947).

² Lotfi A. Zadeh, *Outline of a New Approach to the Analysis of Complex Systems and Decision Processes*, *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. SMC-3, No. 1, 28-44, 30 (1973).

³ See, e.g., *infra* at notes 12, 13, & 47 (demonstrating extensive application of fuzzy logic).

⁴ See, e.g., Bart Kosko and Satoru Isaka, *Fuzzy Logic - The binary logic of modern computers often falls short when describing the vagueness of the real world. Fuzzy logic offers more graceful alternatives*, 269 SCI. AM. 76, 76 (1993) (listing everyday examples where Fuzzy Control Theory is used). The authors state, "[I]n many fields [one] may find that fuzzy common sense models are more useful or accurate than are standard mathematic ones." *Id.*

⁵ See BART KOSKO, *FUZZY THINKING: THE NEW SCIENCE OF FUZZY LOGIC* XVI (1993) (stating that "fuzzy world even extended as much to culture and philosophy as it did to science and math."); DANIEL MCNEILL & PAUL FREIBERGER, *FUZZY LOGIC: THE REVOLUTIONARY COMPUTER TECHNOLOGY THAT IS CHANGING OUR WORLD* (1993) (surveying applications of fuzzy logic theory).

⁶ See, e.g., Jack F. Williams, *The Fallacies of Contemporary Fraudulent Transfer Models as Applied to Intercorporate Guaranties: Fraudulent Transfer Law as a Fuzzy System*, 15 CARDOZO L. REV. 1403, 1406 (1994) (noting that fuzzy logic offers "more graceful alternative in which to view, analyze, and speak about fraudulent transfer law."); Leif M. Clark, *Fuzzy Thinking and Legislating Logically*, 12 AM. BANKR. INST. J. 14, 14 (1994) (discussing effects of fuzzy logic on bankruptcy legislation); Leif M. Clark, *Some Fuzzy Thoughts*, 11 AM. BANKR. INST. J. 14, 14 (1993) (contemplating role of fuzzy logic in evolution of bankruptcy law); Charles M. Yablon, *On the Allocation of Burdens of Proof in Corporate Law: An Essay on Fairness and Fuzzy Sets*, 13 CARDOZO L. REV. 497, 497 (1991) (discussing effect of fuzzy logic on burden of proof allocating theory); David A. Schum, *Probability and the Processes of Discovery, Proof, and Choice*, 66 B.U. L. REV. 825, 862-66 (1986) (applying fuzzy logic theory to evidence and inference).

either emphasize the logic and generalize about the law⁷ or vice versa.⁸ This paper attempts to bridge the two fields more effectively by analyzing the elements of fuzzy logic and suggesting ways in which they may be used to model and manipulate the building blocks of bankruptcy law.⁹

Parts I and II of this paper address "fuzzy sets," which heighten the sensitivity of traditional "true-false" logic by introducing different degrees of "truth" to reflect variations in the objects, people, or situations under examination. These sets and their combinations can be used to refine concepts currently undefined or vaguely described by the Bankruptcy Code.

Part III examines "fuzzy decision theory," which addresses simultaneously two different forms of ambiguous criteria relevant to decision-making. This process may prove useful not only for judges and lawyers but for also for legislators.

In Part IV, uses overlapping sets of "fuzzy algorithms"— simple "if-then" rules incorporating varying degrees of indeterminacy, in model situations whose actual intricacy would be difficult to reduce to equations or even verbal descriptions. Through systems of such rules, fuzzy logicians attempt to "emulate the approximation of the human reasoning process, quantify imprecise information, and make decisions based on vague and incomplete data. Applying a 'defuzzification' process, they attempt to arrive at definite conclusions."¹⁰

At this stage, fuzzy logic has come full circle. Where it initially added color to the black and white palette of classical logic, its fuzzy rules now separate and simplify the rich hues found in the spectra of complex functions. Yet paradoxically, "by moving away from the use of quantified variables and toward the use of the type of linguistic descriptions employed by humans, we acquire a

⁷ See KOSKO, *supra* note 5, at 178-80, 262 (portraying law as fuzzy system of rules and principles and stating, "Law is the set of fuzzy moral claims that society or the state backs up with force.").

⁸ See Williams, *supra* note 6, at 1448-59 (rejecting "binary system" of existing fraudulent transfer law in favor of fuzzy system; reviewing conflict between rules and standards).

⁹ See Clark, *supra* note 6, at 14 (proposing fuzzy logic model for analyzing effectiveness of chapter 11); Edward S. Adams, et al., *Understanding Secured Credit as a Fuzzy System*, 80 VA. L. REV. 2233, 2237-46 (1994) (discussing effects of fuzzy logic on evolution of bankruptcy law).

¹⁰ OFFICE OF COMPUTERS AND BUSINESS EQUIPMENT, INTERNATIONAL TRADE ADMINISTRATION, U.S. DEPARTMENT OF COMMERCE, *FUZZY LOGIC: A KEY TECHNOLOGY FOR FUTURE COMPETITIVENESS* (1991) [hereinafter, *FUTURE COMPETITIVENESS*], at vii.

This report listed "several reasons why fuzzy logic technology is crucial to U.S. national interests": (1) it can be used to create very advanced massively parallel computers; (2) it would help the United States to maintain its position as a world leader in all types of software; (3) there is expected to be future growth in the lucrative information systems market; (4) fuzzy logic can accelerate progress in the development of image processing, pattern recognition, voice recognition, and machine vision, which could help maintain the leadership of the United States industry in information technology. *See id.* at 8.

capability to deal with systems which are much too complex to be susceptible to analysis in conventional mathematical terms."¹¹

Part V briefly discusses the promise of "fuzzy expert systems," which, by combining factual databases with fuzzy legal models, would be able to offer instant and informed legal guidance.

I. FUZZY SETS

A. *Blurring Classical Boundaries*

From its origins in the mid-1960's,¹² Lotfi A. Zadeh renamed the "multivalued logic" of his predecessors as "fuzzy logic" and began reinvigorating it in his seminal paper, *Fuzzy Sets*.¹³ Fuzzy logic crystallized around the proposition that "in sharp contrast to the notion of a class or set in mathematics, most of the classes in the real world do not have crisp boundaries which separate those objects that belong to a class from those that do not."¹⁴ Instead, "everything is a matter of degree."¹⁵

Every day we encounter the ambiguity of "fuzzy" classes, such as the class of "tall people." In a given crowd it may be easy to identify the tallest person, and slightly more difficult to spot "all people (if any) over six feet tall," but how many of the group can simply be called (*i.e.*, are members of the class of) "tall people"?¹⁶ We may all be able to agree that a seven-foot tall person is "tall," and a four-foot tall person is not; but where between is the dividing line between these two extremes?

A related inquiry was posed by Greek philosophers as *sorites* or "the paradox of the heap": at what point does a pile of sand from which grains are removed one by one until nothing remains stop being a "pile?"¹⁷

¹¹ Zadeh, *supra* note 2, at 29.

¹² See MCNEILL & FREIBERGER, *supra* note 5, at 28-34. The field has its roots in the work of America's Charles Sanders Peirce (1839-1914), Poland's Jan Lukasiewicz (1878-1955), and England's Harry Black (1907-1988).

¹³ Lotfi A. Zadeh, *Fuzzy Sets, Information and Control* 8, (3), June 1965, pp. 338-353.

¹⁴ R.E. Bellman and Lotfi A. Zadeh, *Decision-Making in a Fuzzy Environment*, MGMT. SCI. Dec. 1970, at B-141. See also LOTFI A. ZADEH, *Toward a Theory of Fuzzy Systems*, in ASPECTS OF NETWORK AND SYSTEM THEORY 470 (R.E. Kalman & N. DeClaris eds., 1971) [hereinafter ZADEH, *FuzzySystems*].

¹⁵ See KOSKO, *supra* note 5, at 18 (identifying this statement as "the fuzzy principle").

¹⁶ Here and in the examples below this paper does not take into account possible variation of the definition of "tall" and "slim" between sexes. For example, because of differences in the typical men and women, the height of a "tall" woman might be seen as the height of an "average" man. See also Schum, *supra* note 6, at 866-67 (noting that most of our everyday reasoning, events or sets have boundaries that are fuzzy).

¹⁷ MCNEILL & FREIBERGER, *supra* note 5, at 26-27.

1. Truth Values

a. Classical

Classical or "binary" logic aggravates this ambiguity by restricting the "truth value" of any statement to one of two alternatives. For example, "Chief Justice Rehnquist is a tall person" or "On this table is a pile of sand" can either have a "truth value" equal to 1 (when the statement is true) or 0 (when the statement is false).

b. Fuzzy

By contrast to the traditional "crisp" definition of truth, in fuzzy logic the truth value of a statement can occupy any value in the range between (and including) 0 and 1, depending on the extent to which its subject (Chief Justice Rehnquist or a particular collection of sand grains) belongs to the specified class (of tall people, or of piles of sand). A value of 0 indicates that the individual in question does not belong to the class at all; a value of 1 indicates full belonging.

2. Set Theory

The truth and falsity of logical statements are often expressed through set theory.¹⁸ For example, for each statement about members of a group or "set," there will be a subgroup or "subset" of none (the "empty set") or more members for which the statement is true.

a. Classical

In terms of set theory, the classical approach leads to "crisp" sets. An individual member of the "universe" (of all people, or of all piles of sand) either does or does not belong fully to a given set: Chief Justice Rehnquist either is tall or is not, and the sand grains on the table either constitute a pile or do not.

To this extent, traditional logic rounds off, or approximates, variations among members of a class. For instance, the natural distribution of heights among a large population, which generally takes the shape of a "bell curve," would be reduced to a "step function," as in **Figure 1**.

¹⁸ See RICHARD JEFFREY, *FORMAL LOGIC: ITS SCOPE AND LIMITS* 38 (2d ed. 1981) (indicating "[i]t is useful to picture logical relationships among statements as relationships among sets of cases in which those statements are true.").

b. Fuzzy

Fuzzy logic, however, involves "fuzzy sets" which allow individuals to be "partial members."¹⁹ A fuzzy set is a class "in which the transition from membership to non-membership is gradual rather than abrupt."²⁰

Thus, variations in heights would be more closely reflected, as in **Figure 2**.

Notably, admitting such degrees of belonging also allows an individual to be described as simultaneously a member (of value v , between 0 and 1) of one set (say, tall people) and a member (of value $1 - v$) of the "complementary" set of all universe members who are not in the first set ("people who are not tall"). To the extent that fuzzy logic admits that a statement and its opposite can both be (partially) true, it rejects formal logic's "law of the excluded middle" and embraces the more holistic perspectives of Taoism²¹ and Zen Buddhism.²² In examining

¹⁹ ZADEH, *Fuzzy Systems*, *supra* note 14, at 470 (explaining concept of fuzzy sets whose blurred boundaries allow partial members).

²⁰ Zadeh, *supra* note 2, at 28.

²¹ Indeed, one of the central principles of Taoism is that apparent opposites are not only define but also contain each other. The primary text of this philosophy, which is thought to have been authored by the legendary Lao Tzu and to have appeared in written form about 240 B.C., observes that:

[T]ruly Being and Not-being grow out of one another;
 Difficult and easy complete one another;
 Long and short test one another;
 High and low determine one another.
 Pitch and mode give harmony to one another.
 Front and back give sequence to one another.

THE WAY AND ITS POWER: A STUDY OF THE TAO TE CHING AND ITS PLACE IN CHINESE THOUGHT 143 (Arthur Waley trans., Grove Press 1958) (footnotes omitted). The traditional date for this collection of 81 brief passages is the sixth century B.C. See WING-TSIT CHAN, *A SOURCE BOOK IN CHINESE PHILOSOPHY* 137 (1963) which asserts that while some scholars have assigned fourth or third century B.C. date, there is sufficient evidence to uphold sixth century B.C. date.

A translator of the writings of another Taoist sage has identified that the "key to Chuang Tzu's thought is complementary opposites." THOMAS MERTON, *THE WAY OF CHUANG TZU* 30 (1965). The unification of apparent opposites is evoked most strongly in the following passage:

Tao [*i.e.*, the natural Way of life and harmony] is obscured when men understand only one of a pair of opposites, or concentrate only on a partial aspect of being. Then clear expression also becomes muddled by mere word-play, affirming this one aspect and denying all the rest.

Hence the wrangling of Confucians and Mohists; each denies what the other affirms, and affirms what the other denies. What use is this struggle to set up "No" against "Yes," and "Yes" against "No"? Better to abandon this hopeless effort and seek true light!

.....

Right turns into wrong and wrong into right— the flow of life alters circumstances and thus things themselves are altered in their turn. But disputants continue to affirm and to deny the same things they have always affirmed and denied, ignoring the new aspects of reality presented by the change in conditions.

Zen's unique combination of mental and physical apprehension, one scholar has

The wise man therefore . . . sees that on both sides of every argument there is both right and wrong. He also sees that in the end they are reducible to the same thing, once they are related to the pivot of Tao.

When the wise man grasps this pivot, he is in the center of the circle, and there he stands while "Yes" and "No" pursue each other around the circumference.

The pivot of Tao passes through the center where all affirmations and denials converge. He who grasps the pivot is at the still-point from which all movements and oppositions can be seen in their right relationship. Hence he sees the limitless possibilities of both "Yes" and "No." Abandoning all thought of imposing a limit or taking sides, he rests in direct intuition.

Id. at 42-43.

²² See ZEN BUDDHISM: SELECTED WRITINGS OF D.T. SUZUKI 60-61 (William Barrett ed., Doubleday Anchor Books 1956). Zen, the mingling of India's Buddhism and China's Taoism, is traditionally dated from the arrival in China of the legendary Indian master Bodi-Dharma around 500 A.D. This school of thought (or, more literally, of "no-thought") emphasizes direct perception, rather than a merely intellectual understanding, of reality. Techniques used to achieve "enlightenment" include confronting apparent dilemmas, such as in the contemplation of paradoxical *koans* (e.g., What is the sound of one hand clapping?). *Id.* at 134.

The abandonment of opposites is a central feature of Zen:

Tai-Hui was a great [Zen master] of the twelfth century. . . . He used to carry a short bamboo stick which he held forth before an assembly of monks, and said: "If you call this a stick, you affirm; if you call it not a stick, you negate. Beyond affirmation and negation what would you call it?"

Id. at 141.

The classic commentary on this *koan* indicates that it was posed by a master who: intends to use [it] as a means to drive his disciples to the abyss of dualistic contradictions, and expects them to find their way out, to be reborn with a completely new point of view.

....

Be that as is may, so long as you live within the world of ordinary dualistic logic, the question . . . can never be answered. Zen demands that you transcend this contradiction, find a clear and definite way out, and be a person of real freedom.

ZENKEI SHIBAYAMA, ZEN COMMENTS ON THE MUMONKAN 300 (Sumiko Kudo, trans., Harper & Row 1974).

However, the famed "zen stick" is applied more directly to students whose display of such wisdom is seen as premature:

When Yamaoka was a brash young student, he visited the master Dokuon. Wanting to impress the master, he said:

"There is no mind, there is no body, there is no Buddha. There is no better, there is no worse. There is no master and there is no student; there is no giving, there is no receiving. What we think we see and feel is not real. All that is real is Emptiness. None of these seeming things really exists."

Dokuon had been sitting quietly smoking his pipe, and saying nothing. Now he picked up his staff, and without warning gave Yamaoka a terrible whack. Yamaoka jumped up in anger.

"Since none of these things really exists," said Dokuon, "and all is Emptiness, where does your anger come from? Think about it."

ZEN BUDDHISM; AN INTRODUCTION TO ZEN, WITH STORIES, PARABLES AND KOAN RIDDLES TOLD BY THE ZEN MASTERS 59-60 (Peters Pauper Press 1959).

observed that the stick displays Zen's "persistent and often violent opposition to words and then to the intellect which deals exclusively in words."²³ Indeed, formal logic itself can be seen as a special case of fuzzy logic, in which truth values are restricted to 0 and 1.

i. Fuzzy Membership Function

In mathematical terminology, a fuzzy subset A of the "universe" or collection X of individual member x's is characterized by their "membership function," A(x). For each x in X, A(x) is equal to the degree of membership in of x in A.

To continue our example, we will let T be the fuzzy subset "tall people" of the universe P of all people. For each person p in the universe P, the membership function T(p) will produce the value of p's membership in the subset T.

Let us consider a smaller universe V of three judges (Judge 1, Judge 2, and Judge 3), and assume that the membership function T(p) would produce the following values for members of S:

$$\begin{aligned} T(\text{Judge 1}) &= 0.8 \\ T(\text{Judge 2}) &= 0.7 \\ T(\text{Judge 3}) &= 0.5. \end{aligned}$$

B. Bankruptcy Applications

Fuzzy logic provides a flexible means of defining in practice, concepts whose numerical parameters are not specified by the Bankruptcy Code ("Code"). Variations in local practice may easily be incorporated, even if their own contours are not exact.

1. Definitions

Compared to such statutes as the Internal Revenue Code, the Code can be considered relatively "fuzzy." In keeping with the equitable nature of the bankruptcy court,²⁴ the Code grants bankruptcy judges great discretion²⁵ and other than specifying various deadlines, imposes few absolute mathematical constraints (e.g., the specific dollar amounts of federal exemptions in section 522(d)) on

²³ Daisetz Teitaro Suzuki, *Zen: A Reply to Hu Shih*, 3 PHILOSOPHY EAST AND WEST 25, 36 (1953).

²⁴ See, e.g., *Atlas Roofing Co. v. Occupational Safety and Health Review Comm'n*, 430 U.S. 442, 454 n.11 (1977) (stating in exercising its summary jurisdiction, bankruptcy court is "a specialized court of equity"); *Katchen v. Landy*, 382 U.S. 323, 327 (1966) (noting bankruptcy courts are "essentially courts of equity"); *Pepper v. Litton*, 308 U.S. 295, 304 (1939) (explaining Supreme Court holding that "courts of equity are essentially courts of equity, and their proceedings inherently proceedings in equity.").

²⁵ See, e.g., 11 U.S.C. § 105(a) (1994) (authorizing bankruptcy court to "issue any order, process, or judgment that is necessary or appropriate to carry out the provisions of this title.").

debtors and creditors).²⁶ However, the Code does contain many relative constraints, whose exact parameters are set by the factual situation at issue. For example, without specifying dollar amounts, section 547(b) generally enables the trustee or debtor-in-possession to avoid "preferential" transfers that enable the transferee to receive *more than* the transferee would have received in a chapter 7 liquidation. Clearly, that amount, and thus the preference determination, is fact-sensitive.

Because the Code does not formally define many of its key expressions and concepts,²⁷ courts seeking to use such terms have applied a range of hermeneutic methods. They refer variously to: the use of the term or related terms in the law of

²⁶ If time constraints (*i.e.*, "within 90 days") are omitted, the Code's statutory language includes relatively few absolute mathematical specifications, either in terms of actual dollar limits (as in section 522(d)'s specification of federal exemption allowances), non-monetary numbers (as in section 705's limitation of creditor committee membership to no less than three nor more than eleven creditors), or specific proportions (as in section 1126(c)'s condition that at least two-thirds in amount and more than one-half in number of claims in a class vote to accept or reject a plan). Other Code sections that use mathematical constraints include:

- § 101(2)- definition of affiliate
- § 101(18)- definition of family farmer
- § 109(e)- eligibility for Chapter 13 protection
- § 303(b)- requirements for involuntary filing
- § 326(a)- limitation on compensation of trustee
- § 330(b)-(c)- compensation of trustee
- § 502(b)(6)(A)- allowance of claim for lease termination
- § 507- extent of priority claims
- § 522(d)- extent of federal exemptions
- § 523 (a) (2) (c) limitation of discharge for debts for "luxury goods and services"
- § 547(c)(8)- restricting trustee's avoidance power on transfers for consumer debts
- § 702- requirements for electing Chapter 7 trustee
- § 705- composition of creditors' committee
- § 727(a)(9)- exceptions to discharge where debtor filed a Chapter 12 or 13 plan within previous six years.
- § 747(2)- subordination of certain customer claims in stockbroker liquidation
- § 923- publication of notice of commencement of municipal bankruptcy proceeding
- § 1102(b)- composition of creditors' committee
- § 1104(c)(2)- conditions for appointment of examiner
- § 1112(e)- information to be provided for conversion to Chapter 7 case
- § 1114(l)- exception to application of Code provisions concerning payment of insurance benefits to retired employees
- § 1126(c)-(d)- requirements for acceptance of plan

²⁷ See Walter Effros, *The Picture's Colorful, But Lacks Definition: An Undictionary of the Code*, in NEW JERSEY INSTITUTE FOR CONTINUING LEGAL EDUCATION, *THE BANKRUPTCY CODE: FIFTEEN YEARS LATER* 371-76 (1996) (listing alphabetically undefined terms, with accompanying citations from Circuit Courts of Appeal).

the relevant state²⁸ or in other sections of the Code than those at issue;²⁹ the use of the term in other statutes;³⁰ the legislative history of the Code section³¹ or the court's inference regarding what Congress meant;³² the test³³ or "traditional equitable definition"³⁴ applied by other courts; and the term's "established meaning" from non-bankruptcy decisions.³⁵

²⁸ See, e.g., *WJM, Inc. v. Massachusetts Dep't of Pub. Welfare*, 840 F.2d 996, 1007 (1st Cir. 1988) (finding that although Code does not define "property," whether something is property shall be determined by state law); *Billings v. AVCO Colo. Indus. Bank (In re Billings)*, 838 F.2d 405, 406 (10th Cir. 1988) (defining "purchase money security interest," in context of section 522(f) of Code, "the courts have uniformly looked to the law of the state in which the security interest is created."); *Avellino & Bienes v. M. Frenville Co., Inc. (In re M. Frenville Co., Inc.)*, 744 F.2d 332, 337 (3d Cir. 1984) (noting Code does not define when right to payment arises, "absent overriding federal law, is to be determined by reference to state law.").

²⁹ See, e.g., *First City Beaumont v. Durkay (In re Ford)*, 967 F.2d 1047, 1051 (5th Cir. 1992) (noting because Code does not define "contingent claim" for purposes of section 502(c)(1), court looks to judicial definitions of "contingent" for other sections of the Code); *Ray v. City Bank and Trust Co. (In re C-L Cartage Co., Inc.)*, 899 F.2d 1490, 1494 (6th Cir. 1990) (explaining although "transferee" is not defined by section 550(a)(1) of Code, "transfer" is broadly defined in section 101(40) to include any disposition of an interest in property); *Begier v. I.R.S.*, 878 F.2d 762, 769 n.13 (3d Cir. 1989) (stating Code does not define phrase, "property of the debtor" found in section 547(b), courts have looked to whether transferred property may be defined as "property of the estate" under section 541).

³⁰ See, e.g., *United States v. Mansfield Tire & Rubber Co. (In re Mansfield Tire & Rubber Co.)*, 942 F.2d 1055, 1058 (6th Cir. 1991) (looking to Internal Revenue Code for definition of "tax" or "excise tax," which are not defined in Code).

³¹ See, e.g., *University Med. Ctr. v. Sullivan (In re University Med. Ctr.)*, 973 F.2d 1065, 1075 n. 13 (3d Cir. 1992) (understanding although Code does not define "executory contract" in section 365(a), its legislative history states that term "generally includes contracts on which performance remains due to some extent on both sides"); *Georgia Pac. Corp. v. Sigma Serv. Corp.*, 712 F.2d 962, 967 n.4 (5th Cir. 1983) (referring to legislative reports explaining section 541, and to Restatement of Trusts, 2d, § 2 in defining "constructive trust" because term is not defined neither in Code nor by prior courts' interpretations of Code).

³² See, e.g., *Budget Serv. Co. v. Better Homes of Va., Inc.*, 804 F.2d 289, 292 (4th Cir. 1986) (discussing section 362(h) as incorporating chapter 11 proceeding and concluding that, although Code does not define "individual" it is unlikely Congress intended to restrict remedies under this section to personal debtors); *Montello Oil Corp. v. Marin Motor Oil, Inc. (In re Marin Motor Oil, Inc.)*, 740 F.2d 220, 226 (3d Cir. 1984) (noting since neither Code nor Uniform Commercial Code defines "demand," and in the absence of "[any] other source of guidance, we must look to the policy behind section 546(c) to decide what the rule should be"); *McCannon v. Marston*, 679 F.2d 13, 16 (3d Cir. 1982) (stating "in our view Congress cannot have intended . . . an interpretation" of section 544(a) that equates "knowledge," a term undefined in Code, with "notice"); *Foster v. Heitkamp (In re Foster)*, 670 F.2d 478, 489 (5th Cir. 1982) (noting term "provided for by the plan" in section 1325(a)(5) is not defined by Code or its legislative history, "the intended meaning seems clear enough").

³³ See, e.g., *Rimell v. Mark Twain Bank (In re Rimmell)*, 946 F.2d 1363, 1365 (8th Cir. 1991) (adopting Seventh Circuit's objective test for existence of "bona fide dispute" under section 303(b)(1)); *Bartmann v. Maverick Tube Corp.*, 853 F.2d 1540, 1543-44 (10th Cir. 1988) (adopting objective test for existence of "bona fide dispute" under section 303 (b)(1)).

³⁴ See *Willemain v. Kivitz (In re Willemain)*, 764 F.2d 1019, 1023 (4th Cir. 1985) (examining concept of "good faith purchaser" in section 363(m) and adopting "traditional equitable definition" of this term).

³⁵ See, e.g., *Andrews University v. Merchant (In re Merchant)*, 958 F.2d 738, 740-41 (6th Cir. 1992) (explaining since term "loan" not defined by Code, court looks to establish definition of term as adopted by other circuit courts of appeal).

Indeed, bankruptcy courts have themselves defined such equitable terms as the "other cause" that justifies reopening a case under section 350(b).³⁶ Alternatively, as with "shopping center" under section 365(b)(3), they have found that "the proper definition of [an undefined term] is 'left to case-by-case interpretation.'"³⁷

As discussed below,³⁸ the application of the Code's terms, and possibly the terms themselves, can be clarified by the principles of fuzzy logic. Central to this inquiry is the determination of an appropriate membership function for the term in question. In addition to taking into account the factors above, accurate calibration of the membership function should include consideration of local practice.

a. Local Legal Culture

In a land whose population was markedly taller or shorter on average than people elsewhere, both the crisp and the fuzzy definitions of "tall person" might change. For example, in a culture where everyone was taller than was average elsewhere, a "tall person" might be naturally defined as even taller than a "tall person" elsewhere.

Similarly, local legal customs and practices can shift the parameters of concepts critical to bankruptcy law. A growing body of empirical scholarship,³⁹ particularly in the area of consumer bankruptcy, has explored the genesis and persistence of "local legal cultures," defined by Professors Teresa Sullivan, Elizabeth Warren and Jay Westbrook as:

systematic and persistent variations in local legal practices as a consequence of a complex of perceptions and expectations shared

³⁶ See *In re Shondel*, 950 F.2d 1301, 1304 (7th Cir. 1991) (noting decision to reopen a case for "other cause" under section 350(b) lies within discretion of bankruptcy court).

³⁷ See *In re Joshua Slocum Ltd.*, 922 F.2d 1081, 1086 (3d Cir. 1990) (quoting *In re Goldblatt Brothers, Inc.*, 766 F.2d 1136, 1140 (7th Cir. 1985)).

³⁸ See Part II, *infra*.

³⁹ The seminal empirical work on consumer bankruptcy briefly discussed local legal culture in its analysis of data gathered from a sample of those who had filed for bankruptcy in 1981. See TERESA SULLIVAN, ELIZABETH WARREN & JAY L. WESTBROOK, *AS WE FORGIVE OUR DEBTORS: BANKRUPTCY AND CONSUMER CREDIT IN AMERICA* (1989). In an updated version of their study, considering filings made in 1991, the same team concluded that:

The degree of stability evidenced by these data, in the face of a three-fold increase in the number of debtors seeking bankruptcy during an economically volatile decade of debt-driven boom and bust, is stunning. The rates of filing and the chapters chosen within each of our [sample] districts, as well as nationally, remained almost exactly the same. These facts seem to us to lend strong support to a theme we sounded in the earlier study: that chapter choice is strongly affected by lawyer preferences, that lawyers and judges are strongly influenced by a local legal culture in each district, and that these local legal cultures are remarkably stable over long periods of time.

Teresa Sullivan, Elizabeth Warren & Jay L. Westbrook, *Consumer Debtors Ten Years Later: A Financial Comparison of Consumer Bankrupts 1981-91*, 68 AM. BANKR. L.J. 121, 143 (1994).

by many practitioners and officials in a particular locality, and differing in identifiable ways from the practices, perceptions, and expectations existing in other localities subject to the same or a similar formal legal regime.⁴⁰

For example, this team observed significant differences, apparently unrelated to the economic circumstances of debtors in different areas, among the states, in the number of personal bankruptcy petitions filed per 100,000 population;⁴¹ among the bankruptcy districts, in the choice of chapter 13 as an alternative to chapter 7;⁴² and among the districts, in the choice and extent of repayment in chapter 7 (by reaffirmation agreements) and chapter 13 (by repayment plans).⁴³

These differences have been attributed to the local influence of judges (who set attorneys' fees, as well as the procedures, for the two types of cases), lawyers (who influence clients through their own advertising, beliefs, and degree of cooperation or competition with other lawyers), trustees (who set procedures and administer proceedings), local consumer credit counseling agencies (who help borrowers decide if and when, and under what chapter, to file), and creditors themselves (who select relatively aggressive or passive local lawyers, and who determine when to object to different aspects of bankruptcy proceedings).⁴⁴

The influence of local culture adjusts such core, yet undefined, concepts as that of "good faith." For example, because chapter 13 plans must provide for payment to unsecured creditors for at least the amount that they would have received in the chapter 7 liquidation of the debtor,⁴⁵ if a "no-asset" debtor has no non-exempt assets to distribute in a chapter 7 proceeding she can theoretically provide a "0%" plan, which would not pay general unsecured creditors at all.⁴⁶

However, Professor Jean Braucher, who studied bankruptcy filings in Austin, Dayton, Cincinnati and San Antonio, has noted that as a practical matter "chapter 13 trustees and judges in the[se] four cities effectively deter 0% plans and keep most plans above a [widely varying] floor percent that is known to local practitioners. The lawyers then respond by rarely or never submitting plans with

⁴⁰ Teresa A. Sullivan, et al., *The Persistence of Local Legal Culture: Twenty Years of Evidence From the Federal Bankruptcy Courts*, 17 HARV. J.L. & PUB. POL'Y 801, 804 (1994).

⁴¹ See *id.* at 818-22.

⁴² See *id.* at 822-30.

⁴³ See *id.* at 830-33.

⁴⁴ *Id.* at 839-57. See also Jean Braucher, *Lawyers and Consumer Bankruptcy: One Code, Many Cultures*, 67 AM. BANKR. L.J. 501, 503 (1993) (exploring effect on local legal culture of pursuit of four goals by lawyers representing consumer debtors: serving their clients' and their own financial interests and attempting to fulfill some version of appropriate social role playing on the parts of their clients and themselves).

⁴⁵ See 11 U.S.C. § 1325(a)(4) (1994).

⁴⁶ See, e.g., *In re Greer*, 60 B.R. 547, 554 (Bankr. C.D. Cal. 1968) (noting fact that plan provided for nominal or zero payment to general unsecured creditors would not justify denial of chapter 13 plan).

less than the specified percentage [I]n all four cities, lawyers can sometimes get plans confirmed below the floor, if they convincingly establish inability of the debtor to pay more."⁴⁷ She speculates that this *de facto* rule might be justified as "a presumptive case of 'good faith,'" under the Code.⁴⁸

In such a jurisdiction, a lawyer can define "good faith" in this context by a function GF that operates over the range (0 percent to 100 percent) of possible repayments to undersecured general creditors. Just as the tallness function T(p) rose in figure 1 from 0 at 0 feet tall to 1 at six feet tall and remained at 1 for people taller than six feet, so the function GF would generally rise from 0 for 0 percent repayment to 1 at the local target percentage of such Chapter 13 repayments and then remain at 1 for any percentage above that.

II. FUZZY SET OPERATIONS

It might be argued at this point that the apparatus of fuzzy logic has been assembled for no greater purpose than to illustrate the variability possible among interpretations of Code terms, which certainly could have been accomplished without mathematical considerations.

However, the true value of fuzzy sets lies in their use in analyzing definitions that themselves incorporate several fuzzy terms. This level of application involves the union and intersection of fuzzy sets.

A. Unions and Intersections

1. Classical

Chief among the operations performed in the conventional theory of sets are union and intersection.

a. Union

The union of two sets is the set containing all the members of each of the two sets. For example, in the "universe" V of all people, the union of the set T of all tall people and the set S of all slim people, expressed by the notation $T \cup S$, is the set containing all people who are either tall or slim.

⁴⁷ See Braucher, *supra* note 44, at 532.

⁴⁸ *Id.* The floor percentages for routine confirmation in Austin, Dayton, Cincinnati, and San Antonio were, as noted by the respective chapter 13 standing trustees, 25-33 percent, 10 percent, 70 percent, and 100 (!) percent. *See id.*

b. *Intersection*

On the other hand, the intersection of two sets is the set containing those items that are members of both sets. The intersection of the set of all tall people and the set of all slim people, expressed as $T \wedge S$, is the set of all people who are both tall and slim, i.e., the set of all tall slim people.

The "crispness" of these operations results from the sharp classification of any person as either "tall" or "not tall," and as either "slim" or "not slim." For these purposes, the members of any one of the resulting sets, for example, "tall slim people," are further indistinguishable from each other by degrees of height or slimness. The two sets and their union and intersection can thus be depicted as in **Figure 3**.

If, in our example, we considered the "universe" consisting of Judge 1, Judge 2, and Judge 3, and found that only Judge 1 and 2 were "tall" and only Judges 2 and 3 were "slim," we could construct the diagram in **Figure 4**.

2. Fuzzy⁴⁹

By contrast, as discussed above, fuzzy logic distinguishes degrees of tallness and slimness by the membership functions $T(p)$ and $S(p)$, respectively.

For each person p in the universe V , there thus corresponds the descriptive list or "ordered pair" $D(p)$ of the two measurements, $T(p)$ and $S(p)$:

$$\text{For each person } p, D(p) = [T(p), S(p)]$$

For example, if the respective slimnesses of Judge 1, Judge 2, and Judge 3 are taken as 0.6, 0.4, and 0.9, respectively, so that:

$$\begin{aligned} T(\text{Judge 1}) &= 0.8 & S(\text{Judge 1}) &= 0.6 \\ T(\text{Judge 2}) &= 0.7 & S(\text{Judge 2}) &= 0.3 \\ T(\text{Judge 3}) &= 0.5 & S(\text{Judge 3}) &= 0.9 \end{aligned}$$

We then have:

$$\begin{aligned} D(\text{Judge 1}) &= [0.8, 0.6] \\ D(\text{Judge 2}) &= [0.7, 0.3] \\ D(\text{Judge 3}) &= [0.5, 0.9]. \end{aligned}$$

⁴⁹ See Bellman & Zadeh, *supra* note 14, at B-143-45 (providing brief introduction to fuzzy sets).

By representing $T(p)$ on the x-axis and $S(p)$ on the y-axis,⁵⁰ we can chart the respective representations of Judge 1, Judge 2 and Judge 3. See **Figure 5**.

Clearly, an individual person p could, depending on her degrees of membership in classes T and S , be represented by a point anywhere within the box bordered by the ordered pairs $[0,0]$, $[0,1]$, $[1,0]$ and $[1,1]$. However, classical logic would restrict each individual to one of these four points (corresponding to people who are, respectively, short and heavy, short and slim, tall and heavy, and tall and slim).⁵¹

In conventional set theory, unions and intersections are performed on the sets containing the members p themselves (e.g., tall people, slim people), and result in other collections of individuals. However, in fuzzy logic unions and intersections operate on the corresponding $D(p)$'s, and result in new ordered pairs.

a. Fuzzy Union

The fuzzy union of $D(\text{Judge 1})$, $D(\text{Judge 2})$ and $D(\text{Judge 3})$ is the class containing the greatest degree of tallness and the greatest degree of slimness from among these three judges, even if these degrees belong to different people. Since the greatest degree of tallness among the judges is 0.8 (from Judge 1), and the greatest degree of slimness is 0.9 (from Judge 3), the union of the three Judges is $[0.8, 0.9]$. Among them, the judges best satisfy the "tallness" criterion to the degree 0.8 and the "slimness" criterion to the degree 0.9.

b. Fuzzy Intersection

Conversely, the fuzzy intersection of the three Judges represents the greatest common degree to which the Judges belong to each of the fuzzy sets in question. The intersection of the three judges would be 0.5 for tallness (Judge 3) and 0.3 for slimness (Judge 2), or $[0.5, 0.3]$.

C. Optimization

1. Classical

To find "tall slim judges" from among the limited set considered, in classical logic we just isolated the tall and slim subsets of this "universe" and found their intersection: that is, which members belonged to both.

⁵⁰ Of course, we could reverse this order and put $T(p)$ on the y-axis and $S(p)$ on the x-axis.

⁵¹ See KOSKO, *supra* note 5, at 29-34 (exploring this concept for three dimensions).

2. Fuzzy

In fuzzy logic, however, the intersection of the universe members' degrees of membership in the categories of "tall people" and "slim people" produced an ordered pair which corresponded to none of the three judges. What, then, is the optimal or "crisp" answer, which one judge maximizes the combination of "tallness" and "slimness?"

One "straightforward"⁵² means of choosing the optimal answer is to identify the member of the universe that contributed the largest of the degrees of membership to the intersection set. Similarly, another fuzzy logic textbook proposes that "If the decision maker wants to have a 'crisp' decision proposal, if seems appropriate to suggest to him the dividend which has the highest degree of membership in the fuzzy set 'decision,'" as above.⁵³ That is, since Judge 3 contributed 0.5 (for tallness) and Judge 2 contributed only 0.2 (for slimness), and 0.5 is greater than 0.2, Judge 3 is the closest to a "tall slim person" among the judges.⁵⁴

3. Fuzzy Refinements

a. Additional Factors

Like classical logic, fuzzy logic can accommodate additional factors, through the inclusion of additional membership functions to the description function.

For instance, the judges could be distinguished by their golfing ability by the addition of the membership function $g(p)$, whose value for any person p takes on a value in the range from 0 (poor golfer) to 1 (excellent golfer). The description function:

$$D(p) = [T(p), S(p)]$$

would be expanded to become:

$$D'(p) = [T(p), S(p), G(p)].$$

⁵² See GEORGE J. KLIR & TINA A. FOLGER, *FUZZY SETS, UNCERTAINTY, AND INFORMATION* 256 (1988). However, "[s]ince this method ignores information concerning any of the other alternatives, it may not be desirable in all situations. Methods that [take such information into account] may therefore be used instead." *Id.*

⁵³ H.J. ZIMMERMANN, *FUZZY SET THEORY— AND ITS APPLICATIONS* 217 (1985). See also B.R. Gaines, *Foundations of Fuzzy Reasoning*, 8 *INT. J. MAN-MACHINE STUDIES* 623, 637-38 (1976) (discussing same principle).

⁵⁴ Presumably, if the highest degree of membership were shared by two or more judges, fuzzy logic would consider which of those judges had the highest degree of membership in any of the remaining categories.

Of course, the inclusion of none, one, two, or three of the judges in the class of "good golfers" could be accomplished by adding an additional circle to the Venn diagrams of classical logic. (In fuzzy logic, the new criterion would literally add a dimension to the analysis. The judges would be represented by points along three axes, the mutually perpendicular x-, y-, and z-axis).

Clearly, with enough time, paper, and patience, an unlimited number of factors can be added to the Venn diagram depiction, which itself remains two-dimensional. By contrast, beyond three dimensions the fuzzy logic representations of the judges could not be rendered graphically, since they would appear in "n-space," or an imaginary "space" with the dimension n of factors in use.

*b. Weighting Factors*⁵⁵

If the n criteria that the decision-maker applies are of unequal importance to her, weighting factors, whose aggregate total must be 1, can be applied to each criterion function. For example, if in the situation above the judges' tallness was of prime importance and their slimness of relatively little significance the function $D(p) = [T(p), S(p)]$ could be recalibrated as:

$$D'(p) = (0.8) T(p) + (0.2) S(p)$$

As opposed to the optimization procedure above, the optimal choice of Judge p will be the judge for which $D'(p)$ is the greatest, or, in this case, Judge 1:

$$D'(\text{Judge 1}) = (0.8)(0.8) + (0.2)(0.6) = .72$$

$$D'(\text{Judge 2}) = (0.8)(0.7) + (0.2)(0.3) = .62$$

$$D'(\text{Judge 3}) = (0.8)(0.5) + (0.2)(0.9) = .58$$

C. Bankruptcy Applications

The Code contains multi-prong tests, all of whose factors must be satisfied for the test to be satisfied,⁵⁶ as well as multi-alternative tests, only one of whose

⁵⁵ See Bellman & Zadeh, *supra* note 14, at 149-50.

⁵⁶ A classic example of a multi-prong test is the definition of a preference. Section 547(b) of the Code allows the trustee to avoid any transfer of an interest of the debtor in property

- (1) to or for the benefit of a creditor;
- (2) for or an account of an antecedent debt owed by the debtor before such transfer was made;
- (3) made while the debtor was insolvent;
- (4) made-
 - (A) on or within 90 days before the date of the filing of the petition; or

factors must be satisfied.⁵⁷ In addition, many of these factors are themselves multi-prong or multi-alternative tests.

Generally, any single factor may be susceptible to fuzzy analysis. For example, the multi-prong tests with one or more fuzzy factors can be analyzed as intersections of fuzzy sets; and multi-alternative tests with one or more fuzzy factors can be analyzed as unions of fuzzy sets.

1. "Undue Hardship"

An example of a fuzzy definition that can be expressed as the intersection of fuzzy sets is the concept of "undue hardship" in the context of section 523(a)(8)(B),⁵⁸ which prohibits the discharge of an educational loan unless "exempting such debt from discharge . . . will impose an undue hardship on the debtor and the debtor's dependents."⁵⁹

a. Eighth Circuit Test

Four Courts of Appeals have endorsed two separate tests for determining "undue hardship."⁶⁰ First, the Eighth Circuit found that:

(B) between 90 days and one year before the date of the filing of the petition, if such creditor at the time of such transfer was an insider; and
 (5) that enables such creditor to receive more than such creditor would receive if—
 (A) the case were a case under chapter 7 of this title;
 (B) the transfer had not been made; and
 (C) such creditor received payment of such debt to the extent provided by the provisions of this title.

11 U.S.C. § 547 (b) (1994).

⁵⁷ See Section 363(f) allowing:

[t]he trustee may sell property . . . free and clear of any interest in such property of an entity other than the estate, only if—

(1) applicable nonbankruptcy law permits sale of such property free and clear of such interest;

(2) such entity consents;

(3) such interest is a lien and the price at which such property is to be sold is greater than the aggregate value of all liens on such property;

(4) such interest is in bona fide dispute; or

(5) such entity could be compelled, in a legal or equitable proceeding, to accept a money satisfaction of such interest.

11 U.S.C. § 363(f).

⁵⁸ 11 U.S.C. § 523(a)(8)(B).

⁵⁹ *Id.* (emphasis added).

⁶⁰ See *Cheeseman v. Tennessee Student Assistance Corp.* (*In re Cheeseman*), 25 F.3d 356, 359 (6th Cir. 1994) (stating test was met and student loan was to be discharged); *In re Roberson*, 999 F.2d 1132, 1136-37 (7th Cir. 1993) (stating that minimal conditions of living must be met and considered as part of test to ascertain there is undue hardship); *Brunner v. New York Higher Educ. Servs. Corp.*, 831 F.2d 395, 396 (2d Cir. 1987) (stating that conclusion regarding legal effect of bankruptcy court's finding is needed to

each bankruptcy case involving a student loan must be examined on the facts and circumstances surrounding that particular bankruptcy for the Court to make a determination as to "undue hardship." The bankruptcy court must determine whether there would be anything left from the debtor's estimated future income to enable the debtor to make some payment on his/her student loan without reducing what the debtor and his/her dependents need to maintain a minimal standard of living.⁶¹

Clearly, the underlined concepts are fuzzy, in the sense that they are not likely to be taken literally but rather as guidelines for an unenunciated and fact-sensitive range of values. That is, despite the use of the words "anything" and "some," a bankruptcy court presumably might find "undue hardship" even if the debtor's estimated future income would allow the debtor to make a payment of 50 cents, or one dollar, per month on the student loan without reducing the amount needed for a minimal standard of living. Of course, if the debtor could spare \$50 a month for loan repayment without affecting her and her dependents' minimal standard of living, the court would not be so sympathetic.

Instead of finding the exact amount at which the debtor's spare cash becomes so significant as suddenly to constitute "something," fuzzy logic could create a membership function $F(\$)$ to model the substantiality of the debtor's free funds for this purpose, with, say, $F(\$0.50) = 0$ and $F(\$50) = 1$.

Similarly, the question of whether such a payment would reduce the minimal standard of living, and the assessment of the budget for that standard of living itself, would turn on the court's assessment of the proposed plan to support this standard. A lawyer experienced in consumer bankruptcies could probably construct a fuzzy model of the budget for the family in question, by adding fuzzy financial figures representing the costs of various items for each affected person.

Finally, the estimation of the debtor's future income invites ambiguity. The completed fuzzy model of "undue hardship" under the Eight Circuit standard might involve a membership function of the following type:

$$A(\text{plan}) = [F(\text{standard of living}), G(\text{future income}), \\ H(\text{amount payable})]$$

where $A(\text{plan})$ is the acceptability to the court (from 0, unacceptable, to 1, acceptable, of a given plan proposed by the debtor), $F(\text{standard of living})$ is the

determine whether "undue hardship" was imposed); *Andrews v. South Dakota Standard Loan Assistance Corp.* (In re *Andrews*), 661 F.2d 702, 704 (8th Cir. 1981) (explaining that living expenses must be taken into consideration).

⁶¹ *Andrews*, 661 F.2d at 704 (quoting *In re Wegfehrt*, 10 B.R. 826, 830 (Bankr. N.D. Ohio 1981)) (emphasis added).

acceptability to the court of the minimal standard of living identified by the debtor as appropriate, G(future income) is the acceptability of the debtor's estimate of future income, and H(amount payable) is the substantiality of the amount the debtor calculates will be available, with the above variables set as indicated, to repay the loan.

In estimating the possible success of an "undue hardship" argument, the debtor and her attorney will estimate the value of these functions for different values of the variables, attempting to optimize the membership of the plan in the set of acceptable plans.

b. Second Circuit Test

A similarly fuzzy approach, with the addition of functions for "persistence" and "good faith," can be taken to the Second Circuit's three-part test for "undue hardship."⁶² which was advanced six years after the Eighth Circuit's test:

(1) that the debtor cannot maintain, based on current income and expenses, a "minimal" standard of living for herself and her dependents if forced to repay the loans; (2) that additional circumstances exist indicating that this state of affairs is likely to persist for a significant portion of the repayment period of the student loans; and (3) that the debtor has made good faith efforts to repay the loans.⁶³

This test was adopted six years later by the Seventh Circuit, which explicitly rejected a rival three-part test applied by the bankruptcy court and the district court.⁶⁴ Most recently, the Sixth Circuit declined to champion one test over the

⁶² See *Brunner*, 831 F.2d at 396 (adopting standard set forth in district court's unpublished opinion) (emphasis added).

⁶³ *Id.*

⁶⁴ See *Roberson*, 999 F.2d at 1136-37 (7th Cir. 1993) (stating that Seventh Circuit found redundant section 3(a), and improper section 3(b), of the policy test enunciated by the bankruptcy court in *In re Johnson*, 5 Bankr. Ct. Dec. 532, 544 (Bankr. E.D.Pa. 1979):

(1) Mechanical Test: The court must ask: Will the debtor's future financial resources for the longest foreseeable period of time allowed for the repayment of the loan be sufficient to support the debtor and his dependent[s] at a subsistence or poverty standard of living, as well as to fund repayment of the student loan?

....

(2) Good Faith Test: Here, the court asks two questions:

(a) Was the debtor negligent or irresponsible in his efforts to minimize expenses, maximize resources or secure employment?

(b) If "yes," then would lack of such negligence or irresponsibility have altered the answer to the mechanical test?

....

other, since, although the bankruptcy court had not specified which of the tests it was following in finding "undue hardship" to the debtor, the loans in question were dischargeable "under any hardship test the court may have used in reaching its decision."⁶⁵

2. "Plain Language" Yardsticks

Although the Supreme Court has recently appeared to endorse the "plain meaning" of Code sections at issue,⁶⁶ commentators have questioned whether this

(3) . . . Policy Test: The court must ask: Do the circumstances— i.e., the amount and percentage of the total indebtedness of the student loan and the employment prospects of the petitioner indicate:

(a) That the dominant purpose of the bankruptcy petition was to discharge the student debt, or

(b) That the debtor has definitely benefited financially from the education which the loan helped to finance?

⁶⁵ Cheesman v. Tennessee Student Assistance Corp. (*In re Cheesman*), 25 F.3d 356, 359 (1994).

⁶⁶ These decisions include, in chronological order:

See, e.g., *United Savings Ass'n of Texas v. Timbers of Inwood Forest Assocs., Ltd.*, 484 U.S. 365, 371 (1988) (stating "statutory construction . . . is a holistic endeavor"). Justice Scalia's opinion for unanimous Court nonetheless found legislative history and policy considerations of little moment in concluding that undersecured creditors are not entitled to compensation under section 362(d)(1) for delay automatic stay causes in their foreclosing on collateral. *See id.*

U.S. v. Ron Pair Enters., Inc., 489 U.S. 235, 245 (1989) (explaining legislative history is relevant only if Code section is susceptible to reasonable interpretation that clearly conflicts with important state or federal legislation);

Pennsylvania Dep't. of Pub. Welfare v. Davenport, 495 U.S. 552, 557-58 (1990) (stating "statutory interpretation begins with the language of the statute itself" and in determining that restitution payments imposed as conditions of probation in state criminal actions are dischargeable in chapter 13 proceedings);

Toibb v. Radloff, 501 U.S. 157, 160-61 (1991) (finding plain language of Code dictates that chapter 11 protection is available to individual debtors);

Union Bank v. Wolas, 502 U.S. 151, 158 (1991) (explaining fact that Congress may not have foreseen all consequences of Code revision is not sufficient reason for refusing to give effect to its plain meaning);

Dewsnup v. Timm, 502 U.S. 410, 415 (1992) (finding that in "allowed secured claim" of section 506(d) "term-by-term to refer to any claim that is, first, allowed, and, second, secured.");

Taylor v. Freeland & Kronz, 503 U.S. 638, 644 (1992) (stating that plain meaning of Bankruptcy Rules' deadlines on filing objections to exemptions "may lead to unwelcome results, but they prompt parties to act and they produce finality.");

Barnhill v. Johnson, 503 U.S. 393, 406(1992) (reading text of section 547(c), closely, to hold that "date of honor" rather than "date of delivery" rule applies for purposes of calculating preference period for transfer by check);

Patterson v. Shumate, 504 U.S. 753, 755-56 (1992) (finding that under plain language of Code, Employee Retirement Income Security Act of 1974 ("ERISA") constitutes "applicable nonbankruptcy law" for purposes of section 541(c)(2) exclusions);

Pioneer Inv. Servs. v. Brunswick Assocs. Ltd. Partnership, 507 U.S. 380, 395 (1993) (stating "excusable neglect" standard of Bankruptcy Rule 9006(b) for allowing late filing of documents should not be construed rigidly, but in light of circumstances of situation);

Nobelman v. American Sav. Bank, 508 U.S. 324, 331 (1993) (barring lien-stripping in chapter 13 proceedings, under "plausible" interpretation of section 1322(b)(2));

hermeneutic technique is itself dispositive.⁶⁷ Not only does the disagreement among the Courts of Appeals indicate a fundamental ambiguity in the language in question, but the Court's method for obtaining "plain meaning" may itself be inconsistent.⁶⁸

It has been suggested that despite the Court's announced agenda of adhering to the "plain meaning" of specific Code provisions, in actually interpreting subsections the Justices "incorporat[ed] material from six realms of relevance, each progressively more inclusive and farther removed from the literal meaning of the code section in question [and containing] inherent ambiguities and contradictions."⁶⁹

Specifically, the language of a Code subsection can be evaluated in the contexts of:

1. The language itself, "in a vacuum;"
2. The Code section containing the subsection at issue;

Rake v. Wade, 508 U.S. 464, 465 (1993) (indicating that plain language of Code dictates that postpetition interest be allowed to holders of oversecured home mortgages that did not explicitly provide for such interest);

BFP v. Resolution Trust Corp., 511 U.S. 531, 545 (1994) (concluding under section 548(a)(2)(A), "reasonably equivalent value" of property sold at noncollusive, procedurally correct foreclosure sale is foreclosure sale price itself).

⁶⁷ See, e.g., Walter A. Effross, *Grammarians at the Gate: The Rehnquist Court's Evolving 'Plain Meaning' Approach to Bankruptcy Jurisprudence*, 23 SETON HALL L. REV. 1636, 1748 (1993) (stating "[a]lthough ostensibly a significant step towards judicial predictability, the 'plain meaning' approach has proven to be less a cohesive, coherent method of statutory interpretation than a loose collection of principles, which themselves may often be breached."); Susan Block-Lieb, *Bankruptcy Decisions of the Supreme Court of the United States During the 1991-1992 Term*, NORTON BANKR. L. ADVISER 1 (Aug. 1992) ("just when the language of a statute is "plain" is unclear); Charles J. Tabb & Robert M. Lawless, *Of Commas, Gerunds, and Conjunctions: The Jurisprudence of the Rehnquist Court*, 42 SYRACUSE L. REV. 823, 880 (1991) (noting recent emphasis on textualism has led to "the lack of a consistent jurisprudence in the Court's bankruptcy decisions. . . . [T]he Rehnquist Court appears to drift from bankruptcy decision to bankruptcy decision."). Cf. Robert K. Rasmussen, *A Study of the Costs and Benefits of Textualism: The Supreme Court's Bankruptcy Cases*, 71 WASH U. L.Q. 535, 565 (1993) (explaining textualist approach produces better consequences that would overtake use of dynamic interpretation; moreover, changes from textualist to "dynamic interpretation" approach would not necessarily produce dramatic shifts in outcome of Court's bankruptcy decisions); Adam J. Wiensch, Note, *The Supreme Court, Textualism, and the Treatment of Pre-Bankruptcy Code Law*, 79 GEO. L.J. 1831, 1858 (1991) (reasoning that consistent use of textualist approach reduces uncertainty over how bankruptcy courts will decide issues and develops "bright line rules").

⁶⁸ See, e.g., Effross, *supra* note 67, at 1747-58 (identifying six levels of ambiguity in "plain meaning" analysis).

⁶⁹ *Id.* at 1749. Cf. Frankfurter, *supra* note 1, at 529 (stating "I should say that the troublesome phase of [statutory] construction is the determination of the extent to which extraneous documentation and external circumstances may be allowed to infiltrate the text on the theory that they were part of it, written in ink discernible to the judicial eye.").

3. Different Code sections containing (or omitting) the same or similar language;
4. The policies behind the Code;
5. The policies behind other statutes; and
6. Equitable considerations.⁷⁰

A fuzzy model of Code interpretation, which could be used either to estimate the acceptability of a given interpretation or to model the Court's precedent concerning such interpretations, could include an "interpretive strength" function that incorporated six membership functions, each corresponding to the acceptability in one of these six contexts of a given interpretation of a Code subsection. For a given interpretation i , the acceptability of the interpretation could be gauged as, $B(i) = [S1(i), S2(i), S3(i), S4(i), S5(i), S6(i)]$, where $S1(i)$ is the support (from 0, complete contradiction, to 1, complete support) that the first category of interpretation (language in a vacuum) adds to the interpretation, $S2(i)$ is the support added by the second category, and so on.

For example, Rule 9006(b)(1) allows a bankruptcy court "at any time in its discretion" to allow, upon motion, an act to be done or notice to be given past the formal deadline for such act or notice, "where the failure to act was the result of excusable neglect."⁷¹ In *Pioneer Investment Services Co. v. Brunswick Assoc. Ltd. Partnership*,⁷² the Court construed "excusable neglect" broadly, to include circumstances in which the movant had not been the victim of uncontrollable circumstances. The Court held that "the determination is at bottom an equitable one, taking account of all the relevant circumstances surrounding the party's omission,"⁷³ including "the danger of prejudice to the debtor, the length of the delay and its potential impact on judicial proceedings, the reason for the delay, including whether it was within the reasonable control of the movant, and whether the movant acted in good faith."⁷⁴

In affirming the fuzziness of "excusable neglect" and identifying some of its components, the Court cited a collegiate dictionary's definition of "neglect"⁷⁵ and

⁷⁰ See, e.g., Effross, *supra* note 67, at 1754 (discussing class of plain meaning of Code and equitable considerations).

⁷¹ FED. R. BANKR. P. 9006(b)(1).

⁷² 507 U.S. 380, 388 (1993) (stating "Congress plainly contemplated that the courts would be permitted where appropriate to accept late filings caused by inadvertence, mistake, or carelessness, as well as by intervening circumstances beyond the party's control.").

⁷³ *Id.* at 395.

⁷⁴ *Id.*

⁷⁵ *Id.* at 388 (defining neglect as "simple, faultless omissions to act and, more commonly, omissions caused by carelessness").

also the expansive construction of "excusable neglect" in the Federal Rules of Civil Procedure.⁷⁶ Thus, the membership of this interpretation in the set of acceptable interpretations might be seen as

$B(i) = [1, .5, .5, .5, 1, 1]$.

However, the four-Justice dissent cited a standard legal dictionary's entry for "excusable neglect," which focused on the circumstances leading up to the missed deadline rather than the consequences of the failure to meet the deadline.⁷⁷ The dissent bolstered this view with the observation that Rule 9006(b)(1) by its own terms requires that "unless the *failure* to act was *the result* of excusable neglect relief is unavailable."⁷⁸

To reflect this disagreement, a new function, $B'(i)$, could be derived to indicate the acceptance of a given interpretation i by the entire court; in this example, the dissent's opposed view of the Rule's "plain meaning" would lower the first component of $B(i)$ from 1 to some lesser number. (The simplest way would be to assign each Justice a weight of .11, or $1/9$; since the Rule's own language led five Justices to vote for, and four against, the interpretation in question, the first component of $B(i)$ would thus be $5/9$, or .55.)

3. Principles vs. Rules⁷⁹

The fuzzy view of legal decision-making efficiently incorporates the distinction made by Ronald Dworkin⁸⁰ between the strictness of rules ("A debtor may not file a Chapter 7 petition more than once in six years") and the relative flexibility of the underlying principles, which may themselves conflict ("Debtors are entitled to a 'fresh start'"; "Debtors should not be allowed to abuse the bankruptcy process by certain types of repeated filings."). A membership function is rule-like in that it, and its component functions, assign specific values to the satisfaction of specific criteria; yet the effect and interplay of the principles are given effect by the relative weights given to the component functions before the optimization process begins.

⁷⁶ See *id.* at 391-93 (explaining that under FED. R. CIV. PRO. 6(b), 13(f), and 60(b)(6) "excusable neglect" extends to inadvertent delays).

⁷⁷ See, e.g. *Pioneer Inv. Servs.*, 507 U.S. at 402 (noting definition of "excusable neglect" as "a failure to take the proper steps at the proper time, not in consequence of the party's own carelessness, inattention, or willful disregard of the process of the court, but in consequence of some unexpected or unavoidable hindrance or accident, or reliance on the care and vigilance of his counsel or on promises made by the adverse party.") (quoting BLACK'S LAW DICTIONARY 566 (6th ed. 1990) (O'Connor, J., dissenting)).

⁷⁸ *Id.* (emphasis added).

⁷⁹ See 11 U.S.C. § 727(a)(8) (1994) (prohibiting discharge where "the debtor has been granted a discharge under this section, under section 1141 of this title, or under section 14, 371, or 476 of the Bankruptcy Act . . .").

⁸⁰ See generally RONALD DWORKIN, TAKING RIGHTS SERIOUSLY (1977).

Indeed, Bart Kosko sees the legal process not as a "decision tree," in which the interlocking of facts with successive rules leads inexorably to a legal conclusion, but as a set of simultaneous balances, through which the judge fits fuzzy facts to the relevant principles, each of which carries a certain weight. "The judge weighs the principles and cites case precedents to back up the weights."⁸¹ He states that the judge does not give the weights as numbers— at least today a judge does not— but they are a matter of degree."⁸²

III. FUZZY DECISION-MAKING⁸³

Both classical and fuzzy decision theory are geared towards selecting the one that produces the maximum utility from a group of alternatives. If the alternatives are represented on the x-axis and the corresponding utilities on the y-axis, the correspondence of alternatives to utilities can be represented linguistically as "If x, then y."

Classical decision theory, then, seeks to find whichever x maximizes y. However, fuzzy decision theory places an independent utility on the appropriateness of the choice of x itself, and seeks simultaneously to maximize the choice of x and y. This approach, while in its own way creating applications for bankruptcy law, leads to the crucial development of fuzzy rules, examined in Part IV, below.

A. Introduction

Decision theory selects from a set of alternatives, subject to a set of constraints on the choice between different alternatives, the alternative whose utility or performance, as determined by a performance or utility function, is the closest to the specified goal.⁸⁴ One analysis of fuzzy decision-making has concluded that under conditions of risk "there is no longer an optimum policy but a class of optimum policies with varying degrees of risk."⁸⁵

⁸¹ See, e.g., KOSKO, *supra* note 5, at 178-80 (discussing how judges weigh all principals involved in deciding cases).

⁸² See *id.* at 180.

⁸³ See generally Bellman & Zadeh, *supra* note 14, at B-147-50 (describing decision making in fuzzy environment).

⁸⁴ The decisions addressed here are those made under conditions of certainty, where "the decision maker knows which state of nature to expect and he chooses the decision alternative with the highest utility, given the prevailing state of nature," as opposed to the more difficult situation of deciding "under risk," where "he does not know exactly which state will occur, but he only knows a probability function of the states." ZIMMERMANN, *supra* note 53, at 213.

⁸⁵ Sheldon S.L. Chang, *On Risk and Decision Making in a Fuzzy Environment*, in FUZZY SETS AND THEIR APPLICATIONS TO COGNITIVE AND DECISION PROCESSES, 219-226, at 219 (Lotfi Zadeh et al eds.. 1975).

For example, a law student returning to her apartment faces a **goal**: to have the dinner most conducive to her studying for two hours and then going to sleep. She may face the following **constraints** on alternatives: she does not have enough cash to send out for food, she is too tired to go out to the automatic teller machine and then, if she does not go out, the store is too crowded or too far to drive to. Thus, her **alternatives** are confined to what is in her refrigerator: leftover pizza, a cake, and five apples. Deciding that the utility of the pizza (which might keep her from falling asleep easily) and the cake (which might make her too sleepy to study) are low, that eating three of the apples would leave her too hungry and all five, too full, the student decides to eat four apples for dinner.

B. *Conventional Decision Theory*⁸⁶

More generally, many problems in decision theory lend themselves to modeling along x-y axes. The different alternatives, such as an amount of money to invest in a given opportunity are usually represented by various values of x, which occur over an interval (say, between 0 and 10). The relevant performance or utility function matches or "maps" each of these values of x to a corresponding y-value. The goal is usually to find either the alternative (value of x) that will maximize utility (y) or to find the alternative that corresponds to a given performance or utility level (value of y).

A simple example of such an approach involves the degree to which the law student can study for an exam three days away. The set of alternatives, X, consists of all values from 0 (no time spent studying) to 72 hours (100% of the time studying).

The performance function (which will not be specified here), as reflected in the y value associated with each x value, indicates that up to a certain point increasing study provides benefit, but that further study will produce decreasing (and, at some point, as the student foregoes sleep, even lower) benefits.⁸⁷

In this standard approach to decision-making, the constraints on the value of x are non-fuzzy: that is, they are of the form, "x is any amount between 0 and 72." The performance function takes values of x and maps them into values of y. In other words, the problem is reduced to finding the appropriate value (goal) of y (performance or utility) across all relevant x's (alternatives, subject to constraints).

The goal can be crisp, such as, "find the best amount of time [*i.e.*, the amount of time x that produces the greatest utility y] to study [between 0 and 72 hours]." The goal can also be fuzzy: "find an amount of time that I need to study to achieve

⁸⁶ See Bellman & Zadeh, *supra* note 14, at B-147-48 (stating that "[i]n the conventional approach the performance function associated with a decision process serves to define a linear ordering on the set of alternatives.").

⁸⁷ See Figure 6.

a proficiency in the vicinity of 70% of my maximum." In either case, however, conventional decision-making is indifferent to the relative values of choosing among different alternatives (values of x) so long as the appropriate goal (value of y) is achieved.

C. Fuzzy Decision Theory⁸⁸

By contrast, fuzzy decision-making treats not only the goal (y -value) but also the constraints (x -values) as fuzzy. That is, to each alternative x corresponds not only a y -value, $U(x)$, for the utility of x , but also a constraint value, $C(x)$, for the constraint value of x . Thus, there is a function F such that for each alternative x , $F(x) = [U(x), C(x)]$. In other words, goals (restrictions on y) and constraints (restrictions on x) are treated the same, or "symmetrically."⁸⁹

For the student who recognizes that 7 hours of sleep per night is necessary and who has other unavoidable commitments in the week before the examination, the alternative times to study themselves become fuzzy. The student may say, "I want to master about 80% of the material but I also want to study somewhere around 35 hours."

In effect, while the student applying conventional decision theory is solving the single equation, $U(x) = y$ for a specified value of y , the student applying fuzzy logic is finding the x that has highest degree of belonging both to $U(x)$ and $C(x)$, which may appear as in **Figure 7**.

The intersection of these two functions of x will provide the appropriate set of times that could be used to study, and the optimization function here is used to select the best.

D. Fuzzy Decision-Making Refinements

Beyond the limited scope of this paper, there are fuzzy logic techniques for the optimization of choices by means other than the technique described in Part II. B, above. The basis of a popular method, is to take the "mean" or "center of gravity"

⁸⁸ See generally KLIR & FOLGER, *supra* note 53, at 255-57; ZIMMERMANN, *supra* note 53, at 213-57; Bellman & Zadeh, *supra* note 14, at B-148-49 (defining fuzzy decision as "the fuzzy set of alternatives resulting from the intersection of the goals and constraints.").

⁸⁹ See Lotfi A. Zadeh, *The Calculus of Fuzzy Restrictions*, in *FUZZY SETS AND THEIR APPLICATIONS TO COGNITIVE AND DECISION PROCESSES I* (Lotfi A. Zadeh et al, eds., 1975). The author discusses in detail concept of fuzzy restriction [such as "young"]. See *id.* A fuzzy relation that acts as an elastic constraint on the values that may be assigned to a variable is an example. See *id.* Such restrictions appear to play an important role in human cognition, especially in situations involving concept formation, pattern recognition, and decision-making in fuzzy or uncertain environments.

of the set of points in n-dimensional space that correspond to various alternatives available to satisfy n different criteria.⁹⁰

Nor does this discussion consider the substantial literature that has evolved concerning the fuzzy modeling of decisions made by more than one person ("n-person game theories"), beyond noting that a distinction is made between situations in which different decision-makers with different goals consider the same information and independently arrive at different utility rankings of the alternatives ("group theories of decision-making") and situations in which the decision-makers share a goal but each have access to a different subset of the information relevant to the decision ("team theories of decision-making").⁹¹

E. Bankruptcy Applications

As noted, the chief application to bankruptcy law of fuzzy decision theory may be in the construction of fuzzy rules, which are discussed in Point V below.

IV. FUZZY RULES/ALGORITHMS/CONTROLLERS

Fuzzy controllers combine fuzzy rules and fuzzy algorithms, themselves amalgams of fuzzy definitions, fuzzy sets, and fuzzy decision theory, into a coherent system that extracts specificity from overlapping ambiguities.

The chief advantages of fuzzy controllers, which have corresponding benefits to those seeking to model legal rules, are their abilities: (1) to approximate complex mathematical relationships by the conglomeration of relatively simple mathematical rules; (2) to "blend" or average contradictory rules efficiently; and (3) to incorporate more, fewer, or changed rules as the situation and the available information demand.

A. Conventional Control Theory

Traditional control theory simulates the actions of an expert (for instance, an expert temperature regulator) by precise mathematical models. In developing advanced control systems, however, the use of mathematical models can be costly and computationally complex. This analytical approach is even more difficult when dealing with uncertainty such as "too much," "too little," "frequently," or

⁹⁰ See, e.g., KOSKO, *supra* note 5, at 173-74; MCNEILL & FREIBERGER, *supra* note 5, at 112-14.

⁹¹ See KLIR & FOLGER, *supra* note 52, at 257-58; see also L.W. Fung and K.S. Fu, *An Axiomatic Approach to Rational Decision Making in a Fuzzy Environment*, in FUZZY SETS AND THEIR APPLICATIONS TO COGNITIVE AND DECISION PROCESSES 227-256 (Lotfi A. Zadeh et al, eds., 1975) (deducing, from two alternative sets of axioms, conclusions concerning group decision-making).

"quite often," which are commonly handled by human experts in industrial control processes.⁹²

B. Fuzzy Control Theory

1. Basics

By contrast, by employing loosely defined variables as "tall" and "fast," fuzzy controllers enable: "air conditioners, washing machines and other devices [to] judge how fast they should operate or shift from one setting to another even when the criteria for making those changes are hard to define."⁹³

A *fuzzy algorithm* has been defined as:

an algorithm in which some of the instructions are fuzzy in nature. Examples of such instructions are:

(a) Increase x slightly if y is slightly larger than 10; (b) Decrease u until it becomes much smaller than v; (c) Reduce speed if the road is slippery. The sources of fuzziness in these instructions are the underlined words.⁹⁴

A fuzzy system is a collection of fuzzy algorithms. Bart Kosko has described a three-step process for constructing such a system by stating:

First, you pick the nouns or "variables." Call these X and Y. X is the input to the system. Y is the output. . . . If X, then Y. . . . Second, you pick the fuzzy sets. We define fuzzy subsets of the nouns X and Y. . . . Third, you pick the fuzzy rules [, associating various subsets of X with various subsets of Y]. . . .⁹⁵

For example, to automate the operation of a rotary cement kiln, two alternative systems were derived from discussions with human operators: one involved 75

⁹² See FUTURE COMPETITIVENESS, *supra* note 10, at 5. See Figure 10.

⁹³ Kosko & Isaka, *supra* note 4, at 76. These authors also observe that fuzzy controllers have been used in subway trains, washing machines, cameras and camcorders, vacuum cleaners, automobiles, and model helicopters. See *id.* at 78-80. See also FUTURE COMPETITIVENESS, *supra* note 10, at 14-15, 19-22, and 37 (enumerating and discussing fuzzy logic control-based products developed by Japanese industry).

⁹⁴ See ZADEH, *Fuzzy Systems*, *supra* note 14, at 485.

⁹⁵ See KOSKO, *supra* note 5, at 161.

fuzzy rules and 16 fuzzy variables, and the other involved 13 fuzzy rules and 12 fuzzy variables.⁹⁶

2. The Fuzzy Air Conditioner

The prototypical fuzzy controller, described at length by Kosko,⁹⁷ is the fuzzy air conditioner.

a. Variables

In this application, the input, or X, is the temperature of a room, and the output, or Y, is the speed of the air conditioner's motor.

b. Fuzzy Subsets

The fuzzy subsets of temperature are defined for the example as follows: "cold" (50 degrees and cooler), "cool" (40-65 degrees), "just right," (60-70 degrees), "warm" (65-85 degrees) and "hot" (greater than 80 degrees). Graphically, these categories are represented as in **Figure 8**.

The fuzzy subsets of motor speed are: "stop" (0 to 30 revolutions per minute), "slow" (10-50 r.p.m.), "medium" (40-60 r.p.m.), "fast" (50-90 r.p.m.), and "blast" (greater than 70 r.p.m.). These categories are represented as in **Figure 9**.

The fuzzy subsets of both temperature and speed take the form of isosceles triangles with bases spanning the range of temperature or r.p.m. Setting the height of the triangle for any temperature (or r.p.m. setting) corresponds to the degree, from 0 to 1, that that temperature (or setting) is a member of the specified subset. For example, 0, 55, 65, and 75 degrees belong with degree 1 to "cold," "cool," "just right," and "warm," respectively; and 50 and 60 degrees each belong with degree .5 to the fuzzy subset, "cool."

Notably, because the triangles overlap, some temperatures (and r.p.m. settings) are members of two different fuzzy subsets. The r.p.m. setting of 25 belongs with degree 0.3 to "stop" and also with degree 0.6 to "cold."

⁹⁶ See P. Martin Larsen, *Industrial Applications of Fuzzy Logic Control*, in *FUZZY REASONING AND ITS APPLICATIONS* 335, 337-38 (E.H. Mamdani and B.R. Gaines eds., 1981) (describing two fuzzy logic control projects on rotors cement kiln). The first use of a fuzzy controller for the operation of a cement kiln occurred in 1980. See Kosko & Isaka, *supra* note 4, at 76; see also KOSKO, *supra* note 5, at 184-87 (providing "1992 list of fuzzy products in Japan and North Korea," including air conditioners, cameras, copy machines, dishwashers, elevator controls, humidifiers, microwave ovens, refrigerators, televisions, video camcorders, and washing machines).

⁹⁷ See KOSKO, *supra* note 5, at 161-66, 171-75. The following discussion and diagrams of this programming and operation of a fuzzy air conditioner follow Kosko's exposition. See *id.*

c. Fuzzy Rules

The fuzzy rules, which match fuzzy subsets of temperature with fuzzy subsets of speed, are simple:

Rule 1: If the temperature is cold, the motor speed stops.

Rule 2: If the temperature is cool, the motor speed slows.

Rule 3: If the temperature is just right, the motor speed is medium.

Rule 4: If the temperature is warm, the motor speed is fast.

Rule 5: If the temperature is hot, the motor speed blasts.

The rules, which themselves overlap, are represented graphically in **Figure 10**, with temperature (input) on the x-axis and motor speed (output) on the y-axis. Associated with any temperature is either one rule or two overlapping rules (each of which is partially triggered).

C. Bankruptcy Applications

1. Fraudulent Transfers

In examining the circumstances under which intercorporate guaranties constitute fraudulent conveyances, Professor Jack Williams has proposed a fuzzy model of the Code's multi-alternative definition of fraudulent transfer, because:

[t]he binary logic of modern paradigms of the law falls short when describing the vagueness of the real world. A formal approach to fraudulent transfer law has reduced the system to a succession of statements that are either true or false, yes or no. Adequate value is either present, or it is not. A debtor is either insolvent, or it is not. A transfer or obligation is either fraudulent, or it is not . . . The [proposed] model rejects the predominant contemporary fraudulent transfer paradigms because of their bivalent nature.⁹⁸

Professor Williams focused on Section 548(a)(2), which allows the trustee to avoid obligations for which the debtor:

⁹⁸ Williams, *supra* note 6, at 1406.

(A) received less than a reasonably equivalent value in exchange. . . ; and
 (B)(i) was insolvent on the date that . . . such obligation was incurred, or became insolvent as a result of such . . . obligation. . . .⁹⁹

The Uniform Fraudulent Transfer Act¹⁰⁰ and the Uniform Fraudulent Conveyance Act¹⁰¹ are similar provisions.

Of particular interest to our discussion is this commentator's introduction of fuzziness to clarify the concept of insolvency. From the three statutes in question, Williams isolated seven different methods of assessing insolvency of the debtor guarantor, each involving a different permutation of inclusion or rejection of the value of the contingent liability assumed by the debtor (either ignored, included at present or discounted value, and/or included with or without discounting for probability of its occurrence, or included at discounted value limited to asset value of guarantor) and the value of the equitable rights of exoneration, reimbursement, subrogation, and contribution granted to the guarantor (ignored, or included at discounted value, or at present value).¹⁰²

Instead, he proposed a fuzzy new model of calculating insolvency for the purpose of fraudulent transfers:

(1) [I]f the guaranty is absolute and unconditional then, it should be treated as a liability at the face amount of the guaranty less any payments made.¹⁰³

(2) If the guaranty is a type other than absolute and unconditional . . . the court should determine the subjective probability of the guaranty being called based on the facts and circumstances known, or should have been known, by the parties at the time the guaranty obligation was incurred.¹⁰⁴

(a) If the probability that the guaranty will be called is greater than fifty percent, then the full amount of the outstanding indebtedness should be realized as a liability.¹⁰⁵ In this case, the court should conduct a fact-sensitive investigation into the value of

⁹⁹ 11 U.S.C. § 548(a)(2) (1994) (emphasis added); see Williams, *supra* note 6, at 1406, 1411 (describing fraudulent transfer law).

¹⁰⁰ UNIF. FRAUDULENT TRANSFER ACT §§ 4(a)(2), 5(a) (1985); see also Williams, *supra* note 6, at 1411 n.28 (quoting UNIF. FRAUDULENT TRANSFER ACT).

¹⁰¹ UNIF. FRAUDULENT CONVEYANCE ACT §4 (1985), quoted in Williams, *supra* note 6, at 1410 n.24.

¹⁰² See Williams, *supra* note 6, at 1442-45 (explaining different approaches to equitable rights).

¹⁰³ *Id.* at 1463.

¹⁰⁴ *Id.*

¹⁰⁵ *Id.* at 1464.

the equitable rights that the guarantor can count as assets to offset the liability under the guarantee.

(b) If the probability that the guaranty will be called is less than or equal to fifty percent, then the guaranty liability should be valued at zero.¹⁰⁶

The chief advantage claimed for the fuzziness of this model is that it simplifies the prevailing standard, under which the court set a probability value (from 0 percent to 100%) on the contingency's occurring, and then valued the amount of the contingency by multiplying this number by the amount that would be owed by the debtor (whether or not the debtor had assets to pay).¹⁰⁷ Williams also notes:

The contemporary models, however, seek precision at the expense of clarity. In assessing the probability that the guaranty will be called, courts attempt to fix an exact probability— five percent, thirty percent, seventy-eight percent. One court concluded that the probability the guaranty would be called was 1.7%. Why not 1.8%, or 1.6? Courts deceive themselves if they believe they can make such fine distinctions. Moreover, courts mislead themselves if they believe fraudulent transfer law requires the exactitude they seek to attain.¹⁰⁸

2. Plain Language: Heuristics and Hermeneutics

In a 1992 article, (then-)Judge Stephen Breyer identified:

five different circumstances in which courts might turn to legislative history for help in interpreting a statute: (1) avoiding an absurd result; (2) preventing the law from turning on a drafting error; (3) understanding the meaning of specialized terms; (4) understanding the "reasonable purpose" a provision might serve; and (5) choosing among several possible "reasonable purposes" for language in a politically controversial law.¹⁰⁹

¹⁰⁶ *Id.*

¹⁰⁷ See *Covey v. Commercial Nat'l Bank of Peoria*, 960 F.2d 657, 659 (7th Cir. 1992) (modifying previous model by including contingent liability with equitable rights both at discounted value); see also Williams, *supra* note 6, at 1446 (discussing *Covey* decision).

¹⁰⁸ Williams, *supra* note 6, at 1464 (footnote omitted).

¹⁰⁹ Stephen Breyer, *On the Uses of Legislative History in Interpreting Statutes*, 65 S. CAL. L. REV. 845, 860-61 (1992).

If these five categories were seen as (slightly) overlapping fuzzy sets ranged along the spectrum of justifications for the use of legislative history, ranging from least (#1) to most (#5) controversial, they could serve as the "input" for a fuzzy controller whose "output" would be a specified degree of use of legislative history. These fuzzy sets might include the six interpretive contexts identified above, with one added for the use of legislative history. The temperature/r.p.m. axes of Kosko's air conditioner could be replaced by justification/interpretive technique.

This controller would not only allow the Court to model its choice of appropriate technique but would possibly enable it to "control" the interpretation of future questions in the same manner as the air conditioner seeks one stable temperature.

3. Rules vs. Standards

To some extent, the "fuzzy controller" view of law addresses the dissatisfaction expressed by exponents of Critical Legal Studies movement ("CLS") with the loose mix of specific rules and more flexible standards that are applied by courts and lawyers to solve legal problems.¹¹⁰ In fuzzy logic, a rule is a narrow fuzzy set and a standard a wide fuzzy set. Both can be used at the same time, and the dividing line between them is vague.¹¹¹

¹¹⁰ See MARK KELMAN, A GUIDE TO CRITICAL LEGAL STUDIES, 20-49 (1987) (summarizing CLS objections). The CLS objectionism, as enumerated by Kelman, are:

(1) that "the legal system simultaneously embraces doctrines that allow any particular case to be decided as if either the rule-like decision mode or the standard-like one were in force." *Id.* at 20.

(2) that rules can be overinclusive or underinclusive, and that standards can be applied arbitrarily or prejudicially. *Id.* at 40.

(3) that the specificity of rules allows them to be evaded by clever actors, while the flexibility of standards leads to surprise and inefficiency when people are not sure how they will be applied in their case. *Id.* at 40-43.

(4) that "[i]t is possible to establish legal rules, increasingly detailed in covering available cases, that can become mechanically applicable to the vast bulk of actual controversies, but *practice* may well become settled only at the cost of *principled doctrine* becoming chaotic." *Id.* at 46. That is, "citizens cannot conceivably know their precise obligations if they are not apparent from a knowledge of a short list of clear principles." KELLMAN, at 47.

(5) that "while official discretion may be restrained in particular cases, the sense of arbitrariness and horizontal inequity that is supposedly limited by containing discretion runs wild when meaningless factual distinctions govern outcomes." *Id.*

(6) that there is "the possibility that the invocation of a clear rule in one area may not be counterbalanced by a conflicting rule or standard that actors in the system see as arising in the same field, but as 'corrected' in what is generally seen as a separate domain. . . . [and] that complete legal relief depends not simply on the application of a rule but on vague standard as well." *Id.* at 48-49.

¹¹¹ See Williams, *supra* note 6, at 1458-59 providing that:

In fuzzy systems, rules and standards may coexist in the same set. Fine rules allow small patches of discretion. Sloppy rules or standards allow large patches of

V. FUZZY EXPERT SYSTEMS

After suggesting that "in the future we may have good cheap legal advice on a chip," Professor Kosko almost immediately added that "lawyers . . . on a chip would weave a more tangled legal web."¹¹²

Behind this proposal is the idea that law can be reduced to an "expert system," which consists of: a database to which can be added factual information about the situation at hand; a knowledge base, consisting of a number of fuzzy "if—then" rules derived from existing statutes and legal precedents, and an "inference engine" that combines the facts and rules to produce an answer.¹¹³

However, in keeping with the spirit of *direct* experience that underlies fuzzy logic's philosophical forebears, we should remember the Taoist wheelwright who placed little value on books written by "experts" and "authorities." When asked to explain his rejection of such apparent wisdom he simply said:

. . . When I make wheels
 If I go easy, they fall apart,
 If I am too rough, they do not fit.
 If I am neither too easy nor too violent
 They come out just right. The work is what
 I want it to be.
 You cannot put this into words:
 You just have to know how it is¹¹⁴

discretion. Standards are more appropriate where one knows less about a problem. Conversely, where one knows more about a problem, finer rules with more precision may be established. But precision is not a free commodity. Rule-bounded systems tend to lose their fuzziness and their common-sense meaning. The systems may also become unwieldy and difficult to apply. Like any mathematical model, fuzzy logic falls victim to the "curse of dimensionality." The number of fuzzy rules grows exponentially. They become more precise but less useful. At some point, a fuzzy system must contend with a trade-off between numerous rules and standards. Large rule patches, that is, standards, make a system more manageable but also less precise.
 [footnotes omitted]

¹¹² See KOSKO, *supra* note 5, at 262-63.

¹¹³ See MCNEILL & FREIBERGER, *supra* note 5, at 211-21; *see also* FUTURE COMPETITIVENESS, *supra* note 10, at 7.

¹¹⁴ CHUANG TZU (trans. Merton), *supra* note 21, at 83.

APPENDIX

FIGURE 1

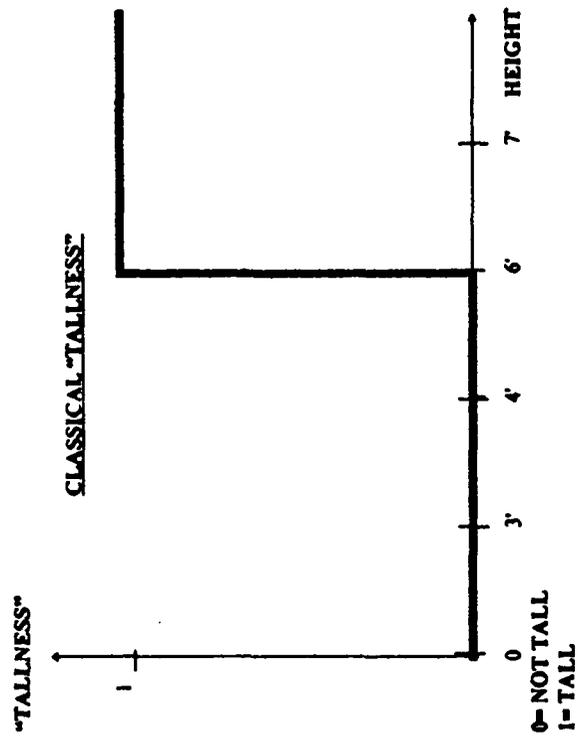
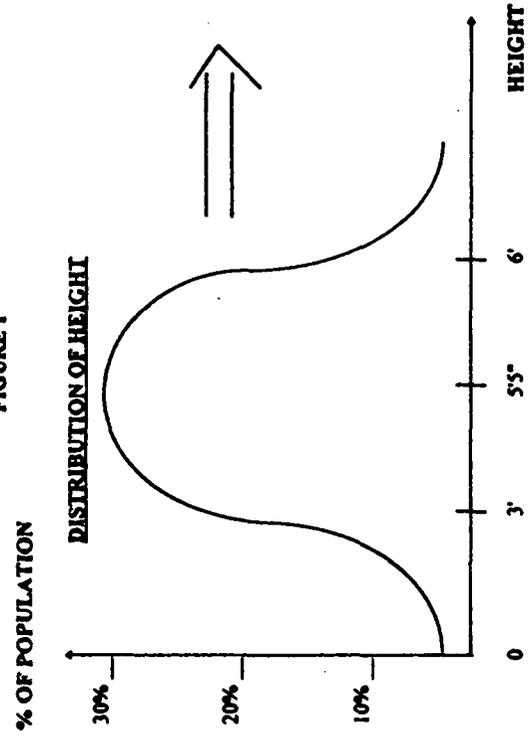
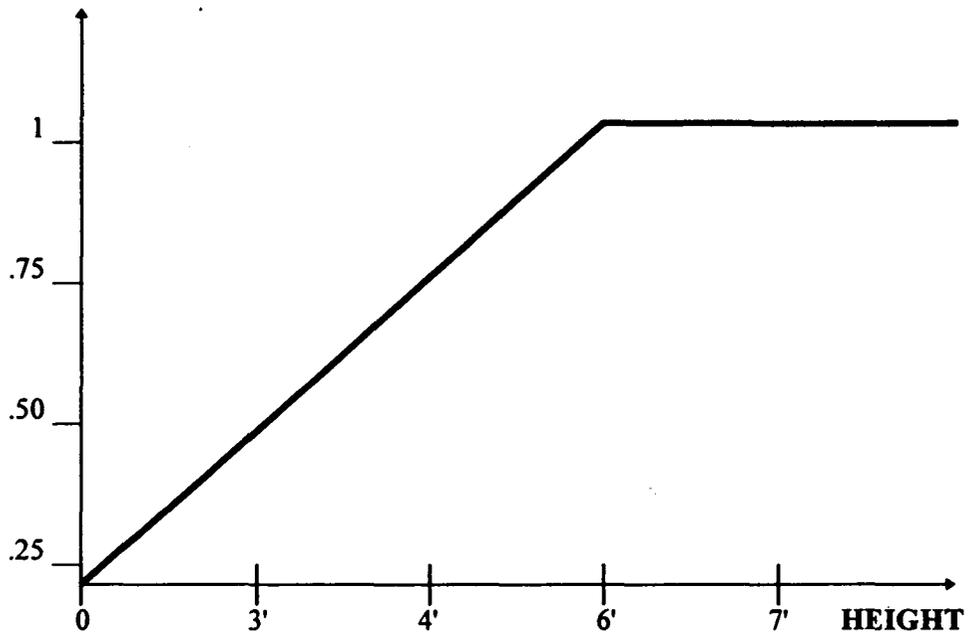


FIGURE 2

"TALLNESS"



0 → 1 = degree of tallness

FIGURE 3

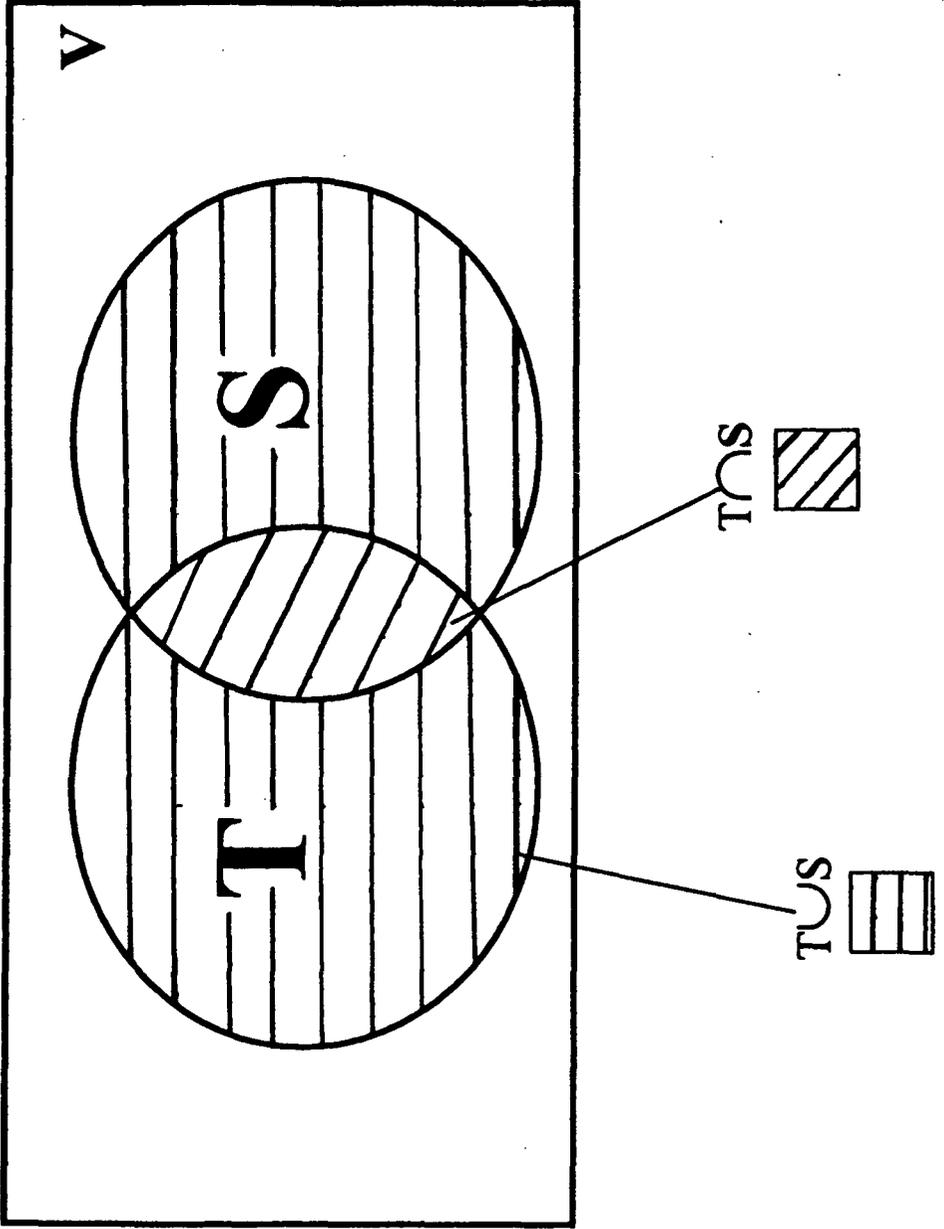


FIGURE 4

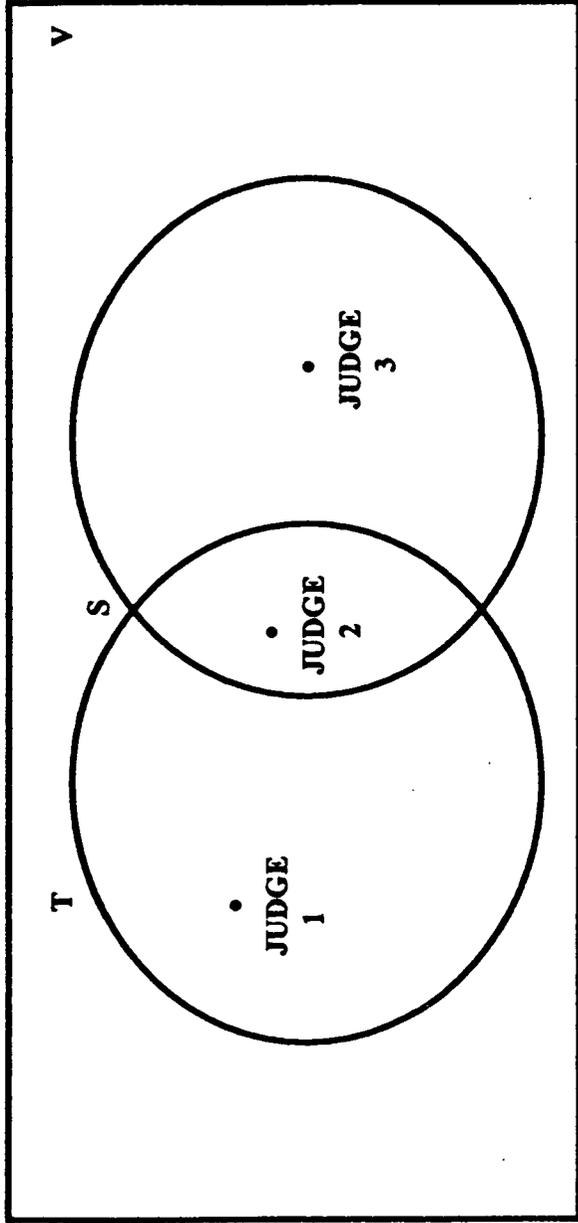


FIGURE 5

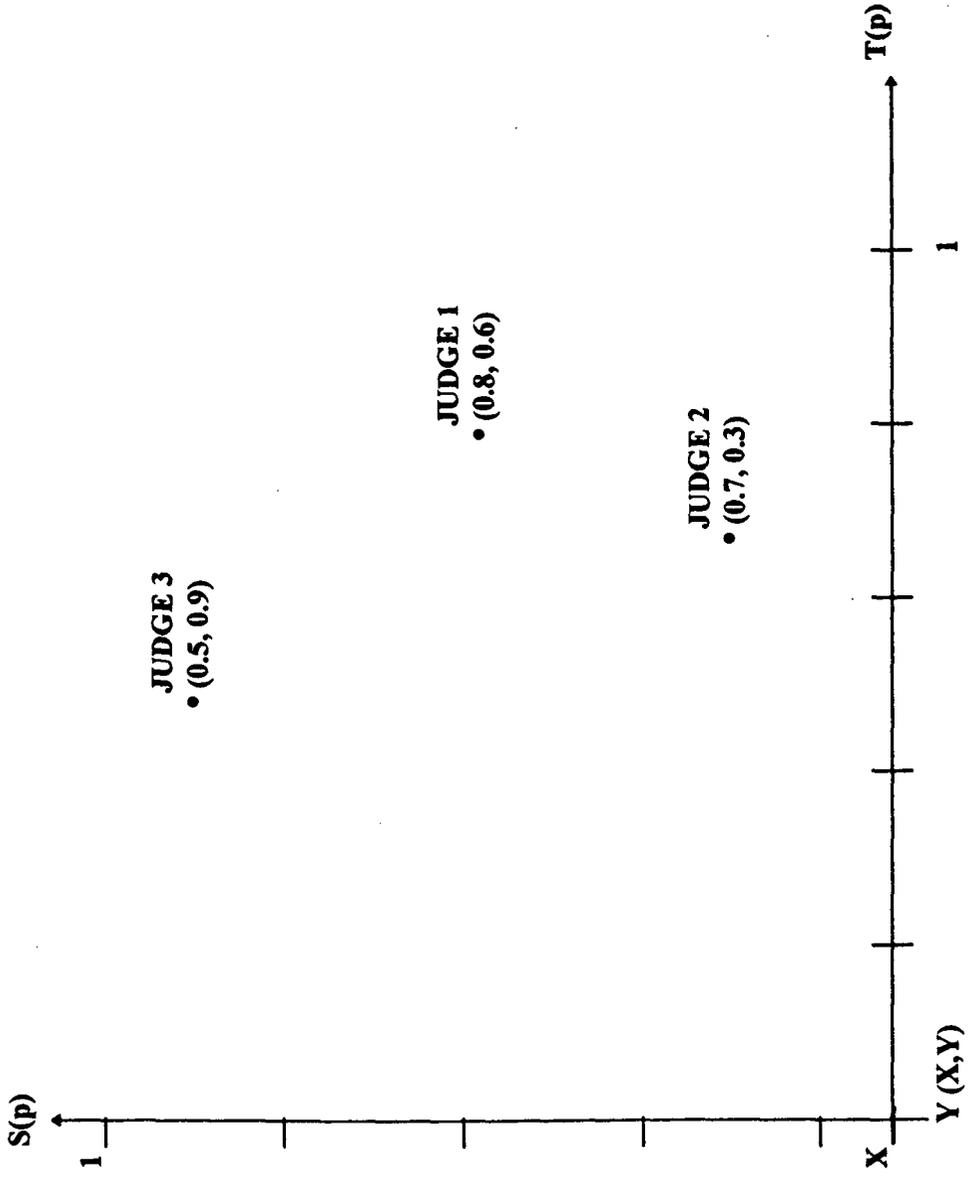


FIGURE 6

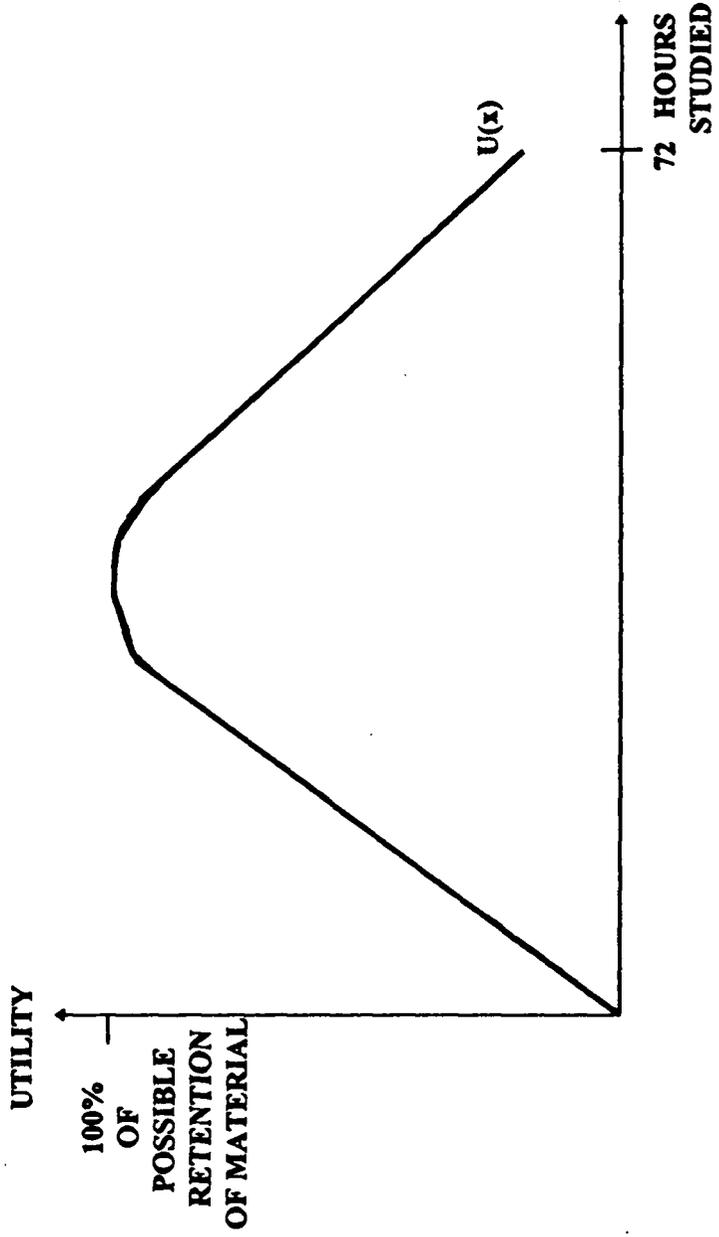


FIGURE 7

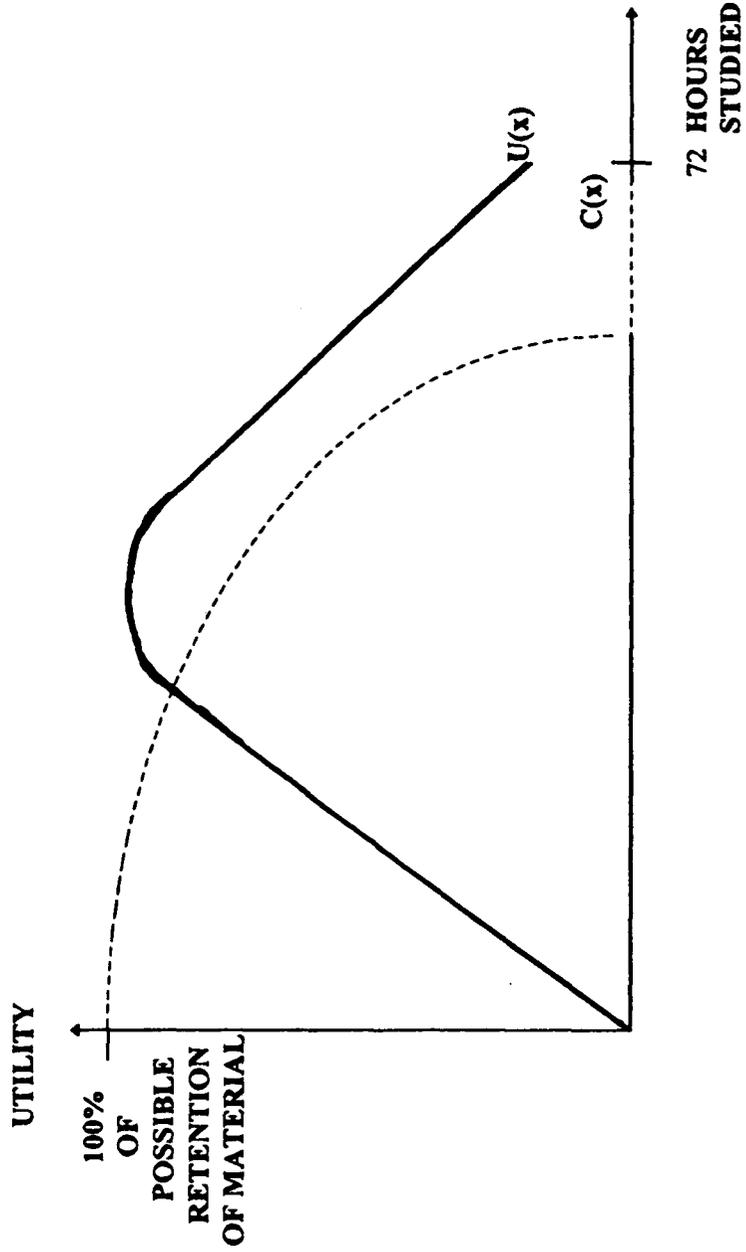


FIGURE 8

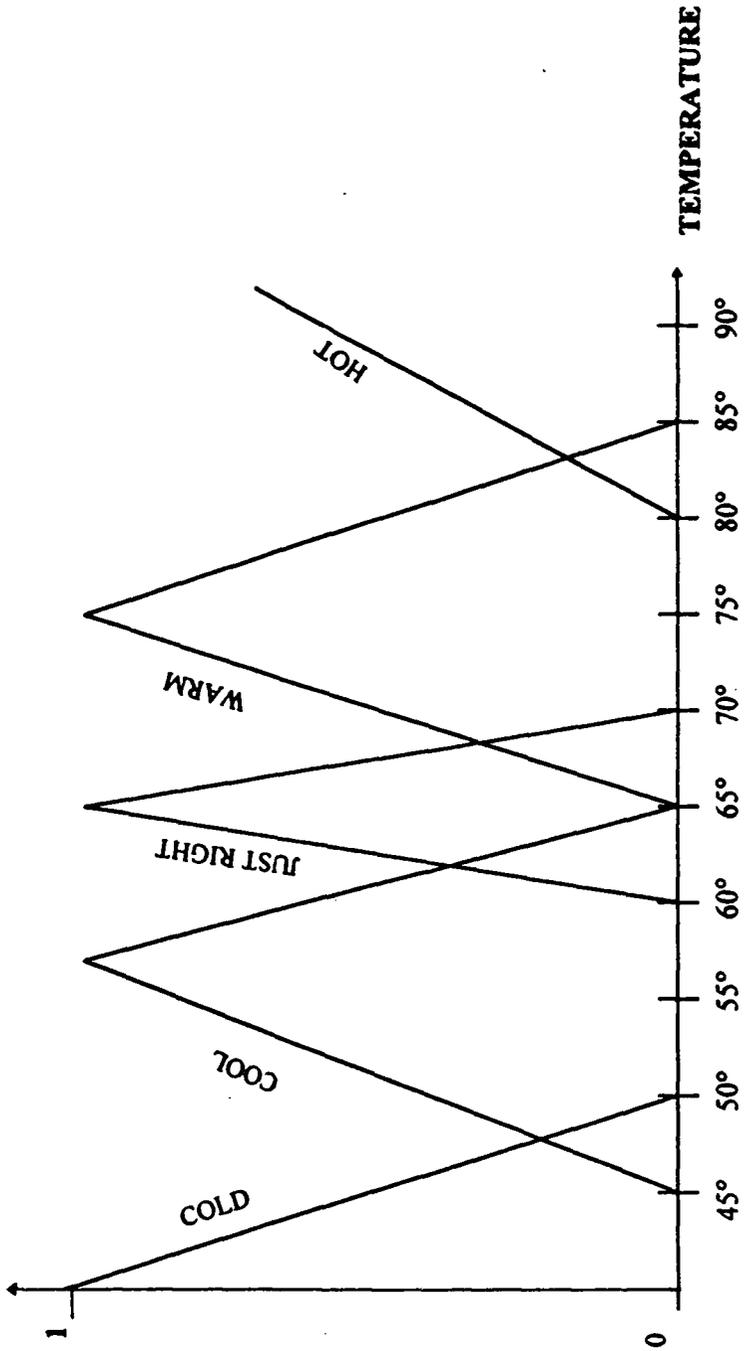


FIGURE 9

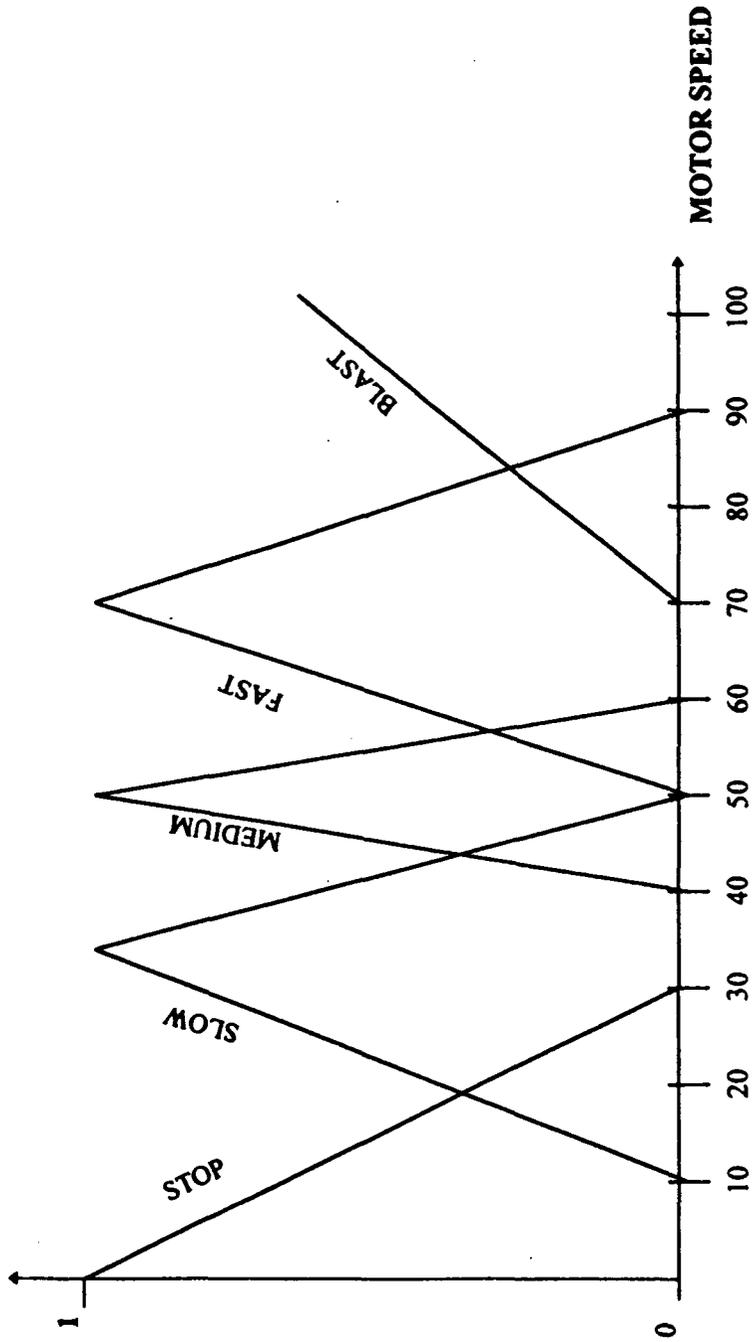


FIGURE 10

