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Laminar Natural Convection in a Discretely Heated Cavity: II—Comparisons of Experimental and Theoretical Results

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Three-dimensional numerical predictions and experimental data have been obtained for natural convection from a 3×3 array of discrete heat sources flush-mounted on one vertical wall of a rectangular cavity and cooled by the opposing wall. Predictions performed in a companion paper (Heindel et al., 1995a) revealed that three-dimensional edge effects are significant and that, with increasing Rayleigh number, flow and heat transfer become more uniform across each heater face. The three-dimensional predictions are in excellent agreement with the data of this study, whereas a two-dimensional model of the experimental geometry underpredicts average heat transfer by as much as 20 percent. Experimental row-averaged Nusselt numbers are well correlated with a Rayleigh number exponent of 0.25 for $Ra_{L_z} \lesssim 1.2 \times 10^8$.

Introduction

Continued miniaturization of integrated circuits on a single computer chip and reduced spacing of chips in an array have contributed to significant improvements in the performance of computer systems. However, increased circuit densities correspond to larger power dissipation rates and complicate thermal control when a principal objective is to maintain components at or below specified maximum service temperatures (Peterson and Ortega, 1990). Currently, thermal control is typically maintained by direct air cooling or by indirect liquid or air cooling (Incropera, 1988; Bar-Cohen, 1991). However, implementation of such cooling schemes for a powerful workstation or desktop computer would require a coolant distribution system, which includes a fan or pump and associated duct or plumbing fixtures. Undesirable noise and vibration may accompany the fan or pump assembly.

In contrast, natural convection affords a means of thermal control, which eliminates the fan or pump and provides a noise and vibration-free environment. However, many previous investigations involving natural convection from a discrete heat source in a cavity assumed that the geometry and resulting flow and heat transfer can be approximated as two dimensional. Therefore, discrete heat sources are approximated as strip heaters such that $L_x = D$. The present study avoids this assumption by using a three-dimensional model to predict flow and heat transfer within a cavity containing discrete heat sources, and experimental data are obtained to validate the model.

Chu et al. (1976) conducted one of the first studies involving two-dimensional, laminar, natural convection from a single heat source mounted on one vertical wall of an air-filled cavity. The opposing wall was cooled, while the top and bottom walls were either adiabatic or cooled. As the Rayleigh number increased, the heater location associated with the maximum Nusselt number shifted downward toward the bottom of the cavity. This trend was experimentally verified by Turner and Flack (1980). The effect of heater location in a two-dimensional cavity with single or dual heat sources mounted to one vertical wall has also been addressed by other investigators (Cesini et al., 1988; Refai and Yovanovich, 1990; Chadwick et al., 1991), revealing similar trends.

Using strip heaters mounted to one vertical wall, Keyhani and co-workers performed various experimental and numerical studies to investigate two-dimensional heat transfer in a discretely heated cavity whose opposing wall was isothermally cooled. Keyhani et al. (1988a) experimentally studied natural convection in a tall cavity ($A_z = 16.5$) with 11 alternating unheated and flush-mounted isoflux sections on one vertical wall. Thermal stratification within the core region was determined to be the primary factor influencing the temperature of each heated section. Reducing the aspect ratio to $A_z = 4.5$, Keyhani et al. (1988b) numerically and experimentally investigated natural convection in a cavity with three alternating adiabatic and flush-mounted isoflux sections of equal height on one vertical wall. Flow visualization in an ethylene glycol-filled cavity revealed a multicellular pattern consisting of primary, secondary, and tertiary cells. Numerically, the flow patterns, temperature fields, and heat transfer rates did not change significantly when the Prandtl number was reduced from 166 to 25. Therefore, experimental correlations developed for ethylene glycol ($105 < \mathrm{Pr} < 166$) were determined to be applicable to other coolants such as fluorocarbon liquids ($\mathrm{Pr} \approx 25$). They also concluded that the core stratification was the primary factor influencing temperatures on the heated wall.

Using the same heater configuration (three alternating adiabatic and isoflux regions), Shen et al. (1989) showed that, by increasing the adiabatic region between heaters, differences among heaters were reduced because more fluid recirculation was allowed in the adiabatic region. Extending this study, Prasad et al. (1990) considered the effects of Prandtl number and aspect ratio on heat transfer. Numerical calculations performed at a modified Rayleigh number of $\mathrm{Ra}_H^* = 2.03 \times 10^9$ for $\mathrm{Pr} = 25$ (fluorocarbon liquid) and $\mathrm{Pr} = 166$ (ethylene glycol) displayed little variation in the streamlines and isotherms, confirming that experimental correlations obtained with ethylene glycol can be applied to other liquids. For the aspect ratios ($1 \le A_z \le 9$) and modified Rayleigh numbers ($10^6 < \mathrm{Ra}_H^* < 10^{10}$) of this study, the average Nusselt number on each heater was insensitive to a Prandtl number decrease from 166 to 25. Further reduction in the Prandtl number to 1 reduced the average Nusselt number by approximately 5 percent.

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The preceding studies focus on two-dimensional flow and heat transfer. In contrast, three-dimensional natural convection in a cavity with discrete heat sources mounted to one vertical wall and an opposing cold wall has received limited attention. One of the first numerical studies addressing this geometry was conducted by Kuhn and Oosthuizen (1986), who determined that the effect of heater location on heat transfer is small if the heated elements are located at least two element heights away from the top or bottom walls. In studying element size effects, the height of a discrete heat source was shown to be more important than its width, unless, of course, the element width was much smaller than its height. Comparisons were also presented between two- and three-dimensional calculations, with the three-dimensional model yielding larger heat fluxes near the heater edges.

A three-dimensional numerical analysis was also performed by Wroblewski and Joshi (1992, 1994), who investigated transient and steady natural convection for a single heat source mounted on one vertical wall of a square enclosure ($A_x = A_z = 1$). Steady-state conditions were characterized by thin boundary layers along the hot and cold walls, stratified temperatures within the core region, and a strong plume above the heat source. Polentini et al. (1993) performed an extensive experimental study of three-dimensional natural convection in a discretely heated cavity containing a 3 × 3 heater array flush-mounted on one vertical wall, with the opposing wall isothermally cooled. Heat transfer was shown to be independent of major aspect ratio ($2.5 \leq A_z \leq 7.5$) and Prandtl number for water ($\mathrm{Pr} \approx 5$) and FC-77 ($\mathrm{Pr} \approx 25$).

Additional studies involving discrete heat sources mounted to one vertical cavity wall with cooled horizontal walls have also been performed (Lee et al., 1987; Carmona and Keyhani, 1989; Keyhani et al., 1991; Joshi et al., 1991; Mukutmoni et al., 1993). With horizontal cold plates, the cavity aspect ratio is an important parameter since heat transfer is influenced by the total cold plate surface area.

Despite the foregoing studies, questions still remain concerning natural convection in discretely heated cavities, particularly in connection with three-dimensional effects in small heat source arrays. In a companion paper (Heindel et al., 1995a), two and three-dimensional predictions were compared for a 3 × 3 array of discrete heat sources flush-mounted to one vertical wall of a liquid-filled cavity, with the opposite wall cooled. In this study, the predictions are compared to experimental results obtained for water and dielectric liquid FC-77 (manufactured by 3M Co.). Empirical heat transfer correlations are developed and compared with correlations obtained by other investigators for similar geometries.

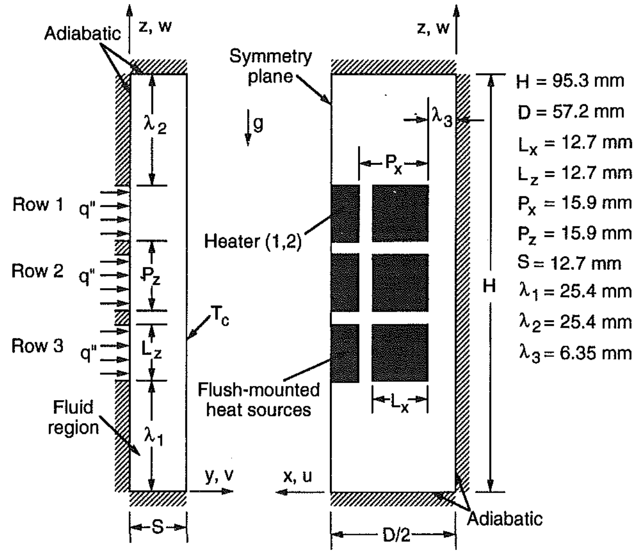

Fig. 1 Cavity nomenclature and associated dimensions

Numerical and Experimental Procedures

Figure 1 displays a half-section of the three-dimensional cavity geometry, which was simulated numerically and for which experimental results were obtained. One vertical wall is composed of a 3 × 3 array of discrete, flush-mounted, isoflux heat sources, with the remainder of the wall approximated as adiabatic in the simulations. The opposite wall is maintained isothermal and the remaining (top and bottom) walls are assumed adiabatic. Since symmetry has been demonstrated in previous three-dimensional calculations (Wroblewski and Joshi, 1992, 1994) a plane of symmetry was numerically prescribed at $x = D/2$ to reduce the computational domain by a factor of two. Details of the mathematical model and related numerical procedures are described in a companion paper (Heindel et al., 1995a).

Test cell dimensions correspond to those outlined in Fig. 1 and a schematic of the test cell is shown in Fig. 2. A 3 × 3 array of copper elements was flush-mounted in a G-10 substrate,

Nomenclature

A_h = heater wetted surface area = $L_x L_z$
A_{htr} = heater aspect ratio = L_x/L_z
A_x = cavity minor aspect ratio = D/S
A_z = cavity major aspect ratio = H/S
D = cavity depth
g = gravitational acceleration
H = cavity height
h = local convection heat transfer coefficient
$\bar{h}$ = average convection heat transfer coefficient
k = thermal conductivity
L_x = heater length in the spanwise (x) direction
L_z = heater length in the vertical (z) direction
Nu_{L_z} = local Nusselt number = hL_z/k_f
$\overline{\mathrm{Nu}}_{L_z}$ = average Nusselt number = $\bar{h}L_z/k_f$
Pr = Prandtl number = ν/α
P_x = heater pitch in the spanwise (x) direction
P_z = heater pitch in the vertical (z) direction
Q = applied power dissipation rate
Q_i = total power crossing a specified solid/fluid interface
q'' = applied heat flux
Ra_{L_z} = Rayleigh number = $g\beta(T - T_c)L_z^3/(\nu\alpha)$
$\mathrm{Ra}^*_{L_z}$ = modified Rayleigh number = $g\beta q'' L_z^4/(k\alpha\nu)$
S = cavity width
T = temperature
W = dimensionless vertical velocity components = wL_z/α
u, v, w = velocity components
x, y, z = coordinate directions
α = thermal diffusivity
β = volumetric thermal expansion coefficient
η = conjugate correction factor
θ = dimensionless temperature = $(T - T_c)/(q''L_z/k)$
λ_i = heater cavity locations (Fig. 1)
ν = kinematic viscosity

Subscripts

c = cold plate
f = film
(i, j) = heater (i, j)
max = maximum
Ri = row i
sur = surface

Superscripts

$\bar{\ }$ = average

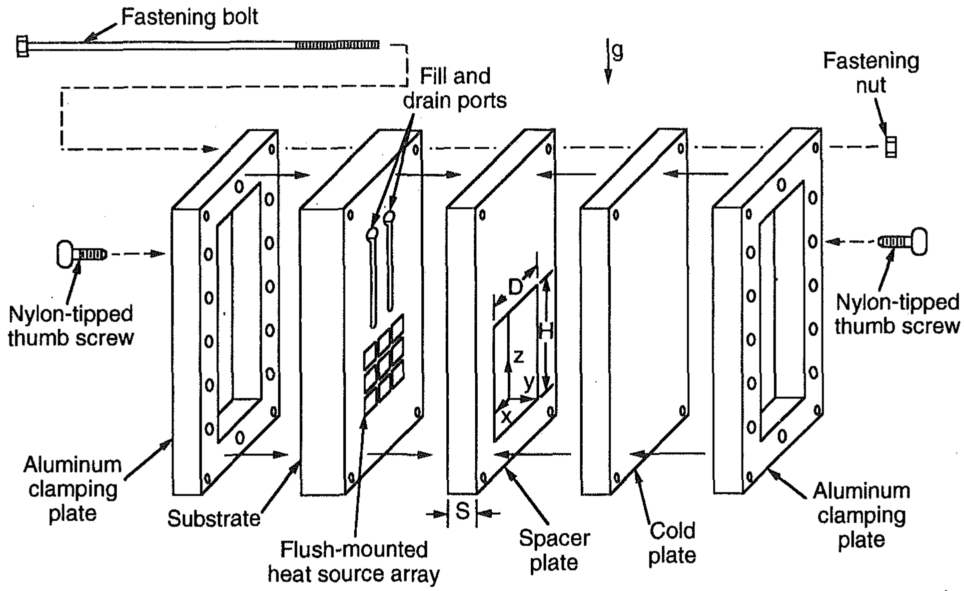

Fig. 2 Schematic of the experimental test cell

which was separated from an opposing copper cold plate by a Lexan spacer plate. The spacer plate was compressed between two aluminum clamping plates by bolts passing through each corner and nylon-tipped thumb screws periodically located about the perimeter. O-rings were used to seal the interfaces between components. Two threaded holes were located near the top of the substrate to allow for filling and draining of the test cell. A length of polyethylene tubing, with the free end open to the atmosphere, was attached to each hole through a tube fitting to allow for fluid expansion.

Each heater was formed by a 12.7 mm × 12.7 mm × 9.5 mm oxygen-free copper block. Two surface-mounted copper-constantan thermocouples were soft-soldered to the copper surface to provide intimate thermal contact. The substrate around each block was beveled and filled with red RTV silicone sealant to provide a seal between the copper block and the substrate. Thick-film resistive elements, each powered individually by a DC source, were soldered to the back of each copper block, forming the discrete heat source. A small cavity behind the heater array was filled with Sylox insulating material ($k = 0.02$ W/mK) to minimize losses to the environment.

A water-cooled, copper cold plate was located opposite the heater array. Due to the large thermal conductivity of copper, this wall was nearly isothermal and maintained at $T_c \approx 15°$C. The plate contained an internal maze of channels through which water from a constant temperature bath could be circulated. The cold plate surface temperature was measured via three copper-constantan surface-mounted thermocouples.

The spacer plate was fixed at $S = 12.7$ mm, providing cavity major and minor aspect ratio of $A_z = 7.5$ and $A_x = 4.5$, respectively. Experiments were conducted with degassed, deionized water and degassed FC-77. All heaters were equally powered, and steady-state temperatures and power inputs were recorded by an HP data acquisition system interfaced to a personal computer.

Data Reduction

All tests were conducted under conditions of equal power dissipation (Q). All fluid properties were evaluated at a local film temperature defined by

$$T_{f_{(i,j)}} = \frac{\bar{T}_{\mathrm{sur},(i,j)} + T_c}{2} \tag{1}$$

where $\bar{T}_{\mathrm{sur},(i,j)}$ and T_c are the average surface temperatures of heater (i, j) and the cold plate, respectively. The data were reduced for each heater in terms of Rayleigh and average Nusselt numbers, defined by

$$\mathrm{Ra}_{L_z(i,j)} = \frac{g\beta(\bar{T}_{\mathrm{sur},(i,j)} - T_c)L_z^3}{\nu\alpha} \tag{2}$$

$$\overline{\mathrm{Nu}}_{L_z(i,j)} = \frac{\bar{h}_{(i,j)}L_z}{k} = \frac{Q_{i,(i,j)}L_z}{A_h(\bar{T}_{\mathrm{sur},(i,j)} - T_c)k} \tag{3}$$

where A_h is the wetted surface area of the flush-mounted heater. The corrected power dissipation rate, $Q_{i,(i,j)}$, represented the heat transfer to the fluid directly from the face of heater (i, j), and accounted for conjugate losses through the substrate. This value was determined from

$$Q_{i,(i,j)} = \eta Q_{(i,j)} \tag{4}$$

where $Q_{(i,j)}$ was the total power dissipated by heater (i, j), as measured from voltage and current readings. The conjugate correction factor, η, was ascertained through a three-dimensional conjugate analysis of the entire test cell (Heindel, 1994). Values of η ranged from $0.78 \lesssim \eta \lesssim 0.91$ for water and $0.58 \lesssim \eta \lesssim 0.74$ for FC-77. A detailed description of these calculations can be found in Heindel et al. (1995b).

Estimated uncertainties in the reported Nusselt and Rayleigh numbers, as computed by the procedure described by Kline and McClintock (1953), were 9.4 and 5.0 percent, respectively. In computing these values, uncertainties in the fluid thermophysical properties were neglected. Therefore, actual uncertainties, especially those associated with FC-77, may be higher.

The discrete heat sources in the experiment are nearly isothermal, but these conditions are not known a priori because each row is at a different isothermal condition. Therefore, isoflux boundaries were numerically modeled to allow for comparisons with the experimental data, based on the energy entering the fluid through the heat sources. A more detailed description of experimental procedures is provided by Heindel (1994).

Results

Numerical Predictions. Three-dimensional numerical calculations were performed over a range of modified Rayleigh

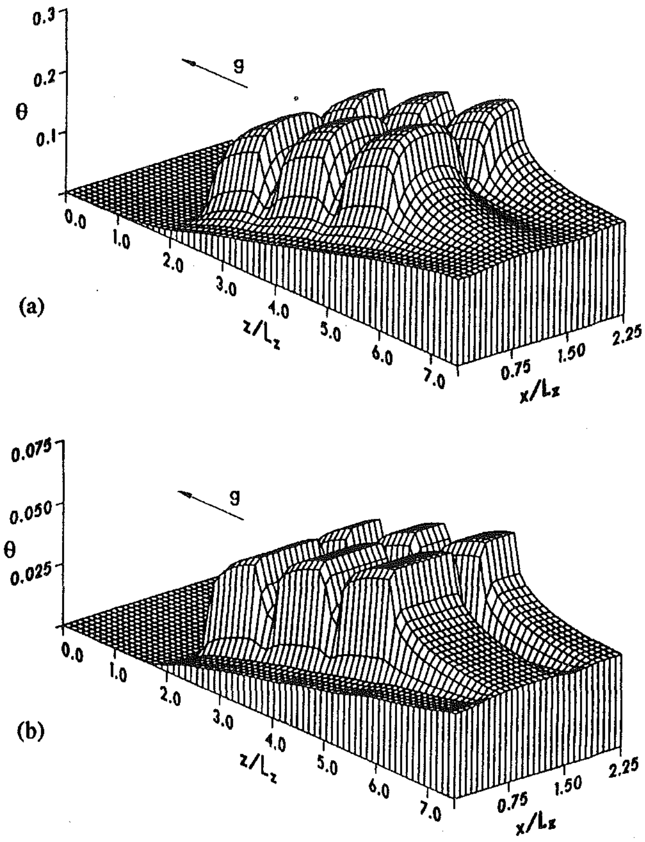

Fig. 3 Distribution of the local dimensionless temperature on the heated wall for Pr = 5.0, A_z = 7.5, and A_{htr} = 1.0: (*a*) $Ra_{L_z}^*$ = 10^5, (*b*) $Ra_{L_z}^*$ = 10^8

numbers ($10^5 \leq Ra_{L_z}^* \leq 10^8$) for fixed values of Pr = 5, A_z = 7.5, and A_{htr} = 1.0. The effect of modified Rayleigh number on the dimensionless temperature distribution along the heated wall is shown in Fig. 3 (note the orientation of the gravity vector). At $Ra_{L_z}^* = 10^5$ (Fig. 3*a*), there is an abrupt increase in θ over the leading and lateral edges of each heater. The maximum temperature on each heat source is located near the heater trailing edge and along the heater midline (x/L_z = 1.0 and 2.25). Due to an increase in the local bulk fluid temperature and thickening of the thermal boundary layer, the maximum temperature increases from row 3 to row 1 (bottom to top of the cavity). Beyond the trailing edge of each heater, the local temperature decreases in a monotonic fashion due to thinning of the thermal boundary layer in the adiabatic region between heaters and dissipation of the boundary layer in the adiabatic region above row 1. The continued increase in surface temperature in the adiabatic region adjacent to the heater columns, with increasing z/L_z, is due to an increase in the local bulk fluid temperature near the cavity wall. Increasing $Ra_{L_z}^*$ (Fig. 3*b*) reduces the predicted dimensionless temperature because the increase in $T - T_c$ is not as large as the increase in q''. Similar trends are observed at $Ra_{L_z}^* = 10^8$, and a major effect of increasing $Ra_{L_z}^*$ is to provide a more uniform temperature across the heater face (in the x direction) and a steeper temperature gradient near the heater edges.

The significance of edge effects becomes more apparent when the local Nusselt number is plotted as a function of position, as in Fig. 4(*a*) for $Ra_{L_z}^* = 10^5$. Beginning with the highest local Nusselt number at the leading edge of row 3 (z/L_z = 2.0), Nu_{L_z} decreases in a monotonic fashion to the heater trailing edge (z/L_z = 3.0). A similar pattern is observed at subsequent downstream heaters, beginning first with a slight increase at the leading edge of each heater. This increase is due to partial dissipation of the thermal boundary layer in the adiabatic region between adjacent heaters and renewal of thermal boundary layer development at the heater leading edge.

Substantial enhancement in the local Nusselt number along the lateral edges of each heater is caused by two factors. First, there is lateral entrainment of relatively cool fluid from the adjacent adiabatic regions, enhancing the local heat transfer coefficient at the edges. Second, there is diffusion from the warm fluid over each heater surface to the relatively cool fluid in the adiabatic regions, which also enhances the local heat transfer rate. Figure 4(*a*) also shows that there are negligible deviations in the local Nusselt number distribution from heater column to heater column. This result suggests that the end walls do not affect local thermal conditions on adjacent heater columns when the wall-to-column spacing is 0.5 heater lengths or larger.

Increasing the modified Rayleigh number to 10^8 (Fig. 4*b*) yields similar trends, but with an increase in the local Nusselt number over the entire surface. This result is due to an increased fluid velocity. With increasing $Ra_{L_z}^*$, the local Nusselt number across the heater face is also more uniform, and edge effects are restricted to a smaller region.

Experimental Results: Water. Figure 5 displays all of the nondimensionalized water data Pr ≈ 5) obtained over the range $5 \times 10^4 < Ra_{L_z} < 6 \times 10^6$ (0.2 W/heater $\lesssim Q \lesssim$ 8.5 W/heater). Due to thermal boundary layer growth, as well as an increase in the local bulk fluid temperature from row 3 to row 1, the heater surface temperature increases from the bottom to the top row. This result is manifested in the decrease in $\overline{Nu}_{L_z}$ with decreasing row number, which was also numerically predicted. The highest Rayleigh numbers are associated with the highest heater surface temperatures and correspond to $\bar{T}_{sur,R1,max} \approx 74°C$, $\bar{T}_{sur,R2,max} \approx 68°C$, and $\bar{T}_{sur,R3,max} \approx 57°C$, where $\bar{T}_{sur,Ri,max}$ is the maximum average surface temperature of row *i*. Consistent with numerical predictions, column-to-column $\overline{Nu}_{L_z}$ variations are negligible. The data reveal that $\overline{Nu}_{L_z}$ increases with Ra_{L_z} at a nearly constant slope, which is close to that for laminar

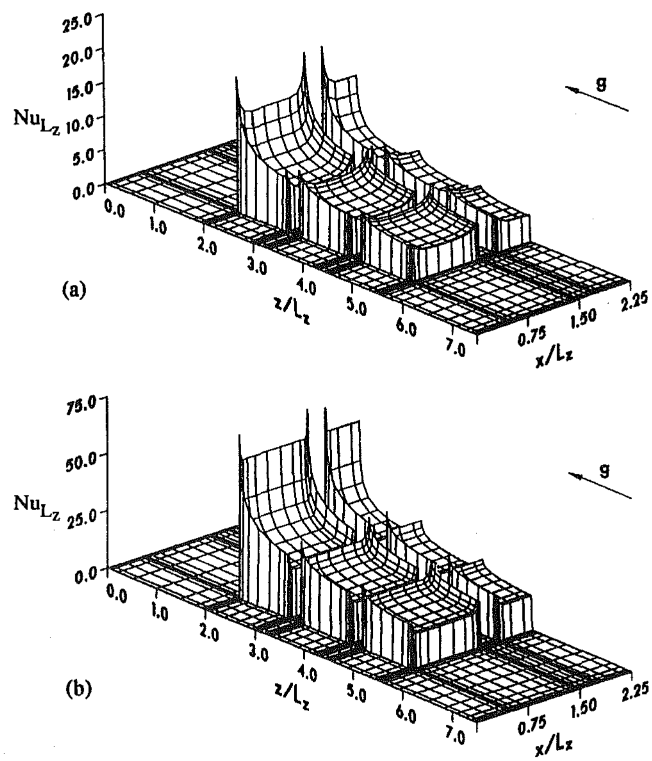

Fig. 4 Local Nusselt number profile on the heated wall for Pr = 5.0, A_z = 7.5, and A_{htr} = 1.0: (*a*) $Ra_{L_z}^*$ = 10^5, (*b*) $Ra_{L_z}^*$ = 10^8

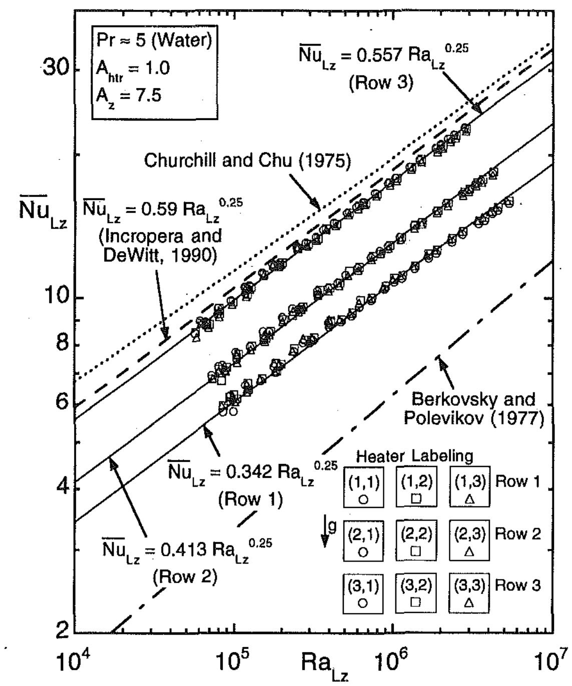

Fig. 5 Experimental variations of average Nusselt number with Rayleigh number for water

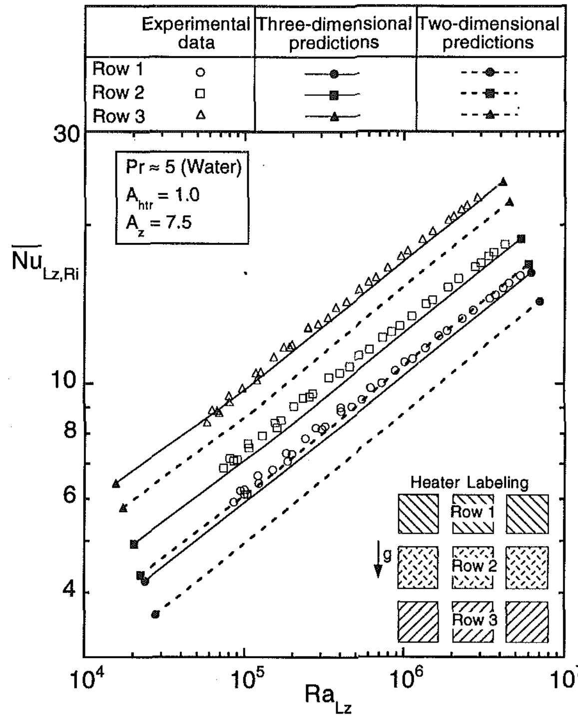

Fig. 6 Comparison of predicted and measured row-averaged Nusselt numbers for water

natural convection over a vertical flat plate (Incropera and DeWitt, 1990). Selecting the Rayleigh number exponent to match that of the flat plate correlation (0.25), the water data are correlated for each row by expressions of the form

$$\text{Row 1:}\quad \overline{\mathrm{Nu}}_{L_z} = 0.342\ \mathrm{Ra}_{L_z}^{0.25} \tag{5a}$$

$$\text{Row 2:}\quad \overline{\mathrm{Nu}}_{L_z} = 0.413\ \mathrm{Ra}_{L_z}^{0.25} \tag{5b}$$

$$\text{Row 3:}\quad \overline{\mathrm{Nu}}_{L_z} = 0.557\ \mathrm{Ra}_{L_z}^{0.25} \tag{5c}$$

These equations are also plotted in Fig. 5. The maximum deviation of the data from these correlations is 5.8 percent, and occurs at the lowest Rayleigh numbers for row 1. Deviations of less than 2 percent are more typical.

The correlation proposed by Churchill and Chu (1975) for natural convection from a vertical flat plate, as well as the correlation of Berkovsky and Polevikov (1977) for natural convection in a differentially heated cavity, are also shown in the figure. The data, particularly for row 3, are in better agreement with the laminar flat plate results than those associated with a differentially heated cavity.

Since column-to-column variations are insignificant, row-averaged Nusselt numbers, $\overline{\mathrm{Nu}}_{L_z,Ri}$, can be used without any loss of generality. The row-averaged Nusselt numbers are compared with the two and three-dimensional numerical predictions for $\mathrm{Pr} = 5.0$ in Fig. 6. The numerically predicted surface temperatures in Fig. 3 indicate that the heater temperatures are fairly uniform from column-to-column and are approximately isothermal (with the exception of the leading edge region), but differ from row to row. This resembles the experimental conditions of nearly isothermal heat sources for each column, which differ from row to row. Therefore, even though the numerical predictions are based on isoflux boundary conditions and the experimental conditions more closely approximate isothermal boundary conditions, the three-dimensional predictions for each row are in excellent agreement with the experimental values (Fig. 6). However, the two-dimensional model underpredicts the data by approximately 20 percent, and differences are due to the omission of lateral edge effects in the model. Therefore, when lateral edge effects influence a significant proportion of the heater area, three-dimensional models are necessary to obtain accurate heat transfer predictions.

Experimental Results: FC-77. To confirm that heat transfer is independent of Prandtl number (Keyhani et al., 1988a, b; Shen et al., 1989; Prasad et al., 1990; Polentini et al., 1993), additional experiments were performed using FC-77 ($\mathrm{Pr} \approx 25$), a fluorocarbon manufactured by 3M Co. Figure 7 shows the nondimensionalized results for $6 \times 10^7 < \mathrm{Ra}_{L_z} < 1.2 \times 10^8$

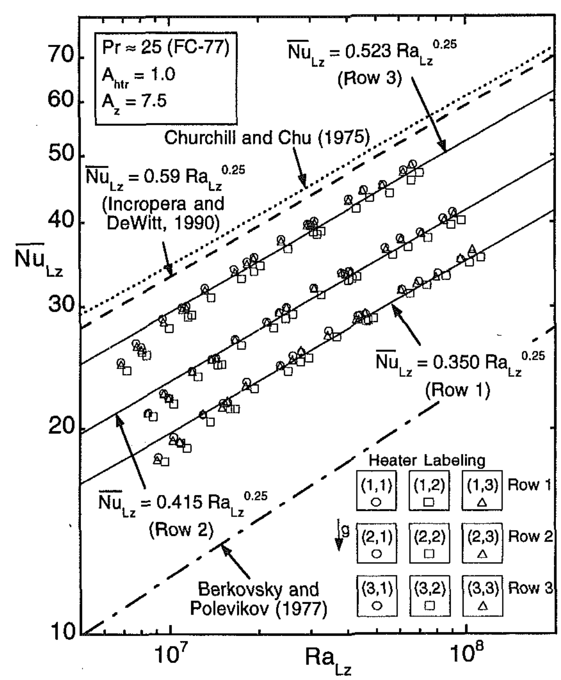

Fig. 7 Experimental variations of average Nusselt number with Rayleigh number for FC-77

(0.2 W/heater $\lesssim Q \lesssim$ 2.7 W/heater). The highest average surface temperatures observed in these experiments were $\bar{T}_{sur,R1,max} \approx 85°C$, $\bar{T}_{sur,R2,max} \approx 78°C$, and $\bar{T}_{sur,R3,max} \approx 66°C$. The trends exhibited by the FC-77 data are similar to those of water. However, there is a slight column-to-column variation in $\overline{Nu}_{L_z}$, with values for the outside heater columns corresponding to one another but exceeding those for the central column. This difference is attributed to stray heat transfer from the environment not considered by the conjugate correction model (Heindel, 1994). However, symmetry about the central column is still observed.

Fixing the Rayleigh number exponent at 0.25, the FC-77 data are correlated by

$$\text{Row 1:} \quad \overline{Nu}_{L_z} = 0.350 \, Ra_{L_z}^{0.25} \tag{6a}$$

$$\text{Row 2:} \quad \overline{Nu}_{L_z} = 0.415 \, Ra_{L_z}^{0.25} \tag{6b}$$

$$\text{Row 3:} \quad \overline{Nu}_{L_z} = 0.523 \, Ra_{L_z}^{0.25} \tag{6c}$$

Maximum deviation of the data from these correlations is 11.4 percent and occurs at the lowest Rayleigh numbers in row 3. These deviations are possibly the result of unrecorded energy entering the system since the heater surface temperatures are slightly less than ambient for these experimental conditions. Typical deviations for $Ra_{L_z} \gtrsim 1.5 \times 10^7$ are less than 5 percent. Differences between the correlations and the experimental values could be reduced by including variations in the Rayleigh number exponent. The two flat plate correlations and the vertical cavity correlation are also included in Fig. 7. Although none of these correlations are in good agreement with the data, the experimental trends are more closely approximated by the flat plate correlations.

In turbulent natural convection, the Nusselt number typically varies with Rayleigh number raised to an exponent of $\frac{1}{3}$ (Incropera and DeWitt, 1990). Since the FC-77 data closely follow a Rayleigh number dependence of $Ra_{L_z}^{0.25}$, heat transfer is considered to remain laminar even at $Ra_{L_z} \approx 10^8$. This premise was verified by visual observation. At $Ra_{L_z} \approx 10^8$, flow in the cavity

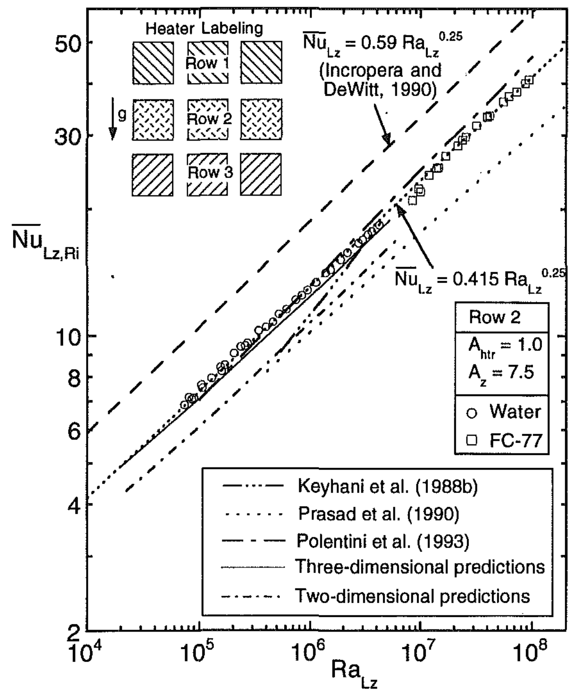

Fig. 9 Comparison of row 2 water and FC-77 data with empirical correlations and numerical predictions

appeared to be steady adjacent to the heat sources. Oscillatory flow was observed, but it was confined to the upper cavity region, downstream of the heat sources, and did not affect conditions on the heater surface.

Additional Comparisons. All of the data are combined to obtain heat transfer correlations that encompass $5 \times 10^4 \lesssim Ra_{L_z} \lesssim 1.2 \times 10^8$ and $5 \lesssim Pr \lesssim 25$. In developing these correlations, the Rayleigh number exponent is again fixed at 0.25 to yield

$$\text{Row 1:} \quad \overline{Nu}_{L_z} = 0.348 \, Ra_{L_z}^{0.25} \tag{7a}$$

$$\text{Row 2:} \quad \overline{Nu}_{L_z} = 0.415 \, Ra_{L_z}^{0.25} \tag{7b}$$

$$\text{Row 3:} \quad \overline{Nu}_{L_z} = 0.530 \, Ra_{L_z}^{0.25} \tag{7c}$$

Maximum deviations of the data from these correlations are 6.7 percent (row 1), 6.9 percent (row 2), and 10.3 percent (row 3), where the largest departure occurs at the lower range of the FC-77 data ($Ra_{L_z} \approx 10^7$). However, typical deviations are much smaller.

The correlations and row-averaged data are shown in Figs. 8–10 for rows 1, 2, and 3, respectively. Two and three-dimensional predictions, the flat plate correlation suggested by Incropera and DeWitt (1990), and additional correlations are also included in these figures. The three-dimensional predictions are in excellent agreement with the data, which are underpredicted by the two-dimensional model. The good agreement between the data and the correlations with a Rayleigh number exponent of 0.25 used for each heater row suggests that flow adjacent to the heat sources remains laminar over the entire Rayleigh number range.

Figures 8–10 also include correlations proposed by Keyhani et al. (1988b) for three isoflux strip heaters, each spaced one heater length apart in an ethylene glycol (105 < Pr < 166) filled enclosure; by Prasad et al. (1990) for the same geometry but with Pr = 25; and by Polentini et al. (1993) for a 3 × 3 array of discrete heat sources immersed in a water or FC-77 filled enclosure. Note that the correlations of Keyhani et al. (1988b) and Prasad et al. (1990), which were presented in

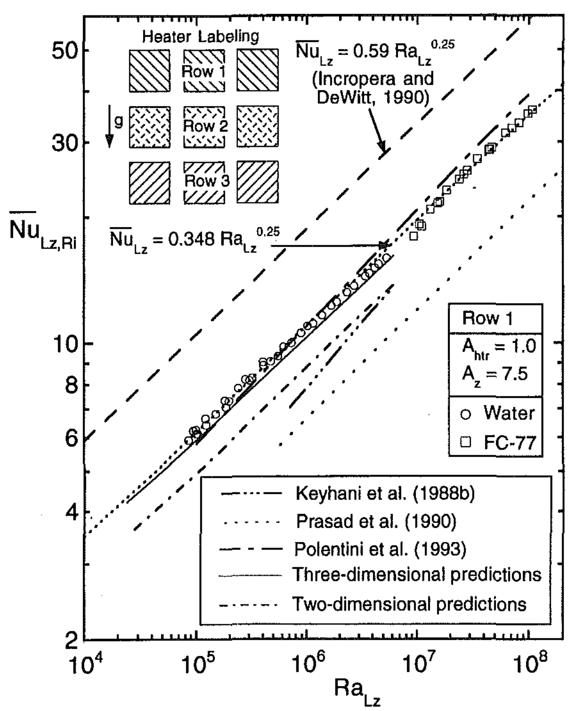

Fig. 8 Comparison of row 1 water and FC-77 data with empirical correlations and numerical predictions

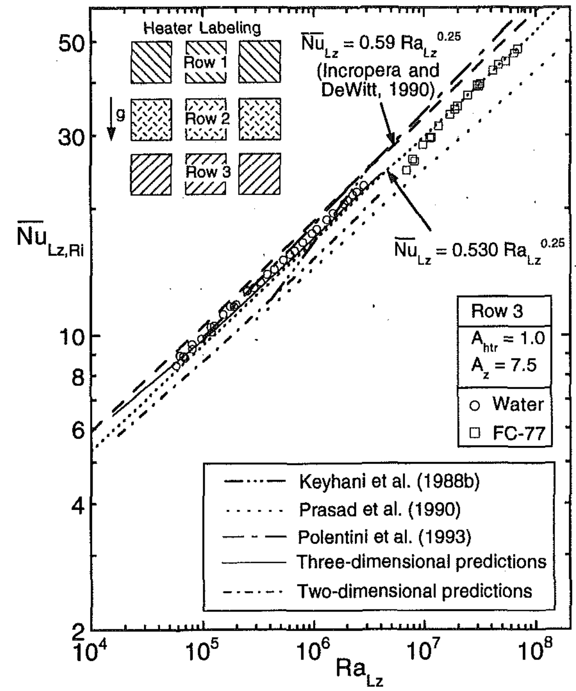

Fig. 10 Comparison of row 3 water and FC-77 data with empirical correlations and numerical predictions

terms of Ra_S^* and Ra_H^*, respectively, have been transformed in terms of Ra_{L_z} to facilitate comparisons.

The Keyhani et al. (1988b) correlations have a steeper slope than the data of this study and underpredict the results of row 1 and 2, while row 3 provides the best comparison. This may be due to the fact that Keyhani et al. (1988b) evaluated the fluid properties at $T_f = T_c + 0.25(T_h - T_c)$ and, as stated by Prasad et al. (1990), the fluid properties of ethylene glycol are very sensitive to temperature. Also note that Keyhani's published correlation for row 3 (Eq. (19a) in Keyhani et al., 1988b) has a correlation constant of $C = 0.292$. However, this correlation does not agree with the given data. The correlation constant should be $C = 0.222$,[2] which agrees with the published data and was used in Fig. 10. The Prasad et al. (1990) correlations were developed using a two-dimensional numerical model and yield a slope that is compatible with the results of this study. However, the correlations underpredict the data, which is consistent with the fact that the correlations do not account for heat transfer enhancement from three-dimensional effects. Additionally, the row 1 correlations of Keyhani et al. (1988b) and Prasad et al. (1990) display the poorest agreement with the data of this study because the row 1 heater in Keyhani's and Prasad's configuration was adjacent to the top adiabatic surface, which was not the case in this study.

The correlations of Polentini et al. (1993) were obtained for geometric conditions equivalent to those of this study (including three-dimensional effects) and are consistent with the data of this study. Discrepancies between the current results and those of Polentini et al. (1993) are attributed to their method of obtaining conjugate correction factors. Their approach provided a reasonable approximation to conjugate effects for low values of the substrate/fluid thermal conductivity ratio (i.e., with water), but underestimated losses when the substrate/fluid thermal conductivity ratio was high (i.e., with FC-77). Therefore, the correlations of Polentini et al. (1993) overpredict the current FC-77 data by a maximum of about 15 percent.

[2] Based on reviewer's comments.

Conclusions

Three-dimensional numerical predictions and experimental data have been presented for natural convection in a rectangular cavity with a 3 × 3 array of discrete, flush-mounted heat sources located on one vertical wall and cooled by the opposing wall. Numerical predictions were obtained for $Pr = 5$, $A_z = 7.5$, $A_{htr} = 1.0$, and $10^5 \le Ra_{L_z}^* \le 10^8$. Increasing the modified Rayleigh number produced more uniform vertical velocity, temperature, and local Nusselt number distributions across the heater face. Heat transfer from the discrete heaters was found to be significantly influenced by edge effects.

Experimental data were obtained for the same cavity geometry with $5 \times 10^4 \lesssim Ra_{L_z} \lesssim 1.2 \times 10^8$ and $5 \lesssim Pr \lesssim 25$. Laminar flow was observed over this Rayleigh number range, and the data were well correlated by a Rayleigh number exponent of 0.25. Three-dimensional predictions of the row-averaged Nusselt numbers were in excellent agreement with the data, while a two-dimensional model underpredicted heat transfer by approximately 20 percent.

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