Winter January 29, 2013

A computational model for full or partial damage of single or multiple adjacent columns in disproportionate collapse analysis via linear programming

Simos Gerasimidis, *University of Massachusetts - Amherst*
C. D. Bisbos, *Aristotle University of Thessaloniki*
C. C. Baniotopoulos, *University of Birmingham*

Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/nsie20

A computational model for full or partial damage of single or multiple adjacent columns in disproportionate collapse analysis via linear programming

Simos Gerasimidis [a], Christos D. Bisbos [b] & Charalampos C. Baniotopoulos [c]

[a] Department of Civil Engineering and Engineering Mechanics, Columbia University, 500 West 120th Street, New York, NY, 10027, USA

[b] Department of Civil Engineering, Institute of Metal Structures, Aristotle University, GR-54621, Thessaloniki, Greece

[c] Chair of Sustainable Energy Systems, Department of Civil Engineering, University of Birmingham, Birmingham, UK

Version of record first published: 29 Jan 2013.

To cite this article: Simos Gerasimidis , Christos D. Bisbos & Charalampos C. Baniotopoulos (2013): A computational model for full or partial damage of single or multiple adjacent columns in disproportionate collapse analysis via linear programming, Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance, DOI:10.1080/15732479.2012.757847

To link to this article: http://dx.doi.org/10.1080/15732479.2012.757847

Structure and Infrastructure Engineering, 2013
http://dx.doi.org/10.1080/15732479.2012.757847

Taylor & Francis
Taylor & Francis Group

A computational model for full or partial damage of single or multiple adjacent columns in disproportionate collapse analysis via linear programming

Simos Gerasimidis[a]*, Christos D. Bisbos[b] and Charalampos C. Baniotopoulos[c]

[a]*Department of Civil Engineering and Engineering Mechanics, Columbia University, 500 West 120th Street, New York, NY 10027, USA;* [b]*Department of Civil Engineering, Institute of Metal Structures, Aristotle University, GR-54621 Thessaloniki, Greece;* [c]*Chair of Sustainable Energy Systems, Department of Civil Engineering, University of Birmingham, Birmingham, UK,*

(*Received 6 January 2012; final version received 18 June 2012; accepted 24 July 2012*)

The evaluation of the sensitivity or insensitivity of structures to local damage has been a major research field during the last decades, mainly provoked due to the series of aging structures and infrastructures. Many researchers have described this property as redundancy, others as the resistance to disproportionate collapse or robustness and still others as the ability of structural systems to display alternate load paths in case of a local damage. In any case, the problem for the evaluation of this property is increasingly alarming since many systems experience similar collapses (American Society of Civil Engineers (2009). *Proceedings of Structures Congress on the first international symposium on disproportionate collapse*. ASCE, Austin, TX). This paper presents the numerical assessment of disproportionate collapse analysis introducing the concept of partial damage of structural elements. Global robustness measures are proposed also for the case of multiple partial losses of adjacent elements. The measures are computed on the basis of a mathematical optimisation problem using collapse load analysis of steel frames with pre-existing damage. Results comparing the cases of partial losses with the full column losses are presented and discussed.

Keywords: disproportionate collapse analysis; progressive collapse; alternate load path method; limit analysis; damaged steel frames; partial damage concept

1. Introduction

One of the most important approaches to the quantification of the property of structural systems to resist disproportionate collapse has been presented by Frangopol and Curley (1987). Terms such as damage or redundancy are explicitly defined and their correlation is realised through measures used to determine degrees of redundancy, reserves of redundancy, residual redundant factors and strength redundant factors. In all these cases, damage is defined on one hand as any deficiency of the structure introduced during the design or construction and on the other hand as any loss of strength effect caused by external loading. A lot of attention is given on the term redundancy which incorporates many structural properties relative to resisting the collapse of structural systems beyond single-element failure.

In the same paper, Frangopol and Curley (1987), a clear separation is made between 'fail-safe' structures, which are structures with the ability of redistributing the loads and engaging multiple load paths, and 'weakest-link' structures, which do not experience this ability because an appropriate alternative load path is not present. It is also highlighted that the degree of indeterminacy is not the appropriate measure of the overall system strength and that system reliability also depends on the existence of these alternative load paths.

A very comprehensive and detailed document presenting many aspects of disproportionate collapse analysis can be found by Ellingwood et al. (2007). Among the many innovative items included in this paper, the distinction of direct and indirect methods of analysis as well as the approach of risk assessment relative to natural or man-made hazards is clarified. Tools regarding the measurement of risk are presented in conjunction with mathematical and decision analysis concepts always in parallel with the identification of potential abnormal load hazards which could trigger disproportionate collapse. Additionally, special focus is provided regarding the direct methods of analysis such as the alternate load path method of analysis while several practical engineering design perspectives are recommended. Ellingwood (2002) also presented many interesting and critical aspects of disproportionate collapse analysis examining the appropriate load combinations that should be used for such approaches.

*Corresponding author. Email: sg2988@columbia.edu

A very important and innovative approach regarding disproportionate collapse global system considerations can be found in Ettouney, Smilowitz, Tang, and Hapij (2006). Important topics regarding the typology of disproportionate (or progressive) collapse have been published by Starossek (2009). The big variety of different types of failures which share the common characteristic of progressiveness and disproportionality is presented by a plethora of examples. Relevant robustness measures are also described and connected with each one of the types of progressive collapse. A clear distinction between the terms disproportionate and progressive collapse of structures can also be found by Starossek (2009).

Several analyses on disproportionate collapse of steel frames have been conducted by Kim and Kim (2009). The dynamic effect of local damage is investigated through the dynamic analysis of steel frames for column removal. Interesting aspects such as catenary action of steel moment frames or the concept of weak storey effect have been analysed through nonlinear dynamic analyses. In the same direction, Foley, Martin, and Schneeman (2007), Foley, Schneeman, and Barnes (2008a), Foley, Schneeman, and Barnes (2008b) present thoroughly the time history method of analysis associated with column removal concepts.

Cavaco, Casas, Neves, and Huespe (2010) present a very comprehensive and detailed literature review regarding the different approaches to the problem. The different robustness measures proposed by many researchers are presented combining the exposure of a structure, the damage induced on the structure, the structural performance and the consequences of the damage. In a way, Cavaco et al. (2010) encircle the problem by different approaches highlighting the advantages and disadvantages of each of the methods used to evaluate structural robustness. The paper adopts structural robustness as a structural property rather than a property of both the structure and the environment.

All the above authors, as well as most of the researchers approaching the problem in a quantitative manner, have accepted the alternate load path method of analysis as the most useful tool related to disproportionate collapse analysis (Baker, Schubert, & Faber, 2008; Izuddin, Vlassis, Elghazouli, & Nethercot, 2008; Dubina, Dinu, & Stratan, 2010; Galal & El-Sawy, 2010; Gerasimidis, Ampatzis, & Bisbos, 2011; Gerasimidis & Baniotopoulos, 2011; Marjanishvili & Agnew, 2011; Tsai & Lin, 2008; Kwasniewski, 2010); the core of this method of analysis is based on the engineering concept of column loss as the initiating event which could cause disproportionate collapse.

However, many remarks regarding the concept of column loss have been made recently putting in doubt its correlation to real structural failure events. The work of Ellingwood (2002) describes why and how the concept of notional element removal is far from being realistic. On one hand, it is explained that it is highly unlikely for a structural element to fail completely in an event of local structural damage and on the other hand even if a situation is dramatic enough to produce full element loss, it is unlikely that only one structural element will be affected by the event. Therefore, although researchers and engineers simplistically assume that a column loss could provide an easy way of introducing damage to the structure, it is not the most accurate way of incorporating damage in a structural element.

This paper presents the numerical assessment of disproportionate collapse analysis, introducing the concept of partial damage of structural elements in the deterministic framework of the alternate load path method. As an advantage of the method, global robustness measures are proposed also for the case of multiple partial losses of adjacent elements which could accommodate a blast event. In such an event, the affected elements in the structural system could be more than just one column and the effect on these elements could be more complicated than the complete removal of one column. The robustness measures are computed on the basis of a mathematical optimisation problem using collapse load analysis of steel frames with pre-existing damage. Parametric studies applied on an example at the end of the paper utilise the method on a steel moment resisting frame for single column partial damage as well as combining partial damage of adjacent columns. Results comparing the cases of partial losses with the full column losses are presented and discussed.

2. Basics of disproportionate collapse – alternate load path analysis for column removal

Based on the definition of disproportionate or progressive collapse as 'the spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure or a disproportionately large part of it', the US Department of Defense (DoD) (2009) suggests the Alternate Path (AP) method as a direct design approach to the problem. In this framework, the (AP) method requires that the structure must be capable of adequately bridging over a missing structural element in order to prevent the spread of the failure and follows the Load and Resistance Factor Design (LRFD) philosophy regarding the design strength of the structure's members. Evidently, redesign must be performed, if necessary.

A central issue of the AP method consists in the definition of the load cases to be considered. The DoD, following the guidance of ASCE 41 (2007), makes a clear distinction between two separate load case categories: the deformation-controlled actions and the force-controlled actions. The intention for the classification of the load cases regards the type of failure of the elements associated with each one of the types of actions. Thus, deformation-controlled actions concern rather ductile failures, while force-controlled actions concern more brittle types of failure, see Figure 1.

Formally, the load case provisions of the DoD can be described in the following way. First, a set of four basic load combinations $\phi_B^{(i)},\ i = 1, \ldots, 4$ is defined and let B be the respective index set:

$$B = \{1, 2, 3, 4\}.$$

The basic combinations include dead, live and snow loads as they are listed in Table 1. Moreover, imperfection lateral loads, equal to $0.002P$ are applied to every floor of the building unidirectionally throughout the height of the building, while P is the sum of gravity loads (dead and live) acting on only that floor.

These four basic load combinations have a global character for the building – without any consideration of column removal and respective effects – and they are used for both deformation- and force-controlled actions.

Assuming that the k-column is removed, each basic combination $\phi_B^{(i)},\ i \in B$ generates the respective actual load cases $\phi_D^{(i)}(k)$ for deformation-controlled actions and $\phi_F^{(i)}(k)$ for force-controlled actions:

$$\phi_D^{(i)}(k) = \phi_B^{(i)} \ + \ (\Omega_D(k) - 1)\ \Delta\phi_B^{(i)}(k) \tag{1}$$

$$\phi_F^{(i)}(k) = \phi_B^{(i)} \ + \ (\Omega_F - 1)\ \Delta\phi_B^{(i)}(k) \tag{2}$$

where $\Delta\phi_B^{(i)}(k)$ is simply the part of $\phi_B^{(i)}$ which corresponds to the floor areas above the line of the removed kth column.

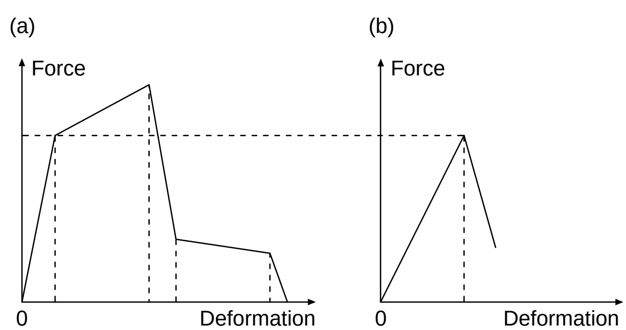

Figure 1. (a) Deformation-controlled and (b) force-controlled actions.

Table 1. Basic load combinations $\phi_B^{(i)}$.

Combination	Dead	Live	Snow	Lateral loading
$i = 1$	0.9	0.5	0	$0.002\Sigma P$
$i = 2$	0.9	0	0.2	$0.002\Sigma P$
$i = 3$	1.2	0.5	0	$0.002\Sigma P$
$i = 4$	1.2	0	0:2	$0.002\Sigma P$

Factors $\Omega_D(k)$ and Ω_F are load increase factors (LIFs) which represent dynamic amplification phenomena. This local overloading scheme is depicted in Figure 2.

The aforementioned LIFs depend on the type of the structure's material and are mainly intended for linear static analysis. If the material is steel, Ω_D depends on the type of the connections between the elements above the removed kth column and this fact is represented by the explicit dependence of this factor on the removed column k. In contrast, factor Ω_F is connection independent.

For nonlinear static analysis, the DoD suggests the use of a dynamic increase factor (DIF) Ω_N and therefore Equation (1) takes the form

$$\phi_D^{(i)}(k) = \phi_B^{(i)} \ + \ (\Omega_N(k) - 1)\ \Delta\phi_B^{(i)}(k). \tag{3}$$

Remarkably, Ω_N is always a smaller number than Ω_D, since Ω_D is defined in such a conservative way in order to capture nonlinear as well as dynamic behaviour, whereas Ω_N accommodates only dynamic effects.

Collapse phenomena are inherently nonlinear due to the plastic stress redistribution within the whole structure. Consequently, in this study factor Ω_N is used for deformation-controlled actions where a nonlinear and ductile response of the structure is captured. Non-ductile, force-controlled action effects are captured by the so-called first section plastification (or elastic limit) problem. Since, in that case, plastic redistribution is limited, factor Ω_F has been correspondingly applied.

3. Limit analysis of damaged steel frames

In this section, a simple formulation of the limit analysis of damaged steel frames is described. Notation follows that of Bisbos and Ampatzis (2008), Skordeli and Bisbos

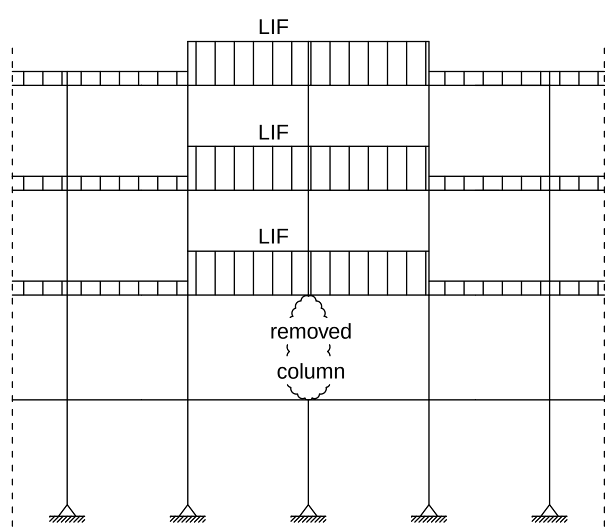

Figure 2. Area of application of LIFs and DIFs.

(2010) and Gerasimidis et al. (2011). Presentation is analytical for the sake of completeness.

3.1 Linear FEM analysis of damaged structures

Let Θ be a damaged structure, discretised within a geometrically linear Finite Element Method (FEM) framework and let NU be the number of free nodal displacements. Θ contains NG numerical integration points (Gauss points), which will be used in this work as stress checking points. Local quantities, always referred to the jth Gauss point of Θ, are denoted by the respective subscript. Damage is introduced via damage indexes δ_j satisfying

$$0 \leq \delta_j \leq 1, \quad j = 1, \ldots, \mathrm{NG}, \tag{4}$$

which determine the local damage degree. Lower bound $\delta_j = 0$ is the local intact state condition (no damage) and upper bound $\delta_j = 1$ characterises the state of full local damage. An element is removed, if the full-damage condition holds for all its Gauss points. We will denote by δ the vector of dimension NG, which lists all δ_j of the structure

The $\mathrm{NU} \times \mathrm{NU}$ linear stiffness matrix $\mathbf{K}$ of the damaged structure reads:

$$\mathbf{K} = \sum_{j=1}^{\mathrm{NG}} w_j \ \mathbf{B}_j^T \ \mathbf{C}_j^d \ \mathbf{B}_j \quad \text{with } \mathbf{C}_j^d = (1 - \delta_j)\mathbf{C}_j$$

where the intact quantities $w_j, \mathbf{C}_j$ and $\mathbf{B}_j$ are the standard integration weight, the modulus matrix and the strain displacement matrix. In the present formulation, the equilibrium matrices $\mathbf{H}_j = w_j \mathbf{B}_j^T$ will also be used.

The linear displacement nodal vector $\mathbf{u}$ and the local elastic stress vectors $\mathbf{s}_j^{(\mathrm{el})}$ under some external nodal load vector ϕ are obtained by the usual FEM equations:

$$\mathbf{K}\mathbf{u} = \phi, \quad \mathbf{s}_j^{(\mathrm{el})} = \mathbf{C}_j^d \ \mathbf{B}_j \mathbf{u}, \quad j = 1, \ldots, \mathrm{NG}. \tag{5}$$

In limit analysis, loading consists of a permanent nodal load vector ϕ_p and of a monotonically growing variable one $\alpha\phi_v$, where scalar α is the load pattern multiplier. Pair (ϕ_p, ϕ_v) constitutes the underlying load case and let $\mathbf{p}_j, \mathbf{v}_j$ be the elastic stresses due to ϕ_p, ϕ_v obtained by Equation (5).

3.2. Piece wise linear yield criteria of damaged sections

Local elastoplastic stresses under loading $\phi_p + \alpha\phi_v$ can be written as the sum of the elastic ones and of the self-equilibrating stresses ρ_j:

$$\mathbf{s}_j = \mathbf{p}_j + \alpha\mathbf{v}_j + \rho_j. \tag{6}$$

In three-dimensional frame analysis, $\mathbf{s}_j$ contains the axial/shearing forces and the twisting/bending moments:

$$\mathbf{s}_j = (N, V_y, V_z, M_t, M_y, M_z)^T$$

and let the respective individual intact plastic capacities $N_{\mathrm{pl}}, V_{\mathrm{pl}.y}, V_{\mathrm{pl}.z}, M_{\mathrm{pl}.t}, M_{\mathrm{pl}.y}$ and $M_{\mathrm{pl}.z}$ in the diagonal entries of a local 6×6 diagonal matrix $\mathbf{N}_j$ be collected. The components of $\mathbf{s}_j$ have to satisfy the local yield criteria, modified by damage, which comprise the individual bounds posed by the individual capacities and the plastic interaction conditions. In this work, N–M_y–M_z interactions are considered, which incorporate the respective individual capacity bounds. Correspondingly, the following partitioning of $\mathbf{s}_j$ is appropriate:

$$\mathbf{s}_j = \mathbf{N}_j(\mathbf{P}_y\mathbf{y}_j + \mathbf{P}_z\mathbf{z}_j), \tag{7}$$

where the dimensionless subvectors $\mathbf{y}_j$, $\mathbf{z}_j$ of $\mathbf{s}_j$ read:

$$\mathbf{y}_j = (m_y, m_z, n)^T, \quad \mathbf{z}_j = (v_y, v_z, m_t)^T$$

and the permutation matrices $\mathbf{P}_y, \mathbf{P}_z$ are given by

$$\mathbf{P}_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{P}_z = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The yield criteria of the damaged j-section are now written in the partitioned form:

$$\mathbf{y}_j \in \mathcal{F}_j, \quad \mathbf{z}_j \in \mathcal{C}_j, \tag{8}$$

where the non-interactive part $\mathbf{z}_j$ satisfies the bounds posed by the individual plastic capacities

$$\mathcal{C}_j = \{\mathbf{z} \in \mathcal{R}^3 : |z_k| \leq (1 - \delta_j), k = 1, 2, 3\}, \tag{9}$$

and the interactive part $\mathbf{y}_j$ must lie within a bounded polyhedron (linearised interaction) with MJ facets:

$$\mathcal{F}_j = \{\mathbf{y} \in \mathcal{R}^3 : \mathbf{L}_j^T\mathbf{y} \ \leq \ (1 - \delta_j) \ \kappa_j\}, \tag{10}$$

where the MJ columns of $\mathbf{L}_j$ are the unit normals to the polyhedron faces and the intact capacity vector κ_j lists the respective positive distances of the facets to the origin.

In this work, we consider two types of $\mathcal{F}_j$: first set $\mathcal{F}_j^{(\mathrm{AISC})}$ contains the damage-modified AISC plastic interaction relations with $\mathrm{MJ} = 16$ facets, written in compact form:

$$\begin{aligned} &\text{AISC}: |n| + (8/9)(|m_y| + |m_z|) \leq 1 - \delta_j, \\ &0.5|n| + |m_y| + |m_z| \leq 1 - \delta_j \end{aligned} \tag{11}$$

and the second criterion $\mathcal{F}_j^{(\text{Rhomb})}$ is a rhombic one with MJ = 8 facets:

$$\text{Rhombic } (EC3): |n| + |m_y| + |m_z| \quad \leq \quad 1 - \delta_j, \tag{12}$$

where

$$n = \frac{N}{N_{\text{pl}}}, \quad m_y = \frac{M_y}{M_{\text{pl},y}}, \quad m_z = \frac{M_z}{M_{\text{pl},z}}$$

whose intact form is accepted as a conservative interaction criterion dictated by the Eurocode EC3 (CEN, 2006).

It is noteworthy that, in the simple formulation of Equations (8)–(12), damage enters only the capacity sides of all criteria inequalities. The criteria original intact form corresponds to $\delta_j = 0$. Respectively, all criteria sets shrink to zero in case $\delta_j = 1$ (full damage).

3.3. *Limit analysis and elastic limit problems*

In a quasi-lower bound framework, the following optimisation problem constitutes the limit analysis problem of the damaged frame Θ:

$$\begin{aligned} &P_{\text{LMT}}(\delta, \phi_p, \phi_v) \quad \text{Maximize } \alpha, \\ &\text{subjected to:} \quad \begin{cases} \sum^{\text{NG}} \mathbf{H}_j \, \mathbf{s}_j = \phi_p + \alpha \phi_v, \\ \mathbf{s}_j = \mathbf{N}_j(\mathbf{P}_y \mathbf{y}_j + \mathbf{P}_z \mathbf{z}_j), \\ \mathbf{y}_j \in \mathcal{F}_j, \quad \mathbf{z}_j \in \mathcal{C}_j, \\ j = 1, \ldots, \text{NG}, \end{cases} \end{aligned} \tag{13}$$

where problem dependence on the given damage vector δ and on the underlying load case was made explicit for the purposes of the present paper. Using Equation (6) yields the following equivalent optimisation problem:

$$\begin{aligned} &P_{\text{LMT}}(\delta, \phi_p, \phi_v) \quad \text{Maximize } \alpha \\ &\text{subjected to:} \quad \begin{cases} \sum^{\text{NG}} \mathbf{H}_j \, \rho_j = 0, \\ \mathbf{p}_j + \alpha \mathbf{v}_j + \rho_j \\ = \mathbf{N}_j(\mathbf{P}_y \mathbf{y}_j + \mathbf{P}_z \mathbf{z}_j), \\ \mathbf{y}_j \in \mathcal{F}_j, \quad \mathbf{z}_j \in \mathcal{C}_j, \\ j = 1, \ldots, \text{NG}. \end{cases} \end{aligned} \tag{14}$$

Setting $\rho_j = 0$ yields the elastic limit problem:

$$\begin{aligned} &P_{\text{ELM}}(\delta, \phi_p, \phi_v) \quad \text{Maximise } \alpha, \\ &\text{subjected to:} \quad \begin{cases} \mathbf{p}_j + \alpha \mathbf{v}_j \\ = \mathbf{N}_j(\mathbf{P}_y \mathbf{y}_j + \mathbf{P}_z \mathbf{z}_j), \\ \mathbf{y}_j \in \mathcal{F}_j, \quad \mathbf{z}_j \in \mathcal{C}_j, \\ j = 1, \ldots, \text{NG}, \end{cases} \end{aligned} \tag{15}$$

which is the problem of first section plastification in frame Θ. P_{LMT} is a problem of elastoplasticity with unlimited ductility and it allows for non-elastic stress redistribution, represented by ρ_j. Problem P_{ELM} can be used for situations with non-ductile behaviour, since non-elastic stress redistribution within Θ is prohibited. In the last case, other intact capacity matrices $\mathbf{N}_j$ could be appropriate, e.g. by including buckling phenomena.

From a numerical point of view, all maximisation problems are linear programming problems which can be solved by appropriate software. The partitioning, defined by Equation (7) allows to take direct advantage of standard software options to treat separately simple variable bounds. Equation (13) is preferable than Equation (14), since the solution of Equation (5) is avoided and damage appears only in the capacity sides of the criteria inequalities. Problem P_{ELM}, defined by Equation (15), is a simple minmax problem.

In the present formulation, the safety factors obtained by the aforementioned limit analysis and elastic limit problems will be denoted by $\alpha^*_{\text{LMT}}(\delta, \phi_p, \phi_v)$ and $\alpha^*_{\text{ELM}}(\delta, \phi_p, \phi_v)$.

4. Collapse load robustness of steel frames with respect to damage

Collapse load analysis for column removal via limit analysis of damaged structures is now straightforward. Let $\mathcal{K}$ be the set of all columns, whose removal is to be considered. Let us assign to each column in $\mathcal{K}$ a global damage vector δ_k:

$$k \in \mathcal{K} \mapsto \delta_k \in \mathcal{R}^{\text{NG}}. \tag{16}$$

This structural damage vector has all entries equal to zero (intact state) except those that correspond to the Gauss points contained within the removed column: their damage indexes are set equal to one (full damage). Actually, Equation (16) is far more general, since it encompasses partial damage. This way, the philosophy and scope of the AP method can be meaningfully extended by assigning a certain partial damage level to the columns, adjacent to the removed one.

Let us now define the abbreviations:

$$\alpha_{\mathrm{LMT}}(i,0) = \alpha^*_{\mathrm{LMT}}(0,0,\phi_B^{(i)}) \tag{17}$$

$$\alpha_{\mathrm{ELM}}(i,0) = \alpha^*_{\mathrm{ELM}}(0,0,\phi_B^{(i)}) \tag{18}$$

$$\alpha_{\mathrm{LMT}}(i,k) = \alpha^*_{\mathrm{LMT}}(\delta_k,0,\phi_D^{(i)}(k)) \tag{19}$$

$$\alpha_{\mathrm{ELM}}(i,k) = \alpha^*_{\mathrm{ELM}}(\delta_k,0,\phi_F^{(i)}(k)) \tag{20}$$

$$A_{\mathrm{LMT}}(i) = \min_{k\in\mathcal{K}} \ \alpha_{\mathrm{LMT}}(i,k) \tag{21}$$

$$A_{\mathrm{ELM}}(i) = \min_{k\in\mathcal{K}} \ \alpha_{\mathrm{ELM}}(i,k) \tag{22}$$

Evidently, Equations (17) and (18) concern the undamaged structure (fully intact without any column removal effect). Condition:

$$A_{\mathrm{LMT}}(i) \ \geq \ 1 \tag{23}$$

represents the 'collapse survival' of the structure for all column removals under the ith deformation controlled loading. Respectively, the 'first plastification' condition of the damaged structure reads

$$A_{\mathrm{ELM}}(i) \ \geq \ 1. \tag{24}$$

By virtue of the safety factor definition

$$\text{Safety factor} = \frac{\text{Load resistance}}{\text{Load demand}},$$

the load multipliers α_{LMT} and α_{ELM} are structural bearing capacities measured in terms of the acting load. In this context, the ratios

$$r_{\mathrm{LMT}}(i,k) = \frac{\alpha_{\mathrm{LMT}}(i,k)}{\alpha_{\mathrm{LMT}}(i,0)} \tag{25}$$

$$r_{\mathrm{ELM}}(i,k) = \frac{\alpha_{\mathrm{ELM}}(i,k)}{\alpha_{\mathrm{ELM}}(i,0)} \tag{26}$$

are actually fractions of the initial bearing capacities of the frame that remain active after the removal of the kth column. Both these dimensionless, residual bearing capacities satisfy the condition

$$0 \ \leq \ r \ \leq \ 1, \tag{27}$$

and the respective bearing capacity loss equals $1-r$.

In this work, quantities $r_{\mathrm{LMT}}(i,k)$ and $r_{\mathrm{ELM}}(i,k)$ are proposed as respective robustness measures. The maximal possible value $r = 1$ indicates full robustness (bearing capacity loss equal to zero) and, respectively, the minimal possible value $r = 0$ characterises zero robustness (capacity loss maximised).

Obviously, the critical robustness measures, which correspond to $A_{\mathrm{LMT}}(i)$ and $A_{\mathrm{ELM}}(i)$ under the I-th loading, are given by

$$R_{\mathrm{LMT}}(i) = \frac{A_{\mathrm{LMT}}(i)}{\alpha_{\mathrm{LMT}}(i,0)} = \min_{k\in\mathcal{K}} \ r_{\mathrm{LMT}}(i,k) \tag{28}$$

$$R_{\mathrm{ELM}}(i) = \frac{A_{\mathrm{ELM}}(i)}{\alpha_{\mathrm{ELM}}(i,0)} = \min_{k\in\mathcal{K}} \ r_{\mathrm{ELM}}(i,k). \tag{29}$$

The ranges of safety factors are completely described by defining the respective upper values:

$$\bar{A}_{\mathrm{LMT}}(i) = \max_{k\in\mathcal{K}} \ \alpha_{\mathrm{LMT}}(i,k) \tag{30}$$

$$\bar{A}_{\mathrm{ELM}}(i) = \max_{k\in\mathcal{K}} \ \alpha_{\mathrm{ELM}}(i,k) \tag{31}$$

and the respective robustness measures are given by

$$\bar{R}_{\mathrm{LMT}}(i) = \frac{\bar{A}_{\mathrm{LMT}}(i)}{\alpha_{\mathrm{LMT}}(i,0)} = \max_{k\in\mathcal{K}} \ r_{\mathrm{LMT}}(i,k) \tag{32}$$

$$\bar{R}_{\mathrm{ELM}}(i) = \frac{\bar{A}_{\mathrm{ELM}}(i)}{\alpha_{\mathrm{ELM}}(i,0)} = \max_{k\in\mathcal{K}} \ r_{\mathrm{ELM}}(i,k). \tag{33}$$

5. Parametric studies on partial column removal

5.1 Structural model

The structural system selected as the base for all the parametric analyses is a simplified version of the 'Structural Steel example' analysed in Appendix E of the DoD; detailed information regarding the geometry and other data can be found in DoD (2009) (indicatively dead load 4.3 kN/m^2, live load 4.8 kN/m^2). The structure is a four-storey, steel moment frame with improved welded unreinforced flange connections.

The model of the structure is a two-dimensional frame, consisting of 726 nodes (see Figure 3). Ground connections are pinned; the FEM mesh contains 356 standard three-node Timoshenko beam-column isoparametric elements with 2 Gauss points per element, (Bathe, 1996), therefore the free degrees of freedom amount to NU = 4356. The total number of Gauss points is NG = 752.

The structure has been analysed with the use of a simple linear FEM code (*cf*. Bathe, 1996 which provides the equilibrium matrices $\mathbf{H}_j$, not available via commercial FEM software. This code provides the necessary data to be used by the mathematical programming software MOSEK (Andersen, Roos, & Terlaky, 2003). Linking has been realised via a simple code, written in *MATLAB version 6.5*

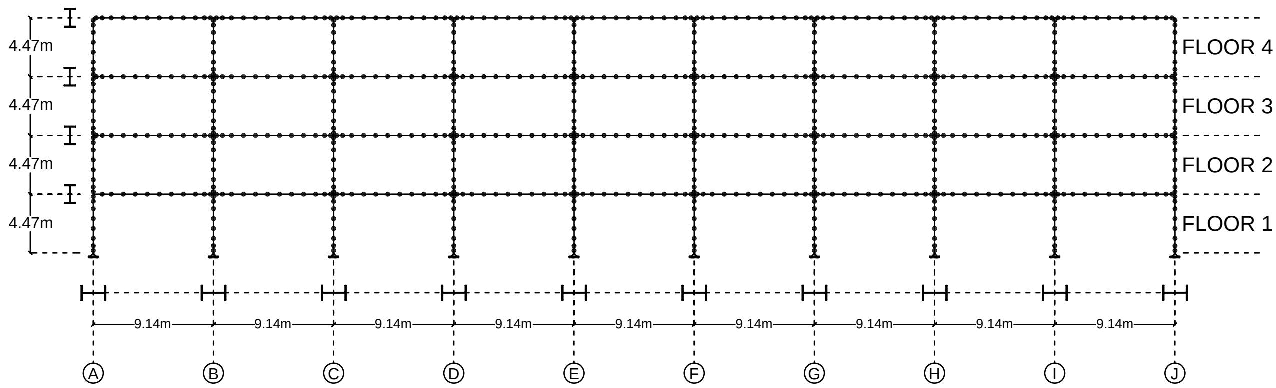

Figure 3. Frame geometry and mesh.

(2003), which provides the possibility of changing the plastic interaction criteria.

The model includes the sections produced by the design of the structure; standard AISC steel sections were used for the frame members (AISC, 2005). All the steel shapes are of ASTM A992 specification and are summarised in Table 2. The orientation of the beam sections is the standard one (flanges horizontal) and the orientation of the columns can be seen in Figure 3.

It is necessary to be mentioned at this point that buckling, lateral torsional buckling of the columns due to bending and axial forces or any second-order effects are not captured in the presented analysis and are expected to be treated in future work already undertaken by the authors. Additionally, the analysed frame of this work is a two-dimensional frame and the analysis is not taking into account out-of-plane flexural buckling (i.e. minor axis flexural buckling of columns).

5.2 Load combinations

In the framework of the DoD, the alternate load path method, based on the concept of element removal, requires that a structural system must be redesigned in order to have the ability to bridge over a missing structural element, preventing the spread of the local failure.

One of the main issues of alternate load path method within the DoD criteria is the definition of the load combinations to be considered. According to the DoD, the appropriate load combinations should combine dead, live, snow loads while imperfection lateral loads, equal to $0.002\Sigma P$, should also be applied to every floor of the building, where ΣP is the sum of gravity loads (dead and live) acting on only that floor. Recent literature has shown that within all the load combinations prescribed in the DoD, the one which produces the governing results for the response of the structure is load combination $i = 3$ as shown in Table 1 and has been, therefore, adopted for all the analyses presented in this paper (Gerasimidis et al., 2011).

Table 2. Steel sections of example frame.

	Corner columns	Middle columns	Beams
Floor 1	W18 × 40	W18 × 55	W24 × 55
Floor 2	W18 × 40	W18 × 55	W24 × 55
Floor 3	W18 × 86	W18 × 86	W24 × 68
Floor 4	W18 × 86	W18 × 86	W24 × 68

5.3 LIFs and DIFs

5.3.1 Full column removal

In the DoD criteria, the concept of column removal is considered as the full column removal and no partial damage scenarios are described. One can easily realise that the event of a full column removal should be analysed as an intense dynamic event and therefore, the corresponding DIFs are recommended to capture the dynamic effects of the phenomenon. For that reason and for the case of full column removal, this work follows the DoD applying Ω_F for the elastic limit (appearance of first plastification in the structure) and Ω_N for the nonlinear collapse analysis (allowing for plastic redistribution). According to the DoD, for the current example, $\Omega_F = 2$ and $\Omega_N = 1.43$.

5.3.2 Partial column damage

However, in the case of partial column damage which does not appear in the DoD, it can be assumed that the dynamic phenomenon is significantly milder and the effect will not be equally dramatic to the full column removal event. Hence, this DIF could be reduced from the aforementioned DoD value, since the remaining part of the damaged element could strongly affect the response of the structure to a more ductile behaviour. In the following analyses of this paper, besides the use of the recommended Ω_N, a slightly lower value for the DIF equal to 1.25 has been also used, based on the engineering perspective of the authors, in order to better capture the dynamic effects of partial column damage.

Consequently, in this study, Ω_F and Ω_N for all the cases of full column removal and Ω_F, Ω_N and $\Omega = 1.25$ for all the cases of partial column removal, in order to produce comparative results are used. The scheme describing the analyses is provided in Table 3.

5.4 Parametric analyses

The goal of the analyses presented in this paper is to compare the case of partially damaged elements over the case of fully damaged elements used by guidelines so far. For this purpose, it was decided to conduct analyses first for the case of single column damage and second for the case of multiple column damage where it would be easier to combine partially damaged columns.

5.4.1 Single column damage

For the case of single column damage, six levels of damage (δ_j) have been studied:

- $\delta_j = 0$ (intact state, no damage)
- $\delta_j = 0.2$
- $\delta_j = 0.4$
- $\delta_j = 0.6$
- $\delta_j = 0.8$
- $\delta_j = 1$ (full damage, column removal).

Damage was introduced to columns belonging to grid lines A, C and E. Due to the symmetry of the structure, the results from these analyses could provide a good insight into the overall behaviour of the structure.

5.4.2 Multiple column damage

For the case of multiple column damage, it was decided to perform analyses for the columns belonging to grid lines A, C and E combined with damage at their adjacent columns. Damage in grid lines A, C and E will be named dominant damage, while damage at adjacent grid lines B, D and F, respectively, will be named secondary damage. For example, in case column A1 (column at grid line A, first floor) experiences damage equal to 80% (leaving 0.2 of the column), it is combined with damage of column B1 (column at grid line B, first floor) equal to 20% (leaving 0.8 of that column). All the different multiple column partial damage cases are summarised in Table 4.

Table 3. Analyses scheme regarding DIFs.

Type of analysis	Full column removal	Partial column damage
Elastic limit (first plastification)	Ω_F	Ω_F or Ω
Limit analysis (collapse analysis)	Ω_N	Ω_N or Ω

5.5. Results

5.5.1 Single column damage

The results for single column damage cases can be found in Tables 5 and 6:

- Table 5 presents the results ($\alpha_{\mathrm{ELM}}(i, k)$ and $\alpha_{\mathrm{LMT}}(i, k)$) of the analyses for single column damage for the elastic limit and limit analysis, respectively. The term (int.) denotes the intact condition of the column and the term (rem.) denotes the complete column removal from the system.
- Table 6 presents the results ($\alpha_{\mathrm{ELM}}(i, k)$ and $\alpha_{\mathrm{LMT}}(i, k)$) for the single column partial damage cases for five different levels of damage taking into account $\Omega = 1.25$, for both elastic limit and limit analysis. The term (int.) denotes the intact condition of the column, the term (rem.) denotes the complete column removal from the system and NA denotes not applicable. It must be mentioned that for the case of full removal, the common engineering perspective as described in previous sections is not applied, since this case is adequately described in the DoD.

Figures 4 and 5 present the limit analysis robustness measures ($r_{\mathrm{LMT}}(i, k)$) for $\Omega_N = 1.43$ and $\Omega = 1.25$, respectively. The double line in the graphs shows the robustness measures when full column removal is included and all the rest of the lines show the robustness factors when partial damage is included.

A first look at the $\alpha_{\mathrm{LMT}}(i, k)$ values of Table 5, which are based on limit analysis, shows that the response of the frame is significantly affected by the level of damage included in the system.

First of all, for the intact system ($\delta_j = 0$), the system of the frame shows that it can stand 3.63 times the load described by the third load combination of the DoD ($i = 3$). For complete column removal (as described in the DoD), the respective values drop dramatically and they range from 0.71 to 0.97, if the column removals of floor 4 are not included. This shows that the frame needs to be redesigned and retrofitted since it does not withstand the loads applied.

However (and this could be described as the most important finding of the current work), even when only

Table 4. Multiple column partial damage cases, δ_j values.

Dominant		Secondary		Dominant		Secondary		Dominant		Secondary	
A1	1	B1	0	C1	1	D1	0	E1	1	F1	0
	0.8		0.2		0.8		0.2		0.8		0.2
	0.6		0.4		0.6		0.4		0.6		0.4
A2	1	B2	0	C2	1	D2	0	E2	1	F2	0
	0.8		0.2		0.8		0.2		0.8		0.2
	0.6		0.4		0.6		0.4		0.6		0.4
A3	1	B3	0	C3	1	D3	0	E3	1	F3	0
	0.8		0.2		0.8		0.2		0.8		0.2
	0.6		0.4		0.6		0.4		0.6		0.4
A4	1	B4	0	C4	1	D4	0	E4	1	F4	0
	0.8		0.2		0.8		0.2		0.8		0.2
	0.6		0.4		0.6		0.4		0.6		0.4

Table 5. $\alpha_{ELM}(i, k)$ and $\alpha_{LMT}(i, k)$ for partial single column damage and $\Omega_F = 2$ (for elastic limit), $\Omega_N = 1.43$ (for limit analysis).

	Column damage, δ_j											
	Elastic limit (F)						Limit analysis (N)					
Damaged column	0 (int.)	0.2	0.4	0.6	0.8	1 (rem.)	0 (int.)	0.2	0.4	0.6	0.8	1 (rem.)
A1	2.67	1.32	1.33	1.09	0.65	0.39	3.63	2.54	2.54	2.41	1.66	0.74
A2	2.67	1.30	1.29	1.11	0.71	0.40	3.63	2.54	2.49	2.35	2.06	0.71
A3	2.67	1.24	1.06	0.86	0.59	0.44	3.63	2.48	2.41	2.35	1.98	0.73
A4	2.67	2.56	2.25	2.25	1.70	0.88	3.63	3.52	3.43	3.33	3.24	1.23
C1	2.67	1.41	1.07	0.73	0.39	0.41	3.63	2.40	2.03	1.64	1.24	0.78
C2	2.67	1.54	1.56	1.07	0.55	0.45	3.63	2.84	2.58	2.04	1.47	0.83
C3	2.67	1.53	1.55	1.24	0.64	0.51	3.63	2.84	2.84	2.35	1.69	0.96
C4	2.67	2.64	2.64	2.64	2.63	1.44	3.63	3.63	3.63	3.63	3.63	2.47
E1	2.67	1.49	1.13	0.77	0.40	0.42	3.63	2.47	2.10	1.70	1.28	0.81
E2	2.67	1.52	1.55	1.12	0.58	0.46	3.63	2.84	2.66	2.11	1.52	0.85
E3	2.67	1.51	1.53	1.30	0.67	0.51	3.63	2.84	2.84	2.43	1.75	0.97
E4	2.67	2.66	2.66	2.66	2.66	1.45	3.63	3.63	3.63	3.63	3.63	2.47

Note: The term (int.) denotes intact condition and (rem.) denotes fully removed element.

Table 6. $\alpha_{ELM}(i, k)$ and $\alpha_{LMT}(i, k)$ for partial single column damage and $\Omega = 1.25$ for both elastic limit and limit analysis.

	Column damage, δ_j											
	Elastic limit (Ω)						Limit analysis (Ω)					
Damaged column	0 (int.)	0.2	0.4	0.6	0.8	1 (rem.)	0 (int.)	0.2	0.4	0.6	0.8	1 (rem.)
A1	2.67	2.13	2.14	1.73	1.04	NA	3.63	2.90	2.90	2.75	1.89	NA
A2	2.67	2.10	2.10	1.73	1.10	NA	3.63	2.90	2.84	2.68	2.35	NA
A3	2.67	1.92	1.64	1.33	0.92	NA	3.63	2.83	2.76	2.69	2.26	NA
A4	2.67	2.59	2.52	2.43	2.07	NA	3.63	3.53	3.44	3.34	3.25	NA
C1	2.67	2.25	1.72	1.17	0.62	NA	3.63	2.75	2.33	1.88	1.42	NA
C2	2.67	2.51	2.48	1.72	0.89	NA	3.63	3.25	2.95	2.34	1.68	NA
C3	2.67	2.50	2.52	2.00	1.04	NA	3.63	3.25	3.25	2.70	1.93	NA
C4	2.67	2.66	2.66	2.66	2.66	NA	3.63	3.63	3.63	3.63	3.63	NA
E1	2.67	2.38	1.81	1.23	0.65	NA	3.63	2.83	2.39	1.94	1.46	NA
E2	2.67	2.48	2.52	1.80	0.93	NA	3.63	3.25	3.04	2.41	1.74	NA
E3	2.67	2.47	2.50	2.09	1.08	NA	3.63	3.25	3.25	2.77	2.00	NA
E4	2.67	2.66	2.66	2.66	2.66	NA	3.63	3.63	3.63	3.63	3.63	NA

Note: The term (int.) denotes intact condition and (rem.) denotes fully removed element.

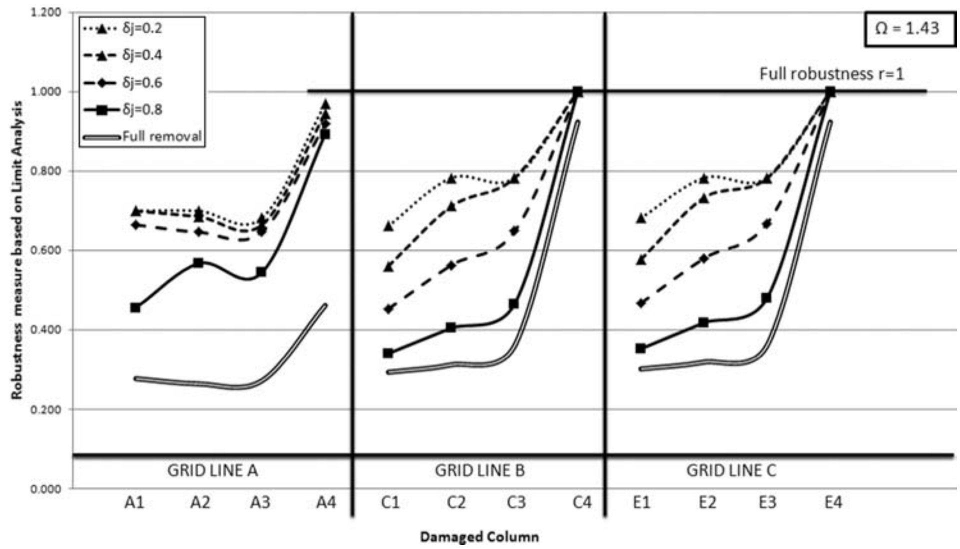

Figure 4. Robustness measures ($r_{\mathrm{LMT}}(i, k)$) based on limit analysis for single column damage and Ω_N.

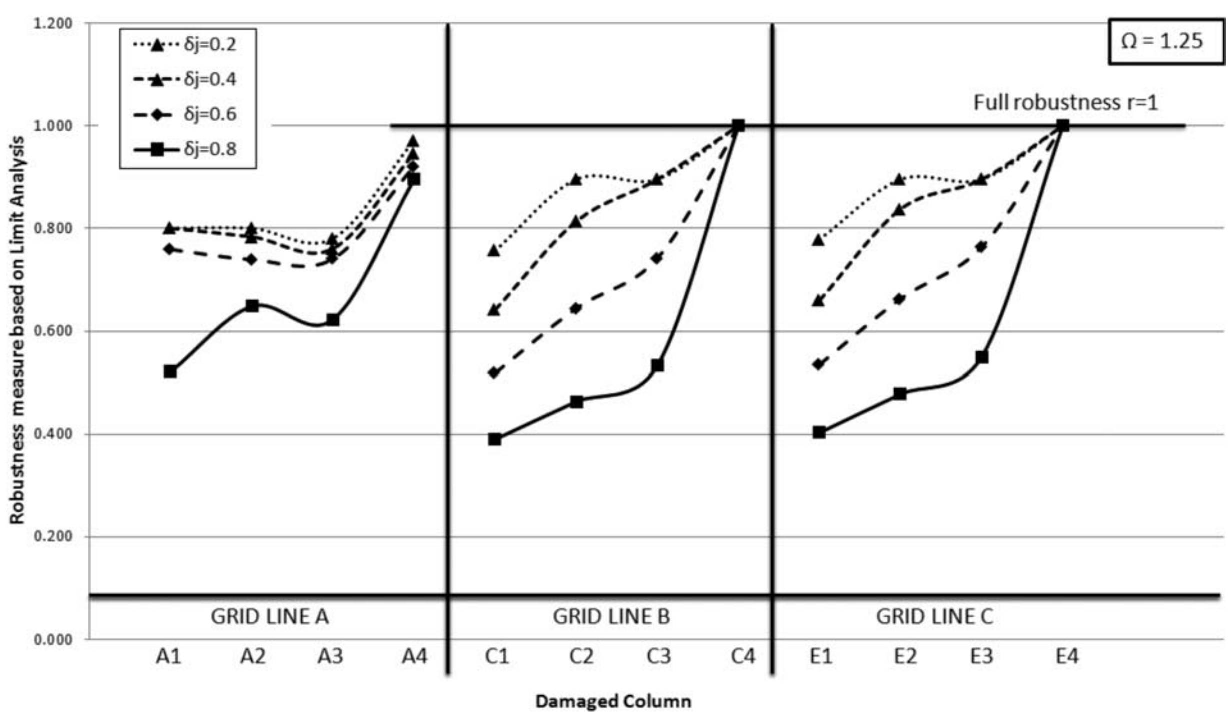

Figure 5. Robustness measures ($r_{\mathrm{LMT}}(i, k)$) based on limit analysis for single column damage and Ω.

20% of the column is still remaining in the system (the case of $\delta_j = 0.8$), all the respective values are higher than 1 which means that the frame is still in the safe region. Additionally, in the case when the damage is less than 20%, the results are even better and the values of the limit analysis factors are much higher.

This trend is depicted in a better way in Figure 4 rather than Table 5, where the respective robustness measures are presented. It is worth mentioning the big difference in the robustness measures between the cases of full column removal and the cases of partial damage, especially for the corner columns (on grid line A). Nevertheless, a clear pattern of significant difference in the robustness of the frame appears when even a small part of the column is considered to be remaining in the structural system.

The same pattern appears also for the case of single column damage when Ω is applied. However, an interesting remark is that the values of Table 6 are slightly higher than the values of Table 5 and this is expected since the DIF is lower. Therefore, a first conclusion that can be drawn from all the above is that the consideration of partial damage in the system (instead of the full element removal) has a favourable effect in the response of the structure.

Another interesting point is also that the response of the structure is affected more if the damage appears in a lower floor. It can be seen from both Figures 4 and 5 that when damage appears in the first floor, the results are worse for the structure. Additionally, the corner columns (grid line A) provide, in most of the cases, the lowest values when compared with the respective columns in the internal grid lines.

Also interesting to note are the values for the cases where damage is inserted for columns A4, C4 or E4. It is worth mentioning that the values for these cases either for the elastic limit or for the limit analysis are very similar, and this can be explained by the fact that when damage appears in these locations of the frame, the triggered mechanism consists only of the above beams or beams. Therefore, for the case of A4, the remaining cantilever is the 'collapse mechanism' of the structure, while for cases C4 and E4, the appearance of hinges in the above two bay long beam is the 'collapse mechanism'.

5.5.2 *Multiple column damage*

The results for multiple column partial damage cases can be found in Tables 7 and 8:

- Table 7 presents the results ($\alpha_{ELM}(i, k)$ and $\alpha_{LMT}(i, k)$ for multiple column partial damage losses for $\Omega_F = 2$ and $\Omega_N = 1.43$, for elastic limit and limit analysis, respectively.
- Table 8 presents the results ($\alpha_{ELM}(i, k)$ and $\alpha_{LMT}(i, k)$) for multiple column partial damage losses for $\Omega = 1.25$, for both elastic limit and limit analysis.

It must be noted that the case where damage is equal to 1/0 coincides with the case of single column full loss, i.e. the case of A1/B1 = 1/0,, coincides with A1 full column loss. In Table 8, the results for that case (full column loss) are not presented since they are not described in any guideline and it would be improper to include full column loss with a different DIF than the one already recommended for that case by the guidelines.

Similar to Figures 4 and 5, Figures 6 and 7 provide the robustness measures ($r_{LMT}(i, k)$) based on the limit analysis factors for $\Omega_N = 1.43$ and $\Omega = 1.25$ respectively. The solid line in Figure 6 shows the robustness factors when full column removal is included and all the rest of the lines show the robustness factors when partial damage is included.

The results presented in the above tables and figures also show the clear trend as in the case of single partial column damage. In the cases when the damage is distributed in multiple elements rather than in a single one, the effect on the response of the structure is mitigated. For example, when the A1 column is completely removed, the limit analysis factor is 0.74. On the other hand, when the damage is distributed as 80% on element A1 and 20% on element B1, then the limit analysis factor is increased to 1.655, an increase of 223%. In any case, it is apparent from the aforementioned figures that even when a small partiality of column loss is included in the analyses instead of full column loss, the results are dramatically different.

In Figures 6 and 7, one can notice again the same patterns like the ones mentioned for single column damage. First of all, the worst case scenario is always the case when damage appears in lower floors while the appearance of damage at higher floors is better for the structure. Therefore, a conclusion that can be drawn is that if an event would happen at a higher floor and affect adjacent columns, it would produce a favourable situation rather than an event in lower floors affecting only one element.

Table 7. Multiple column partial damage, $\Omega_F = 2$ (for elastic limit) and $\Omega_N = 1.43$ (for limit analysis).

		Elastic limit (Ω_F)			Limit analysis (Ω_N)		
Dominent (δ_j)	/Secondary (δ_j)	1/0	0.8/0.2	0.6/0.4	1/0	0.8/0.2	0.6/0.4
A1	/B1	0.39	0.65	1.08	0.74	1.66	2.20
A2	/B2	0.40	0.71	1.10	0.71	2.06	2.35
A3	/B3	0.44	0.59	0.85	0.73	1.98	2.35
A4	/B4	0.88	1.07	2.25	1.23	3.24	3.33
C1	/D1	0.41	0.39	0.73	0.78	1.24	1.64
C2	/D2	0.45	0.55	1.06	0.83	1.47	2.04
C3	/D3	0.51	0.64	1.23	0.96	1.69	2.35
C4	/D4	1.44	2.63	2.64	2.47	3.63	3.63
E1	/F1	0.42	0.40	0.76	0.81	1.28	1.70
E2	/F2	0.46	0.58	1.11	0.85	1.52	2.11
E3	/F3	0.51	0.67	1.29	0.97	1.75	2.43
E4	/F4	1.45	2.66	2.66	2.47	3.63	3.63

Table 8. Multiple column partial damage, $\Omega = 1.25$ for both elastic limit and limit analysis.

		Elastic limit (Ω)			Limit analysis (Ω)		
Dominent (δ_j)	/Secondary (δ_j)	1/0	0.8/0.2	0.6/0.4	1/0	0.8/0.2	0.6/0.4
A1	/B1	NA	1.03	1.70	NA	1.89	2.43
A2	/B2	NA	1.10	1.71	NA	2.35	2.68
A3	/B3	NA	0.92	1.32	NA	2.26	2.69
A4	/B4	NA	2.07	2.42	NA	3.25	3.34
C1	/D1	NA	0.62	1.16	NA	1.42	1.87
C2	/D2	NA	0.90	1.71	NA	1.68	2.34
C3	/D3	NA	1.04	1.97	NA	1.93	2.69
C4	/D4	NA	2.66	2.66	NA	3.63	3.63
E1	/F1	NA	0.65	1.22	NA	1.46	1.92
E2	/F2	NA	0.93	1.78	NA	1.74	2.41
E3	/F3	NA	1.08	2.07	NA	2.00	2.77
E4	/F4	NA	2.66	2.66	NA	3.63	3.63

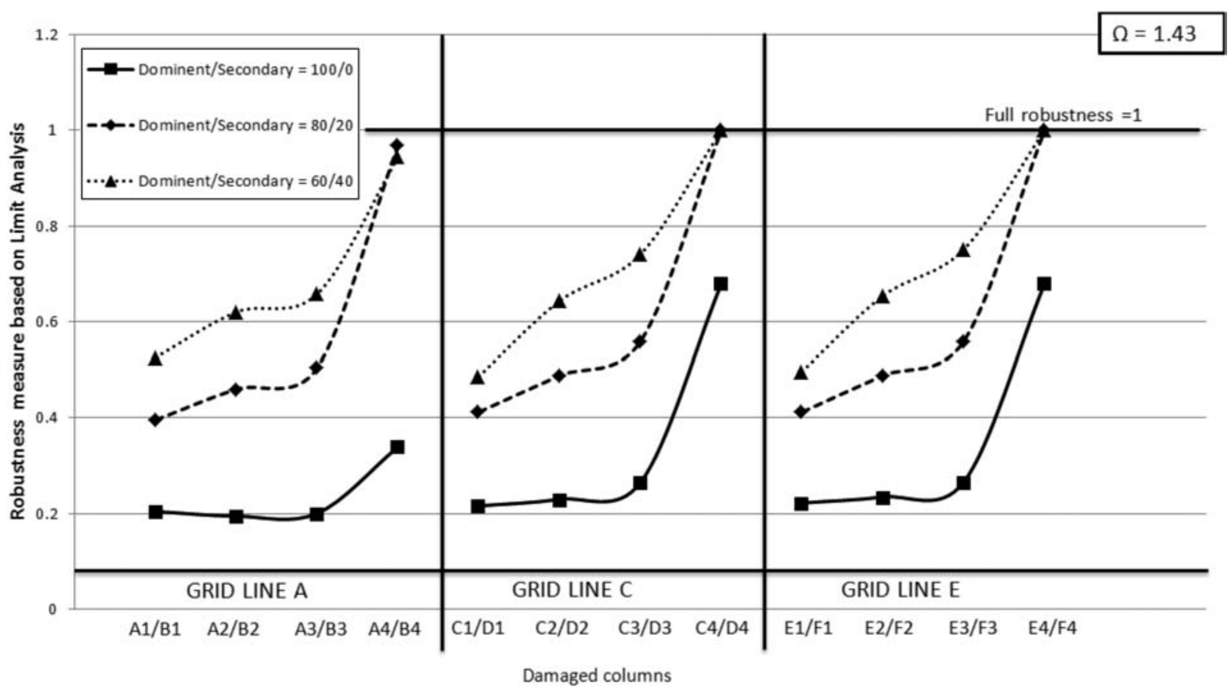

Figure 6. Robustness measures ($r_{\mathrm{LMT}}(i, k)$) based on limit analysis for multiple column damage and $\Omega_N = 1.43$.

6. Conclusions

Many remarks regarding the concept of column loss have been made recently doubting its correlation to real structural failure events and describing why and how the concept of notional element removal is far from being realistic. On one hand, it is stated that it is highly unlikely for a structural element to fail completely in an event of local structural damage and on the other hand even if a situation is dramatic enough to produce full element loss, it is unlikely that only one structural element will be affected by the event. This paper presents the numerical assessment of disproportionate collapse analysis introducing the concept of partial damage of structural elements. Global robustness measures are proposed for the case of single column damage as well as for the case of damage in multiple adjacent elements. The measures are computed on the basis of a mathematical optimisation problem using collapse load analysis of steel frames with pre-existing damage. The most important findings of the current work are the following:

- In the case when partial column damage is included in a structural system, the effect on the structure is much milder than when a complete element removal is considered. This effect is noticed in a steel frame

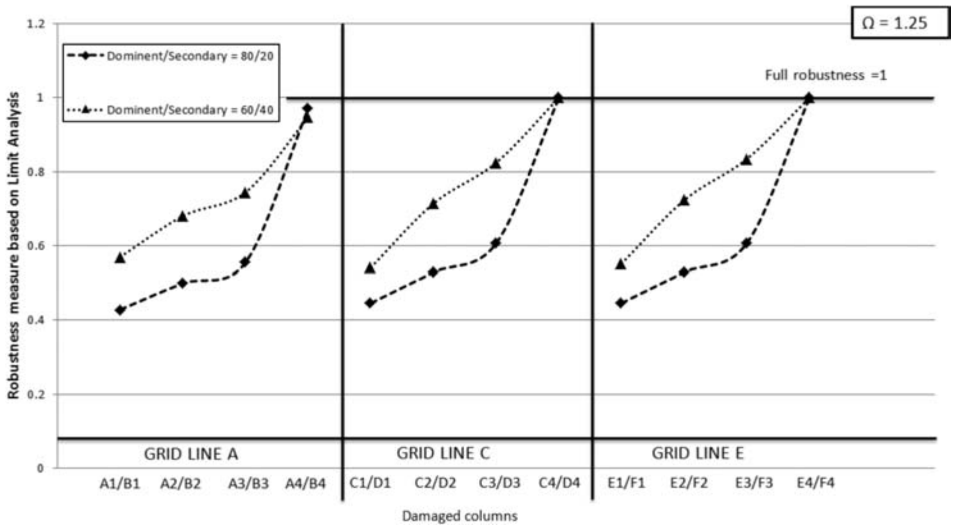

Figure 7. Robustness measures ($r_{\mathrm{LMT}}(i, k)$) based on limit analysis for multiple column damage and Ω.

structure for a series of parametric analyses and several different column locations.

- The significantly milder effect of partial damage (when compared with complete element removal) inside a structural system appears even if damage reaches levels of 60% or −80%.
- Different levels of damage in a system produce different responses. If one is asked to perform progressive collapse analysis, the effect of column removal is not the only way to simulate damage in a system.
- The current work studies also the case where multiple columns are partially damaged. The studies show that the response of the structure is better when damage is distributed in two adjacent elements rather than in only one element. This could be the case for blast events in which several elements will be affected but not completely damaged in order to be considered as removed from the system.
- Further work on the local or global instability of the system should proceed in order to achieve more accurate results of the phenomenon.

Acknowledgements

The preparation of this paper was supported, in part, by an award from the Panhellenic Association of Kappadocian Unions and the scholarship of the family of Filippos Savvopoulos. This support is gratefully acknowledged. However, the views expressed are solely those of the authors and may not represent the positions of the sponsoring organisations.

References

AISC (2005). *Steel Construction Manual*. Chicago, IL: American Institute of Steel Construction.

Andersen, E.D., Roos, C., & Terlaky, T. (2003). On implementing a primal-dual interior-point method for conic quadratic optimization. *Mathematical Programming*, *95*, 249–277, Web site of MOSEK: http://www.mosek.com

ASCE 41 (2007). *Seismic rehabilitation of existing buildings* (1st ed.). Reston, VA: American Society of Civil Engineers.

Baker, J.W., Schubert, M., & Faber, M. (2008). On the assessment of robustness. *Structural Safety*, *30*, 253–267.

Bathe, K.J. (1996). *Finite element procedures*. Upper Saddle River, NJ: Prentice-Hall, Inc.

Bisbos, C.D., & Ampatzis, A.T. (2008). Shakedown analysis of spatial frames with parameterized load domain. *Engineering Structures*, *30*, 3119–3128.

Cavaco, E.S., Casas, J.R., Neves, L.A.C., & Huespe, A.E. (2010). Robustness of corroded reinforced concrete structures – A structural performance approach. *Structure and Infrastructure Engineering*, *9*(1), 1–17.

CEN (2006). *EN 1991-1-3 Design of steel structures*, CEN.

Dubina, D., Dinu, F.L., & Stratan, A. (2010). Tower centre international building in Bucharest Part II: Performance-based seismic evaluation and robustness. *Steel Construction 2010*, *3*, 14–18.

Ellingwood, B.R. (2002). Load and resistance factor criteria for progressive collapse design., Available from http://www.nibs.org

Ellingwood, B.R., Smilowitz, R., Dusenberry, D.O., Dutinh, D., Lew, H.S., & Carino, N. (2007). *Best practices for reducing the potential for progressive collapse in buildings*. Gaithersburg, MD: Department of Commerce.

Ettouney, M., Smilowitz, R., Tang, M., & Hapij, A. (2006). Global system considerations for progressive collapse with extensions to other natural and man-made hazards. *Journal of Performance of Constructed Facilities*, *20*(4), 403–417.

Foley, C.M., Martin, K., & Schneeman, C. (2007). *Robustness in structural steel framing systems* Final report submitted to American Institute of Steel construction. Chicago: Marquette University.

Foley, C.M., Schneeman, C., & Barnes, K. (2008a). Quantifying and enhancing the robustness in steel structures: Part 1-Moment-resisting frames. *Engineering Journal* American

Institute of Steel Construction, (pp. 247–266). Fourth Quarter Chicago, IL.

Foley, C.M., Schneeman, C., & Barnes, K. (2008b). Quantifying and enhancing the robustness in steel structures: Part 2- Floor framing systems. *Engineering Journal* American Institute of Steel Construction, (pp. 267–286). Fourth Quarter Chicago, IL.

Frangopol, D.M., & Curley, J.P. (1987). Effects of damage and redundancy on structural reliability. *Journal of Structural Engineering*, *113*, 1533–1549.

Galal, K., & El-Sawy, T. (2010). Effect of retrofit strategies on mitigating progressive collapse of steel frame structures. *Journal of Constructional Steel Research*, *66*, 520–531.

Gerasimidis, S., & Baniotopoulos, C.C. (2011). Steel moment frames column loss analysis: The influence of time step size. *Journal of Constructional Steel Research*, *67*, 557–564.

Gerasimidis, S., Ampatzis, A., & Bisbos, C.D. (2011). A mathematical programming computational model for disproportionate collapse analysis of steel building frames. *Optimization Letters*, doi:10.1007/s11590-011-0278-2.

Izuddin, B.A., Vlassis, A.G., Elghazouli, A.Y., & Nethercot, D. A. (2008). Progressive collapse of multi-storey buildings to sudden column loss – Part I: Simplified assessment framework. *Engineering Structures*, *30*, 1308–1318.

Kim, J., & Kim, T. (2009). Assessment of progressive collapse-resisting capacity of steel moment frames. *Journal of Constructional Steel Research*, *1*, 169–179.

Kwasniewski, L. (2010). Nonlinear dynamic simulations of progressive collapse for a multistory building. *Engineering Structures*, *32*, 1223–1235.

Marjanishvili, S., & Agnew, E. (2011). Comparison of various procedures for progressive collapse analysis. *Journal of Performance of Constructed Facilities*, *20*, 365–374.

MATLAB version 6.5 (2003). Natick, MA: The MathWorks, Inc.

Skordeli, M.-A.A., & Bisbos, C.D. (2010). Limit and shakedown analysis of 3D steel frames via approximate ellipsoidal yield surfaces. *Engineering Structures*, *32*, 1556–1567.

Starossek, U. (2009). *Progressive collapse of structures*. Hamburg: Thomas Telford.

Tsai, M.H., & Lin, B.H. (2008). Investigation of progressive collapse resistance and inelastic response for an earthquake-resistant RC building subjected to column failure. *Engineering Structures*, *30*, 3619–3628.

The First International Symposium on Disproportionate Collapse (2009). Austin, Texas: ASCE Proceedings of Structures Congress, American Society of Civil Engineers.

US Department of Defense (2009). *United facilities criteria (UFC), design of buildings to resist progressive collapse*. USA: US Department of Defense.