DO SPECULATIVE STOCKS LOWER PRICES AND INCREASE VOLATILITY OF VALUE STOCKS?

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The influence of speculative stocks on value stocks is examined through a set of economics experiments. The speculative asset is designed to model a company involved in a rapidly growing market that will be saturated at some unknown point. Using a control experiment where both assets are similar value stocks, we find statistical support for the assertion that the presence of a speculative stock increases the volatility and diminishes the price of the value stock. In addition, the temporal minimum price of the value stock during the last phase of the experiment is lower in the presence of the speculative stock (when the trading price of the speculative asset is declining sharply). These results indicate that an overreaction in the speculative stock tends to divert investment capital away from other assets. An examination of the relative magnitude of monthly closing price changes confirm strong correlations between the Dow Jones Average and the more speculative Nasdaq index during the time period in 1990 to 2001 and particularly during the two years prior to the peak in March 2000 (0.72 correlation) and the March 2000 to August 2001 decline (0.79 correlation). Supplementary experiments using independent (or legally separate) markets trading the same asset show that a higher price in one market does not lead to a higher one in the other.

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Introduction

During the late 1990’s a conspicuous feature of the US stock market was the schism between the high-flying Internet/high-tech sector, with prices that were astronomical in relation to earnings or revenues, and the more traditional sectors with low price-to-earnings ratios (Dreman [1998] and Shiller [2000]). The latter included many stocks that were favored by value minded investors. In many cases these stocks produced solid earnings for many years, but the stock prices did not soar, in part because the businesses were well understood, and consequently left little room for positive surprises. Many of the Internet stocks on the other hand exhibited no earnings, and sometimes little revenue. Their future earnings and value were thus difficult to assess, thereby facilitating the projections based upon future expectations.

The uncertainty of the future stream of earnings also meant that fairly small changes reported in revenues and earnings, when projected for many years, resulted in large changes in prices that traders would pay for these stocks. Of course, once the large price movements were noted, momentum traders (i.e., those who trade based upon recent price trends) would attempt to buy into the trend, thereby enhancing the volatility (Dreman [1998]). At the same time, the stocks with more predictable earnings attracted mainly those investors who were focused on value, i.e., the purchasing motivated by a discount from the realistic value, and were shunned by investors who based their decisions on momentum. This suggests a negative relation between value stock prices and the high flyers.

This bipolar market of the late 1990’s was not the first such event in American financial history. During the early 1970’s the “two-tier market” was characterized by two sets of stocks: at one extreme were about 50 stocks (called the “nifty fifty”) featuring high price earning ratios, while the remaining stocks languished at low price/earnings (P/E) ratios. The nifty fifty featured ideas that were novel at the time, and included such names as Avon and Polaroid (see Dreman [1977]).
In the 19th century the railroad stocks were perceived to have extremely high growth potential just as the internet stocks did recently. As in the current high tech bubble, the railroad stock attracted much attention at the expense of other companies whose business and growth prospects were regarded as more traditional at the time. One prevailing idea of the time was that the speculation in railroad stocks suppressed the prices of the other stocks.

With a fixed amount of investment capital, if one asset is increasing rapidly, then allocating funds to the growing asset necessarily diverts funds from the other assets. In this way, the price behavior of one stock may have a cascading negative effect on other assets and thus their price volume relationship. The effect could be transmitted through the money supply or liquidity in the market.

In previous experimental studies (see Porter and Smith [1994]) and Caginalp, Porter and Smith [2000]) of asset markets, markets with a single asset were investigated where the structure of the asset and investor expectations resulted in price bubbles and crashes relative to asset fundamental value. In these experiments, the asset had a declining fundamental value and expired after 15 periods. Dividends were paid on the asset after each trading period. The price patterns have been successfully modeled using dynamical systems in which there is a population of both fundamental and momentum investors. This paper complements this previous work by extending the asset market experiments to include multiple assets that have price growth expectations. Furthermore, since we control market liquidity in the experiments, we have a natural mechanism for examining spillover effects from one market to the other. This structure allows us to investigate the features of the stock markets of the 1990’s. We also use multi-asset price theory to test the hypothesis that a speculative asset has substantial influence on the volatility and price of a more stable value asset in an environment that maintains a constant supply of cash.

Our experiments are designed specifically to answer the following questions:
1. Does the presence of a speculative stock increase or decrease the volatility in the value stock?
2. Does the presence of the speculative stock increase or decrease the trading price of the value stock during the time period of very positive earnings updates? Similarly, does the value stock exhibit a lower minimum as the speculative stock has mediocre earnings updates near the end of the experiment?
3. How does excess cash influence volatility?

Experimental Design

We design an experiment that introduces key features of a two-tier market environment by defining two assets whose expected return is updated every two minutes during a 42-minute trading period. The assets have a single payout at the end of this period, but the value of this payout is updated regularly, like earnings reports, in the form of a percentage increase or decrease. For example, if the expected return is 130 francs just before the announcement of an increase of 4%, the new expected return is 130 times 1.04, or 135.20 francs. This process is repeated so that the expected value of the asset evolves in time similar to a stock’s value as quarterly reports are released.

Specifically, we let $V_A(t)$ be the value of asset A at time t with an expected growth rate of $\beta_A$, then over the fixed horizon $[0,T]$ the expected terminal value of the asset would be $V_A(0) [(T - 0) (1 + \beta_A)]$. Suppose the actual growth rate of the asset is uncertain with distribution $f_A(.)$ for each “earnings” announcement period. At each earnings announcement a draw is made from $f_A(.)$ and the current value of the asset is the previous value times the growth value $(1+\beta_A(t))$ drawn at announcement t. In our experiment design $\beta_A(t)$ is non-zero except on a finite set (namely on even minutes of the trading period). Then the current value at announcement time t would be:

$$V_A(t) = V_A(0) (1+\beta_A(0))(1+\beta_A(1)) . . (1+\beta_A(t)). \tag{1}$$
This current value remains constant until the next announcement. In a two asset case (assets A and B) we say that asset B is more speculative than asset A if $|\beta_B| \geq |\beta_A|$ and the corresponding variances have the property $\sigma^2_B > \sigma^2_A$.

Using this basic structure, the experimental economic environment had the following intuitive arrangement: the nature of randomness can be described in terms of drawing different colored balls from an urn. If the fraction of balls of each color is known at each time, then one has a precise knowledge of the expected values. On the other hand, if one is told merely that there are three colors present in the urn, and asked to make inferences as the draws are made, then the situation is much more complicated. For example, if there are finitely many balls (though exact numbers are unknown) and one particular color ball (representing a good earnings period, for example) is not replaced upon being drawn, then one has to guess not only the probability of choosing that color, but also how the probabilities appear to be changing as the remaining number of such balls diminishes. This results in a higher level of uncertainty, particularly in a relatively short number of repetitions.

The heart of the experimental setup involves the nature of assets to be traded. In an attempt to understand the effect of a speculative stock on a value stock, we performed two types of experiments. In ten of the experiments, asset A was the value stock, while asset B was the speculative. In the set of four control experiments, both A and B were very similar value stocks.

Both the value and speculative asset were defined in terms of their expected payout at the end of the trading session. Initially both had a value of 100 francs. Subjects were informed that an update to the expected payout would be visible on the computer screen every two minutes. Each experimental trading session lasted 42 minutes so that there were exactly 20 announcements. For the value asset, subjects were told that each update would alter the expected payout by +4, or -1%. For the

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1 A Franc is the name of the experimental currency used in the experiments. At the end of the experiment, Francs are converted into US Dollars at a rate of 100 to 155 Francs to 1 Dollar.
speculative asset the analogous possibilities were +10%, +1% and -6%, with the caveat that the number of good draws of +10% would be fewer than the total number of draws. This latter feature was intended to simulate rapidly increasing profits for a company involved in a consumer market that would be saturated at some undetermined point in the future.

Throughout the experiment and instructions, the terms “value” and “speculative” were not used, and the analogy with recent markets was not described. A set of random draws was selected for the updates for the speculative asset and the value asset, as shown in Figures 1 and 2, and used in all of the experiments. In the case of the control experiments, where both assets were “value” stocks, the second asset, B, was given a similar, but not identical, set of draws.
Figure 1

Current Value Path Value-Speculative Treatment

Figure 2

Current Value Path Value-Value Treatment
In addition to the value versus speculative asset treatment, we also examined the effect of liquidity in the markets. In particular, we define the cash endowment per person, or money supply per capita, $K$, which is calculated as

$$K := \frac{M}{N} \quad (2)$$

where $M$ is the initial total cash endowment (money supply) in the experiment and $N$ is the total number of traders in the experiment. Note that in each experiment there are 10 times as many shares as participants (five of each asset).

In each of the experiments there were between 38 and 65 shares of each of the two assets, A and B, i.e., five shares of each asset per average participant. The amount of cash was varied in the experiments of both types (value-value and value-speculative), since the excess cash in a market is known from previous work to be an important factor in the magnitude of a market bubble\(^2\) (see Caginalp, Porter and Smith [2000]). We use the designation $S = 1$ to denote the value-speculative experiment and $S = 0$ the value-value, and the per capita money supply, $K$, as defined above. Table 1 lists each experiment and the associated treatments.

**Table 1: Experimental Design**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Date</th>
<th>$K$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Feb19</td>
<td>500</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Feb20(1)</td>
<td>1105</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Feb20(2)</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Feb21</td>
<td>1000</td>
<td>1</td>
</tr>
</tbody>
</table>

\(^2\) The experiments with two value assets exhibited little variation across different groups, so fewer of these have been conducted. Since the effects of excess cash are well understood from previous experiments, a broad spectrum of cash levels were implemented in order to assure robustness of the conclusions and the interaction between excess cash and the presence of a speculative asset. In particular, the value asset exhibits a lower trading price in the presence of the speculative asset even when there is a somewhat greater supply of cash per trader (see below).
Our basic strategy for understanding the influence of a speculative asset on a value asset within a two asset model is to compare the trading history of the asset labeled A, which is always the value asset, under the two conditions: Asset B is speculative, or asset B is also a value asset. A pair of typical experiments is shown in Figures 3a and 3b where the price evolution of asset A is shown for two experiments: one that is juxtaposed with a speculative asset and another with a similar value asset, respectively. In each pair of experiments it is evident that the price volatility of asset A is much higher in the presence of the speculative stock. This issue will be examined below within a precise definition and statistical setting. A related question that will be examined is whether the presence of the speculative stock lowers the trading prices of the value stock.

Figure 3a

Graphical representation of the fundamental values of assets A and B through time and the contract price of each transaction observed in the two markets in a representative value-value treatment (data from April 23rd)
The time evolution graphs for the other experiments are similar in that the value asset A exhibits more volatility (as defined below) and a lower trading price (see Results).

As with most laboratory experiments, participants executed trades through a computer network after receiving computerized instructions describing the asset and trading rules (see Davis and Holt [1993] and Smith [1982] for general discussion of experimental markets). Such experimental asset markets have attracted much interest, particularly in terms of financial bubbles that have been generated under robust conditions (see, for example, Smith, Suchanek and Williams [1988], Lei, Noussair and Plott [1994] and Porter and Smith [1998]).
The experiments were conducted at the University of Arizona during February to April of 2001 with undergraduates, numbering 8 to 13 per experiment. In each of 14 experiments, the participants were informed that there would be two assets that they could trade continuously throughout a 42-minute trading period. The participants are given an allotment of the assets and “francs” that are converted to US dollars at the end of the experiment. Average subject earnings for each 90 minute experiment was $25 with a range of [7,55]. The full set of instructions are posted at http://economic.gmu.edu/Asset/Instructions.htm.

Results

There are several measurements we are interested in analyzing from our experiments. First, we define volatility $\lambda$, or relative magnitude of the price change between successive trades of the same asset (e.g. A) as

$$\lambda_A(t) = \text{absolute value} \left[ \frac{P_A(t) - P_A(t-1)}{P_A(t)} \right]$$

For each experiment and trade t we calculate the following measurement:

(i) the transaction price, $P(t)$,
(ii) the corresponding volatility, $\lambda(t)$,
(iii) the other asset in the experiment (value or speculative) designated by $S = 0$ or 1,
(iv) the cash in the experiment per person, or the per capita money supply $K=M/N$, with $M$ as the total cash in the experiment and $N$ as the total number of participants.

We break-up the observations of each experiment into three time periods to aggregate our measurements. The first is the initial five minutes during which trading is somewhat erratic (for both assets) as participants become accustomed
to the draws. It is only near the end of this period that it becomes evident from
experience that the speculative asset may have greater growth in the near term
(see Figure 1). From the sixth minute to the 26th minute the speculative asset
receives mainly good draws and has an expected payout value that exceeds the
value asset. In the final period, from the 27th minute to the 42nd the expected
payout of the speculative asset declines after a brief plateau. For our purposes,
the most interesting aspect of the experiment in terms of the questions above is
the middle part.3

Table 2 lists for each experiment the mean price, \( (P_A^{\text{mean}} \text{ and } P_B^{\text{mean}}) \) for each
asset along with the maximum price \( (P_A^{\text{max}} \text{ for } t>5 \text{ and } t<26) \) for the value asset up to the 26th
minute and its the minimum price \( (P_A^{\text{min}} \text{ for } t>26) \) after the 26th minute for asset A (value).
Note that the “good” draws have dried up for the speculative asset near the 26th
minute.

Table 2

Average prices observed in both markets in each experiment and the
maximum price observed up to the 26th minute and the minimum price
observed after the 26th minute in the market for asset A.

<table>
<thead>
<tr>
<th>EXP</th>
<th>K</th>
<th>S</th>
<th>( P_A^{\text{mean}} )</th>
<th>( P_B^{\text{mean}} )</th>
<th>( P_A^{\text{max}} \text{ for } t&gt;5 \text{ and } t&lt;26 )</th>
<th>( P_A^{\text{min}} \text{ for } t&gt;26 )</th>
<th>( P_B^{\text{max}} \text{ for } t&gt;5 \text{ and } t&lt;26 )</th>
<th>( P_B^{\text{min}} \text{ for } t&gt;26 )</th>
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</thead>
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<tr>
<td>1</td>
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<td>1</td>
<td>35.7</td>
<td>43.4</td>
<td>44</td>
<td>31</td>
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<td>20</td>
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<tr>
<td>2</td>
<td>1105</td>
<td>1</td>
<td>113.5</td>
<td>133.8</td>
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<td>78.2</td>
<td>66</td>
<td>57</td>
<td>95</td>
<td>91</td>
</tr>
<tr>
<td>5</td>
<td>2250</td>
<td>1</td>
<td>115.9</td>
<td>159.1</td>
<td>127</td>
<td>106</td>
<td>212</td>
<td>165</td>
</tr>
<tr>
<td>6</td>
<td>2250</td>
<td>1</td>
<td>111.7</td>
<td>137.8</td>
<td>225</td>
<td>65</td>
<td>420</td>
<td>90</td>
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<tr>
<td>7</td>
<td>1500</td>
<td>0</td>
<td>121.0</td>
<td>117.7</td>
<td>128</td>
<td>124</td>
<td>128</td>
<td>119</td>
</tr>
</tbody>
</table>

3 This structure was selected to be analogous to the time interval 1995 to the beginning of 2000 during
which Nasdaq soared 550 percent, while the Dow Jones Industrial average increased by a more moderate
200 percent. The final part of the experiment is analogous to the period between April 2000 and September
2001 when Nasdaq plummeted by about 70% while the Dow declined by 11%.
We consider first the middle part of each experiment (6th to 26th minutes). For each of the quantities, $\lambda(t)$ and $P(t)$, we perform a linear regression using the mixed effects model. This is an effective procedure to deal with the fact that many trades throughout the set of experiments are executed by the same persons, and are thus not independent of one another. An ordinary linear regression would treat each of the trades as an independent event, and the statistical significance would be much higher under this assumption. The mixed effects model adjust for the lack of independence by examining the variances within each group (which is each experiment for our purposes. With $P_A(t)$ as the dependent variable and $S$ and $K$ as the independent variables, we estimate the coefficients of the linear regression

$$P_A(t) = \alpha_0 + \alpha_1 S + \alpha_2 K,$$

with the result $\alpha_0 = 72.79$, $\alpha_1 = -27.24$, $\alpha_2 = 0.0264$ and p-values 0.001, 0.10 and 0.06 respectively. (Number of observations is 1171 with 14 groups. The degrees of freedom are 1157 for the constant term and 11 each for the two variables. See Appendix for remaining statistical features.)

The statistical results imply that the presence of a speculative asset during the boom period tends to suppress the trading prices of a value asset. The additional result that higher overall excess cash raises trading prices is consistent with earlier work with a single asset. Performing a similar study for the volatility, and writing the coefficients directly, we have,

$$\lambda_A(t) = 0.039 + 0.089 S - 0.00001 K.$$
The p-values for the coefficients of S and K are 0.233 and 0.862 respectively. This provides some support for the assertion that the presence of a speculative stock increases the volatility of the value stock. There is no statistical evidence that excess cash leads to greater volatility.

Performing a similar analysis for the last part of the experiments (27th minute to end) we obtain

\[ P_A(t) = 74.79 - 17.53S + 0.0392K, \]

with p-values 0.21 and 0.05 respectively for the coefficients of S and K, respectively. Thus excess cash remains an important factor while there is some support that the presence of the speculative asset lowers the price of the value asset. For the volatility, we obtain

\[ \lambda_A(t) = 0.174 + 0.289S - 0.00011K, \]

with p-values of 0.21 and 0.56 for S and K, respectively. Hence, there is support for the assertion that the presence of the speculative asset increases the volatility for the value asset during this period. Note that a smaller number of trading minutes compared to the middle part has a negative effect on the statistical significance.

For the entire experiment, the data provide almost the same statistical confidence for the assertion that the speculative asset suppresses the trading price of the value asset. There is somewhat less support regarding the volatility, since prices trade more erratically during the first few minutes of most experiments when there is limited information on both the earnings updates and other traders. In particular, the p-value for the coefficient of the S variable is still 0.1078 in the regression for the price, but 0.5648 for the volatility.

Next, we examine two key features of the trading price for the value asset in each experiment:
(i) the mean price, $P_A^{\text{mean}}$

(ii) the minimum price after the 26$^{\text{th}}$ minute, $P_A^{\text{min}}$.

The latter quantity is defined as the lowest trading price of asset A between the 27$^{\text{th}}$ minute through the 42$^{\text{nd}}$ minute for a particular experiment. We will examine whether this minimum for the value stock, A, is lower or higher when the other asset is speculative. With each of these as the dependent variables, respectively, and $S$ and $K$ as the independent variables, we perform a linear regression for the mean trading price of the form

$$P_A^{\text{mean}} = \alpha_0 + \alpha_1 S + \alpha_2 K.$$ 

The resulting coefficients are $\alpha_0 = 71.43, \alpha_1 = -22.19, \alpha_2 = 0.028$, with p-values of 0.002, 0.11 and 0.02 respectively. Hence the presence of a speculative stock lowers the mean price of the asset by 22.19 francs. If the cash endowment per share is 150 francs (a neutral cash/asset situation) the mean of the value asset without the speculative asset is $71.43 + (0.28)(150) = 113$; with the speculative asset it is $71.43 - 22.19 + (0.28)(150) = 113 - 22$. Thus, the speculative asset lowers the mean price of the value asset by $22/113 = 19.4\%$. The same computation with a lower cash endowment of 100 francs yields a 22% decrease in the mean trading price of the value asset. Further statistical details for these regressions are in the Appendix.

Similarly, the regression for the minimum price yields

$$P_A^{\text{min}} = 69.19 - 41.69 S + 0.0276 K,$$

with p-values of 0.006, 0.099 and 0.177, respectively.

Hence, these results indicate that the mean price and the minimum price of the value asset are lower when there is a speculative asset, consistent with results discussed above. For a neutral cash endowment per share of 150 francs, the mean trading price without the presence of the speculative asset is $69.19 + (0.276)(150) = 111$. In the presence of the speculative asset the minimum trading price of the value asset is $69.19 + (0.276)(150) - 41.69 = 111 - 41.69$. Hence the minimum trading price of the
value asset is lower by $\frac{41}{111} = 38\%$ due to the presence of the speculative asset. For a lower cash endowment per share of 100 francs the comparable percentage is 43%.

The large negative coefficient for $S$, which is more than half of the constant term in magnitude, indicates that the observed minimum trading price of the value asset is significantly lower in the presence of the speculative asset, which is undergoing a decline during this period. We discuss the significance of this finding in the Conclusion.

In summary, the data are compatible with the following aggregate results:

1. The value stock exhibits lower prices overall and a lower minimum during the last phase of the experiment, and higher volatility (as defined above) in the presence of a speculative asset. The results are consistent with the argument that the asset drains money away from bids for the value asset (the excess cash argument). They are also consistent with other arguments that would suggest that the possibility of a higher return diminished bids for the value asset. An example would be the anchoring principle that may lead traders to find the value asset less appealing after learning of a higher potential growth in the speculative asset.

2. The fact that there is a lower minimum suggests that the observation of the rapidly declining speculative stock has a negative effect on the bids for the value asset (the affect heuristic argument).

In other words, as the speculative asset reaches a peak, there should be less money available for bidding on the value asset according to the excess cash argument. This is confirmed in the experiment. During the final phase of the time period, when the earnings updates for the speculative stock indicate decreasing expected payout, the value asset may be benefiting from the excess cash factor since the alternative is clearly undesirable. The owner of the value asset observes a declining value of the
speculative asset (which he may also own) while also observing a steady rise in the fundamentals of his value asset holdings. Thus, the data indicate that the heuristic affect, or the impact of observing declining prices in a related investment, is more significant than the effect of excess cash during a collapse of a related asset. A review of the literature by Slovic et al (2001) discusses numerous experiments in which a clearly visible negative image distorts the risk involved in making a decision in which the probabilities are not altered by the image. In the final part of the experiment there is no change in the frequency of positive draws for the value asset. Yet there is a very substantially lower minimum in the presence of the speculative asset that is declining rapidly at this stage. The average maximum price before the 26th minute for all the experiments is 137.85 and the average minimum price after the 27th minute is 81.14.

Figure 4
Graphical representation of the price evolution of asset A through time in the two markets from a pair of representative value-speculative and value-value experiments (data from Feb 21st and Apr 23rd) with K = 100.
Next we examine these results in view of current economic theory and recent ideas in behavioral finance. For a multi-asset market classical economics stipulates that assets trade in accordance with portfolio theory (Markowitz [1991]) in equilibrium, which would correspond to a single-period experiment. These results cannot be explained by adapting portfolio theory to a multi-period setting. Portfolio theory is based on the primary assumption of risk aversion. (The greater volatility of the speculative asset should diminish the demand for this asset and increase demand for the less volatile asset, particularly when that asset presents a better value, according to this theory.) Specifically, for a simple one period case, if an investor is considering investing in a portfolio \( P \in S \) where \( S \) represents the set of all feasible portfolios, the expected rate of return on this portfolio with weights \( w_i \) in each included security is:

\[
E(r_p) = \sum_{i=1}^{n} w_i E(r_i)
\]

where \( E(r_i) \) is the expected rate of return on security \( i \). The risk of \( P \) is represented by the variance of the portfolio's rate of return which is

\[
\sigma_p^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} w_j w_i \text{Cov}(r_p, r_j).
\]

(Basically, this is \( \text{Var}(\sum_{i=1}^{n} w_i E(r_i)) \).) The assumption is that the investor chooses the optimal expected return-risk characteristics of the portfolio. In order to do that the investor is assumed to have a utility function in \( E(r_p) \) and \( \sigma_p^2 \) such that it is strictly increasing in the former and strictly decreasing in the latter. The purpose of this utility function is to rank the different possible portfolios on the basis of their desirability. An example of such a utility function is

\[
U = E - Ar^2
\]

where \( A > 0 \) is an index of the investor's risk aversion.

In a multi-period setting such as the one of the current experiments the model has to be modified in such a fashion so that the expected return and variance take into account the time dimension. However, it is obvious from the results we obtained that
the subjects do not behave according to the predictions of this theory. Even though the subjects are not given any specific information about the underlying distributions as the experiments unfold they have enough information to make reasonable assumptions about the expected return and variance of the two assets. Nevertheless they invest in portfolios that are clearly undesirable (from the portfolio theory perspective) compared to other portfolios that are also possible. In particular, when the only alternative to the value stock is a highly overvalued stock, optimizing portfolios would lead to selling the speculative stock and buying the value stock. The fact that the value stock is lower priced (see Table 2) under these conditions indicates that traders are choosing a balance that is not optimal from the perspective of portfolio theory. This comes as no surprise to any practitioner who has followed the markets closely during the recent high-tech bubble and subsequent collapse. However, it is a clear departure from modern portfolio theory that is widely accepted among academicians.

On the other hand, these results are consistent with the predictions based upon differential equations incorporating excess cash and momentum (see for example, Caginalp and Balenovich [1999]) and earlier experimental work involving both of these concepts (Caginalp, Porter and Smith [2000] and references therein). The differential equations imply that the excess cash is a key factor in raising the prices above the current or fundamental value. As in previous experiments involving a single asset we see that this is consistently verified under multi-asset conditions, i.e., the price of asset A is higher if the cash level is higher and all other conditions are identical. These works also suggest that momentum, or the tendency to buy due to rising prices alone, plays a key role in further price rises. As the favorable “earnings reports” or draws materialize early on for the speculative asset the rise in prices attracts the attention of the momentum, or trend-based traders, and prices soar. In this two-asset environment, the available cash for bidding has two alternative, asset A or asset B. When asset B is speculative and its price soars due the momentum trading, the multi-asset adaptation of the differential equations leads to the conclusion that cash is drained away from the value asset, thereby suppressing its price. The model also suggests that the bubble becomes more fragile as a large portion of potential capital is already invested leaving a surplus of sellers and a
shortage of buyers. Thus even a small change in the supply/demand balance can lead to a sharp drop at this point. Once the decline begins the momentum trading then has a negative effect on prices, and the supply/demand picture is further aggravated by the absence of value investors at this stage. Prices in the speculative asset thus fall precipitously. The excess cash and momentum arguments (as well as portfolio theory) however, do not explain the lower minimum due to the presence of the speculative asset (see Figure 4).

As noted above, the last phase of the experiment, during which the speculative asset declines in value and price, suggests an important role for the affect heuristic and other psychological processes that bias decision making (Hilton article [2001]). The psychology involved in observing a sharply declining asset is different from observing a soaring asset. It appears that the affect heuristic argument is more powerful than the excess cash argument for the declining speculative asset and less so for the rising asset. The decline in the value asset during the last phase of the experiment is not consistent with portfolio theory or excess cash and momentum. It appears to be a consequence of observing a decline in another asset held and changing trading investment strategy as a result. For example, the investor who sees his speculative stock declining sharply may become more risk averse and not willing to bid as high for the value stock.

**Supplementary Experiments: Trading A and B in Independent Markets**

The results discussed above illuminate an important feature of markets featuring multiple assets. There may be an intrinsic competition between the roles of excess cash (or even desire to balance portfolios) and the bias generated by one asset whose price is changing rapidly. If a speculative stock is rising in price, the increase in resources for bidding for this asset diminishes funds for the value asset, resulting in lower prices for the latter. However, there may be a tendency to raise bids and
asks based upon the cognitive effects in observing much higher prices for another asset. If both of these competing effects are, in fact, present they would be difficult to isolate and distinguish as they vie for dominance throughout a boom-bust cycle. One way of dealing with this issue is to examine the cognitive effects in the absence of excess cash considerations.

Toward this end we conducted eight experiments during the same time period and within the same software and auction structure. Instead of two different assets within the same market and money supply, however, we now use two independent markets trading an asset with the same expected payout. The traders were divided into two groups: those trading asset A, and those trading asset B. The setup of the asset was similar to those described earlier, with the updates of the current value for both A and B defined as in the speculative case above. All traders were able to see the trading in their own group as well as the trading in the other group, but they were allowed to trade only the asset in their own group.

Our basic strategy was to maintain identical conditions for the A market in all eight experiments while imposing either neutral or high cash endowment on the B market. In the neutral case, the cash level in the B market was identical to that of the A market. In the high case situation, the cash endowment of the B market was about 2.6 times that of the A market. Our expectation was that asset B would exhibit higher trading prices overall, including a higher mean and maximum. If this expectation is fulfilled, the more interesting question from the perspective of the affect heuristic is whether this increased price in B will result in a similar higher price and volatility for the A market even though the two markets are completely separate. In other words, for the A market all of the liquidity and valuation conditions are identical in the full set of experiments. If there is an increase in either price or volatility in A, it can only be attributed to the difference in the trading that is observed in the B market.

Specifically, the experiments featured at least eight participants in each of the A and B markets. The cash endowment per trader, $K_A$ and $K_B$, were each approximately 1600 in the four “control” experiments in which A and B markets had the same
endowment. In the other four experiments the A market cash level was unchanged while the B market was endowed with cash at approximately \( K_B = 4200 \).

In the sequel, we will use \( K_B \), i.e., the cash level per participant for the B market, as the only variable since \( K_A \) is constant throughout the eight experiments for the A market.

**Results of Identical Assets in Independent Markets**

An examination of the prices in the B market indicates that there is strong support for the excess cash argument that a larger cash endowment does indeed lead to higher prices (as described below). The trading prices of the A markets indicate, however, that there is no support for the argument that observing higher prices in an identical but separate market leads to higher prices. There was some support for increased volatility in the A market. The key statistics are presented in Table 3 below.

**Table 3**

**Key statistics in experiments with identical assets trading in independent markets.** Average prices observed in both markets in each experiment and the maximum price observed up to the 26\(^{th}\) minute and the minimum price observed after the 26\(^{th}\) minute in the market for asset A.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Average Price of A</th>
<th>Cash Level in B Market</th>
<th>Maximum price of A (6(^{th}) to 26(^{th}) minute)</th>
<th>Minimum price of A (after 26(^{th}) minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>129.4435146</td>
<td>High</td>
<td>200</td>
<td>132</td>
</tr>
<tr>
<td>2</td>
<td>38.53703704</td>
<td>High</td>
<td>45</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>71.8164794</td>
<td>High</td>
<td>150</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>60.7721519</td>
<td>High</td>
<td>83</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>143.9204545</td>
<td>Neutral</td>
<td>181</td>
<td>140</td>
</tr>
<tr>
<td>6</td>
<td>116.1374046</td>
<td>Neutral</td>
<td>123</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>63.00900901</td>
<td>Neutral</td>
<td>85</td>
<td>55</td>
</tr>
<tr>
<td>8</td>
<td>147</td>
<td>Neutral</td>
<td>188</td>
<td>140</td>
</tr>
</tbody>
</table>
A statistical analysis of the prices of the B market can be performed using the mixed effects model with the cash level per participant, K, as the only independent variable. As in the studies above, we consider the time interval between the 6th and 26th minute. The linear regression for the price in terms of K leads to
\[ P_B(t) = 38.3 + 0.025 K. \]

The coefficient of K has a standard deviation of 0.01 with t=2.5 and p=0.046. (Note that the coefficient is a large number so this term is a significant part of the price.) Hence there is excellent support (as expected from previous works) for the assertion that a high cash endowment in a market leads to higher trading prices. Next we consider the mean trading price throughout each experiment. Comparing the four means under high K with the four means in the low K experiments (see Table 3), we perform a two sample t-test and a Mann-Whitney test. Both tests indicate that the means of the high K experiment are higher than those of the low K experiments with a statistical confidence at a level of about p=0.05.
Hence the trading prices of the B market are clearly higher in the experiments in which the B market had a higher cash endowment per participant, and the traders in the A market observed the trading in the B market. We examine whether the observation by the A market participants of higher prices in the B market has led to higher prices in the A market. We perform a linear regression for the mean price of A with the mean price of B as the independent variable. The result is

$$P_A^{\text{mean}} = 129 - 0.305 P_B^{\text{mean}}$$

where the constant term has a standard deviation of 41.39 and a p-value of 0.02 and t=3.12. The coefficient of $P_A^{\text{mean}}$ is 0.358 corresponding to p=0.42 and t= -0.36. Hence there is no support for the hypothesis that higher prices in the B market lead to higher prices in the A market. The correlation between the two sets of means is -0.33, confirming the same result.

Considering the middle interval (6th to the 26th minute) once again we use a mixed effect model for a linear regression of the trading price of A in terms of the cash level of the B market as the only variable. In particular we obtain

$$P_A(t)=895.6 -0.504 K,$$

with standard errors of 476 and 0.30 respectively (corresponding to p=0.06 and p=0.14). This also indicates that there is no support for the assertion for that observing higher prices in the B market leads to higher prices in the A market.

Despite the fact that the mean trading price in the B market is about 60% higher under the high cash conditions, and the participants in the A market are aware of the trading in an identical asset, the prices in A are slightly lower under these conditions. In these experiments, then, there is no support for the assertion that observing higher prices of an identical asset in a separate market should lead to higher prices. It is possible that this difference of 60% is not adequate to generate this effect, and that a difference of perhaps several times this magnitude in the B market would have an impact on the A market.
We use a mixed effects model regression on the volatility of asset A for trades in the middle period again, with the result,

\[ \lambda_A(t) = -0.914 + 0.0006K. \]

Neither of these coefficients is significant with \( p=0.54 \) and \( p=0.50 \) respectively. Hence the presence of excess cash in the B market has no significant impact on the volatility of the A asset. Legally separated markets in which the same asset trades in two markets are utilized by a number of countries, such as China, where the A asset is only traded by domestic investors and the B asset by foreign investors. This division is believed to have several benefits. Often, the sum of the B asset shares represents less than 50% ownership of the company, so this prevents takeover by foreign interests. Also, there may be exogenous liquidity issues with foreign traders that lead to large price changes in the B shares. Since the foreigners are not allowed to own the A shares, this may offer some additional protection. On the other hand, if the market in B shares is small (in total volume of traded shares) it may exhibit large price swings and become highly overvalued or undervalued. There is then the potential for domestic investors to become influenced by this trading price in the B market and to similarly trade the A asset at prices that are far from realistic value. The experiments above are an indication that this does not happen, at least when the changes in price are not very extreme. If these results are confirmed by further experiments, it could be an indication that legally separated markets are not subject to large price changes as a result of similar price swings in the market reserved for foreign investors.

It is possible, however, that the affect heuristic regarding A/B shares as described above is more significant when the B asset is falling dramatically in terms of trading price. Further experimentation on this issue is needed to resolve this issue.

**Conclusion**
During the late 1990’s the soaring prices and valuations of the speculative high-tech sector raised the question: do such speculative stocks affect the trading prices of the companies with solid valuation and earnings? For example, a news article (The Economist [1999]) speculated as to whether a collapse in the high tech bubble would lead to a collapse of the broader market. Similar issues had been raised in other eras both domestic and abroad. We examine this question in an experimental context by defining two assets, one with consistent growth, and the other with more volatile growth. The latter experiences rapid growth in expected value during most of the experiment, but diminishing fortunes near the end.

The microeconomic theory for price dynamics involving two different assets consists mainly of the theory of random fluctuations about the expected return. Portfolio theory, which is essentially an equilibrium idea, stipulates that risk aversion and growth of assets lead to balancing a portfolio. Thus we consider this neoclassical explanation along with more recent paradigms.

At the outset of the experiments, one might consider various explanations for the possible outcomes. First, there is the null hypothesis that the price of the value asset is not influenced by the presence of the speculative asset. The portfolio balancing idea suggests that as the speculative asset rises and becomes more volatile, traders have an excessive amount of the speculative asset and would seek to replace it by more of the value asset if it is at a lower price. This would inhibit substantial decline in the value asset, compared to the control experiments. Thus portfolio theory, as well as the efficient market hypothesis, would imply the first alternative. Second, there is the hypothesis, based upon the “affect heuristic” that the value asset trades higher when the speculative asset is in the boom period and lower during the stalling phase. Third, the excess cash and momentum ideas suggest lower prices for the value asset as prices of the speculative asset rise. The momentum of the speculative asset induces more to buy into the rally, and leaves fewer assets to bid for the value asset. Analogously, one would expect higher prices during the decline of the speculative asset.
Both the excess cash/momentum and the heuristic affect would suggest that volatility increases in the presence of a speculative asset.

Our results indicate that the trading prices of asset A are generally lower and more volatile when asset B is a speculative stock. In fact there is a decrease of almost 20% in the mean trading price when B is a speculative stock instead of another value stock. The minimum price in the last phase displays an even more striking difference of about 38% in this respect. These figures are calculated for neutral cash/asset ratios, and are even more pronounced when there is a lower cash/asset ratio.

During the period of rapidly increasing prices for the speculative asset, the data suggests that the excess cash arguments outweigh the psychological arguments. However, the minimum value of the asset during the final part of the experiment are also lower (see Table 2), suggesting that the psychological impact of rapidly falling prices in the speculative asset lead to more conservative bidding and aggressive selling of the value asset, even though the latter does not suffer negative earnings updates during this period. Hence, it appears that the image of decline in a related asset overshadows the excess cash considerations. There are other possible explanations beyond the three that we have considered explicitly. For example, the fact that there is another asset defined with a possible earnings update of 10% rather than a maximum of 4% may have an anchoring effect that diminishes the appeal of the value asset. In other words, in the experiments for which S=0, both assets offer the maximum possible earnings of 4%, so that 4% does not appear to be a small number. In the S=1 experiments (i.e. the other asset is speculative) the 10% possible earnings update may serve as an anchor for a good earnings update, thereby diminishing the appeal of 4% earnings update, and leading participants to place lower bids on the value asset as a consequence. Hence more experimentation is needed to further our understanding of the precise mechanism that lead to these observations.
Investors in value stocks during the late 1990’s have often lamented the attention devoted to the speculative high-tech stocks and the influx of funds into these extremely overvalued sectors. A general sentiment is that many stocks with more solid value languished in the process. By many measures, the volatility of most stocks, even the less speculative ones, increased during this period.

Using the Dow Jones Industrial Average as a proxy for value, and the Nasdaq average as representative of speculative high-tech stocks, we examine volatility which we define in terms of the absolute value of the relative change in successive daily closes. The Dow and Nasdaq diverged in terms of valuation (with respect to P/E ratios, etc.) during the late 1990’s, despite the addition of several Nasdaq stocks near the end of the decade. As a rough approximation, one might regard both the Dow and the Nasdaq as “value” investments during the early 1990’s. In the late 1990’s, however, the majority of Nasdaq had become sufficiently inflated by the standards of most value investors so that it could only be regarded as speculative. Comparing the early 1990’s with the late 1990’s then gives us an opportunity to examine the volatility aspect of the experiments in the context of US markets. We first list (see Table 4) the calculations of the volatility for the two indexes during successive two year periods. The first period after the Gulf War (March 1992- March 1994) exhibits a volatility of 0.0046. This increases in each two year period until it nearly doubles, at 0.009, during 1998-2000 which coincides with the height of the speculative bubble in Nasdaq. By considering the monthly averages of volatility, we monitor the correlations between the Nasdaq and Dow volatility during the 1990’s. One might expect a lower correlation in volatility as a result. We find, however, that the correlations are among the highest during the height of the bubble from March 1998 to March 2000, when the differences in the two groups were most pronounced, and also during the huge decline in Nasdaq after March 2000 (with correlations of 0.79 and 0.72 respectively). This data indicate that a similar effect could be present in the US markets, suggesting that investors of value stock are subject to substantial additional risk due to the presence of a speculative bubble.
### Table 4

**Calculated volatility of the DJIA and NASDAQ indices for specific time periods and the correlations between the two sets of volatility measures**

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Average Volatility of DJIA</th>
<th>Average Volatility of NASDAQ</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 1990 - Aug. 2001</td>
<td>0.006893</td>
<td>0.010268</td>
<td>0.736</td>
</tr>
<tr>
<td>Mar. 2000 - Aug. 2001</td>
<td>0.009519</td>
<td>0.024032</td>
<td>0.792</td>
</tr>
<tr>
<td>Mar. 1998 - Mar. 2000</td>
<td>0.009034</td>
<td>0.014561</td>
<td>0.717</td>
</tr>
<tr>
<td>Mar. 1996 - Mar. 1998</td>
<td>0.007207</td>
<td>0.008053</td>
<td>0.753</td>
</tr>
<tr>
<td>Mar. 1994 - Mar. 1996</td>
<td>0.004962</td>
<td>0.006233</td>
<td>0.419</td>
</tr>
<tr>
<td>Mar. 1992 - Mar. 1994</td>
<td>0.004595</td>
<td>0.005737</td>
<td>0.651</td>
</tr>
<tr>
<td>Mar. 1990 - Mar. 1992</td>
<td>0.006833</td>
<td>0.007188</td>
<td>0.844</td>
</tr>
</tbody>
</table>

The central limitation of retrospective data analysis is the inability to examine what would have happened under different conditions. Since numerous other events occurred during the same time period, it is difficult to argue such causality based upon data analysis alone. Experimental asset markets, on the other hand, enable the testing of hypotheses under varying conditions, so that a causal explanation is possible.

Our previous experiments involving a single asset under a variety of auction mechanisms and asset descriptions have consistently demonstrated that the cash level per participant plays a crucial role in the level of trading prices. These experiments confirm this result when more than one asset is traded. They lend further support to the liquidity arguments by suggesting that money flowing to one asset tends to suppress the trading price of the other. Our complementary set of experiments featuring identical assets trading in separate markets (but visible to all...
traders) suggests that the psychological effect of observing higher prices of an equivalent asset are not significant in this case.

Speculative manias have occurred in many different countries and eras. The most recent internet/high-tech bubble has been one of the largest, and presents convincing evidence that these phenomena are not a feature of new markets, or those with inadequate flow of information. Rather, they are intrinsic to markets that satisfy a number of conditions (Caginalp, Porter and Smith [2000] and Miller [2002])).

Since financial bubbles cannot be explained by classical economics, efficient market theorists often address the problem by minimizing their importance – often using the word “anomaly” to describe such phenomena. However, the results of our experiments remove even that level of comfort by indicating that a speculative bubble has a profound effect on the remaining assets that are not the subject of the speculation. The recent high-tech bubble is estimated to have resulted in losses of 4.5 to 5.5 trillion dollars. If the experimental effect we have observed is present in the US markets, the suppression of prices in value stocks may result in even greater losses.

Acknowledgements: The authors are grateful to Mr. David Dreman for suggesting this line of experimentation. Grants from the Dreman Foundation and Fred Maytag Family Foundation are very much appreciated.

References


The Economist. "When the bubble bursts," January 30, 1999 (print edition)


Viscusi, W.K., “Alarmist Decisions with

Appendix: Statistical Methodology

We present the statistical details of the linear regressions above. There are more than a thousand trades that are not statistically independent since they are generated by 14 groups of participants. As explained above the Mixed Effects Model with stratification by experiment is ideally suited for this problem.

The Mixed Effects Model using stratification by experiment was used for the middle time period (6th through 26th minutes) of 14 experiments.

For the linear regression formula
$$P_A(t)=\alpha_0 + \alpha_1 S + \alpha_2 K,$$
the full set of statistics are stated below.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>72.79604</td>
<td>21.71035</td>
<td>1157</td>
<td>3.353057</td>
<td>0.0008</td>
</tr>
<tr>
<td>K</td>
<td>0.0026412</td>
<td>0.0012736</td>
<td>11</td>
<td>2.073872</td>
<td>0.0624</td>
</tr>
<tr>
<td>S</td>
<td>-27.2408</td>
<td>15.36354</td>
<td>11</td>
<td>-1.77308</td>
<td>0.1039</td>
</tr>
</tbody>
</table>

Number of Observations: 1171
Number of Groups: 14

Similarly for the volatility, the full set of statistics for the linear regression formula
$$\lambda_A(t)=\alpha_0 + \alpha_1 S + \alpha_2 K$$
are stated below.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>0.039109</td>
<td>0.100042</td>
<td>1143</td>
<td>0.390928</td>
<td>0.6959</td>
</tr>
<tr>
<td>K</td>
<td>0.00000104</td>
<td>0.00000586</td>
<td>11</td>
<td>0.177549</td>
<td>0.8623</td>
</tr>
<tr>
<td>S</td>
<td>0.08946</td>
<td>0.070888</td>
<td>11</td>
<td>1.261984</td>
<td>0.2331</td>
</tr>
</tbody>
</table>

Number of Observations: 1157
Number of Groups: 14

Next we present the full statistics for the Mixed Effects Model using stratification by experiment for the final time period (27th minute to end). The regression

$$P_A(t)=\alpha_0 + \alpha_1 S + \alpha_2 K$$
leads to the estimates:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>74.78868</td>
<td>18.97446</td>
<td>795</td>
<td>3.941545</td>
<td>0.0001</td>
</tr>
<tr>
<td>K</td>
<td>0.0039251</td>
<td>0.0011142</td>
<td>11</td>
<td>3.522693</td>
<td>0.0048</td>
</tr>
</tbody>
</table>
The regression for the volatility

\[ \lambda_A(t) = \alpha_0 + \alpha_1 S + \alpha_2 K \]

leads to the estimates:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>0.174393</td>
<td>0.310238</td>
<td>781</td>
<td>0.562128</td>
<td>0.5742</td>
</tr>
<tr>
<td>K</td>
<td>-0.0000109</td>
<td>0.00001827</td>
<td>11</td>
<td>-0.59745</td>
<td>0.5623</td>
</tr>
<tr>
<td>S</td>
<td>0.288746</td>
<td>0.218535</td>
<td>11</td>
<td>1.32128</td>
<td>0.2132</td>
</tr>
</tbody>
</table>

Number of Observations: 795
Number of Groups: 14

For the entire trading period the Mixed Effects using stratification by experiment is used to perform a regression of the form:

\[ P_A(t) = \alpha_0 + \alpha_1 S + \alpha_2 K \]

The estimates of the coefficients and errors are given by the following:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>71.27176</td>
<td>17.80127</td>
<td>2381</td>
<td>4.003747</td>
<td>0.0001</td>
</tr>
<tr>
<td>K</td>
<td>0.0028115</td>
<td>0.0010446</td>
<td>11</td>
<td>2.691571</td>
<td>0.021</td>
</tr>
<tr>
<td>S</td>
<td>-22.0516</td>
<td>12.59492</td>
<td>11</td>
<td>-1.75083</td>
<td>0.1078</td>
</tr>
</tbody>
</table>

Number of Observations: 2395
Number of Groups: 14

Respectively for the volatility we have for the linear regression

\[ \lambda_A(t) = \alpha_0 + \alpha_1 S + \alpha_2 K \]

the estimates below:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>0.2284</td>
<td>0.189106</td>
<td>2367</td>
<td>1.20779</td>
<td>0.2272</td>
</tr>
<tr>
<td>K</td>
<td>-0.0000047</td>
<td>0.00001111</td>
<td>11</td>
<td>-0.42255</td>
<td>0.6808</td>
</tr>
<tr>
<td>S</td>
<td>0.079398</td>
<td>0.133758</td>
<td>11</td>
<td>0.593598</td>
<td>0.5648</td>
</tr>
</tbody>
</table>

Number of Observations: 2381
Number of Groups: 14
We note that the Mixed Effects Model the entire time period but now without using stratification by experiment leads to the linear regression

\[ P_A(t) = \alpha_0 + \alpha_1 S + \alpha_2 K \]

with the following error estimates.

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>72.16368</td>
<td>3.212223</td>
<td>2392</td>
<td>22.46534</td>
</tr>
<tr>
<td>K</td>
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<td>0.00018932</td>
<td>2392</td>
<td>13.99268</td>
</tr>
<tr>
<td>S</td>
<td>-21.3065</td>
<td>2.995621</td>
<td>2392</td>
<td>-7.11253</td>
</tr>
</tbody>
</table>

Number of Observations: 2395
Number of Groups: 1

Hence, the statistical confidence level is extremely high without stratification.

Next, we examine the mean trading price of asset A as a function of S and K. The statistical independence of the data set of means is assured since we are using only one data point per experiment. Hence, we perform an ordinary linear regression for \( P_A^{\text{mean}} \) with the results:

\[ P_A^{\text{mean}} = 71.43 - 22.19 S + 0.0028 K. \]

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>71.43332</td>
<td>17.84134</td>
<td>4.00381</td>
</tr>
<tr>
<td>K</td>
<td>0.0028113</td>
<td>0.00104681</td>
<td>2.685573</td>
</tr>
<tr>
<td>S</td>
<td>-22.188</td>
<td>12.62381</td>
<td>-1.75763</td>
</tr>
</tbody>
</table>

In a similar way we consider the minimum trading price of asset A during the final phase of the experiment from the 27th minute to the 42nd minute. Again, there is only one data point per experiment so that an ordinary linear regression can be used. This leads to the linear regression

\[ P_A^{\text{min}} = 69.19 - 41.68 S + 0.00277 K \]

with the following errors:

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>69.19295</td>
<td>32.72261</td>
<td>2.11453</td>
</tr>
<tr>
<td>K</td>
<td>0.00276779</td>
<td>0.00191995</td>
<td>1.441591</td>
</tr>
<tr>
<td>S</td>
<td>-41.685</td>
<td>23.1532</td>
<td>-1.8004</td>
</tr>
</tbody>
</table>

**Identical Assets in Separate Markets**

The Mixed Effects Model using stratification by experiment was used for the middle time period (6th through 26th minutes) of 8 experiments. The complete results are as follows.
\[ P_B(t) = 38.3 + 0.025 \, K. \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>38.31324</td>
<td>32.17747</td>
<td>896</td>
<td>1.190685</td>
</tr>
<tr>
<td>K</td>
<td>0.02543</td>
<td>0.01027</td>
<td>6</td>
<td>2.476809</td>
</tr>
</tbody>
</table>

Number of Observations: 904
Number of Groups: 8

\[ P_A(t) = 895.6 - 0.504 \, K \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>895.6187</td>
<td>476.2757</td>
<td>1070</td>
<td>1.880463</td>
</tr>
<tr>
<td>K</td>
<td>-0.5044</td>
<td>0.2997</td>
<td>6</td>
<td>-1.682888</td>
</tr>
</tbody>
</table>

Number of Observations: 1078
Number of Groups: 8

\[ \lambda_A(t) = -0.914 + 0.0006 \, K \]

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.9141867</td>
<td>1.500708</td>
<td>1070</td>
<td>-0.6091701</td>
</tr>
<tr>
<td>K</td>
<td>0.0006803</td>
<td>0.000945</td>
<td>6</td>
<td>0.7201603</td>
</tr>
</tbody>
</table>

Number of Observations: 1078
Number of Groups: 8