Price Dispersion and the Costs of Inflation

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Abstract

The new Keynesian literature typically makes the assumption that firms always have to satisfy demand. Because this assumption is at odds with profit-maximizing behavior under Calvo pricing when long-run inflation is positive, we present a new Keynesian model that relaxes this assumption. Our model predicts that inflation causes a substantially smaller loss in effective aggregate productivity compared to a benchmark model without the possibility of rationing. Moreover, under positive inflation, firms choose smaller markups over marginal costs in our model than in the benchmark model. As a result, our analysis suggests that the standard new Keynesian model may exaggerate the welfare costs of inflation. We also show that the effects are plausible to be quantitatively important.

Keywords: trend inflation, optimal inflation target, new Keynesian model, welfare costs of inflation.

JEL: E31, E50.
1 Introduction

What makes inflation costly? How large are the social costs of inflation? These core questions in monetary economics are not only interesting in their own right. They also have important implications for monetary policy-making as the answers to them determine the level of the inflation target that central banks should try to achieve.

The mainstream new Keynesian model implies that inflation is costly because, in combination with staggered price-setting, it leads to distortions in relative prices and thereby to an inefficient allocation of economic resources (see e.g. Woodford (2003)). Our paper shows that this mechanism is substantially weaker once one relaxes the implicit assumption typically made in new Keynesian models that firms always satisfy demand. While this behavior is optimal when inflation is zero and prices are set above marginal costs, profit-maximizing firms would not satisfy demand completely after sufficiently long spells of fixed nominal prices when inflation is positive.

Despite the fact that the fraction of firms that profit from rationing demand is typically small, we show that the aggregate consequences may be non-negligible. In our benchmark new Keynesian model without the possibility to ration demand, an increase in the inflation rate from 0% to 7% causes inefficient price dispersion that results in a decrease in effective productivity by 36%. By contrast, the effective productivity decrease induced by an inflation rate of 7% is less than one percent in our main model that allows firms to choose their output optimally in every period.

The intuition is straightforward. In the standard new Keynesian model under positive inflation, too many resources are allocated to firms that have not adjusted their prices for long periods of time and therefore have low relative prices. If these firms have the possibility to constrain production to some extent, this will alleviate the inefficiencies caused by inflation.

There is also another effect that leads to lower social costs of inflation when firms have the possibility to ration demand. Without this possibility, it is risky for firms to choose small markups when inflation is positive, as the corresponding nominal price may be
fixed for a considerable time period, which would result in a very low relative price and possibly high losses. The possibility to ration demand alleviates this risk and thus induces firms to choose smaller markups, which leads to more moderate distortions stemming from monopolistic competition.

To obtain these findings, we compute the stationary equilibria of a new Keynesian model with Calvo (1983) pricing where firms can ration demand but which is completely standard otherwise. Moreover, we utilize this framework to analyze the long-term relationship between inflation and other economic variables. In particular, we derive the following findings. First, we show analytically that, under positive inflation, price dispersion, which is inversely related to effective aggregate productivity, is always lower in our model with demand rationing compared to the standard new Keynesian model where firms always have to satisfy demand.

Second, we prove that our model has an equilibrium for arbitrary levels of inflation. This contrasts with the well-known finding that the standard new Keynesian model does not have an equilibrium for sufficiently high inflation rates (see Ascari and Sbor-done (2014), for example).¹ For high inflation rates, the inefficiencies due to distorted relative prices are so severe in the standard new Keynesian model such that no positive quantity of aggregate output can be produced. By contrast, the harmful effect of inflation on effective productivity is significantly smaller in our model, which explains why an equilibrium exists for arbitrarily high inflation rates.

Third, we derive the conditions under which firms ration demand. For severe deflation, firms ration demand for some time after re-setting their prices. For moderate deflation, firms never ration demand. For positive inflation, rationing occurs for firms that have not adjusted their prices for some time, as their relative prices are comparably low and the demands for their goods are high.

Fourth, we prove that, in our main model with rationing, the distortions due to the interaction of positive inflation rates with monopolistic competition are less severe than in our benchmark model. More precisely, when inflation is positive, firms always choose

¹This is true under Calvo (1983) pricing but would hold under Rotemberg (1982) pricing as well.
smaller markups over marginal costs in our main model compared to the benchmark new Keynesian model without rationing.

Fifth, together with the finding that the effective productivity loss caused by inflation is considerably smaller in our main model, this effect entails that the standard new Keynesian model exaggerates the welfare costs of inflation. For example, the standard model predicts that 7% inflation leads to a 36% consumption-equivalent welfare loss over and against zero inflation. In our main model, the respective loss is only 2%.

While, to the best of our knowledge, this paper is the first to study the possibility of rationing demand in the new Keynesian model and the consequences for price-setting as well as the costs of inflation, it is related to a large literature on the costs of inflation and the socially desirable level of inflation in the long term (see Schmitt-Grohé and Uribe (2010) and Diercks (2017) for surveys). The traditional view about the costs of inflation stresses that higher inflation is associated with higher nominal interest rates and thereby larger opportunity costs of holding money. As a consequence, the Friedman rule, i.e. permanent deflation that eliminates the opportunity costs of holding money, is optimal. By contrast, zero inflation is typically optimal in the standard new Keynesian model in the limiting case where real money balances are zero, as it alleviates the distortions in relative prices under staggered price setting. In a model where both channels are at work, Khan et al. (2003) find optimal rates of inflation that are slightly negative.²

Recently, the standard mechanism for the costs of inflation in the new Keynesian model has come under fire by Nakamura et al. (2018). With the help of a new data set on prices that also covers the Great Inflation of the 1970’s, they argue that empirical support for a link between price dispersion and inflation rates is wanting. Our paper is complementary to theirs, as we provide a theoretical argument for why the standard new Keynesian model may exaggerate the costs of inflation.

²Adam and Weber (2017) and Lepetit (2017) show that different forms of heterogeneity may entail a positive optimal rate of inflation. Blanco (2018) demonstrates that a positive inflation target can be optimal in a model with menu costs, idiosyncratic shocks to firms’ productivities, and the zero lower bound.
In the aftermath of the 2007/2008 global financial crisis in particular, the literature has examined to which degree the so-called zero lower bound on interest rates and the possibility of a lower natural real rate of interest provide a rationale for higher inflation targets. This question is addressed by Reifschneider and Williams (2000), Coibion et al. (2012), and Andrade et al. (2018), among others. As our paper concentrates on analyzing the steady state for different inflation rates, we abstract from the zero lower bound, which appears more relevant in analyses that study how shocks can temporarily push interest rates to zero.

While much of the new Keynesian literature considers models that are log-linearized around a zero-inflation steady state, there are several authors who allow for positive trend inflation (Ascari (2004), Hornstein and Wolman (2005), Yun (2005), Amano et al. (2007), Ascari and Ropele (2007), Ireland (2007), Kiley (2007), Levin and Yun (2007), Cogley and Sbordone (2008), Coibion and Gorodnichenko (2011), Ascari et al. (2015), and Kurozumi (2016)). This literature, which is surveyed in Ascari and Sbordone (2014), examines, among other things, how positive trend inflation affects equilibrium determinacy as well as the equilibrium dynamics in response to shocks.\footnote{Yun (2005) analyzes how optimal policy depends on the initial degree of price dispersion.} None of these papers examine how the welfare costs of inflation depend on the assumption that firms always have to satisfy demand.

This paper is organized as follows. In Section 2, we present a benchmark new Keynesian model and our main model, in which we allow firms to ration demand. Section 3 solves the benchmark model and identifies the situations in which rationing would be profitable to firms. In the subsequent Section 4, we solve our main model and determine several properties of the equilibrium analytically. Our main results regarding price dispersion and the costs of inflation are presented in Section 5. Section 6 concludes.

2 Model

Our model is based on a textbook macroeconomic model (see Woodford (2003)) with price setting à la Calvo (1983) and positive trend inflation. We will analyze two variants
of this model. In Section 3, we will examine our benchmark model, i.e. the standard case where each firm always satisfies the demand for its goods. In Section 4, we examine our main model, where firms can supply smaller quantities than demanded if that is profitable to them. In line with our objective to examine the costs of inflation in the long run, we consider stationary equilibria of both economies for different inflation rates.

Time is continuous and denoted by \( t = [0, \infty) \).\(^4\) The model is populated by households, a representative, perfectly competitive final-goods producer, monopolistically competitive intermediate-goods producers, and a central bank. We provide details about each of these groups in turn.

### 2.1 Households

There is a continuum of households that own identical shares of all firms and receive firms’ profits as dividends. Each household has the following instantaneous utility function

\[
U(c(t), n(t)) = \frac{c(t)^{1-\sigma} - 1}{1 - \sigma} - \frac{\Psi n(t)^{1+\varphi}}{1 + \varphi},
\]

where \( \sigma, \varphi, \) and \( \Psi \) are positive parameters, \( n(t) \) stands for the household’s supply of labor and \( c(t) \) is consumption of a final good.

Utility in future periods is discounted at a positive rate \( \rho \). In all periods \( t \), the nominal budget constraint is

\[
P(t)c(t) + \dot{B}(t) = I(t)B(t) + P(t)w(t)n(t) + P(t)T(t),
\]

where \( P(t) \) denotes the price of the final good, \( B(t) \) bond holdings, \( I(t) \) the nominal interest rate, \( w(t) \) the economy-wide real wage, and \( T(t) \) a real transfer, which includes the profits of firms and the government’s seigniorage revenues. Throughout the paper,

\(^4\)The assumption of continuous time is mathematically convenient in our main model. As we will see, there is typically a specific price spell for which firms start rationing demand (or stop rationing demand). For continuous time, this price spell is a continuous and differentiable function of the initial price chosen by an intermediate firm. As a consequence, the solution to the firm’s profit maximization problem can be obtained by solving a standard first-order condition.
a dot on top of a variable stands for the derivative with respect to time. Bonds are in zero net supply.

As is well-known, the household’s optimization problem leads to the following consumption Euler equation and condition for the optimal supply of labor:

\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1} (I(t) - \pi(t) - \rho) \tag{3}
\]

\[
\Psi n(t)^{\sigma} = w(t)(c(t))^{-\sigma}, \tag{4}
\]

where \( \pi(t) := \frac{P(t)}{P(t)} \) is the inflation rate.

### 2.2 Final-Goods Producers

Perfectly competitive final-goods producers assemble the different intermediate goods \( y_j(t) \ (j \in [0,1]) \) to produce a final good according to the production function

\[
y(t) = \left[ \int_0^1 (y_j(t))^\frac{\theta-1}{\sigma} \, dj \right]^{\frac{\sigma}{\theta-1}}, \tag{5}
\]

where \( \theta \ (\theta > 1) \) stands for the elasticity of substitution between the differentiated goods. The resulting demand for intermediate good \( j \) is

\[
y_j^d(t) = \left( \frac{P_j(t)}{P(t)} \right)^{-\theta} y(t). \tag{6}
\]

In the benchmark model, intermediate-goods producers always satisfy the demand for their goods. In our main model, these firms may ration demand if that is profitable.

### 2.3 Intermediate-Goods Firms

There is a continuum of monopolistically competitive intermediate-goods producers, indexed by \( j \in [0,1] \). The production function of firm \( j \) is of the form

\[
y_j(t) = An_j(t)^{\gamma}, \tag{7}
\]
where \( A > 0, \gamma \in (0, 1) \) and \( n_j(t) \) is the labor input of firm \( j \) at time \( t \). As \( \gamma < 1 \), there are decreasing returns to scale.\(^5\) One possible interpretation of this case is that output is given by a standard Cobb-Douglas production function with fixed capital.\(^6\)

In line with Calvo (1983), intermediate-goods firms can only adjust their prices if they receive a shock, which arrives with constant rate \( \delta \).\(^7\) Thus we do not consider indexation or rule-of-thumb pricing for firms that are not able to re-optimize their prices. These assumptions are sometimes made in the literature (see Yun (1996), for example), but are incompatible with the microeconomic evidence that prices are fixed for considerable periods (see Klenow and Kryvtsov (2008) or Nakamura and Steinsson (2008)).

As has been mentioned before, we distinguish between two cases. First, as is standard in the new Keynesian literature, we assume in our benchmark model that firms always have to satisfy demand at the current price even if this entails losses in this period. Second, in our main model we adopt the assumption that firms are able to ration demand at the current price. It is straightforward to derive that, in the case where firm \( j \) could choose its output freely, its optimal output, for given price \( P_j(t) \), would be

\[
y^*_j(t) = \left( \gamma \frac{A^{\frac{1}{\gamma}} P_j(t)}{w(t) P(t)} \right)^{\frac{\gamma}{1-\gamma}}.
\]

Figure 1 illustrates schematically how output is determined in the two model variants. In the benchmark model, output is always given by demand (see the left panel). While this is a perfectly reasonable assumption under zero inflation rates because monopolistically competitive firms set prices above the market-clearing level, this assumption is more problematic under positive inflation rates and rigid prices. If a price \( P_j(t) \) has not been adjusted for some time, the relative price \( \frac{P_j(t)}{P(t)} \) may drop below the market-clearing price.

\(^5\)Decreasing returns to scale are frequently used in the literature. In our case, they have the convenient implication that firms that choose to ration demand still supply positive quantities of intermediate goods. For constant returns to scale, firms would stop production completely when relative prices drop below marginal costs.

\(^6\)It would be easy to include capital into our model and to examine the consequences of different inflation rates for the capital stock in the long run. Here we follow the large part of the literature that disregards capital accumulation (see the discussion on p. 724 in Ascari and Sbordone (2014)). At any rate, our main findings are plausible to hold in a model with endogenous capital as well.

\(^7\)It might be interesting to endogenize \( \delta \). We discuss this issue in more detail in Section 6.
level. In this case, \( y^s_j(t) < y^d_j(t) \). As a consequence, the main model (illustrated in the right panel) involves that output is supply-determined in this case.

In our main model, firm \( j \)'s optimal choice in a period \( t \) in which it can change its price is given by the solution to the following maximization problem:

\[
\max \mathbb{E}_t \left[ \int_t^{\infty} \frac{\lambda(\tau)}{\lambda(t)} e^{-(\rho+\delta)(\tau-t)} \left( \frac{P_j(t)}{P(\tau)} y_j(\tau) - w_\tau \left( \frac{y_j(\tau)}{A} \right)^{1/\gamma} \right) d\tau \right] \\
\text{s.t. } y_j(\tau) = \min\{y^d_j(\tau), y^s_j(\tau)\}, \forall \tau \geq t,
\]

(9)

where \( \lambda(t) = c(t)^{-\sigma} \). The optimization problem in the benchmark case without the possibility of rationing is identical, except that the constraint is \( y_j(\tau) = y^d_j(\tau) \), \( \forall \tau \geq t \), in this case.

### 2.4 Monetary Policy

We close the model by assuming that the central bank follows a simple Taylor rule:

\[
I(t) = \rho + \pi^* + \phi(\pi(t) - \pi^*),
\]

(10)

where \( \phi > 1 \) and \( \pi^* \) is the central bank's inflation target.
3 Benchmark Model

Before exploring the possibility that firms ration demand, it will be instructive to solve the benchmark case where intermediate-goods firms always have to satisfy demand.\(^8\)

In a stationary equilibrium, equations (3) and (4) simplify to:

\[
I_B = \pi^* + \rho, \tag{11}
\]

\[
\Psi(n_B)^\varphi = w_B(y_B)^{-\sigma}, \tag{12}
\]

where we have introduced the subscript \(B\) to denote equilibrium values for the benchmark economy and have used \(c_B = y_B\).

Equation (5) can be rewritten as

\[
y_B = \delta \left[ \int_0^\infty e^{-\delta \tau} \left( q_B^{-\theta} e^{\theta \pi^*} y_B \right)^{\frac{\theta+1}{\theta}} d\tau \right]^{\frac{1}{\theta-1}}, \tag{13}
\]

where \(q_B\) is a firm’s reset price in relation to the price index \(P(t)\), i.e. \(q_B = \frac{P_j(t)}{P(t)}\) at the time when it adjusts its price. Computing the integral in (13) and re-arranging yields\(^9\)

\[
q_B = \left( \frac{\delta}{\delta - (\theta - 1)\pi^*} \right)^{\frac{1}{\theta-1}}. \tag{14}
\]

We note that \(q_B\) satisfies \(q_B = 1\) for \(\pi^* = 0\) and increases with inflation. The fact that \(q_B > 1\) for \(\pi^* > 0\) has the straightforward interpretation that, under positive inflation, newly adjusted prices are higher than prices that were selected in the past. It is also noteworthy that we have to impose the restriction \(\pi^* < \frac{\delta}{\theta-1}\).\(^10\) Otherwise the integral in (13) cannot be computed and no solution for \(q_B\) exists.

The production function for the final good, (5), can be combined with the demand for intermediate goods, (6), to obtain the following well-known equation for the price level:

\[
P(t) = \left[ \int_0^1 (P_j(t))^{\theta-1} dj \right]^{\frac{1}{\theta-1}} \tag{15}
\]

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\(^8\)A similar analysis for non-zero steady-state inflation can be found in Ascari and Sbordone (2014), for example.

\(^9\)According to the terminology introduced by King and Wolman (1996) and used in Ascari and Sbordone (2014), \(q_B\) corresponds to the inverse of the price-adjustment gap.

\(^10\)It is well-known that inflation must not be too high for the new Keynesian model with Calvo (1983) pricing to have an equilibrium. The respective conditions for the discrete-time case are stated in Ascari and Sbordone (2014), for example.
It may be worth stressing that this equation will not hold automatically in the main model with possible rationing, as an intermediate firm’s output is not necessarily given by the respective demand, (6).

The labor-market clearing condition, \( n_B = \int_0^1 n_j(t) \, dj \) can also be stated as \( n_B = \int_0^1 \left( \frac{y_j(t)}{A} \right)^{\frac{1}{\gamma}} \, dj \). Using (6), \( \frac{P_j(t)}{P(t)} = q_B e^{-\pi \tau} \) for the relative price of a firm \( j \) that adjusted its price \( \tau \) periods ago, and \( \delta e^{-\delta \tau} \) for the density function for the ages of prices, we obtain

\[
\frac{\Delta}{\delta - \frac{\pi^*}{\gamma}} \left( \frac{y_B}{A} \right)^{\frac{1}{\gamma}} (q_B)^{-\frac{\phi}{\gamma}}.
\]

The integral that we evaluated to compute \( n_B \) only exists if \( \pi^* < \frac{\gamma \delta}{\theta} \).

Equations (14) and (16) can be combined to yield

\[
y_B = \frac{A}{s_B} (n_B)^{\gamma},
\]

where

\[
s_B := \left( \frac{\delta}{\delta - \frac{\pi^*}{\gamma}} \right)^{\gamma} \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\phi \gamma - 1}.
\]

In the literature, \( s_B \) is viewed as a measure of price dispersion. In line with (17), we can interpret \( A/s_B \) as a measure of effective aggregate productivity.

As is well-known, \( s_B \) attains its global minimum of 1 at \( \pi^* = 0 \) (see Schmitt-Grohé and Uribe (2007)). Analogously, effective productivity \( A/s_B \) reaches its global maximum of \( A \) at \( \pi^* = 0 \). For \( \pi^* < 0 \), \( A/s_B \) increases monotonically with \( \pi^* \). It decreases with \( \pi^* \) for positive inflation rates. Importantly, as \( \pi^* \) approaches \( \frac{\gamma \delta}{\theta} \), \( s_B \) goes to infinity and effective productivity approaches zero accordingly. In this case, even a very high quantity of labor can produce only minimal aggregate output. For inflation rates equal to or higher than \( \frac{\gamma \delta}{\theta} \), no equilibrium of the standard new Keynesian model exists as a consequence.

\[\text{11} \text{See the survey by Ascari and Sbordone (2014), for example.}\]

\[\text{12} \text{In the limit where prices are perfectly flexible, i.e. for} \delta \rightarrow \infty, \text{price dispersion goes to one as well.}\]
An intermediate firm $j$ that can adjust its price in period $t$ will have the following profits in period $\tau \geq t$ as a function of the relative price chosen in $t$, $q_j(t) = P_j(t)/P(t)$, provided that it will not have been able to change its price in the meantime:

$$\Pi^j_\tau (\tau) := e^{\pi^\tau (\theta - 1)(\tau - t)} y_B - \left( \frac{e^{\pi^\tau (\theta - 1)(\tau - t)} y_B}{A q_j(t)^\theta} \right)^{\frac{1}{\gamma}} w_B,$$

where we have used that $y_j(\tau) = y_j^j(\tau)$ and $P(\tau) = P(t)e^{\pi^\tau (\tau - t)}$. The optimal value of $q_j(t)$ solves the optimization problem

$$\max_{q_j(t)} \int_t^\infty e^{-(\rho + \delta)(\tau - t)} \Pi^j_\tau (\tau) d\tau.$$

We observe that we have to require $\pi^* < \frac{\gamma(\rho + \delta)}{\theta}$ for the integral to exist. Then the equilibrium value of $q_j(t)$, $q_B$, has to satisfy

$$q_B = \left( \frac{\theta}{\theta - 1} \frac{\rho + \delta - \pi^*(\theta - 1)(y_B)^{\frac{1 - \gamma}{\gamma}} w_B}{\gamma(\rho + \delta) - \pi^\theta} \right)^{\frac{1}{\gamma(1 - \gamma)\theta}}. \tag{19}$$

We summarize our results in the following proposition:

**Proposition 1.** For a given level of $\pi^*$ that satisfies $\pi^* < \frac{\gamma(\rho + \delta)}{\theta}$, the equilibrium of the benchmark economy is given by the values of $I_B$, $w_B$, $q_B$, $n_B$, $s_B$, and $y_B$ that satisfy (11)-(14) and (17)-(19). For $\pi^* \geq \frac{\gamma(\rho + \delta)}{\theta}$, no equilibrium exists.

We observe that the condition mentioned in the proposition, $\pi^* < \frac{\gamma(\rho + \delta)}{\theta}$, ensures that the other parameter restrictions stated in this section, $\pi^* < \frac{\gamma(\rho + \delta)}{\theta}$ and $\pi^* < \frac{\delta}{\theta - 1}$, hold automatically. While no equilibrium of the standard new Keynesian model exists for sufficiently high inflation rates, $\pi^* \geq \frac{\gamma(\rho + \delta)}{\theta}$, the model is compatible with arbitrarily low negative rates inflation.

In the next proposition, we identify the situations in which there are incentives for firms to ration demand, i.e. in which firms would profit from lowering output if this were possible.

**Proposition 2.** Consider a fixed level of $\pi^*$ with $\pi^* < \frac{\gamma(\rho + \delta)}{\theta}$. In the equilibrium of the benchmark economy, the incentives to ration demand are as follows:
1. For severe deflation, \( \pi^* < -\frac{\gamma}{1-\gamma} \frac{\rho+\delta}{\theta(\theta-1)} \), each firm would benefit from rationing demand in each period where it adjusts its price and in subsequent periods where the price spell satisfies \( \tau < \hat{\tau} \) with

\[
\hat{\tau} := \gamma \frac{\ln \left( \frac{\gamma\theta}{\theta-1} \frac{\rho+\delta-\pi^*(\theta-1)}{\gamma(\rho+\delta)-\pi^*\theta} \right)}{\pi^*(\gamma + (1-\gamma)\theta)}.
\]

When a time period of at least \( \hat{\tau} \) has elapsed since the last adjustment, rationing would no longer be desirable.

2. For moderate deflation, \( \pi^* \in \left[ -\frac{\gamma}{1-\gamma} \frac{\rho+\delta}{\theta(\theta-1)}, 0 \right] \), firms would never benefit from rationing demand.

3. For positive inflation, \( \pi^* \in (0, \frac{\gamma\delta}{\theta}) \), firms would benefit from rationing demand after not having adjusted their prices for more than \( \hat{\tau} \) periods. They would not benefit from rationing demand before.

The intuition for the proposition, which is proved in Appendix A, is straightforward. For positive inflation, the relative price of a firm decreases over time. At some point, the price will be so low and demand so high such that it would be desirable to supply fewer goods than demanded. For moderate deflation, the relative price increases during a spell of a constant nominal price. Hence the relative price never drops below the point where it would be optimal to ration demand. For severe deflation, each firm chooses a price that, in real terms, is so low initially such that it would be desirable not to satisfy the entire demand at the beginning. As time passes by, the relative price increases under deflation and demand drops by sufficiently much so that rationing demand is no longer attractive.

4 Main Model with Possible Demand Rationing

Having seen that there are situations where firms would benefit from rationing demand in the benchmark model, we now turn to our main model, where firms actually have the possibility to do so. We note that the possibility to ration demand will lead to equilibria that differ in a non-trivial way from those identified in the previous section, as
this possibility will affect firms’ price-setting in general. For example, under positive inflation, one might expect that firms choose lower prices for given marginal costs compared to the benchmark model because they take into account that low relative prices are less harmful after long spells of constant prices when rationing is possible. Later we will show that this is indeed the case.

It will be useful to examine the three regions for the inflation rate specified in Proposition 2 separately. We begin our analysis with the case of severe deflation.

**Proposition 3.** For severe deflation, i.e. \( \pi^* < -\frac{\gamma}{1-\gamma} \frac{\rho^+ \delta}{\theta (\sigma - 1)} \), a unique equilibrium exists in our main model. Each firm rations demand in every period where it adjusts its price and in all subsequent periods until a time period of \( \hat{\tau} \) (see (20)) has elapsed since it last changed its price. If a time period of \( \hat{\tau} \) has elapsed since the last adjustment and in all subsequent periods until the next price adjustment, firms satisfy demand completely.

This proof is given in Appendix B. While markups over marginal costs are different than in the equilibrium of the benchmark economy with severe deflation, we note that the intuition for when rationing is profitable is identical. When deflation is severe, firms choose very low prices initially, as they anticipate their prices to increase over time in relative terms. This makes it optimal to ration demand for newly adjusted prices. If prices have not been changed for some time, they become sufficiently high relative to the other prices such that rationing ceases to be profitable.

Having explored the case of severe deflation, we now turn to moderate deflation:

**Proposition 4.** For moderate deflation, i.e. \( \pi^* \in \left[-\frac{\gamma}{1-\gamma} \frac{\rho^+ \delta}{\theta (\sigma - 1)}, 0\right] \), a unique equilibrium exists in our main model. In this equilibrium, firms never ration demand. The equilibrium is identical to the one for the benchmark economy.

Recall that Proposition 2 has shown that there are no incentives for firms to ration demand in the benchmark scenario under moderate deflation. Thus it appears plausible that the equilibrium of the benchmark economy also constitutes an equilibrium in our main model. This is shown formally in Appendix C.

It remains to analyze the case of positive inflation. In Appendix D, we prove
Proposition 5. For $\pi^* > 0$, a unique equilibrium exists in our main model. Firms do not ration demand for $\hat{\tau}$ periods after adjusting their prices. Afterwards they start rationing demand.

We would like to mention that, like in the case with severe deflation considered in Proposition 3, the equilibrium of our main model with positive inflation involves markups that are different from those in the benchmark case. As firms know that they will have the possibility to ration demand in the future, they choose lower markups over marginal costs. This effect and the resulting consequences for the welfare costs of inflation will be examined in more detail in the next section.

5 The Costs of Inflation

5.1 Theoretical Results

Before stating our main results regarding the costs of inflation in the main model compared to the benchmark model, we analyze how the possibility to ration demand affects the relative prices of firms. In Appendix E we prove

Lemma 1. Consider the range of positive inflation rates for which an equilibrium of the benchmark economy exists, i.e. $\pi^* \in \left(0, \frac{\gamma\delta}{\theta}\right)$. In the model with the possibility to ration demand, the initial relative price set by firms when they are able to adjust their prices, $q_{R}$, is always lower than $q_{B}$, the respective value in the benchmark case without the possibility to ration demand.

Here and henceforth the subscript $R$ stands for the main model where rationing is possible.

The lemma can be interpreted in the following way. As has been mentioned in our discussions of the benchmark model, the relative price chosen by a firm when it adjusts its price, $q_{B}$, is strictly larger than one under positive inflation, and it increases with inflation. This is simply a consequence of the fact that under positive inflation, newly chosen prices are higher than prices that were chosen in the past. It is obvious that this pattern also holds in our main model with rationing. However, it may appear
surprising at first why $q_B > q_R$ holds for a given positive level of inflation, which is implied by Lemma 1. This finding can be explained by noting that, in our main model, prices that have not been adjusted for a long time and where rationing is profitable as a result receive a smaller weight in the price index.

The effect that demand rationing reduces the weights on very low prices is also responsible for a comparably small price dispersion $s$ in our main model. This is demonstrated by the following proposition, which is proved in Appendix F.

**Proposition 6.** Suppose $\pi^* \in \left(0, \frac{\gamma \delta}{\theta}\right)$. In our main model with the possibility to ration demand, price dispersion $s$ is always lower than in the benchmark case without the possibility to ration demand.

As a consequence, effective productivity $A/s$ is always higher in the main model than in the benchmark model under identical, positive inflation rates. In this sense, the benchmark model overstates the costs of inflation that arise from price dispersion.

In the next step, we compare the firms’ markups across the two models. In Appendix G, we show

**Proposition 7.** Consider the range of positive inflation rates for which an equilibrium of the benchmark economy exists, i.e. $\pi^* \in \left(0, \frac{\gamma \delta}{\theta}\right)$. When adjusting their prices, firms always choose a smaller markup over marginal costs in the model with the possibility to ration demand than in the benchmark case without this possibility. For $\pi^* \to 0$, the difference in markups converges to zero.

Thus, for positive inflation rates, markups are lower in our main model than in the benchmark case. Markups are comparably high in the benchmark model, as high markups are a precautionary measure against long price spells, which imply low relative prices and potentially high losses. In the main model, long price spells and the resulting low relative prices are less costly because firms can avoid to sell the large quantities demanded at these low prices.

Proposition 7 identifies a second reason why the assumption that firms cannot supply fewer goods than demanded tends to lead to higher costs of inflation: The possibility
to ration demand entails lower markups over marginal costs and thereby reduces the inefficiencies resulting from imperfect competition.

5.2 Quantitative Results

Our theoretical analysis has shown that the possibility to ration demand always leads to more moderate effects of inflation on price dispersion and effective aggregate productivity compared to the benchmark model where rationing is not possible. Moreover, the possibility to ration demand leads to smaller markups over marginal costs under positive inflation rates. Both effects tend to entail smaller welfare costs of inflation in our main model. In order to be able to assess the plausible magnitudes of these effects, we have to assign numerical values to the parameters of our model.

We normalize aggregate productivity to $A = 1$. Under the interpretation that the production function (7) is of the Cobb-Douglas form with constant capital, we can adopt $\gamma = 2/3$, which is a value that is often found in the literature. $\rho$ is approximately equal to the annual real interest rate, which leads us to choose $\rho = 0.03$. A value of $\theta = 11$ is frequently selected in new Keynesian analyses and would guarantee a steady-state markup of 10% in the absence of inflation. We set the inverse of the intertemporal elasticity of substitution to $\sigma = 1$ and the inverse of the Frisch elasticity of labor supply to the same value, i.e. $\varphi = 1$. Without loss of generality, we choose $\Psi = 1$ for the weight on the disutility of labor in the utility function. The Calvo parameter $\delta$ determines the length of price spells, $1/\delta$. As Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) find that prices typically remain unchanged for roughly three quarters, we select $\delta = 4/3$.

Before comparing the solutions for our two model variants, it may be interesting to compute $\gamma \delta \theta$, the level of inflation, where an equilibrium of the benchmark economy ceases to exist. For our parameter choices, $\gamma \delta \theta \approx 8.1\%$. As a consequence, the benchmark model makes the prediction that effective productivity reaches zero at an annual
inflation rate of \(\exp(0.081) - 1 \approx 8.4\%\). This is clearly implausible as inflation rates of similar magnitude prevailed e.g. in the US during the 1970s. As a next preliminary step, we evaluate the critical level of inflation that separates the moderate deflation scenario and the severe deflation scenario in Propositions 2, 3, and 4. We obtain 
\[- \frac{\gamma}{1 - \gamma} \frac{\rho + \delta}{\theta (\theta - 1)} \approx -2.5\%.
\] As a consequence, rationing does not occur for negative inflation rates above \(-2.5\%\). For lower inflation rates and positive inflation rates, firms find it sometimes desirable to ration demand. One might also ask how strongly firms ration demand in our main model. Our calculations imply that, for an inflation rate of 2\%, the output of intermediate goods is approximately 2\% lower than demand on average.

To illustrate how inflation affects price-setting, we plot the initial markup over marginal costs that is selected by firms that re-adjust their prices as a function of the inflation rate for the benchmark case (dashed line) and the main model (solid line) in the left panel of Figure 2. The figure illustrates the finding of Proposition 7 that, for positive inflation, markups are smaller in the main model than in the benchmark version. Moreover, it reveals that this effect is non-negligible quantitatively even for comparably low inflation rates of a few percent. According to the figure, markups in the main model are smaller than in the benchmark case for severe deflation as well. Finally, the figure confirms that the initial markup of adjusting firms increases with inflation in both model variants.\(^{14}\)

The panel on the right-hand side of Figure 2 shows price dispersion \(s\) for different inflation rates \(\pi^*\). In both model variants, price dispersion is minimal at zero inflation. The figure illustrates the claim of Proposition 6 that, under positive inflation rates, \(s\) is larger in the benchmark case than in the main model. The figure also shows that the effect is quantitatively large. A closer look at our simulation results reveals, e.g., that an annual inflation rate of 7\% leads to an increase of price dispersion by 57\% compared to the zero-inflation scenario. By contrast, our main model predicts a moderate increase

\(^{13}\)Some papers select lower values of \(\theta\). However, even a rather small value of \(\theta = 6\) would imply that no equilibrium exists for inflation rates above 16\%.

\(^{14}\)The fact that the initial markup increases with inflation does not imply that, for all given durations of price spells, markups increase with inflation.
Figure 2: The markup over marginal costs chosen by firms that re-adjust their prices (left panel) and price dispersion (measured as a deviation from one, right panel) as functions of inflation $\pi^*$. Of 0.8% in this case. At this point, it may be worth remembering that increases in price dispersion directly translate into decreases in effective productivity, $A/s$. Thus the benchmark model predicts that effective productivity decreases by 36% percent when inflation is at 7% rather than at zero. Our main model with rationing predicts that inflation rates of 7% lead to drops in effective productivity of only 0.8%

The differences in effective productivities and markups across the two models also lead to differences in output and employment. These differences are shown in Figure 3. The left-hand panel, which shows output for the two models, demonstrates that the output loss caused by inflation is considerably more moderate in our main model than in the benchmark scenario. The right-hand panel reveals that both models predict a very different response of employment. In the benchmark model, higher inflation rates lead to sizable increases in employment, whereas the main model predicts decreases in employment in response to higher inflation.

We conclude this section with a quantitative analysis of the effects of inflation on welfare. For this purpose, Figure 4 plots the relative change in consumption in the zero-inflation steady-state that would make the household indifferent to the steady state with an inflation rate of $\pi^*$ in the benchmark model (dashed line) and the main model (solid line). The figure confirms that the welfare costs of inflation are substantially
higher in the benchmark model. For example, the benchmark model predicts that an inflation rate of 7% leads to a consumption-equivalent welfare loss of 36% compared to the steady-state with zero inflation. In our main model, the welfare loss is only 2% in this case.\textsuperscript{15}

We note that the welfare-maximizing inflation rate is 0.18% and thus slightly positive in the benchmark model.\textsuperscript{16} In our main model, rationing occurs only very rarely for small positive inflation rates, as most prices are re-adjusted before the time period $\hat{\tau}$ has passed.\textsuperscript{17} Hence the welfare-maximizing inflation rate in our main model is effectively indistinguishable from the one in the benchmark model.

Interestingly, in the benchmark model the costs of inflation and deflation are highly asymmetric. For example, a positive inflation rate of 5% is considerably more costly

\textsuperscript{15}For a rather small value of $\theta = 6$, the welfare costs of inflation are lower in both cases. In the benchmark model, the consumption-equivalent welfare loss is 4.9% under 7% inflation; it is 1.6% in our main model. Thus the difference in costs is smaller but the benchmark model still involves welfare costs that are higher by a factor of three. Moreover, such a small value of $\theta$ would imply that firms initially set prices at 46% and 63% over marginal costs in the main model and the benchmark model respectively, which appears rather high.

\textsuperscript{16}The Ramsey-optimal policy would plausibly entail a zero-inflation steady state. However, it is well-known that the welfare-maximizing steady-state is typically different from the steady state under the Ramsey policy (see Schmitt-Grohé and Uribe (2010, p. 696), for example). The welfare-maximizing inflation rate is positive because we do not consider a sales subsidy, which is often considered in papers relying on a log-linear approximation. A positive rate of inflation then mitigates the distortions due to monopolistic competition to some extent.

\textsuperscript{17}On average, the quantity supplied by intermediate-goods producers is only 0.00001% lower than demand at the welfare-maximizing inflation rate of 0.18%.
than a negative inflation rate of -5%. The costs of inflation and deflation are substantially less asymmetric in our main model. In fact, the pattern is reversed qualitatively: A positive rate of inflation is always socially more desirable than a negative inflation rate of the same magnitude.

6 Conclusions

This paper has pointed out that the large costs of inflation in the new Keynesian literature rely on the implicit assumption that firms always have to satisfy the demands for their goods, even in cases where doing so entails negative profits. Relaxing this assumption leads to substantially more moderate costs of inflation and empirically more plausible relationships between inflation and other economic variables like employment and output.

While this paper has focused on stationary equilibria, it would be interesting to study the dynamics of our economy in response to shocks in future work. In such an extension to our framework, one could assess whether the results in the literature about the impact of trend inflation on the short-term dynamics of the new Keynesian model and equilibrium determinacy continue to hold in a framework where firms can choose the quantities they produce optimally.\footnote{This literature was discussed in the Introduction.}
In addition, one might ask how an endogenous frequency of price adjustment would affect our results. To address this question, one could follow the approach of Levin and Yun (2007) and assume that firms also choose the mean duration of the price spell when they adjust their prices. While a full-fledged analysis is beyond the scope of this paper, it would plausibly involve the following effects. First, in both models price stickiness would decrease with inflation. Second, in our main model with rationing, this decrease is likely to be smaller because the possibility to ration demand makes long price spells less costly. As a consequence, the aggregate direct costs of price adjustment would be smaller in our main model. Third, it appears likely that the social costs of inflation would be smaller in our main model in the case where the frequency of price adjustment is endogenous compared to the case considered in this paper. In this sense, our main finding that the costs of inflation may be lower than implied by the standard new Keynesian model with exogenous frequency of prices would be strengthened.

Finally, it might be interesting to speculate how the mechanism outlined in this paper would affect the optimal level of the inflation target when shocks may push interest rates towards the zero lower bound from time to time. As our mechanism is plausible to reduce the costs of inflation also in a richer model that features the zero lower bound on nominal interest rates, it might provide an argument in favor of higher inflation targets.

However, caution may be advisable with respect to raising inflation targets as there may be other costs of inflation that are not captured by the standard new Keynesian model and the model in this paper. At any rate, this paper supports the point made by Nakamura et al. (2018) that the specific mechanism that is responsible for the costs of inflation in new Keynesian models may be less relevant than previously thought.

\[^{19}\text{An early paper that considers endogenous price stickiness is Ball et al. (1988).}\]
A Proof of Proposition 2

Let us assume, without loss of generality, that a particular firm $j$ has adjusted its price in period 0. Let $\hat{\tau}$ be the duration of the price spell where the firm is indifferent between rationing demand or not. With the help of $q$, which is the initial relative price chosen by a firm when it adjust its price, we can state the following condition for $\hat{\tau}$

$$
\left( \frac{A^{1/\gamma}}{w} q e^{-\pi \hat{\tau}} \right)^{1/\gamma-1} = q^{-\theta} e^{\pi \hat{\tau}} y,
$$

(21)

where we have used $y^*_j(\tau) = y^d_j(\hat{\tau})$, (6), (8), as well as the observation that, $\hat{\tau}$ periods after the relative price of firm $j$ has been set to $q$, it is $e^{-\pi \hat{\tau}} q$. Solving (21) for $\hat{\tau}$ yields:

$$
\hat{\tau} = \frac{\log(q)}{\pi^*} + \frac{\log \left( \frac{q^\gamma A^{1/\gamma}}{y^{1-\gamma}} \right)}{k/\pi^*},
$$

(22)

where

$$
k := \gamma + (1 - \gamma) \theta.
$$

(23)

Using (19) to substitute for $q$ in (22), we obtain the expression for $\hat{\tau}$ stated in the proposition. It is obvious from this expression that $\hat{\tau}$ is strictly positive for $\pi^* > 0$. It is also clear that, for positive inflation rates, the relative price of the intermediate firm’s good, $qe^{-\pi \tau}$, decreases over the duration of the price spell. For positive inflation rates, we therefore conclude that the price is sufficiently high for $\tau < \hat{\tau}$ such that $y^*_j(\tau) > y^d_j(\tau)$ and that it is so low for $\tau > \hat{\tau}$ such that $y^*_j(\tau) < y^d_j(\tau)$. This implies that rationing is desirable iff $\tau > \hat{\tau}$.

For $\pi^* \in \left[\frac{-\gamma}{1-\gamma} \frac{\theta+\delta}{\theta(\theta-1)}, 0\right]$, no solution for $\hat{\tau}$ exists in Equation (20), and $y^*_j(\tau) \geq y^d_j(\tau)$ holds for all $\tau \geq 0$. Hence rationing would never be profitable for intermediate firms. For $\pi^* < -\frac{\gamma}{1-\gamma} \frac{\theta+\delta}{\theta(\theta-1)}$, $\hat{\tau}$, given by (20), is strictly positive. Moreover, we note that the relative price of the firm’s good increases over time under deflation. As a consequence, $y^*_j(\tau) < y^d_j(\tau)$ for $\tau < \hat{\tau}$ and $y^*_j(\tau) > y^d_j(\tau)$ for $\tau > \hat{\tau}$. Thus rationing would be profitable initially after an intermediate firm has adjusted its price. After $\hat{\tau}$ periods, rationing demand would cease to be attractive. \qed
B Proof of Proposition 3

As has been shown in Proposition 2, an equilibrium for $\pi^* < -\frac{\gamma}{1-\gamma} \frac{\rho+\delta}{\theta(\theta-1)}$ would necessarily involve that intermediate firms sometimes ration demand. Assume, again without loss of generality, that firm $j$ adjusts its price in period 0. Then it has to choose its relative initial price $q$ to maximize its profits over the duration of the price spell,

$$
\int_0^{\hat{\tau}} \left( q e^{-\tau \pi^*} y_j^\delta(\tau) - w \left( \frac{y_j^\delta(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau \\
+ \int_{\hat{\tau}}^{\infty} \left( q e^{-\tau \pi^*} y_j^d(\tau) - w \left( \frac{y_j^d(\tau)}{A} \right)^{\frac{1}{\gamma}} \right) e^{-(\delta+\rho)\tau} d\tau,
$$

(24)

where $\hat{\tau}$ is given by (22), the relative price in period $\tau$ is $q e^{-\tau \pi^*}$, and we have taken into account that the intermediate firm’s output is determined by (8) until period $\hat{\tau}$ and by (6) afterwards. It is very tedious but straightforward to show that the first-order condition can be rewritten as

$$
\xi^{1-\gamma + \frac{\rho+\delta}{\pi^*}} = 1 - \frac{\left( (\rho + \delta) + \theta(\theta - 1)\pi^* \frac{1-\gamma}{\gamma} \right) \left( \rho + \delta + \frac{\pi^*}{1-\gamma} \right)}{\left( \rho + \delta + \frac{\pi^*}{1-\gamma} \right) \left( \rho + \delta - (\theta - 1)\pi^* \right)},
$$

(25)

where, with the help of $\kappa$ (see (23)), we have introduced

$$
\xi := q \left( \frac{\gamma}{w} \frac{A}{y^{1-\gamma}} \right)^{\frac{1}{\pi}}.
$$

(26)

As a next step, we analyze the fraction on the right-hand side of (25) more closely. As $\pi^* < 0$, the denominator is strictly positive. The first term in the numerator, $(\rho + \delta) + \theta(\theta - 1)\pi^* \frac{1-\gamma}{\gamma}$, is strictly negative for the values of $\pi^*$ under consideration. For the remaining term in the numerator, $\rho + \delta + \frac{\pi^*}{1-\gamma}$, two cases need to be distinguished. First, it may be strictly positive. In this case, the right-hand side of (25) is strictly larger than one. As the exponent of $\xi$ on the left-hand side is strictly negative for $\rho + \delta + \frac{\pi^*}{1-\gamma} > 0$, we can conclude that, for $\rho + \delta + \frac{\pi^*}{1-\gamma} > 0$, (25) has a unique solution for $\xi$, which is positive and smaller than one. Second, $\rho + \delta + \frac{\pi^*}{1-\gamma} < 0$ may hold, which implies that the right-hand side of (25) is strictly smaller than one.\footnote{For the sake of brevity, we omit the analysis of the knife-edge case where $\rho + \delta + \frac{\pi^*}{1-\gamma} = 0$, as it is straightforward and does not deliver additional insights.}

It is easy to
see that the right-hand side of (25) is also strictly positive. As the exponent of $\xi$ on the left-hand side is positive in the constellation under consideration, we can conclude again that a unique solution for $\xi$ exists and that this solution is positive and smaller than one.

We note that $\xi$ and $\hat{\tau}$ are linked via

$$\hat{\tau} = \frac{\log(\xi)}{\pi^*}. \quad (27)$$

As $\pi^* < 0$ and $\xi < 1$, we obtain that $\hat{\tau}$ is strictly positive.

Having analyzed the first-order condition for an intermediate firm’s profit-maximization problem, we now turn to the condition that links aggregate output and the output levels of intermediate goods, i.e. Equation (5). As in our case, the intermediate output of a firm that adjusted its price fewer than $\hat{\tau}$ periods ago is given by $y^*_j(\tau)$ (see (8)) and by $y^d_j(\tau)$ (see (6)) otherwise, (5) can be stated as

$$y^a_j = \delta \left( \gamma A^\frac{1}{\theta} w q \right)^{\frac{\gamma - \theta - 1}{\theta}} \int_0^{\hat{\tau}} e^{-\left(\delta + \frac{\gamma - \theta - 1}{\theta} \pi^*\right)\tau} d\tau + \frac{\delta}{\pi^{\theta - 1}} y^a_j \int_{\hat{\tau}}^{\infty} e^{-\left(\delta - \pi^*(\theta - 1)\right)\tau} d\tau. \quad (28)$$

Using (22), (23), as well as (26) and re-arranging yields

$$q = \left( \frac{\delta \xi^a_j}{\pi^* \left(\theta - 1\right) \frac{\gamma - \theta - 1}{\theta}} + \frac{\pi^* \left(\theta - 1\right) \kappa \delta \xi^a_j}{\left(\delta - \pi^*(\theta - 1)\right) \left(\delta (1 - \gamma) \theta + \pi^* \gamma (\theta - 1)\right)} \right)^{1 \pi^{\theta - 1}}. \quad (29)$$

With the help of $0 < \xi < 1$, (23), and $\pi^* < -\frac{\gamma - \theta - 1}{\theta (\theta - 1)}$, it is possible to show that the term in parentheses on the right-hand side is strictly positive. Hence (29) always defines a unique value of $q$.

Having examined the intermediate-goods firm’s first-order condition and the final-goods producers’ production function, it remains to analyze the labor-market clearing condition $n = \int_0^1 n_j(t) \, dj = \int_0^1 \left( \frac{y^j(t)}{A} \right)^{\frac{1}{\theta}} \, dj$. Using that $y_j(\tau) = y^*_j(\tau)$ for firms with price spells $\tau < \hat{\tau}$ and $y_j(\tau) = y^d_j(\tau)$ for $\tau > \hat{\tau}$, where $y^d_j(\tau)$ and $y^*_j(\tau)$ are given in (6) and (8), we obtain

$$n = \frac{\delta}{A^\frac{1}{\theta}} \left( \gamma A^\frac{1}{\theta} w q \right)^{\frac{1}{\theta}} \int_0^{\hat{\tau}} e^{-\left(\delta + \frac{\gamma - \theta - 1}{\theta} \pi^*\right)\tau} d\tau + \frac{\delta}{A^\frac{1}{\theta} q^{1 \pi^{\theta}}} y^1 \int_{\hat{\tau}}^{\infty} e^{-\left(\delta - \pi^*(\theta - 1)\right)\tau} d\tau. \quad (30)$$
After several steps, this expression can be shown to be equivalent to

\[ n = \frac{1}{A \gamma q^\theta} \delta y^\gamma \frac{\xi^{\gamma (1-\gamma)}}{(1-\gamma)(\gamma \delta - \pi^* \theta)} \left( 1 + \frac{\xi^{-\delta} - \frac{1}{\gamma (1-\gamma)} \kappa \pi^*}{(1-\gamma)(\gamma \delta - \pi^* \theta)} \right). \]  

(31)

The measure of price dispersion, \( s = (An^\gamma)/y \) can thus be written as

\[ s = \left[ \left( \frac{\delta}{\delta + \frac{\pi^*}{1-\gamma}} \right) \left( 1 + \frac{\xi^{-\delta} - \frac{1}{\gamma (1-\gamma)} \kappa \pi^*}{(1-\gamma)(\gamma \delta - \pi^* \theta)} \right) \right]^{\gamma \frac{\kappa}{1-\gamma}} \frac{\xi^{\gamma \kappa}}{q^\theta}. \]  

(32)

It is immediate to verify that the expression in brackets is always positive. Hence, for given positive values of \( \xi \) and \( q \), a unique positive value of \( s \) exists.

The uniqueness of the equilibrium follows from our previous finding that (25), (29), and (32) admit unique positive solutions for \( \xi \), \( q \), and \( s \). Together with \( s = (An^\gamma)/y \) and (26), (11) and (12) can be used to determine the equilibrium values of \( I \), \( n \), \( y \), and \( w \). The value of \( \hat{\tau} \) can be easily derived with the help of (27).

\[ \square \]

C Proof of Proposition 4

As shown in Proposition 2, if \( q \) is set as in the benchmark equilibrium, then rationing demand would always make firms worse off. It remains to verify that firms would not benefit from setting \( q \) to a value sufficiently low so that rationing is optimal for some time. However, our analysis in Appendix B has shown implicitly that this is not the case for \( \pi^* \in \left[ -\frac{\gamma}{1-\gamma} \frac{\rho+\delta}{\theta(\theta-1)}, 0 \right] \).

\[ \square \]

D Proof of Proposition 5

We begin our analysis of the economy with possible rationing of demand for \( \pi^* > 0 \) by looking at the profit-maximization problem of an intermediate firm. As before, we assume without loss of generality that the firm adjusts its price in period 0. Let us use \( q \) again to denote the relative price that the firm chooses in period 0. As the relative price of the firm, \( q e^{-\pi^* \tau} \), will fall over the duration of a price spell, it is clear that there will be a price duration, \( \hat{\tau} \), after which rationing demand is strictly profitable. \( \hat{\tau} \) is
strictly positive, as selecting a value of $q$ that is so low such that rationing demand is always desirable cannot be profitable.\textsuperscript{21}

The firm’s discounted profits over the duration of the price spell that starts in period 0 are

$$
\int_0^\tau \left( qe^{-\pi^*} y_j^d(\tau) - w \left( \frac{y_j^d(\tau)}{A} \right)^\frac{q}{\gamma} \right) e^{- (\delta + \rho) \tau} d\tau
+ \int_\tau^\infty \left( qe^{-\pi^*} y_j^s(\tau) - w \left( \frac{y_j^s(\tau)}{A} \right)^\frac{q}{\gamma} \right) e^{- (\delta + \rho) \tau} d\tau,
$$

(33)

where $\hat{\tau}$ is given by (22). Additionally, we have taken into account that the relative price in period $\tau$ is $qe^{-\pi^*}$ and that output is determined by $y_j^d(\tau)$ until period $\hat{\tau}$ and by $y_j^s(\tau)$ afterwards. Utilizing (6), (8), and (27), the resulting first-order condition can be simplified to

$$
f(\xi) := - (\theta - 1) \frac{\xi^{\alpha+\delta} (\theta - 1) - 1}{\rho + \delta - (\theta - 1) \pi^*} + \theta \left( \frac{\xi^{\alpha+\delta} (\theta - 1) - 1}{\rho + \delta - \frac{\pi^* \alpha}{\gamma}} + \frac{1}{\rho + \delta + \frac{\pi^*}{1 - \gamma}} \right) = 0.
$$

(34)

We note that $\hat{\tau} \geq 0$ requires $\xi \geq 1$, as $\hat{\tau} = \log(\xi)/\pi^*$. In order to establish that (34) has a unique solution for $\xi$ with $\xi \geq 1$, we highlight several properties of $f(\xi)$. First, $f(\xi)$ is a continuous and differentiable function. Second, $f(1) = \frac{1}{\rho + \delta + \frac{\pi^*}{1 - \gamma}} > 0$ holds. Third, we note that $f'(1) = \frac{1}{\pi^*}$. Fourth, $f'(\xi) = 0$ has a unique solution for $\xi > 1$. It is given by $\xi = \left( \frac{\theta}{\theta - 1} \right)^{\frac{\pi^*}{\theta - 1}}$. Fifth, $\lim_{\xi \to \infty} f(\xi) = -\infty$. Taken together, these properties imply that $f(\xi) = 0$ has a unique solution for $\xi$, which satisfies $\xi > \left( \frac{\theta}{\theta - 1} \right)^{\frac{\pi^*}{\theta - 1}} > 1$.

As a next step, we evaluate the condition that relates aggregate output and intermediate inputs, i.e. (5). For positive inflation rates, the intermediate output of a firm that adjusted its price fewer than $\hat{\tau}$ periods ago is given by $y_j^d(\tau)$ (see (6)) and by $y_j^s(\tau)$ (see (8)) otherwise. Consequently, (5) can be written as

$$
y^{\theta - 1}_{\theta - 1} = \delta - \frac{\rho - 1}{\theta - 1} y^{\theta - 1}_{\theta - 1} \int_0^\tau e^{- (\delta - \pi^*(\theta - 1)) \tau} d\tau + \delta \left( \frac{A^{\frac{\gamma}{\theta}}}{w} \right)^{\frac{\gamma}{1 - \gamma} \frac{\theta - 1}{\theta}} \int_\tau^\infty e^{- (\delta + \frac{\gamma}{1 - \gamma} \frac{\theta - 1}{\theta} \pi^*) \tau} d\tau.
$$

(35)

\textsuperscript{21}Note that per-period profits, under the assumption that $y_j^s(t) < y_j^d(t)$, decrease with the relative price in the respective period.
Using (22), (23), and (26) and re-arranging yields

\[ q = \frac{\delta}{\delta - (\theta - 1)\pi^*} \left( 1 - \frac{\pi^*(\theta - 1)\kappa\xi - \frac{\delta}{\pi^*} + \theta - 1}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} \right)^{\frac{1}{\theta - 1}}. \tag{36} \]

One can easily demonstrate that the expression in square brackets is always positive. Thus (34) has a solution, which is unique for a given level of \( \xi > 1 \).

The labor-market clearing condition,

\[ n = \int_0^1 n_j(t) \, dj = \int_0^1 \left( \frac{y_j(t)}{A} \right)^{\frac{1}{\gamma}} \, dj, \]

can be shown to be equivalent to

\[ s = q^{-\theta} \left[ \frac{\delta}{\delta - \pi^*\frac{\theta}{\gamma}} \left( 1 - \frac{\pi^*\kappa\xi - \frac{\delta}{\pi^*} + \frac{\theta}{\gamma}}{\gamma(1 - \gamma) + \pi^*} \right) \right]^\gamma, \tag{37} \]

where \( s = (An^\gamma)/y \).

The uniqueness of the equilibrium follows from the fact that (35), (36), (37) admit unique positive solutions for \( \xi, q, \) and \( s \). Together with \( s = (An^\gamma)/y \) and \( \xi = q \left[ \left( \frac{A}{w} \right)^{\gamma} \frac{A}{y^{1 - \gamma}} \right]^\frac{1}{\gamma} \), (11) and (12) can be used to determine the equilibrium values of \( I, n, y, \) and \( w \). The value of \( \hat{\tau} \) follows from (27).

\[ \square \]

E Proof of Lemma 1

To show the lemma, we have to compare \( q \) in the benchmark case, which is given by (14), with the respective value for the model with the possibility of rationing, i.e. (36).

For \( \pi^* > 0 \), it is clear that

\[ \left( 1 - \frac{\pi^*(\theta - 1)\kappa\xi - \frac{\delta}{\pi^*} + \theta - 1}{\delta(1 - \gamma)\theta + \pi^*\gamma(\theta - 1)} \right)^{\frac{1}{\theta - 1}} < 1, \tag{38} \]

which proves the statement of the lemma. \[ \square \]
F Proof of Proposition 6

To establish that $s$ is smaller in the economy with the possibility of rationing than in the benchmark case, we combine (36) and (37) to yield the following expression for $s$ in the main model with positive inflation:

$$ s_R = \left( \frac{\delta}{\delta - \pi^* \theta} \right)^{\gamma} \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\frac{\theta}{\theta - 1}} \left( \frac{1 - \pi^* \kappa \xi^{-\frac{\delta}{\pi^* + \theta}}}{\gamma(\delta(1 - \gamma) + \pi^*)} \right)^{\gamma} \left( \frac{1 - \pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^* + \theta}} - 1}{\delta(1 - \gamma)\theta + \pi^* \gamma(\theta - 1)} \right)^{\frac{\theta}{\theta - 1}}. $$

(39)

For the benchmark case, it has been shown before that $s$ amounts to

$$ s_B = \left( \frac{\delta}{\delta - \pi^* \theta} \right)^{\gamma} \left( \frac{\delta - (\theta - 1)\pi^*}{\delta} \right)^{\frac{\theta}{\theta - 1}}. $$

(40)

Proving the proposition therefore amounts to establishing that

$$ \left( 1 - \frac{\pi^* \kappa \xi^{-\frac{\delta}{\pi^* + \theta}}}{\gamma(\delta(1 - \gamma) + \pi^*)} \right)^{\gamma} < \left( 1 - \frac{\pi^*(\theta - 1)\kappa \xi^{-\frac{\delta}{\pi^* + \theta}} - 1}{\delta(1 - \gamma)\theta + \pi^* \gamma(\theta - 1)} \right)^{\frac{\theta}{\theta - 1}}. $$

(41)

As $\gamma < \frac{\theta}{\theta - 1}$, it is sufficient to show

$$ \frac{\xi^{-\frac{\delta}{\pi^* + \theta}}}{\gamma(\delta(1 - \gamma) + \pi^*)} > \frac{(\theta - 1)\xi^{-\frac{\delta}{\pi^* + \theta}} - 1}{\delta(1 - \gamma)\theta + \pi^* \gamma(\theta - 1)}. $$

(42)

Next we note that, due to $\xi > 1$, we have $\xi^{-\frac{\delta}{\pi^* + \theta}} > \xi^{-\frac{\delta}{\pi^* + \theta}} - 1$. Hence it suffices to prove

$$ \delta(1 - \gamma)\theta + \pi^* \gamma(\theta - 1) > (\theta - 1)\gamma(\delta(1 - \gamma) + \pi^*), $$

(43)

or equivalently,

$$ \delta(1 - \gamma)\theta > (\theta - 1)\gamma(1 - \gamma). $$

(44)

As this inequality holds, we have shown the claim of the proposition.

\[\square\]

G Proof of Proposition 7

We can define firm $j$’s marginal cost of production in period $t$ as

$$ MC_j(t) := w(t) \frac{dn_j(t)}{dy_j(t)}. $$

(45)
In the period where the price is set, we can use the equation for demand (6) and firm $j$'s production function (7) to write the marginal cost as

$$MC = \frac{w \left( q^{-\theta} y \right)^{1-\gamma}}{A^{\frac{1}{\gamma}}}.$$  \hfill (46)

With the help of (46), we can reformulate (19) as

$$q_B = \frac{\theta}{\theta - 1} \frac{\gamma (\rho + \delta - \pi^*(\theta - 1))}{\gamma (\rho + \delta) - \pi^* \theta} MC.$$  \hfill (47)

In the special case where $\pi^* = 0$, we obtain $q_B = \frac{\theta}{\theta - 1} MC$, which is the standard result that firms choose a markup of $\frac{\theta}{\theta - 1}$ over marginal costs.

Using (26), and (46), we obtain that the optimal price is given by $q_R = \xi \frac{\xi}{\gamma} MC$ in the main model, where $\xi$ is the solution to $f(\xi) = 0$ (see (34)). The claim of the proposition, $q_B > q_R$ for fixed $MC$, is thus equivalent to

$$x > \xi,$$  \hfill (48)

where

$$x := \left( \frac{\gamma \theta}{\theta - 1} \frac{\rho + \delta - \pi^*(\theta - 1)}{\gamma (\rho + \delta) - \pi^* \theta} \right)^{\frac{1}{\gamma}}.$$  \hfill (49)

As $f(\xi)$ decreases monotonically for values of $\xi$ larger than the one implied by $f(\xi) = 0$, it is sufficient to show

$$f(x) < 0.$$  \hfill (50)

It is tedious but straightforward to derive

$$f(x) = -\frac{\pi^*(\rho + \delta) \kappa^2}{(\rho + \delta - \pi^*(\theta - 1)) (\gamma (\rho + \delta) - \pi^* \theta) (\pi^* + (1 - \gamma)(\rho + \delta))}.$$  \hfill (51)

This expression is strictly negative for $\pi^* \in \left(0, \frac{\gamma \delta}{\theta}\right)$, which proves the claim of the proposition. \hfill $\Box$
References


