Determination of Plate Source, Detector Separation from One Signal

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Abstract We address the problem of locating a transient source, such as an acoustic emission source, in a plate. We apply time-frequency analysis to the signals detected at a receiver. These highly dispersive and complex waveforms are measured for source-receiver separations ranging from 40 to 180 plate thicknesses and at frequencies such that ten to twenty Rayleigh-Lamb branches are included. Re-assigned, smoothed, pseudo-Wigner-Ville distributions are generated that exhibit the expected sharp ridges in the time-frequency plane, lying along the predicted frequency-time-of-arrival relations. The source-receiver separation can be determined from such plots.

Keywords Time-frequency analysis, acoustic emission, source location, Wigner-Ville distribution, spectrogram, source-detector separation, plate modes, wave dispersion, ultrasound

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1 Introduction

Acoustic emission measurements are often used to determine the location of an acoustic source in a structure. Conventional source location algorithms rely on the measurement of the arrival time of one or more wave modes at a number of sensors, and the inversion of the relative arrival time data to determine the location and time of activation of the source. Only one type of information, one signal arrival time is extracted from a waveform and with triangulation, the source can be located, provided that the speed of propagation of the signal is known. It also possible to identify in a waveform two features whose speed of propagation is known and different. This forms the basis of an earlier patent [4]. In contrast, we describe here a method which utilizes the entire waveform, viewed in the time-frequency domain, to determine both the time of signal activation as well as the source-detector separation. The extraction of this distance from a single detected signal will permit the use of fewer transducers in source location measurements.

Our method relies on the identification of guided modes propagating in a plate. The generated modes are governed by the
characteristics of the source. Their evolution results from differences in speeds of propagation of various wave modes present in a signal, their damping, and the effects of geometrically induced wave dispersion. The arrival times of dispersive modes can be displayed in time-frequency space. We compare in this paper the measured arrival times for dispersive modes with values calculated using an elastodynamic model. The free parameters in the model include \( t_0 \) the instant of source activation and \( d \), the source-detector separation. These are adjusted such that the model optimally matches the measured data.

2 Time-frequency analysis of waveforms

Figure 1 shows the measurement geometry and a sample synthetic waveform from a normal force step-excitation in a steel plate 10 mm thick, with a source-receiver separation of 1.80 m. The first step in processing is to compute a time-frequency distribution. Such a distribution localizes a signal in both time- and frequency-domains. That is, it provides simultaneous time resolution and inversely proportional frequency resolution. For example, a gated burst has position in both time and frequency, but the conventional time and frequency representations present only one aspect. A time-frequency distribution combines both time and frequency information into a single representation. There are a large number of possible time-frequency distributions, however we will focus only on the two which are most often used. These are the spectrogram and the Wigner-Ville distribution.

The spectrogram is based on the Short Time Fourier Transform (STFT) with a sliding window. One example is

\[
\Psi(t, f) = \left| \int_{-\infty}^{\infty} x(\tau) e^{\frac{(t-\tau)^2}{2T^2}} e^{-j2\pi f \tau} d\tau \right|^2
\]  

Eq. (1) gives the definition of a spectrogram with a Gaussian window function of half-width \( T \). It is the power spectrum of a signal which corresponds to the squared magnitude of the Fourier transform of the windowed signal.

The Wigner-Ville distribution provides increased resolution relative to the spectrogram, but it has interference terms. The definition of the Wigner-Ville distribution is given by the following equation

\[
\Psi(t, f) = \int_{-\infty}^{\infty} x \left( t + \frac{\tau}{2} \right) x^* \left( t - \frac{\tau}{2} \right) e^{-j2\pi f \tau} d\tau
\]  

When \( \tau \) is near zero, \( x(t + \frac{\tau}{2}) \) and \( x^*(t - \frac{\tau}{2}) \) are coherent and their product contributes to the integral. However, when \( \tau \) is
large, \( x(t + \frac{\tau}{2}) \) and \( x^*(t - \frac{\tau}{2}) \) have incoherent phases and thus average to zero. The Wigner-Ville distribution can be thought of as being a squared Fourier transform centered about a point.

The Wigner-Ville distribution is quadratic in \( x \), so if \( x \) is a sum \((a + b)\), the Wigner-Ville distribution of \( x \) contains an interference term \( 2ab \) in addition to the desired quantity \((a^2 + b^2)\). These interference terms result in an increased noise level of the Wigner-Ville distribution relative to the spectrogram. In practice, these interference terms can be dramatically reduced by smoothing in time and frequency. The result is the smoothed-pseudo Wigner-Ville distribution (SPWVD) which is defined by

\[
\Psi(t, f) = g(t) \ast \left( \int_{-\infty}^{\infty} h(\tau) \left( x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) \right) e^{-j2\pi f \tau} d\tau \right)
\]  

(3)

Here \( \ast \) is defined as a convolution with respect to time \( t \). The function \( g(t) \) is the smoothing function in time and \( h(\tau) \) restricts the range of the integral in \( \tau \). Restricting the range in \( \tau \) is equivalent to smoothing in frequency. The SPWVD reduces to the conventional Wigner-Ville distribution when \( h(\tau) = 1 \) and \( g(t) = \delta(t) \). The SPWVD is the Wigner-Ville distribution we used to obtain the results described here [1].

A reassignment algorithm is used to sharpen the features in the time-frequency distribution. The reassignment algorithm moves energy “uphill” in time-frequency-amplitude space, thus sharpening the image. Auger, et al. [1] provide a detailed discussion of the reassignment process. Figure 2 depicts the reassigned Wigner-Ville distribution of the signal from Fig. 1.

## 3 Matching the time-frequency distribution with computed curves

The \( \omega \) vs. \( k \) dispersion curves depicted in Fig. 3(a) for the allowable plate modes can be readily calculated given the material properties [2, 3]. The group velocity shown in Fig. 3(b) is determined as a function of frequency from the dispersion relation by \( \frac{dw}{dk} \). The arrival times of various modes are determined from \( \frac{d}{v_g} \). A sample result is shown in Fig. 3(c). The computed arrival curves can then be visually matched with the ridges in the measured time-frequency distribution shown in Fig. 3(d).

To effect the match, two parameters are selectively varied: the source-detector separation \( d \), and the time shift \( t_0 \). The adjustment of the \( d \) parameter is made so that the spacing in time of the different computed arrivals matches the measured data. The time shift \( t_0 \) is determined by shifting the position of the calculated arrivals so that they best match the time-frequency distribution. The lowest anti-symmetric mode (denoted by \( A_0 \)), the lowest symmetric mode (denoted by
Figure 3: Dispersion, group velocity, and time-frequency arrival curves for steel; source-receiver separation of 180h
Figure 4: Wave modes useful for matching ridges with computed arrival curves

S0), and the Rayleigh modes, along with the arrival tails, usually provide the best anchors for an accurate match. This is depicted in Fig. 4. The number of modes which are easily identifiable in a representation varies greatly and depends on source and sensor characteristics as well as the coupling of the sensor to the structure.

4 Results: Synthetic and measured waveforms

The analysis of synthetic waveforms provides a best-case demonstration of what can be accomplished using such time-frequency analysis. Synthetic waveforms corresponding to a normal force and the normal displacements were generated for given material properties and source-receiver separations. With an infinite signal-to-noise ratio, such signals represent the ideal case. Not surprisingly, the proposed time-frequency analysis works better at large source-receiver separations. The increased temporal spacing of arrivals at larger separations enhances the delineation between the different modes in time-frequency space.

To demonstrate the utility of this approach, we carried out a number of experiments using a capillary-break on the surface of a glass plate as a normal force, step-source. Although the experimental data exhibit less detail and fewer arrivals than the corresponding synthetic data, the time-frequency distribution of the detected signals exhibited sufficient detail for solving the inverse source-receiver separation problem.

We have also compared the results obtained using the Wigner-Ville distribution and the spectrogram. We found that the Wigner-Ville distribution has a higher background noise level although it provides a slightly sharper image. The source of this background noise level is likely the \((2ab)\) interference terms in the distribution.

By following the matching procedure outlined above, we have found that we can solve the inverse problem of determining source-receiver offset from a single measured signal. Figure 5 shows a sample result. The source-detector separation \(d\) is most easily determined from the temporal maxima and minima of the arrival curves in the time-frequency plane. It is at these points that the signal is particularly concentrated in time; it is also at these points that modes suddenly appear or
Figure 5: Sample inverse problem in which source-receiver separation was determined

suddenly disappear [3]. Unfortunately, the number and quality of such identifiable arrivals is sometimes limited. The complexity of the matching procedure precludes simple automation. We have found that we can obtain accurate measures of the source-receiver separation by manually matching calculated arrival curves to the entire time-frequency distribution.

5 Conclusions

We have demonstrated a method for the time-frequency analysis of plate modes. This method has been applied to the inverse problem of determining the source-detector separation. We have demonstrated that this separation can be determined from a single signal.

References


