Trapezoidal phase-shifting method for 3D shape measurement

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ABSTRACT

We propose a novel structured light method, namely trapezoidal phase-shifting method, for 3-D shape measurement. This method uses three patterns coded with phase-shifted, trapezoidal-shaped gray levels. The 3-D information of the object is extracted by direct calculation of an intensity ratio. Theoretical analysis showed that this new method was significantly less sensitive to the defocusing effect of the captured images when compared to the traditional intensity-ratio based methods. This important advantage makes large-depth 3-D shape measurement possible. If compared to the sinusoidal phase-shifting method, the resolution is similar, but the processing speed is at least 4.5 times faster. The feasibility of this method was demonstrated in a previously developed real-time 3-D shape measurement system. The reconstructed 3-D results showed similar quality as those obtained by the sinusoidal phase-shifting method. However, since the processing speed was much faster, we were able to not only acquire the images in real time, but also reconstruct the 3-D shapes in real time (40 fps at a resolution of 532 x 500 pixels). This real-time capability allows us to measure dynamically changing objects, such as human faces. The potential applications of this new method include industrial inspection, reverse engineering, robotic vision, computer graphics, medical diagnosis, etc.

Keywords: 3-D shape measurement, structured light, trapezoidal phase shifting, intensity ratio.

1. INTRODUCTION

3-D shape measurement techniques have applications in such diverse areas as industrial inspection, reverse engineering, robotic vision, computer graphics, medical diagnosis, etc. A number of techniques have been developed, among which structured light is one of the most popular techniques. In a structured light system, a projector is commonly used to project certain coded patterns onto the object being measured. The images of the object with the projected patterns are captured by a camera. The depth information is extracted based on triangulation after decoding.

Binary coding, which uses only two illumination levels (0 and 1), is one of the most widely used techniques to code structured light patterns. Multiple patterns are typically required to extract the depth information of the object.1-4 Every pixel has its own codeword formed by 0's
and 1’s from multiple images. 3-D information can be obtained by decoding the codewords and then applying triangulation. Since only 0’s and 1’s are used, this method is more robust when the images are noisy. However, its resolution is relatively low because the stripe width must be larger than 1 pixel.

Structured light techniques based on sinusoidal phase-shifting methods have the advantage of pixel-level resolution. These techniques use a set of phase-shifted sinusoidal fringe patterns to extract the phase values. Depth information is included in the phase map and can be obtained based on triangulation, similar to the binary coding method. Another advantage of the sinusoidal phase-shifting methods lies in its large dynamic range, since image defocus does not affect significantly the measurement results. However, the processing speed is relatively low due to the need to compute the arctangent function for phase calculation. To improve the processing speed, Fang et al. proposed a linear-coding method, which combined the use of sawtooth-like patterns with the concept of phase shifting. Unfortunately, the coding method is highly sensitive to image defocus.

Codification based on linearly changing gray levels, or the so-called intensity-ratio method, has the advantage of fast processing speed because it requires only a simple intensity-ratio calculation. Usually two patterns, a ramp pattern and a uniform bright pattern, are used. Depth information is extracted from the ratio map based on triangulation. However, this simple technique is highly sensitive to camera noise and image defocus. To reduce measurement noise, Chazan and Kiryati proposed a pyramidal intensity-ratio method, which combined this technique with the concept of hierarchical stripes. Later Horn and Kiryati developed piecewise linear patterns in an attempt to optimize the design of projection patterns for best accuracy. To eliminate the effect of illumination variation, Savarese et al. developed an algorithm that used three patterns, flat low, linearly changing, and flat high. The flat low image was regarded as the background and subtracted from the other two, which were then used to calculate the ratio. However, this technique is still very sensitive to camera noise and image defocus. Moreover, its resolution is low unless periodical patterns are used, which then introduces the ambiguity problem.

In this paper, we describe a novel coding method, the trapezoidal phase-shifting method, which combines the advantages of the high processing speed of the intensity ratio based methods and high resolution of the sinusoidal phase-shifting methods. Compared to the traditional intensity-ratio based methods, this method is also far less sensitive to image defocus, which significantly reduces measurement errors when the object has a large depth. Section 2 explains the trapezoidal phase-shifting method. Section 3 analyzes the potential error sources of the system, in particular the image defocus error. The experimental results are presented in Section 4 and conclusions are given in Section 5.

2. TRAPEZOIDAL PHASE-SHIFTING METHOD

Intensity-ratio based methods for 3-D shape measurement have the advantage of fast processing speed because the calculation of the intensity ratio is rather simple. However, these methods usually show large measurement noise, which limits their applications. To reduce measurement
noise, one has to repeat the ramp pattern to create the so-called triangular or pyramidal patterns. The smaller the pitch of the pattern is, the lower the noise level will be. However, the periodical nature of the pattern introduces the ambiguity problem, which causes errors when objects with discontinuous features are measured. Another major problem with the use of a triangular or pyramidal pattern is that the measurement is highly sensitive to the defocusing of the image. This can cause problems when objects with a relatively large depth are measured and the projector or the camera does not have a large enough depth of focus.

In this research, we propose to use a new coding method called the trapezoidal phase-shifting method to solve the problems of the conventional intensity-ratio methods while preserve its advantages. This method can reduce the noise level by 6 times without introducing the ambiguity problem. For even lower noise level, the pattern can also be repeated. This introduces the ambiguity problem but to a lesser degree. Another advantage of the trapezoidal method is that the measurement is much less sensitive to image defocus.
The proposed trapezoidal phase-shifting method is very similar to the three-step sinusoidal phase-shifting method, only that the cross-sectional shape of the patterns has been changed from sinusoidal to trapezoidal. To reconstruct the 3-D shape of the object, three patterns, which are phase-shifted by 120 degrees or one-third of the pitch, are needed. Figure 1 shows the cross sections of the three patterns. Their intensities can be written as follows:

\[
I_1(x, y) = \begin{cases} 
I'(x, y) + I''(x, y) & x \in [0, T/6) \text{ or } [5T/6, T] \\
I'(x, y) + I''(x, y)(2 - 6x/T) & x \in [T/6, T/3) \\
I'(x, y) & x \in [T/3, 2T/3) \\
I'(x, y) + 2I''(x, y)(6x/T - 4) & x \in [2T/3, 5T/6)
\end{cases},
\]

(1)

\[
I_2(x, y) = \begin{cases} 
I'(x, y) + I''(x, y)(6x/T) & x \in [0, T/6) \\
I'(x, y) + I''(x, y) & x \in [T/6, T/2) \\
I'(x, y) + I''(x, y)(4 - 6x/T) & x \in [T/2, 2T/3) \\
I'(x, y) & x \in [2T/3, T]
\end{cases},
\]

(2)

\[
I_3(x, y) = \begin{cases} 
I'(x, y) & x \in [0, T/3) \\
I'(x, y) + I''(x, y)(6x/T - 2) & x \in [T/3, T/2) \\
I'(x, y) + I''(x, y) & x \in [T/2, 5T/6) \\
I'(x, y) + I''(x, y)(6 - 6x/T) & x \in [5T/6, T]
\end{cases},
\]

(3)

where \(I_1(x, y), I_2(x, y)\) and \(I_3(x, y)\) are the intensities for the three patterns respectively, \(I'(x, y)\) and \(I''(x, y)\) are the minimum intensity and intensity modulation at position \((x, y)\) respectively, and \(T\) is the pitch of the patterns. Each pattern is divided evenly into six regions that can be identified by knowing the sequence of the intensity values of the three patterns. The intensity ratio can be computed by

\[
r(x, y) = \frac{I_{\text{med}}(x, y) - I_{\text{min}}(x, y)}{I_{\text{max}}(x, y) - I_{\text{min}}(x, y)},
\]

(4)

where \(I_{\text{min}}(x, y), I_{\text{med}}(x, y),\) and \(I_{\text{max}}(x, y)\) are the minimum, median, and maximum intensities of the three patterns for point \((x, y)\). The value of \(r(x, y)\) ranges from 0 to 1. Figure 2 shows the cross section of the intensity ratio map. The triangular shape can be removed to obtain a ramp by using the following equation:

\[
r(x, y) = 2 \times \text{round}\left(\frac{N - 1}{2}\right) + (-1)^{N+1} \frac{I_{\text{med}}(x, y) - I_{\text{min}}(x, y)}{I_{\text{max}}(x, y) - I_{\text{min}}(x, y)},
\]

(5)

where \(N = 1, 2, \ldots, 6\) is the region number, which is determined by comparing the three intensity values at each point. After the removal of the triangular shape, the value of \(r(x, y)\) now ranges from 0 to 6, as shown in Figure 3. If multiple fringes are used, the intensity ratio is wrapped into this range of 0 to 6 and has a sawtooth like shape. A process similar to phase unwrapping in the traditional sinusoidal phase-shifting method needs to be used. The Depth information can be obtained from this intensity ratio based on an algorithm similar to the phase-to-height conversion algorithm used in the sinusoidal phase-shifting method.\(^{6,7}\)
3. ERROR ANALYSIS

In this research, a digital-light-processing (DLP) video projector was used to project the trapezoidal fringe patterns to the object. The images are captured by a CCD camera. The major potential error sources are the image defocus error due to limited depth of focus of both the projector and the camera and the nonlinear gamma curve of the projector. The nonlinear gamma curve can be corrected by software. However, the residual nonlinearity can still cause errors that cannot be ignored. The following sections discuss the effects of these two error sources.

3.1. Image defocus error

In the sinusoidal phase-shifting method, image defocus will not cause major errors because a sinusoidal pattern will still be a sinusoidal pattern when the image is defocused, even though the fringe contrast may be reduced. However, in the trapezoidal phase-shifting method, image defocus will blur the trapezoidal pattern, which may cause errors that cannot be ignored. In order to quantify this error, we use a Gaussian filter to simulate the defocusing effect. By changing the size of the filter window, we can simulate the level of defocus and calculate the corresponding error. Following is the equation for the intensity ratio \( r(x, y) \) when the fringe images are defocused:

\[
r(x, y) = \frac{I_{\text{med}}(x, y) \otimes G(x, y) - I_{\text{min}}(x, y) \otimes G(x, y)}{I_{\text{max}}(x, y) \otimes G(x, y) - I_{\text{min}}(x, y) \otimes G(x, y)},
\]  

\[ \text{Equation (6)} \]
Figure 4. Maximum error due to image defocus.

where symbol $\otimes$ denotes the convolution operation and

$$G(x, y) = \frac{1}{2\pi \sigma^2} e^{-\frac{(x-\bar{x})^2 + (y-\bar{y})^2}{2\sigma^2}},$$

is a 2-D Gaussian filter, which is a 2-D normal distribution with standard deviation $\sigma$ and mean point coordinate $(\bar{x}, \bar{y})$.

To simplify the analysis without losing its generality, we consider only a 1-D case (along $x$-axis) within regions $N = 1$ and $N = 2$. In this case, the ratio function in Eq. (6) becomes

$$r_{def}(x, y) = \begin{cases} \frac{\sum_{n=-N}^{T/6-x} [6(x+n)/T] G(n) + \sum_{n=T/6-x}^{N} G(n) \left[2-6(x+n)/T\right]}{\sum_{n=-N}^{T/6-x} G(n) + \sum_{n=T/6-x}^{N} G(n)} & x \in [0, T/6) \\ \frac{\sum_{n=-N}^{T/6-x} G(n) + \sum_{n=T/6-x}^{N} G(n) \left[2-6(x+n)/T\right]}{\sum_{n=-N}^{T/6-x} [6(x+n)/T] G(n) + \sum_{n=T/6-x}^{N} G(n)} & x \in [T/6, T/3) \end{cases},$$

where

$$G(n) = \begin{cases} e^{-\frac{n^2}{2\sigma^2}} / \sum_{n=-N}^{N} e^{-\frac{n^2}{2\sigma^2}} & n \in [-N, N] \\ 0 & \text{otherwise} \end{cases},$$

is a 1-D discrete Gaussian filter with standard deviation $\sigma$. The ratio error due to the defocusing effect is then

$$\Delta r_{def}(x, y) = r_{def}(x, y) - r(x, y).$$
This error depends on the window size of the filter or the level of defocus. Figure 4 is a plot that shows the relationship between the maximum error and the window size of the filter. From this figure, we can see that the error due to defocus increases with the window size when the window size is less than about 0.7T. After that, the error is stabilized to a relatively small value of 0.6%. This phenomenon is due to the fact that as the window size, or the defocus level, is increased, the trapezoidal pattern becomes increasingly like a sinusoidal pattern and once becoming a sinusoidal pattern, the error does not change anymore even if it is further defocused. Figure 5 shows how the trapezoidal pattern is blurred for different window sizes. Clearly, when the window size is increased to T, the pattern is already like a sinusoidal pattern.

To understand why for such dramatic defocusing of the fringe pattern, the error is still limited to only about 0.6%, we can look at the transitional area between regions N = 1 and N = 2, which is shown in Figure 6. We can see that the cross sections of $I_1(x, y)$ and $I_2(x, y)$ are symmetrical with respect to the borderline of the two regions. Even when the fringe patterns are defocused, this symmetry is maintained. This results in similar drops in $I_1(x, y)$ and $I_2(x, y)$ in the regions close to the borderline, which reduces the error in the calculation of the ratio.
\[ I_1(x, y)/I_2(x, y) \]. At the borderline of the two regions, the ratio, which still equals to one, does not change after defocusing.

In summary, even though the trapezoidal phase-shifting method is still sensitive to the defocusing effect (unlike the sinusoidal phase-shifting method), the resulting error is small, in particular when compared to conventional intensity ratio based methods. For example, for a filter window size of \(0.1T\), the method proposed by Savarese et al.\(^{15}\) will have an error of more than 53\%, while the error of the trapezoidal phase-shifting method will only be less than 0.2\%, which is dramatically smaller. Therefore, the trapezoidal phase-shifting method is capable of measuring objects with large depth with limited errors.

3.2. Nonlinearity error

The relationship between the input grayscale values and the grayscale captured by the camera should be linear. Otherwise it will result in errors in the final measurement results. Since the gamma curve of the projector is usually not linear, we use a software compensation method to linearize this relationship. However, the relationship after compensation may still not be exactly linear. This nonlinearity directly affects the measurement accuracy. In fact, the shape of the ratio curve in each region, which should be linear ideally, is a direct replica of the gamma curve, if no defocusing effect is considered. Therefore, reducing the nonlinearity of the gamma curve is critical to the measurement accuracy.

4. EXPERIMENTS

To verify its performance, we implemented the trapezoidal phase-shifting method in a previously developed real-time 3-D shape measurement system.\(^{16}\) In this system, the patterns are generated by a personal computer and projected to the object by a modified DLP projector. The images are captured by a CCD camera. Before measurement, the system was calibrated so that the ratio map could be converted to the depth map. Figure 7 shows one example result. To increase the image resolution, we used periodic patterns with a pitch of 30 pixels. The 3-D result was obtained after removing the periodical discontinuity by adding or subtracting multiples of 6, which is similar to phase unwrapping in the sinusoidal phase-shifting method. The measurement resolution is comparable to that of the sinusoidal phase-shifting method. The advantage lies in the processing speed of the fringe patterns, thanks to the simple intensity-ratio calculation as opposed to the phase calculation with an arctangent function in the sinusoidal phase-shifting method. This enables potential real-time 3-D shape measurement for objects with dynamically changing surface geometry. Figure 8 shows the measured result of a live human face. At an image size of 532×500 pixels, it took approximately 4.6 ms to obtain the ratio map, but 20.8 ms to compute the phase map with a P4, 2.8 GHz personal computer. These experiments confirmed that the proposed trapezoidal phase-shifting method could potentially be used to measure the 3-D surface shapes of slowly moving objects in real time.
5. CONCLUSIONS

We described a novel structured light method, trapezoidal phase-shifting method, for 3-D shape measurement in this paper. Compared to the traditional sinusoidal phase-shifting methods, this method has the advantage of a faster processing speed because it calculates a simple intensity ratio rather than phase, which is a computationally more time-consuming arctangent function. The depth resolution is similar. The disadvantage is that image defocus may cause some errors, even though they are quite small. Compared to the traditional intensity-ratio based methods, this method has a depth resolution that is six times better. It is also significantly less sensitive to image defocus, which allows it to be used to measure objects with large depth variations. No obvious disadvantages can be thought of. Experimental results demonstrated that the newly proposed method could be used to provide 3-D surface shape measurements for both static and dynamically changing objects.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under grant CMS-9900337 and National Institute of Health under grant No. RR13995.

REFERENCES

Figure 7. 3-D shape measurement of a plaster sculpture. (a)-(c) are the captured fringe images. (d)-(f) are the reconstructed 3-D model with different display methods.

Figure 8. 3-D shape measurement of a live human face. (a)-(c) are the captured fringe images. (d)-(f) are the reconstructed 3-D model with different display methods.