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ABSTRACT

Structured light system using a digital video projector is increasingly used for a 3-D shape measurement because of its digital nature. However, the nonlinear gamma of the projector causes the projected fringe patterns to be non-sinusoidal, which results in phase error therefore shape measurement error. Previous work showed that, by using a small look-up-table (LUT), this type of phase error can be reduced significantly for a three-step phase-shifting algorithm. In this research, we prove that this type of phase error compensation method is not limited to a three-step phase-shifting algorithm. It is generic for any phase-shifting algorithm. The phase error compensation algorithm is able to theoretically eliminate the phase error caused by the gamma of the projector completely. It is based on our finding that in phase domain, the phase error due to the projector’s gamma is preserved for arbitrary object’s surface reflectivity under arbitrary ambient light condition. The phase error can be pre-calibrated and stored into a LUT for phase error reduction. In this research, we captured a set of fringe images of a uniform flat surface board for such a calibration. The phase error of the flat board is analyzed and stored in a LUT for phase error compensation. Our experimental results show that by using this method, the measurement error can be reduced by at least 13 times for any phase-shifting algorithm.

Keywords: Phase error, gamma, projector, structured light, non-sinusoidal waveform, 3-D shape measurement, phase shifting, look-up-table.

1. INTRODUCTION

Optical non-contact 3-D surface profile methods, including stereo vision, laser scanning, and time or color-coded structured light, have been developed to obtain 3-D contour. Structured-light-based methods have the advantages of fast 3-D shape measurement and high-accuracy, among which various phase-shifting methods have been widely used due to their accuracy and efficiency. For phase-shifting methods, the non-sinusoidal waveform error and the phase shift error are two major error sources.

With the development of the digital display technologies, commercial digital video projectors are more and more extensively employed in 3-D shape measurement systems. A structured-light-based system differs from a stereo-based system by replacing one of the camera of the stereo system with a projector, and projects coded structured patterns which help to identify the correspondence. The structured light system based a on binary-coding method has the disadvantages of spatial resolution and measurement speed since the stripe width must larger than one pixel. However, it is robust to the nonlinear gamma of the projector since only two levels (0s and 1s) are used for this methods. On the other hand, a system based on a phase-shifting method has the advantages of the measurement speed and resolution because it can obtain the coordinates of the object pixel-by-pixel. The drawback of such a system is that they are very sensitive to the nonlinear gamma of the projector, any nonlinear effect deforms the ideal sinusoidal fringe pattern to be nonsinusoidal and brings error. The phase error caused by this nonsinusoidal waveform is the single dominant error source of the measurement because the phase shift error does not appear in this method due to the digital fringe generation nature. For accurate measurement, the nonlinear gamma of the projector is not desirable. However, the gamma of any commercial projector is purposely distorted to be nonlinear to have better visual effect. To obtain better measurement accuracy, this type of phase error has to be reduced. Previously proposed methods, including double three-step phase-shifting algorithm, and direct correction of the nonlinearity of the projector’s gamma, demonstrated significant phase error reduction, however, the residual error remains non-negligible. Guo et. al proposed a gamma correction method using a simple one-parameter gamma function technique by statistically analyzing the fringe images. This technique significantly reduces the phase error due to the nonlinear gamma. However, the one-parameter assumption for the gamma function of the projector is too strong. The actual gamma of the projector is very complicated. Therefore, this method cannot completely remove the error due to the projector’s nonlinear gamma.
Previous work showed that, by calibrating the nonlinear gamma of the projector, a small look-up-table (LUT) can be created to reduce this type of error significantly for a three-step phase-shifting algorithm. In this research, we prove that this type of phase error compensation method is not limited to the three-step phase-shifting method. It is generic for any phase-shifting method. The phase error compensation algorithm is able to theoretically eliminate the phase error caused by the projector’s gamma completely. It is based on our finding that in phase domain, the phase error due to the gamma of the projector is preserved for arbitrary objects surface reflectivity under arbitrary ambient lighting condition. The phase error can be pre-calibrated and stored into a LUT for phase error reduction. In this research, we captured a set of fringe images of a uniform flat surface board for such a calibration. The phase error of the flat board is analyzed and stored in a LUT for phase error compensation. Our experimental results show that by using this method, the measurement error can be reduced by at least 13 times for any phase-shifting algorithm.

Section 2 explains the theoretical background of the this error compensation method. Section 3 explains the LUT creation procedures. Section 4 shows experimental results. Section 5 discusses the advantages and limitations of the method and Section 6 concludes the paper.

2. PRINCIPLE

2.1. Least Square Algorithms

Phase-shifting method has been adopted extensively in optical metrology to measure 3-D shapes of objects at various scales. Many different sinusoidal phase-shifting algorithms have been developed, the general form of phase-shifting algorithms is the least square algorithms. The intensity of the $i-$th images with a phase shift of $\delta_i$ are as follows:

$$I_i(x,y) = I'(x,y) + I''(x,y)\cos(\phi(x,y) + \delta_i),$$  \hspace{1cm} (1)

where $I'$ is the average intensity, $I''$ the intensity modulation, and $\phi$ the phase to be solved.

Solving Eqs. (1) simultaneously by least square algorithm, we obtain,

$$\phi(x,y) = \tan^{-1}\left(-\frac{a_2(x,y)}{a_1(x,y)}\right),$$  \hspace{1cm} (2)

and

$$\gamma(x,y) = \frac{I''(x,y)}{I'(x,y)} = \frac{[a_1(x,y)^2 + a_2(x,y)^2]^{1/2}}{a_0(x,y)},$$  \hspace{1cm} (3)

where

$$\begin{bmatrix} a_0(x,y) \\ a_1(x,y) \\ a_2(x,y) \end{bmatrix} = A^{-1}(\delta_i)B(x,y,\delta_i),$$  \hspace{1cm} (4)

here

$$A(\delta_i) = \begin{bmatrix} N & \sum \cos(\delta_i) & \sum \sin(\delta_i) \\ \sum \cos(\delta_i) & \sum \cos^2(\delta_i) & \sum \cos(\delta_i) \sin(\delta_i) \\ \sum \sin(\delta_i) & \sum \cos(\delta_i) \sin(\delta_i) & \sum \sin^2(\delta_i) \end{bmatrix},$$  \hspace{1cm} (5)

and

$$B(x,y,\delta_i) = \begin{bmatrix} \sum I_i \\ \sum I_i \cos(\delta_i) \\ \sum I_i \sin(\delta_i) \end{bmatrix}.$$  \hspace{1cm} (6)

Where $I'(x,y) = a_0(x,y)$ can be used to obtain the flat image of the measured object. Eq. (2) provides the so-called modulo $2\pi$ phase at each pixel whose values range from 0 to $2\pi$. If multiple fringe pattern is used, a continuous phase map can be obtained by phase unwrapping. The continuous phase map can be further converted to coordinates through calibration.
2.2. Phase Error Correction

The images captured by the camera is formed through the procedures illustrated in Fig. 1. Let us assume that the projector’s input sinusoidal fringe images generated by a computer have the intensity as,

\[ I_i^p(x, y) = b_1 \{ 1 + \cos[\phi(x, y) + \delta_i] \} + b_0, \]

where \( b_1 \) is the dynamic range of the fringe images, \( b_0 \) the bias. After being projected by the projector, the output intensity of the fringe images becomes,

\[ I_i^o(x, y) = f_i(I_i^p), \]

where \( f_i(I_i^p) \) is a function of \( I_i^p \), which represents the real projection response of the projector to the input intensity for \( i-th \) image. If we assume that the projector projects light onto a surface with reflectivity \( r(x, y) \) and the ambient light is \( a_1(x, y) \), the reflected light intensity is

\[ I_i^r(x, y) = r(x, y)[I_i^p(x, y) + a_1(x, y)], \]

The reflected image is captured by a camera with a sensitivity of \( \alpha \), here we assume the camera is linear response to input light intensity, namely, \( \alpha \) is a constant. Then the intensity of the image captured by the camera is,

\[
\begin{align*}
I_i^c(x, y) &= \alpha[I_i^p + a_2(x, y)], \\
&= \alpha r(x, y)I_i^p + \alpha r(x, y)a_1(x, y) + \alpha a_2(x, y), \\
&= c_1I_i^p + c_2
\end{align*}
\]

where \( a_2(x, y) \) represents ambient light entering the camera. \( c_1 = \alpha r(x, y) \), and \( c_2 = \alpha r(x, y)a_1(x, y) + \alpha a_2(x, y) \).

If we assume

\[ C = A^{-1} = \begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix}, \]

then,

\[
\begin{align*}
a_2(x, y) &= C_{20} \sum I_i^p + C_{21} \sum I_i^p \cos(\delta_i) + C_{22} \sum I_i^p \sin(\delta_i), \\
&= C_{20} \sum (c_1 I_i^p + c_2) + C_{21} \sum (c_1 I_i^p + c_2) \cos(\delta_i) + C_{22} \sum (c_1 I_i^p + c_2) \sin(\delta_i), \\
&= c_1 \left[ C_{20} \sum I_i^p + C_{21} \sum I_i^p \cos(\delta_i) + C_{22} \sum I_i^p \sin(\delta_i) \right] + c_2 \left[ C_{20} N + C_{21} \sum \cos(\delta_i) + C_{22} \sum \sin(\delta_i) \right].
\end{align*}
\]

From \( C = A^{-1} \), i.e., \( CA = identity \). From Eq.(5) and Eq. (13) for the definition of \( A \) and \( C \) matrix, the third row, first column element in the resultant matrix is zero, we have

\[ C_{20} N + C_{21} \sum \cos(\delta_i) + C_{22} \sum \sin(\delta_i) = 0. \]

If we substitute this constraints into Eq. (17), we obtain

\[ a_2(x, y) = c_1 \left[ C_{20} \sum I_i^p + C_{21} \sum I_i^p \cos(\delta_i) + C_{22} \sum I_i^p \sin(\delta_i) \right]. \]
Similarly, we can prove that
\[
a_1(x,y) = c_1 \left[ C_{10} \sum I_i^p + C_{11} \sum I_i^p \cos(\delta_i) + C_{12} \sum I_i^p \sin(\delta_i) \right].
\]
(19)

Based on the least square algorithms, phase \( \phi(x,y) \) can be calculated as follows:
\[
\phi(x,y) = \tan^{-1} \left\{ \frac{-a_2(x,y)}{a_1(x,y)} \right\},
\]
(20)
\[
= \tan^{-1} \left\{ \frac{-c_1 \left[ C_{20} \sum I_i^p + C_{21} \sum I_i^p \cos(\delta_i) + C_{22} \sum I_i^p \sin(\delta_i) \right]}{c_1 \left[ C_{10} \sum I_i^p + C_{11} \sum I_i^p \cos(\delta_i) + C_{12} \sum I_i^p \sin(\delta_i) \right]} \right\},
\]
(21)
\[
= \tan^{-1} \left\{ \frac{-C_{20} \sum I_i^p + C_{21} \sum I_i^p \cos(\delta_i) + C_{22} \sum I_i^p \sin(\delta_i)}{C_{10} \sum I_i^p + C_{11} \sum I_i^p \cos(\delta_i) + C_{12} \sum I_i^p \sin(\delta_i)} \right\}.
\]
(22)

From this equation we can see that phase \( \phi(x,y) \) is independent of the response of the camera, the reflectivity of the object and the intensity of the ambient light. This indicates that the phase error due to non-sinusoidal waveforms depends only on the nonlinearity of the projector’s gamma. Therefore if the projector’s gamma is calibrated and the phase error due to the nonlinearity of the gamma is calculated, a LUT that stores the phase error can be constructed for error compensation.

Since the wrapped phase \( \hat{\phi}(x,y) \) for ideal sinusoidal fringe images with vertical stripes is linearly increasing from 0 to \( 2\pi \) horizontally within one period. However, once the fringe images are distorted to be nonsinusoidal due to the nonlinear effects of the system, e.g., the gamma of the projector, the real wrapped phase \( \phi(x,y) \) is nonlinear. The difference between the wrapped phase and the ideal linear one is
\[
\Delta(\phi(x,y)) = \phi(x,y) - kx,
\]
(23)
here, \( k = 2\pi/N \) is the slope of the line ranges from 0 to \( 2\pi \), \( N \) is the number of points sampled. \( \Delta \) is a function of the computed real phase value \( \phi(x,y) \). The error LUT is therefore constructed as a map \( (\phi, \Delta) \). Figure 2 shows the phase error creation method. In this figure, we used a 312 sampling points to show the concept. Figure 2(a) plots the real wrapped phase values and the ideal phase values for each sampling points. We subtract the real phase value by the ideal phase value for each point and obtain its phase error. The phase error corresponding to the real wrapped phase value is plotted in Fig. 2(b). From this plot, the phase error LUT can be created.
Zhang and Huang proposed a method by calibrating the gamma of the projector, pre-computing the phase error that can be stored into a LUT.\textsuperscript{10} The authors demonstrated that the proposed method can reduce the error to 10 times smaller. However, this method requires to calibrate the projector’s gamma, which is a time consuming procedure. Moreover, it requires the projectors gamma to be monotonic, so that the phase error LUT can be created. In this research, we propose a method that does not require to calibrate the gamma of the projector directly and does not require the gamma to be monotonic. Instead, we use a calibration board, image it and analyze the fringe images to obtain the phase error LUT. A uniform flat surface is utilized such purpose.

To study the characteristics of the phase error, we project four phase-shifted fringe patterns with an arbitrary phase shift of $\delta_1 = 0^\circ$, $\delta_2 = 270^\circ$, $\delta_3 = 130^\circ$, and $\delta_4 = 220^\circ$ onto the calibration board, the phase error is shown in Fig. 3. In this figure, the fringe pitch, the number of pixels per fringe period, used is 120. This figure shows the relationship between the phase error and wrapped phase (range from 0 to $2\pi$). We used 10-row points in the central of the image. It can be seen clearly that the phase error is fairly stable with regard to the wrapped phase.
We further analyze the phase error using two different fringe pitches, 60 and 120, respectively. The phase errors of 10 rows on different points from the center of the image are shown in Fig. 4. It can be seen that for different fringe pitches, the phase error is also stable with respect to the wrapped phase. From this figure we can also see that the phase error is independent of the projected fringe pitch. To verify this, we image the same board and obtain the phase error from a series of fringe image sets for different fringe pitches. The result is shown in Fig. 5. The fringe pitch numbers used in this experiment range from 60 to 240 with an interval of 30. This figure shows that the phase error with respect to the wrapped phase is well maintained with different fringe pitches. These experiments demonstrated that the phase error is independent of the fringe pitch number used in the measurement. Therefore, the phase error LUT can be built based on one pitch fringe image set.

In this research, we use the fringe pitch of 120 as an example to create the LUT. We project four fringe images with the same phase shift as previously specified, capture the four fringe images with our camera, and compute the phase error. Figure 6 shows the plot of 101 rows with respect to the wrapped phase. The phase error map can be quantized and stored.
Figure 7. Phase-shifted fringe images for the flat board. (a) $I_1 (\delta_1 = 0^\circ)$. (b) $I_2 (\delta_2 = 270^\circ)$. (c) $I_3 (\delta_3 = 130^\circ)$. (d) $I_4 (\delta_4 = 220^\circ)$.

into a LUT. In this research, we use a 256 array to store the phase error. The phase error LUT is created using the following equation:

$$LUT_k = \frac{h/2+50}{w} \sum_{i=h/2-50}^{h/2} \sum_{j=1}^{w} PE(i, j)/N \quad 2\pi/256 \times k < PE(i, j) < 2\pi/256 \times (k+1).$$

(24)

Where $LUT_k$ is the $k$-th item of the phase error LUT. $PE$ is the phase error map of the whole image with $(i, j)$ being the pixel of $i$-th row, $j$-th column pixel. $h$ is the image height, $w$ is the image width, $N$ is the total number belong to the range, which is 101 rows in the center area. The LUT data is plotted in the solid curve in Figure 6.

4. EXPERIMENTAL RESULTS

To demonstrate the performance of the proposed phase error compensation algorithm, we built a 3-D shape measurement system using a digital video projector (Optoma EP739) and a CCD digital camera (Dalsa CA-D6-0512). The digital video projector projects four phase-shifted fringe images with a phase shift of $\delta_1 = 0^\circ$, $\delta_2 = 270^\circ$, $\delta_3 = 130^\circ$, and $\delta_4 = 220^\circ$, the CCD camera captures the reflected fringe images by the object. Figure 7 shows all the fringe images of the flat board. Figure 8 shows the 3-D measurement results before and after phase error correction. The phase error before error reduction is approximately root-mean-squared (RMS) 0.16 radians and reduces to RMS 0.012 radians once the phase error compensation algorithm is applied. The error is approximately 13 times smaller.

We also implemented the error compensation algorithm in our real-time 3-D shape system using a three-step phase-shifting method.14–16 Figure 9 shows the measurement results of the flat board using the algorithm proposed in this paper and that developed in the previous paper.10 In comparison with the previously proposed method, the error compensation method proposed in this paper is much simpler and faster, since no projector’s gamma calibration is required, although the accuracy loses slightly.

In addition, we measured a more complex plaster model, Zeus Head. Figure 10 shows the reconstructed 3-D models before and after error compensation. The reconstructed 3-D geometric surface after error compensation is much smoother and has better visual effects. These experimental results confirmed that the error correction algorithm successfully improved the accuracy of measurement significantly.

5. DISCUSSIONS

The phase error compensation method discussed in this paper has the following advantages:

- **Simple.** The compensation algorithm introduced in this paper is simple since the phase error can be easily corrected using small LUT.
- **Accurate.** In theory, the phase error due to a nonlinear gamma curve can be completely eliminated as long as the projection response functions can be determined accurately by calibration.
• Universal. The method discussed in this paper can correct the phase errors due to the nonsinusoidal waveform for any phase-shifting algorithm.

On the other hand, the algorithm is based on the assumption that the camera is a linear device. This assumption is true for most of the cameras, although some cameras may be nonlinear. If the camera is nonlinear, it has to be calibrated before applying this algorithm.

6. CONCLUSIONS

This paper introduced an error compensation method for 3-D shape measurement system using a digital video projector. We proved that the phase error caused by the nonlinear gamma of the projector can be theoretically completely eliminate.
Figure 9. Comparison of the proposed method in this paper and that proposed in previous work. (a) The phase error before compensation (RMS: 0.16 radians). (b) The phase error after error compensation with the algorithm proposed in this paper (RMS: 0.012 radians). (c) The phase error after error compensation using the previously proposed algorithm (RMS: 0.006 radians).

Figure 10. 3-D measuring results of sculptures before and after error compensation. (a) 3-D result without error compensation. (b) 3-D result after phase error correction.

for any phase-shifting algorithm. It is based on our finding that the phase error caused by the projector’s nonlinear gamma is preserved for arbitrary object’s surface reflectivity under arbitrary ambient lighting condition. A simple phase error LUT creation method was introduced that uses a uniform calibration board and fringe image analysis technique. The experimental results demonstrated that by utilizing a small LUT (256 elements in this research), the phase error can be reduced to at least 13 times smaller. Theoretical analysis and experimental findings were also presented.

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