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ABSTRACT

This paper applies two well-known structural dynamics computational algorithms to the problem of disproportionate collapse of steel moment frames applying the alternate load path method. Any problem of structural dynamics strongly depends on the accuracy and the reliability of the analysis method since the parameters involved in the selection of the appropriate algorithm are affected by the nature of the problem. Disproportionate collapse is herein simulated via a time history analysis used to "turn off" the effectiveness of an element to the structure. For this kind of problem the time step size of the computational algorithm is of major importance for the accuracy of the method and thus, remains a variable throughout the present analyses. Two plane steel moment frames are used for the numerical examples, while all the analyses are performed independently. Firstly the β-Newmark method is applied and secondly the linear Hilbert–Hughes–Taylor a-method is applied and the respective results are compared and discussed in the last part of the paper.

1. Introduction

Since the first report of the Institution of Structural Engineers about the collapse of flats at Ronan Point in 1968 due to a gas explosion [1], a lot of progress has been made towards the analysis of the problem of disproportionate collapse resistance of structures. Additionally, the upward trend of appearance of related events has attracted the interest of many researchers trying to quantify the problem [2–12].

Among many approaches, the report of Ellingwood et al. [13] provides a very detailed review of the available methods used to mitigate disproportionate collapse while it proposes practical design methods to prevent the phenomenon. Additionally, the report presents in a thorough manner the loading patterns associated with the event of disproportionate collapse, relatively to the appearance of abnormal loading on structures. The report also emphasizes on the factors that contribute to structural vulnerability such as the lack of continuity within the structural system or lack of ductility in materials, members or connections.

In this framework, the property of disproportionate collapse resistance, called by many researchers as robustness, is not simply defined as the inherent hyperstatic level of the structure, but is instead directly related to much more complicated internal load paths which are activated in case of a damaged structural system. Nevertheless, Leyendecker and Burnett [14] have estimated that around 15%–20% of building collapses develop in this manner.

To this day, the outcome of the research efforts towards the quantification of disproportionate collapse has been acknowledged and depicted in relevant guidelines such as the General Services Administration guidelines [15] and the Department of Defense Criteria [16]. These documents so far include several different design methods; the indirect methods such as the tie force method and the direct methods such as the specific local resistance method and the alternate load path method.

For building structures ([17–27]), the alternate path method incorporates the event of a vertical element failure tuning the structure such that it can bridge over the failed element through the redistribution of the load to the remaining structure. Therefore, the critical elements of such structures mainly include columns or load bearing wall elements. The method employs three analysis procedures: linear static, nonlinear static and nonlinear dynamic.

In this framework, Izzuddin et al. [28] have studied the phenomenon of disproportionate collapse of a multi-storey building due to a sudden column loss, simulating the event with the sudden application of the gravity loads to the remaining structure after the removal of the column. This technique is used to determine the maximum dynamic response of the structure which is of the highest interest regarding the collapse resistance of the structure in such an event. Additionally, Kim and Park [29] applied a plastic design method for the design of steel moment-resisting structures against disproportionate collapse using a dynamic method of analysis.

This paper applies two structural dynamics computational algorithms, the β-Newmark method and the linear Hilbert–Hughes–Taylor a-method to two different plane steel moment frames for the event of column loss. The intention of this study is to apply a

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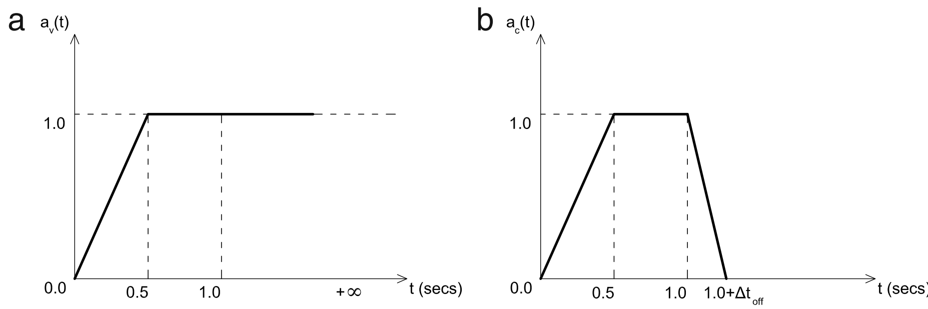

Fig. 1. Time history analysis functions.

linear dynamic method of analysis based on the fact that practitioners performing progressive collapse studies are more likely to use linear static analysis or linear dynamic analysis regardless of the inaccuracy of the procedure. The event is simulated herein via a linear static and a linear time history analysis "turning off" the effectiveness of the column. The accuracy of these problems, researched in this study, is mainly dependent on the time step size of the algorithm and that is why it is the key variable of the analyses.

2. Method of analysis

2.1. General

The attempt to quantify the resistance of a structure to disproportionate collapse using the alternate load path method initially includes the computational simulation of the elimination of selected key elements of the structure. During this effort, several different analysis approaches have been investigated so far by the researchers, such as the complete or partial removal of elements and the static analysis of the remaining structure or the dynamically gradual removal of the element until its complete ineffectiveness.

A simple but rather conservative way of analyzing a structure under the scenario of a failed element is using a linear static method. This method is described today in all the relevant guidelines [15,16] as an acceptable way of analyzing the problem. However, it involves the application of the so-called load increase factors or dynamic amplification factors which are used to simulate the dynamic effect of the phenomenon. Since a static analysis cannot include this way of failure, the load increase factors are used in order to compensate this behavior.

However, it can be easily proven that in the case of disproportionate collapse, the triggering event which could be the failure of an element, happens in a rather dynamic manner rather than being simply removed from the structure. Evidently, the loading patterns associated with disproportionate collapse scenarios usually initiated by gas explosions, blast or terrorist attacks or other similar conditions are mainly short lived loads but with very high values, requiring thereby the application of dynamic analysis methods.

Therefore, when dynamic tools are applied to the event of disproportionate collapse, load increase factors are not needed for the analysis. The failure of an element can be simulated in a more detailed way such that the response of the structure can be correctly assessed without them. In this framework, the most appropriate method of analysis is the time history analysis through which the loads of the structure can be described, relatively to the event of the triggering failure of the key element.

2.2. Time history analysis

The process by which an element is "*turned off*" from a structure can be simulated by a time history analysis. The calculation steps of using time history functions initially require the removal of the critical element from the structural model and the application of the element's forces in the opposite direction to the remaining structure. These forces $(a_c(t))$ follow the time history function shown in Fig. 1(b) while all the rest of the loads applied on the structure $(a_v(t))$ follow the time history function shown in Fig. 1(a).

Fig. 1 presents the time history analysis recommended by most relevant documents so far. The structure is loaded from 0 to 0.5 s until it reaches a point of equilibrium. The time frame between 0.5 and 1.0 s is used in order to let the structure balance after any possible dynamic response which could affect its later response due to the element removal. Then, at $t = 1.0$ s the element gradually starts to become ineffective until it is completely removed at $t = 1.0 + \Delta t_{\text{off}}$. Therefore, the critical duration of the element death is represented by Δt_{off}.

Nevertheless, the correct estimation of the duration of the element death still remains under research while several studies on the issue can be found in recent literature (see [30,31]).

2.3. Computational algorithms

For the analytical solution of the semi-discrete equation of motion, numerical time-stepping methods for the integration of differential equations are applied. Among the plethora of available methods, the careful selection of the appropriate algorithm remains a critical point in the solution of the problem. For the purposes of this paper, two methods are briefly presented and applied in the numerical examples, the β-Newmark method and the Hilbert–Hughes–Taylor a-method ([32–36]).

2.3.1. β-Newmark method

Let us say that M is the mass matrix of a structure, C is the viscous damping matrix, K is the stiffness matrix, Φ is the external load matrix and $u, \dot{u}, \ddot{u}$ are the displacement, velocity and acceleration vectors respectively. The equation of motion is defined as follows:

$$M\ddot{u} + C\dot{u} + Ku = \Phi.$$

Newmark equations in their standard form are the following:

$$u_t = u_{\Delta t-t} + \Delta t \dot{u}_{\Delta t-t} + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{u}_{\Delta t-t} + \beta \Delta t^2 \ddot{u}_t$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \ (1-\gamma)\, \Delta t \ddot{u}_{t-\Delta t} + \gamma \Delta t \ddot{u}_t.$$

The stability of the Newmark method depends on the integration parameters while for zero damping, the method is considered unconditionally stable if:

$$2\beta \geq \gamma \geq \frac{1}{2}.$$

2.3.2. Hilbert–Hughes–Taylor (HHT) a-method

On the other hand, the a-method is a generalization of the Newmark method and uses the following modified equation of

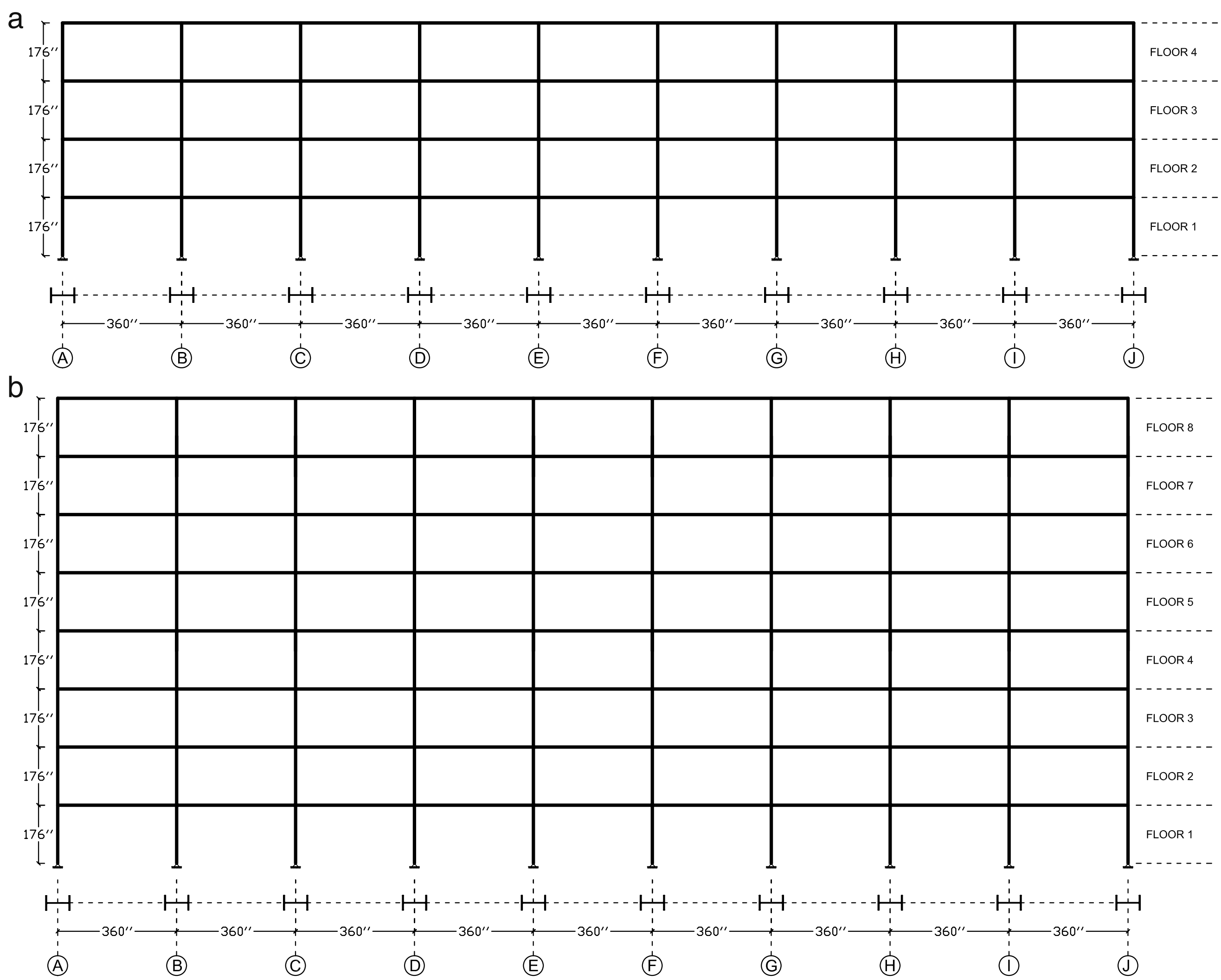

Fig. 2. Geometry of the two frames.

Table 1
Frame sections of the 4-storey frame.

	Corner columns	Middle columns	Beams
Floor 4	W18 × 40	W18 × 55	W24 × 55
Floor 3	W18 × 40	W18 × 55	W24 × 68
Floor 2	W18 × 86	W18 × 86	W24 × 68
Floor 1	W18 × 86	W18 × 86	W24 × 68

motion:

$$M\ddot{u}_t + (1+a)\,C\dot{u}_t + (1+a)\,Ku_t = (1+a)\,\Phi_t - a\Phi_t + aC\dot{u}_{t-\Delta t} + aKu_{t-\Delta t}.$$

In case $a = 0$, the method is clearly reduced to the β-Newmark.

3. Numerical applications

3.1. Description of the structures

For the purposes of the present research, two simple plane steel moment frames were selected as the numerical examples of the method. The first one is taken from [16] and consists of a 4-storey frame as shown in Fig. 2(a). Standard AISC steel sections were selected for the frame members. Both beam and column sections for all the floors are summarized in Table 1:

The respective section data can be found in [37]; all the steel shapes are ASTM A992. The orientation of the beam sections is the standard one (flanges horizontal) and the orientation of the columns is shown in Fig. 2(a).

Table 2
Frame sections of the 8-storey frame.

	Corner columns	Middle columns	Beams
Floor 8	W18 × 40	W18 × 55	W24 × 55
Floor 7	W18 × 86	W18 × 55	W24 × 68
Floor 6	W18 × 86	W18 × 86	W24 × 68
Floor 5	W18 × 86	W18 × 86	W24 × 68
Floor 4	W18 × 86	W18 × 55	W24 × 68
Floor 3	W18 × 86	W18 × 55	W24 × 68
Floor 2	W18 × 86	W18 × 86	W24 × 68
Floor 1	W18 × 86	W18 × 86	W24 × 68

The second numerical example is produced by the first and consists of an 8-storey frame as shown in Fig. 2(b). The beam and column sections for all the floors are summarized in Table 2:

3.2. Loading

The initial design of the structure was performed under gravity and lateral loading. Earthquake load, snow loads and rain loads are not included in the analysis and are assumed not to control the design of the structure.

The dead load includes the self weight of the members plus a weight of 75 psf for the slab, a 3 psf allowance for deck and a roof metal deck of 5 psf. The live loads are 80 psf + 20 psf

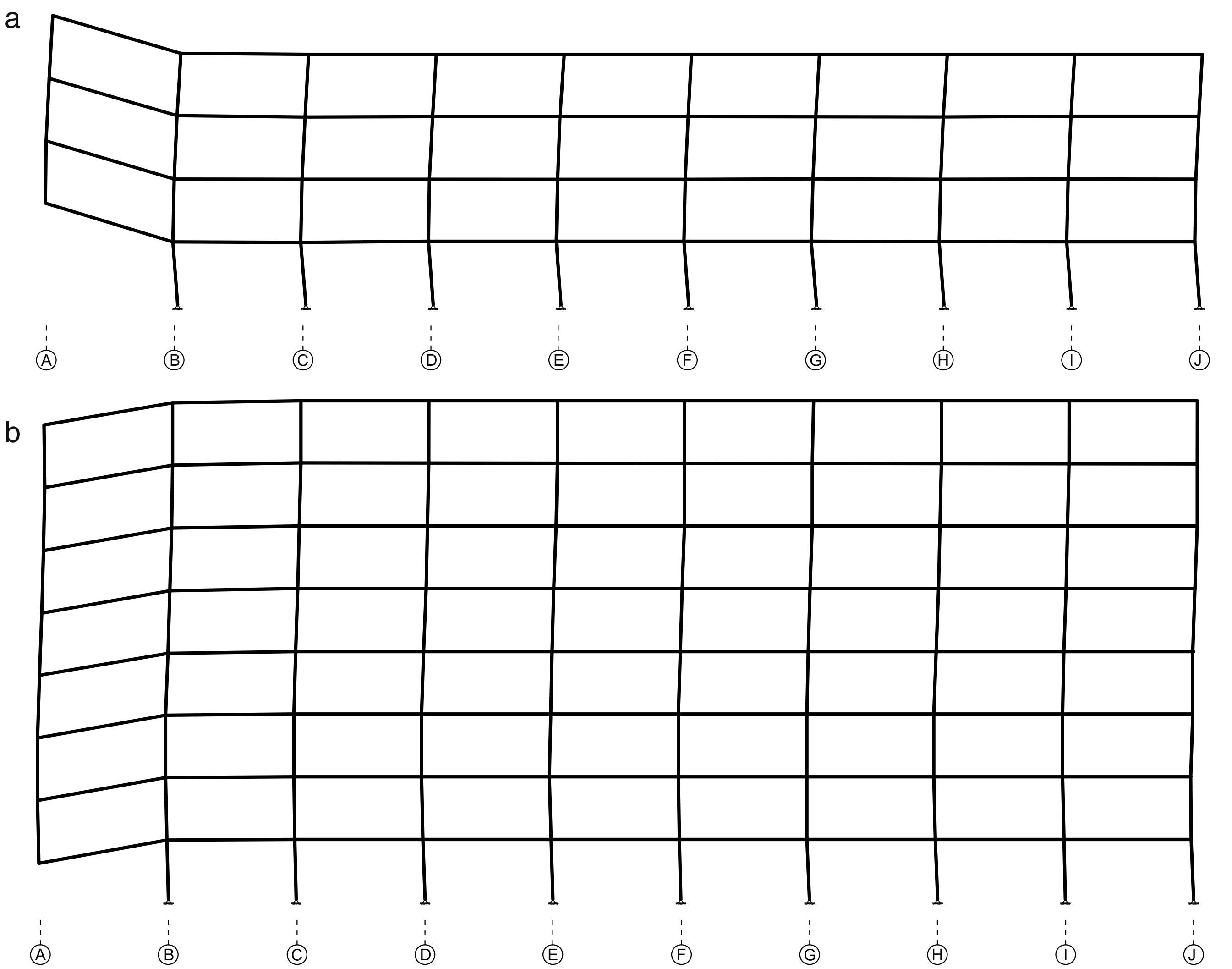

Fig. 3. Cantilever mode of deformation of frames associated with the column removal.

Table 3
Load combination used in the analyses.

	Dead	Live	Snow	Lateral load
Loadcase	1.2	0.5	0	$0.002\Sigma P$

Table 4
The periods of the two frames for the corner column removal of the first floor.

	4-storey	8-storey
Period (s)	0.15246	0.15154

allowance for partitions for floors 1, 2, 3 for the 4-storey building, 1–7 for the 8-storey building and 20 psf for the roof. The wind loads are according to IBC 2006 using 110 mph, exposure = B and importance factor 1.15 (further description of the loads can be found in Appendices E 2.1 and E 2.2 of [16]).

The critical load case was assumed to be the one described in Table 3. This assumption is compatible with the recommendations of [16].

The lateral load is applied to every floor, while ΣP is the sum of gravity loads (dead and live) acting on only that floor [16].

3.3. Corner column loss—estimation for Δt_{off}

For the sake of simplicity, the key element chosen to be removed for both frames was the corner column of floor 1. Relevant research [29] has shown that the corner column of the frame structure would produce more indicative results than any other column of the structure.

Regarding the correct estimation of the time interval during which the column becomes ineffective for the structure, already published guidelines were followed. The recommendation of [16], is to fix Δt_{off} to 1/10 of the period of the structure associated with the structural response mode for the element removal. For that reason a modal analysis was performed for both examples and the periods of the frames for the corner column removal were found as shown in Table 4. It must be noted here that the vertical (cantilever) mode of deformation of the frames, shown in Fig. 3, due to the corner column removal is responsible for the same periods of the two frames.

Thus, Δt_{off} was fixed at 0.015 s.

3.4. Time step size

In order to study the influence of the time step size to the response of the structure, the selection of several different values of time step sizes were adopted. The values were defined relatively to the element death duration Δt_{off} and range from Δt_{off} to $\Delta t_{\mathrm{off}}/300$, as shown in Table 5. It must be noted here that small values of time step size quickly escalate the computational time as is shown in Table 5.

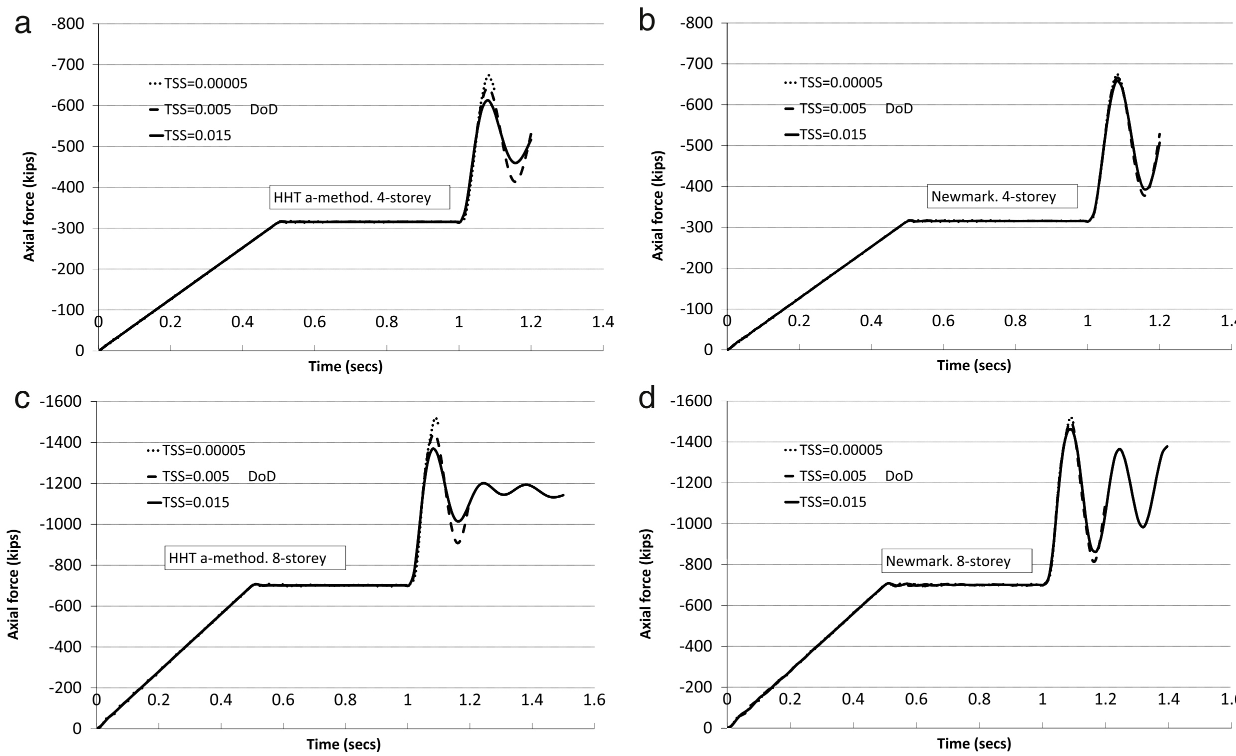

Fig. 4. Response of the B1 column as a function of time.

Table 5
The boundary values of the time step size and the corresponding computational times.

	Time step (s)	# of steps	Computational time (s)
Maximum value $= \Delta t_{off}$	0.015	80	12
Δt_2	0.014	90	14
Δt_3	0.013	95	15
Δt_4	0.012	100	15
Δt_5	0.011	110	16.5
Δt_6	0.010	120	18
Δt_7	0.009	135	21
Δt_8	0.008	150	23
Δt_9	0.007	175	26.5
Δt_{10}	0.006	200	30
Δt_{11}	0.005	240	36
Δt_{12}	0.004	300	45
Δt_{13}	0.003	400	60
Δt_{14}	0.002	600	90
Δt_{15}	0.001	1 200	180
Δt_{16}	0.0005	2 400	360
Δt_{17}	0.0002	6 000	900
Δt_{18}	0.0001	12 000	1800
Minimum value $= \Delta t_{off}/300$	0.00005	24 000	3600

3.5. *Selection of the algorithms' parameters*

During the analysis with both algorithms, it was decided not to introduce numerical damping because it is under the scope of this research to always use the instantaneous maximum values of forces and displacements which appear shortly after the complete removal of an element, so the effect of damping was going to be very small.

For the β-Newmark method, both examples were analyzed fixing parameter $\gamma = 0.6$ and $\beta = 0.3025$ while the Hilbert–Hughes–Taylor a-method was utilized fixing parameter $\alpha = -0.3$. This way, only the algorithmic damping is applied to the method, in order to eliminate the effect of the highest order eigenmodes which cannot be taken into consideration during the numerical integration due to the nature of the method.

Table 6
Maximum instantaneous axial force values of column B1 (kips).

Dynamic analysis				
TSS (s)	β-Newmark		HHT a-method	
	4-storey	8-storey	4-storey	8-storey
0.015	658	1456	613	1369
0.005 (DoD)	665	1500	643	1443
0.00005	675	1522	674	1521

4. Results

The application of time history analysis for the event of disproportionate collapse mainly aims at the extraction of the maximum instantaneous response of the structure which occurs shortly after the removal of the column. To achieve the same result, a linear static procedure would use load increase factors to compensate the dynamic effect of the column loss.

Fig. 4 presents the axial force of the column just next to the removed one, on line B, as a function of time. Fig. 4(a) and (b) show the response of the column for the 4-storey building while Fig. 4(c) and (d) show the response of the column for the 8-storey building for the Hilbert–Hughes–Taylor a-method and the β-Newmark method respectively. All the figures include the results for three different time step sizes used in the algorithms, 0.015, 0.005, 0.00005 s. It can be noticed that the response of the column depends on the solution algorithm as well as on the selected time step size.

Table 6 shows the instantaneous maximum values that are represented in Fig. 3, in kips. It can be easily shown that the differences between the analyses of the two computational algorithms produce significant divergences. The results of the instantaneous maximum axial force values for all the time step sizes are depicted in Fig. 5(a) for the 4-storey building and in Fig. 5(b) for the 8-storey building. Surprisingly, only for very low values of the time step size the two methods converge. It must be noted here that the DoD recommends the application of time step

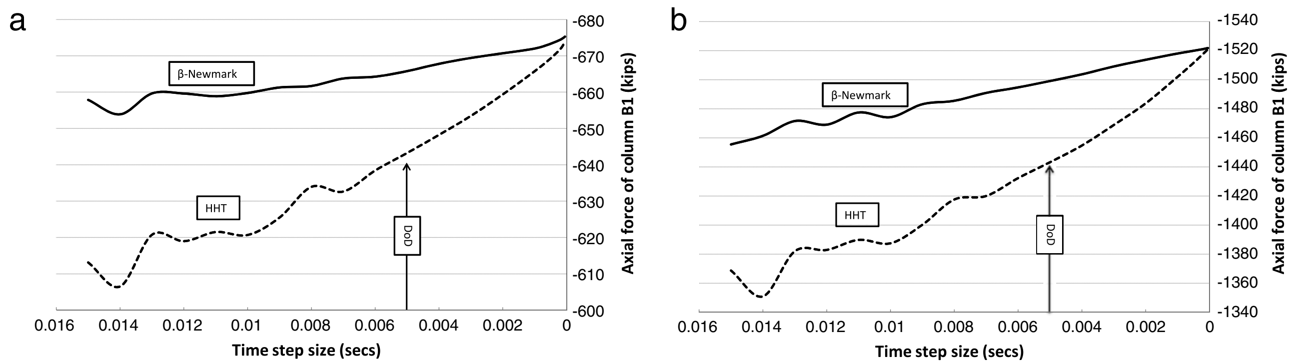

Fig. 5. Maximum instantaneous axial force values of column B1 for all TSS ranging from 0.015 to 0.00005 s, (a) for the 4-storey frame and (b) for the 8-storey frame.

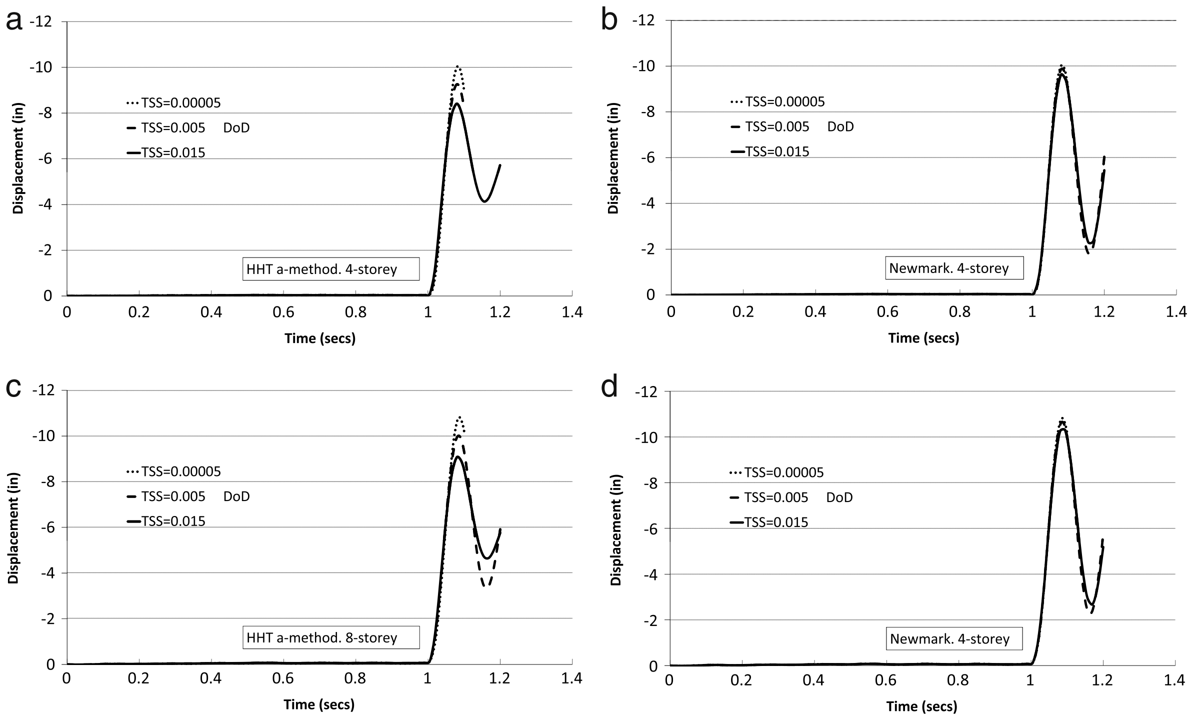

Fig. 6. The displacement above the removed column as a function of time.

Table 7
Axial force values of column B1 from linear static analysis (kips).

Static analysis		
Load increase factor	4-storey	8-storey
No factor	507	1135
2	852	1903

size of 0.005 s and that is why this value is chosen to be shown in the tables. However, as it is shown in Fig. 5, a smaller value would be more accurate for both algorithms, since 0.005 s of time step size produce different results for the two algorithms while the value 0.00005 s produces identical results. Evidently, it is important to use low values in the time step size of the solution algorithm, no matter which is the solution algorithm.

Table 7 shows the results of the linear static analysis for two different cases. The first one regards the static analysis without any load increase factors (LIFs) and the second with the factors set to 2, as described in [16]. It is worth to note that when the LIFs are set to 2, the results seem to be very conservative compared to the results from the linear dynamic analysis.

Table 8
Maximum instantaneous displacements of the node above the removed column (in.)

Dynamic analysis				
TSS (s)	β-Newmark		HHT a-method	
	4-storey	8-storey	4-storey	8-storey
0.015	9.62	10.3	8.4	9.1
0.005 (DoD)	9.9	10.7	9.24	10
0.00005	10.1	10.8	10	10.8

It is noteworthy that the forces shown in the graphs are produced by the linear elastic dynamic analysis of the structure without including any plastic redistribution of the forces.

Figs. 6 and 7, represent the results regarding the displacement of the top joint of the removed column. It is clear that the trend is the same as in the case of the axial force of the column, although the results seem to be less different in this case.

Table 8 shows the instantaneous maximum values that are represented in Fig. 3, in inches. Similarly, Table 9 shows the results from the linear static analysis. Contrary to the results of the axial force, the deflection produced by the linear static analysis is lower than the one by the dynamic analysis.

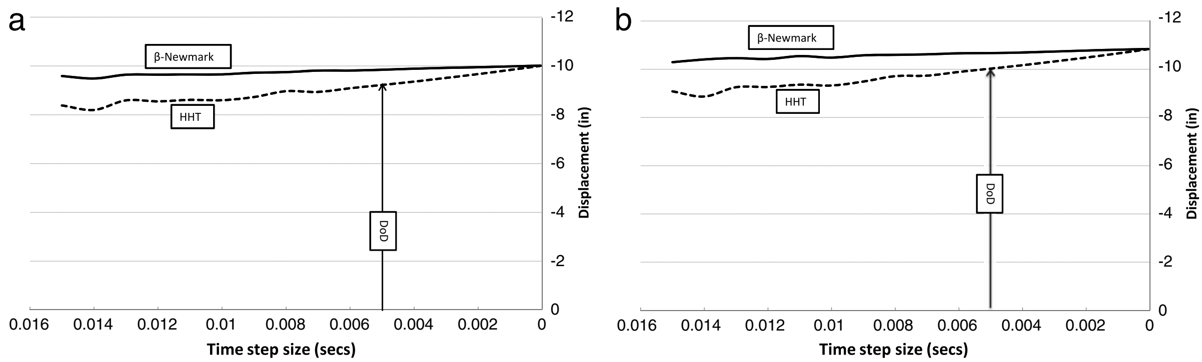

Fig. 7. Maximum instantaneous displacement values of the node above the removed column for all TSS ranging from 0.015 to 0.00005 s, (a) for the 4-storey frame and (b) for the 8-storey frame.

Table 9
Axial force values of column B1 from linear static analysis (kips).

Static analysis		
Load increase factor	4-storey	8-storey
No factor	5.47	6.14
2	9.13	10.28

5. Conclusions

This study focused on the problem of disproportionate collapse of steel moment frames, under the event of a column loss. It should be noted that the collapse of a single column in a frame structure, as described in [16], is highly unlikely to happen since the surrounding elements will also be affected by the triggering event. Nevertheless, already undertaken future work will allow the authors to examine the effect of a column collapse with the partial damage of surrounding elements.

The objective of this study is to examine the influence of the time step size of the solution algorithm to the linear elastic response of the structure: the nonlinear effects were deliberately not accounted in the study in order to highlight the results. Nevertheless, the nonlinearity of the phenomenon, which plays a major role in the response of the structure, is included in upcoming publications by the authors.

It is common practice for engineers to apply linear static or dynamic methods of analysis when doing this type of study unaware of the parameters involved in the accuracy of the methods. Throughout the analyses of the numerical examples, many of the recommendations of the DoD guidelines have been incorporated, regarding the loading parameters of the structures, the simulation of the process by which the column is "turned off" from the structure and the time interval during which the column becomes gradually inactive.

Although this work does not examine the broad range of structures covered by the DoD guidelines, the study tried to highlight the importance of the solution algorithm regarding the dynamic problem of time history disproportionate collapse analysis of steel moment frames and also the influence of the selection of the appropriate time step size relevant to the solution algorithm using a linear elastic dynamic method of analysis. For the better comparison of the results, one of the frames studied is identical with the example included in [16].

In any case the time step size is of major importance and should be treated by the codes recommending the associated method of analyses. Additionally, depending on the properties of every structural system or failure, the time step size should be calculated independently and applied appropriately. This process requires a detailed investigation of the structural system and its modal analysis so that the algorithm and its' parameters are appropriately applied in order not to introduce any numerical errors in the solution.

Two numerical examples were analyzed for the purposes of this paper, a 4-storey and an 8-storey steel moment frame. For both frames two solution algorithms were applied for the solution of the column loss: the β-Newmark and the Hilbert–Hughes–Taylor a-method. The analysis showed that the response of the structure differs when the solution algorithm changes. This divergence is higher when the time step size of the methods is also high especially when it is close to the time interval of the column removal. Additionally, the results show that the response of the structure is underestimated when the time step sizes are not low enough to produce safe results. This underestimation could lead to false conclusions such as the identification of an incorrect collapse mechanism or the wrong assessment of the mitigation actions.

However, as the time step size approached values close to zero (0.00005 s) both algorithms produced almost identical results, showing that low time step size values are crucial for the reliability of any algorithm method. As a general conclusion from the analyses, for any solution algorithm, the time step size must be selected significantly lower than the time interval of the column removal. For the present examples, the results showed that better results are achieved when the time step size is close to $\Delta t_{\mathrm{off}}/300$.

Nevertheless, the selection of the appropriate solution algorithm is an issue than needs further examination, as it is of major importance for the reliability of the analyses. In addition, the nature of every problem and structural system requires detailed treatment in order to achieve safe results.

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