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Decomposing Poverty Change: Within- and Between-group Effects

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Introduction

This Slide is to explain the method of

[Decomposing Poverty Change: Deciphering Change in Total Population and Beyond](), *Review of Income and Wealth*

See related blog, [Growth, Inequality and Population Effects on Poverty Reduction]()

Motivation

- Poverty reduction could be decomposed into
  - Within Group Effects
    - Growth/income effect (all benefit/lose proportionately).
    - Inequality effect (redistribution on account of various welfare schemes or other economy-wide changes).
    - Population effect (increase/decrease of population will have negative/positive implications, respectively).
  - Between Group Effect
    - Sectoral/migration effect because of shifts in population shares where, in the absence of status quo, there has to be at least one group with a reduction in share/effect and at least one group with an increase in share/effect.

Preliminaries

- $\mathcal{I}$ is identification
- i is i^{th} individual identified (or under $\mathcal{I}$)
- x is individual income (also per capita income)
- X is total income of the population
- $\mathcal{L}$ is inequality (in the form of Lorenz ratio or Gini coefficient)
- n is population
- $m=(X/n)$ is mean (or per capita) income of population
- $\mathcal{P}$ is poverty (head count ratio)
- There are two groups/sectors (k: rural, r; urban, u)
- There are two time periods: $t, \tau=1, 2; t\neq\tau$
- Poverty line: $\mathcal{Z}$ =35; if $x_i<35$ then i is poor
- b is share of population (time and individual/group-specific)
- $\lambda_v= (X_\tau/X_t), (n_t/n_\tau), (m_\tau/m_t); v=X, n, m$
- $\gamma_\tau= X_\tau, n_\tau, m_\tau$
- $j=X, \mathcal{L}, n$ (income/growth, inequality and population, respectively)

Counting Poor

- In the next two slides, we will give examples of counting poor for two periods.
- One could consider the examples as sample observations or even actual population at two different time periods.
- One could also consider the two examples as two situations/locations, but we will keep that aside for the time being.

Counting Poor, Period 1

$\mathcal{I}_1$	x_1	$\mathcal{P}_1$
Su	10	1
Ma	20	1
El	30	1
Ki	40	0
n_1=4	25	3/4

Counting Poor, Period 2

$\mathcal{I}_2$	x_2	$\mathcal{P}_2$
Bh	20	1
Tu	30	1
Ka	40	0
Li	50	0
Ch	60	0
$n_2=5$	30	2/5

Change in Poverty

Using the previous two slides, total poverty change is

$\mathcal{P}_2$-$\mathcal{P}_1$ =(2/5)-(3/4)

=40%-75%

=-35%

Decomposing Poverty Change

- Poverty change can be decomposed to within- and between-group effects.
- Within-group has three broad effects: growth, inequality and population.
- The within-group effects will depend on the base period (we use period 2 as base).
- Given base, there will be six possible ways to compute three within-group effects (depends on sequence of each).

Six possible Sequences

- Growth-Inequality-Population
- Growth-Population-Inequality
- Inequality-Growth-Population
- Inequality-Population-Growth
- Population-Growth-Inequality
- Population-Inequality-Growth

These six possible sequences have 12 possible specific attributions.

Specific Attributions and their weights

Effect	Attribution	Weight	Components, $\mathcal{P}_2$ base
Growth	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|X_1}$
	Given inequality	One-sixth	$\mathcal{P}_{2\|\mathcal{L}_1} - \mathcal{P}_{1\|n_2}$
	Given population	One-sixth	$\mathcal{P}_{2\|n_1} - \mathcal{P}_{1\|\mathcal{L}_2}$
	Given inequality and population	One-third	$\mathcal{P}_{1\|X_2} - \mathcal{P}_1$
Inequality	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|\mathcal{L}_1}$
	Given growth	One-sixth	$\mathcal{P}_{2\|X_1} - \mathcal{P}_{1\|n_2}$
	Given population	One-sixth	$\mathcal{P}_{2\|n_1} - \mathcal{P}_{1\|X_2}$
	Given growth and population	One-third	$\mathcal{P}_{1\|\mathcal{L}_2} - \mathcal{P}_1$
Population	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|n_1}$
	Given inequality	One-sixth	$\mathcal{P}_{2\|\mathcal{L}_1} - \mathcal{P}_{1\|X_2}$
	Given growth	One-sixth	$\mathcal{P}_{2\|X_1} - \mathcal{P}_{1\|\mathcal{L}_2}$
	Given inequality and growth	One-third	$\mathcal{P}_{1\|n_2} - \mathcal{P}_1$

Note: In the paper , the components given in Table 1 have $\mathcal{P}_1$ as base.

Counting poor with conditions

To obtain the 12 components indicated in the previous slide, we need the following:

$\mathcal{P}_1$	Poverty in period 1
$\mathcal{P}_{1\|X_2}$	Poverty in period 1 with income of period 2
$\mathcal{P}_{1\|\mathcal{L}_2}$	Poverty in period 1 with inequality of period 2
$\mathcal{P}_{1\|n_2}$	Poverty in period 1 with population of period 2
$\mathcal{P}_2$	Poverty in period 2
$\mathcal{P}_{2\|X_1}$	Poverty in period 2 with income of period 1
$\mathcal{P}_{2\|\mathcal{L}_1}$	Poverty in period 2 with inequality of period 1
$\mathcal{P}_{2\|n_1}$	Poverty in period 2 with population of period 1

Note: $\mathcal{P}_{1|\mathcal{L}_2} = \mathcal{P}_{2|m_1}$; $\mathcal{P}_{2|\mathcal{L}_1} = \mathcal{P}_{1|m_2}$ (see proposition 4 in paper).

Controlling growth and population

- For growth effect, one imposes the total income of the other period by increasing each individual’s income proportionately (by the same growth rate).
- For population, a proportionate change can be interpreted as a change in the multiplier or adult equivalence scale that each observation represents.

Note on controlling inequality

- For controlling inequality, one has to ensure that shares of population and shares of income are that of the other period. We do this by using the observation of the other period with total income and total population of the current period.

Computing Poverty with Conditions

$$\mathcal{P}_{t|\gamma_\tau} = \sum_{i_t=1}^{n_t} b_{i_t}\left(\frac{z - \lambda_v x_{i_t}}{z}\right)$$

$\lambda_v = (X_\tau/X_t), (n_t/n_\tau), (m_\tau/m_t)$
$\gamma_\tau = X_\tau, n_\tau, m_\tau$

Counting poor with conditions, period 1

$\mathcal{I}_1$	$\boldsymbol{x}_1$	$\mathcal{P}_1$	$x_{1\|X_2}$	$\mathcal{P}_{1\|X_2}$	$x_{1\|n_2}$	$\mathcal{P}_{1\|n_2}$	$x_{1\|m_2}$	$\mathcal{P}_{1\|m_2}$
Su	10	1	20	1	8	1	16	1
Ma	20	1	40	0	16	1	32	1
El	30	1	60	0	24	1	48	0
Ki	40	0	80	0	32	1	64	0
n_1	25	3/4	50	1/4	20	1	40	1/2
Total	100	75%	200	25%	100	100%	200	50%

Note: $x_{1|m_2} = x_{2|\mathcal{L}_1}$; $\mathcal{P}_{1|m_2} = \mathcal{P}_{2|\mathcal{L}_1}$ (see proposition 4 in paper).

Counting poor with conditions, period 2

$\mathcal{J}_2$	x_2	$\mathcal{P}_2$	$x_{2\|X_1}$	$\mathcal{P}_{2\|X_1}$	$x_{2\|n_1}$	$\mathcal{P}_{2\|n_1}$	$x_{2\|m_1}$	$\mathcal{P}_{2\|m_1}$
Bh	20	1	10	1	25.0	1	12.50	1
Tu	30	1	15	1	37.5	0	18.75	1
Ka	40	0	20	1	50.0	0	25.00	1
Li	50	0	25	1	62.5	0	31.25	1
Ch	60	0	30	1	75.0	0	37.50	0
n_2	40	2/5	20	1	50.0	1/5	25.00	4/5
Total	200	40%	100	100%	200	20%	100	80%

Note: $x_{2|m_1}$ = $x_{1|\mathcal{L}_2}$; $\mathcal{P}_{2|m_1}$ = $\mathcal{P}_{1|\mathcal{L}_2}$ (see proposition 4 in paper).

Attribution-specific Effects

Effect	Attribution	Weight	Components, $\mathcal{P}_2$ base		
Growth	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|X_1}$	-1/5	-11/20
	Given inequality	One-sixth	$\mathcal{P}_{2\|\mathcal{L}_1} - \mathcal{P}_{1\|n_2}$	-1/12	
	Given population	One-sixth	$\mathcal{P}_{2\|n_1} - \mathcal{P}_{1\|\mathcal{L}_2}$	-1/10	
	Given inequality and population	One-third	$\mathcal{P}_{1\|X_2} - \mathcal{P}_1$	-1/6	
Inequality	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|\mathcal{L}_1}$	-1/30	-1/40
	Given growth	One-sixth	$\mathcal{P}_{2\|X_1} - \mathcal{P}_{1\|n_2}$	0	
	Given population	One-sixth	$\mathcal{P}_{2\|n_1} - \mathcal{P}_{1\|X_2}$	-1/120	
	Given growth and population	One-third	$\mathcal{P}_{1\|\mathcal{L}_2} - \mathcal{P}_1$	1/60	
Population	Alone	One-third	$\mathcal{P}_2 - \mathcal{P}_{2\|n_1}$	1/15	9/40
	Given inequality	One-sixth	$\mathcal{P}_{2\|\mathcal{L}_1} - \mathcal{P}_{1\|X_2}$	1/24	
	Given growth	One-sixth	$\mathcal{P}_{2\|X_1} - \mathcal{P}_{1\|\mathcal{L}_2}$	1/30	
	Given inequality and growth	One-third	$\mathcal{P}_{1\|n_2} - \mathcal{P}_1$	1/12	
Total	(Growth + Ineq + Popn)		$\mathcal{P}_2 - \mathcal{P}_1$		-7/20

Note: In the paper , the components given in Table 1 have $\mathcal{P}_1$ as base.
The values given in column 5 are after applying weights of column 3.

Bringing Subgroups – rural and urban

- Now, we bring in groups – rural and urban.
- In period 1: Su, Ma and El are rural; Ki is urban.
- In period 2: Bh, and Tu are rural; Ka, Li and Ch are urban.
- We now count poor with conditions for the rural and urban subgroups.

Counting poor: sector-specific, period 1

$\mathcal{J}_1$	x_1	$\mathcal{P}_1$	$x_{1\|X_2}$	$\mathcal{P}_{1\|X_2}$	$x_{1\|n_2}$	$\mathcal{P}_{1\|n_2}$	$x_{2\|\mathcal{L}_1}$	$\mathcal{P}_{2\|\mathcal{L}_1}$
Su	10	1	13.3	1	15	1	20	1
Ma	20	1	26.7	1	30	1	40	0
El	30	1	40.0	0	45	0	60	0
n_{1r}	20	1	26.7	2/3	30	2/3	40	1/3
Tot_{1r}	60	100%	80	67%	60	67%	80	33%
Ki	40	0	120	0	13.3	1	35	0
n_{1u}	40	0	120	0	13.3	1	35	0
Tot_{1u}	40	0%	120	0%	40	100%	105	0%
n_1*	25	3/4	50	1/2	20	13/15	30	2/15
Tot_1*	100	75%	200	50%	100	87%	150	13%

Note: * weighted averages.

Counting poor: sector-specific, period 2

$\mathcal{J}_2$	x_2	$\mathcal{P}_2$	$x_{2\|X_1}$	$\mathcal{P}_{2\|X_1}$	$x_{2\|n_1}$	$\mathcal{P}_{2\|n_1}$	$x_{1\|\mathcal{L}_2}$	$\mathcal{P}_{1\|\mathcal{L}_2}$
Bh	30	1	22.5	1	20.0	1	15	1
Tu	50	0	37.5	0	33.3	1	25	1
n_{2r}	40	1/2	30	1/2	26.7	1	20.0	1
Tot_{2r}	80	50%	60	50%	80	100%	60	100%
Ka	20	1	6.7	1	60	0	20	1
Li	40	0	13.3	1	120	0	40	0
Ch	60	0	20.0	1	180	0	60	0
n_{2u}	40	1/3	13.3	1	180	0	40	1/3
Tot_{2u}	120	33.3%	40	100%	120	0%	40	33%
n_2*	40	2/5	20	4/5	50	3/4	25	5/6
Tot_2*	200	40%	100	80%	200	75%	100	83%

Note: * weighted averages.

Attributions: sector-specific

Effect	Attribution	Weight	Rural	Urban	Combined
Growth	Alone	One-third	0	-2/9	-2/15
	Given inequality	One-sixth	-1/18	-1/6	-11/90
	Given population	One-sixth	0	-1/18	-1/72
	Given inequality and population	One-third	-1/9	0	-1/12
Inequality	Alone	One-third	1/18	1/9	4/45
	Given growth	One-sixth	-1/36	0	-1/90
	Given population	One-sixth	1/18	0	1/24
	Given growth and population	One-third	0	1/9	1/36
Population	Alone	One-third	-1/6	1/9	-7/60
	Given inequality	One-sixth	-1/18	0	-11/180
	Given growth	One-sixth	-1/12	1/9	-1/180
	Given inequality and growth	One-third	-1/9	1/3	7/180

Note: The values given in columns 4-6 are based on slides 20 and 21 after applying weights indicated in column 3.

Decomposing poverty change by sectors

Effects	Rural	Urban	Combined
Growth	-1/6= (-16.7%)	-4/9= (-44.4%)	-127/360= (-35.3%)
Inequality	1/12= (8.3%)	2/9= (22.2%)	53/360= (14.7%)
Population	-5/12= (-41.7%)	5/9= (55.6%)	-13/90= (-14.4%)
Total	-1/2= (-50.0%)	1/3= (33.3%)	-7/20= (-35.0%)

Note: Computed from slide 22. The value for combined are weighted averages using population shares of the sectors (rural & urban). It differs from the unweighted values in slide 18.

Combining within- & between-groups

- Average of the group-specific population shares for the two periods multiplied by attribution-specific poverty change will give the within-group effects.
- Average of the group-specific poverty values for the two periods multiplied by group-specific change in population shares will give the between-group effect.

Within- and between-group effects: formula

$$\Delta\mathcal{P} = \left(\sum\nolimits_j \sum\nolimits_{k=\mathcal{r},\mathcal{u}} \left(\frac{b_{k_1} + b_{k_2}}{2}\right) \Delta\mathcal{P}_{jk}\right) + \left(\sum\nolimits_{k=\mathcal{r},\mathcal{u}} \left(\frac{\mathcal{P}_{k_1} + \mathcal{P}_{k_2}}{2}\right) \Delta b_k\right); j = X, \mathcal{L}, \mathcal{n}$$

Within- and between-group effects: result (preliminary)

Effect	Attribution	Rural	Urban	Combin-ed (r+u)
Within	Growth	$\frac{23}{40}*\frac{-1}{6}$	$\frac{17}{40}*\frac{-4}{9}$	$\frac{-41}{144}$
	Inequality	$\frac{23}{40}*\frac{1}{12}$	$\frac{17}{40}*\frac{2}{9}$	$\frac{41}{288}$
	Population	$\frac{23}{40}*\frac{-5}{12}$	$\frac{17}{40}*\frac{5}{9}$	$\frac{-1}{288}$
	Total within	$\frac{-23}{80}$	$\frac{17}{120}$	$\frac{-7}{48}$
Between	Sectoral shift	$\frac{3}{4}*\frac{-7}{20}$	$\frac{1}{6}*\frac{7}{20}$	$\frac{-49}{240}$
Total (within + between)		$\frac{-11}{20}$	$\frac{1}{5}$	$\frac{-7}{20}$

Within- and between-group effects: result (per cent)

Effect	Attribution	Rural	Urban	Combined $(r+u)$
Within	Growth	-9.6%	-18.9%	-28.5%
	Inequality	4.8%	9.4%	14.2%
	Population	-24.0%	23.6%	-0.3%
	Total within	-28.8%	14.2%	-14.6%
Between	Sectoral shift	-26.3%	5.8%	-20.4%
Total (within + between)		-55.0%	20.0%	-35.0%

Within- and between-group effects: result for India (2004-05 and 2009-10)

Effect	Attribution	Rural	Urban	Combined ($r + u$)
Within	Growth	-9.45%	-4.32%	-13.77%
	Inequality	-0.40%	0.53%	0.13%
	Population	4.13%	2.39%	6.51%
	Total within	-5.72%	-1.41%	-7.13%
Between	Sectoral shift	-0.63%	0.39%	-0.25%
Total (within + between)		-6.35%	-1.02%	-7.37%

Note: Table 4 of the paper.

Results: India (2004-05 and 2009-10)

- Poverty reduced by –7.37 percentage points
- Within-group effects contributed to 97% reduction
  - Growth effect contributed to 187% reduction
  - Inequality effect contributed to 2% increase
  - Population effect contributed to 88% increase
- Between-group effect contributed to 3% reduction (through a population shift from rural to urban)
- Rural contributed to 86% reduction
- Urban contributed to 14% reduction

For a detailed discussion see the paper

[Decomposing Poverty Change: Deciphering Change in Total Population and Beyond], *Review of Income and Wealth* (available under open access).

See related blog, [Growth, Inequality and Population Effects on Poverty Reduction]

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