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A mathematical programming computational model for disproportionate collapse analysis of steel building frames

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ORIGINAL PAPER

A mathematical programming computational model for disproportionate collapse analysis of steel building frames

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Abstract Disproportionate collapse analysis aims to assure that frames, a common structural system of buildings, can survive unforeseen local events and a central modeling tool of such abnormal deterioration is the concept of column loss. This paper formulates an appropriate computational model on the basis of mathematical optimization, using the collapse load analysis problem of steel frames with pre-existing damage. A respective collapse load robustness measure is proposed. The model has the ability to consider both full and partial column/node removals. It renders to be a linear programming model, if the US steel design regulations are used. A practical example is presented and several aspects are discussed.

Keywords Disproportionate collapse analysis · Limit analysis · Damaged steel frames · Robustness · Linear programming

1 Introduction

Engineers have two categories of methods at their disposal for the assessment of collapse analysis of structures: the finite element (FEM) nonlinear step-by-step methods and the limit analysis methods of plasticity (cf. [1]). The first category is more general, but it can be very time-consuming, if, e.g. repeated analyses are to be considered. Limit analysis methods are focused on the determination of the global safety margins of the structure. They lead to the solution of a convex optimization problem with data provided by a linear FEM analysis (cf. [2,3]).

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In engineering analysis, situations are often encountered, where multi-level losses occur, which are disproportionate to the triggering event (cf., e.g. [4]). Among them, disproportionate (or progressive) collapse analysis plays a prominent role [5–7]. Disproportionate collapse resistance of structural systems has been commonly accepted as the property of a structural system to maintain its integrity even in cases of unforeseen local failures which can occur due to abnormal events such as gas explosions, terrorist attacks, etc. During the last decades, there has been an upward trend of failures in this pattern; among the most prominent of all are the structural failure of the 22-storey building in Ronan point in 1968, the partial collapse of the Alfred P. Murrah building in Oklahoma City in 1995 and the WTC in New York City in 2001. An important issue within disproportionate collapse analysis consists in quantifying the robustness of the whole structural system against these unforeseen local triggering events.

A plethora of related research activity has emerged throughout the structural engineering community in an attempt to assess the problem and its consequences (cf., e.g. [5,8–15]). The outcome of these research efforts has been instilled in normative regulations such as the Department of Defense (DoD) criteria [6] and the General Services Administration (GSA) guidelines [7]. These two documents have dominated so far the field of regulative disproportionate collapse analysis and design and are commonly accepted as such.

The difficulties in the calculation of the collapse resistance properties of structures derives from the fact that the unforeseen events, usually related to disproportionate collapse events, are impossible to be incorporated in a computational model via usual engineering loading concepts since any data for them are absent and the associated uncertainties are significant. Therefore, among others, an inverse way of simulation is commonly recommended by the guidelines and used by researchers to assess the phenomenon. An appropriate modeling concept used in that direction is the so-called column loss which suggests the analysis of the structural system incorporating the absence of an important part of the structure such as a column, implying that the specific element is completely damaged due to its exposure to an abnormal event. Local overloading above the removed column is introduced in order to take into account dynamic amplification effects. Evidently, disproportionate collapse analysis must be performed in an enumerative way, i.e. by assuming that all the columns of a building structure are consecutively removed one-by-one and the structure is checked for all the respective cases regarding its capacity to survive under the given loading.

Building frames can be viewed as graphs and columns as graph edges (cf., e.g. [16]). In this context, disproportionate collapse analysis can be considered as a specific engineering problem of critical edge detection, exhibiting some qualitative similarities with the critical node detection problem in graphs (cf. [17]).

This work exploits plastic limit analysis to propose a mathematical programming computational model for the treatment of ductile failures in the disproportionate collapse analysis of steel building frames. An appropriate collapse load robustness measure is proposed. This computational model renders to be a linear programming (LP) model if the yield criteria of the American Institute of Steel Construction (AISC) [18] are used.

Simple concepts from damage mechanics are, respectively, exploited (cf. [19–22]). Local Kachanov damage parameters are introduced across the whole frame and column loss is considered as a special case, determined by specific values of these damage parameters. A real-world numerical example is presented.

This way, the treatment of other important unforeseen events is straightforward: important practical examples are the partial loss of pairs of adjacent columns or unforeseen damage concentrated in the vicinity of a node of the building frame.

2 Model formulation

2.1 Limit analysis of damaged steel structures

Let Θ be a steel building structure, discretized within a geometrically linear FEM framework and let NU be the number of free nodal displacements. In this work we follow the notation of [23,24].

Θ contains NG numerical integration points (Gauss points), which will be used in this work as stress evaluation points. Local quantities, always referred to the jth Gauss point of Θ, are denoted by the respective subscript. E.g. $\mathbf{s}_j \in \mathbb{R}^{NS}$ denotes the respective local stress vector. If Θ is a spatial frame, then $NS = 6$.

In a quasi-lower bound framework, our starting point is the following convex optimization problem with unknowns the local stress vectors $\mathbf{s}_j$ and the safety factor α, which constitutes the limit analysis problem of the intact structure Θ:

$$P_{\mathrm{LMT}}(\boldsymbol{\phi}, 0) : \max\ \alpha, \quad \text{s.t.:}$$
$$\sum^{NG} \mathbf{H}_j\, \mathbf{s}_j\ = \alpha \boldsymbol{\phi}, \quad \mathbf{s}_j \in \mathcal{S}_j\ (\boldsymbol{\kappa}_j), \quad j = 1, \ldots, NG. \tag{1}$$

whose data are the nodal load vector $\boldsymbol{\phi} \in \mathbb{R}^{NU}$, the local matrices $\mathbf{H}_j \in \mathbb{R}^{NU \times NS}$ and the sets $\mathcal{S}_j \subset \mathbb{R}^{NS}$. Data $\boldsymbol{\phi}$ and $\mathbf{H}_j$ are provided by a linear FEM analysis.

The local yield criteria sets $\mathcal{S}_j$ are convex, compact and they contain the origin. Their volume (but not their shape) is determined by the positive local parameters (capacity vectors) $\boldsymbol{\kappa}_j$, which depend on the material grade and the shape and dimensions of the steel sections used. The boundaries of the yield criteria sets can be plane or curved (cf., e.g. [24,25]). In the first case the sets are polytopes described by:

$$\mathcal{S}_j(\boldsymbol{\kappa}_j) = \{\mathbf{x} \in \mathbb{R}^{NS}\ :\ \mathbf{L}_j^T \mathbf{x} \leq \boldsymbol{\kappa}_j\} \tag{2}$$

The most important class of nonlinear yield criteria are quadratic, which are derived from the von Mises yield criterion. Usually, they take the form:

$$\mathcal{S}_j(\boldsymbol{\kappa}_j) = \{\mathbf{x} \in \mathbb{R}^{NS}\ :\ \mathbf{x}^T \mathbf{Q}\, \mathbf{x} \leq \boldsymbol{\kappa}_j^2\} \tag{3}$$

Problem $P_{\mathrm{LMT}}(\boldsymbol{\phi}, 0)$ provides the safety factor against collapse of the structure without any deterioration due to eventual existing damage. Now, we introduce damage via Kachanov indexes δ_j satisfying:

$$0 \leq \delta_j \leq 1, \quad j = 1, \ldots, NG \tag{4}$$

which determine the local damage degree. The respective capacity vectors read:

$$\boldsymbol{\kappa}_{dj} = (1 - \delta_j)\boldsymbol{\kappa}_j, \quad j = 1, \ldots, NG \tag{5}$$

Lower bound $\delta_j = 0$ is the local intact state condition (no damage) and upper bound $\delta_j = 1$ characterizes the state of full local damage (zero volume of set $\mathcal{S}_j$). An element (e.g. a column) is removed, if the full-damage condition holds for all its Gauss points. We will denote by $\boldsymbol{\delta}$ the vector of dimension NG, which lists all δ_j of the structure. Now, Eq. (1) takes the form:

$$\begin{aligned} P_{\mathrm{LMT}}(\boldsymbol{\phi}, \boldsymbol{\delta}) : &\max\ \alpha, \quad \text{s.t.:} \\ &\sum^{NG} \mathbf{H}_j\, \mathbf{s}_j\ = \alpha\boldsymbol{\phi}, \\ &\boldsymbol{\kappa}_{dj} = (1 - \delta_j)\boldsymbol{\kappa}_j, \quad \mathbf{s}_j \in \mathcal{S}_j\ (\boldsymbol{\kappa}_{dj}), \quad j = 1, \ldots, NG. \end{aligned} \tag{6}$$

Let $\alpha^*_{\mathrm{LMT}}(\boldsymbol{\phi}, 0)$ and $\alpha^*_{\mathrm{LMT}}(\boldsymbol{\phi}, \boldsymbol{\delta})$ denote the objective function optimal values of the $P_{\mathrm{LMT}}(\boldsymbol{\phi}, 0)$ and $P_{\mathrm{LMT}}(\boldsymbol{\phi}, \boldsymbol{\delta})$, respectively. The ratio:

$$r_{\mathrm{LMT}}(\boldsymbol{\phi}, \boldsymbol{\delta}) = \alpha^*_{\mathrm{LMT}}(\boldsymbol{\phi}, \boldsymbol{\delta})\ /\ \alpha^*_{\mathrm{LMT}}(\boldsymbol{\phi}, 0) \tag{7}$$

expresses the damage effects on the safety margins of the structure under the loading $\boldsymbol{\phi}$. This ratio is of primary engineering importance and will be called, in the sequel, as the collapse load robustness.

2.2 Disproportionate collapse analysis

According to the DoD regulations [6], disproportionate collapse analysis must ensure that the building structure survives several alternative column losses (see Sect. 1). DoD recommends several direct and indirect methodologies to assure the survivability of the structure and the most widely used direct one is the alternate load path method. Furthermore, a clear distinction is made between deformation-controlled and force-controlled actions. The concept of the distinction lies upon the encapsulation of ductile as well as brittle failures, respectively. In the first case, plastic redistribution of the stresses within the whole structure is allowed for and, consequently, the limit analysis methods of plasticity can be applied. Within this framework, survivability of the structure means that the respective safety factor α remains greater than unity.

Collapse load analysis for column loss via limit analysis of damaged structures is straightforward. Let $\mathcal{K}$ be the set of all columns, whose removal is to be considered. Let us assign to each column in $\mathcal{K}$ a global damage vector $\boldsymbol{\delta}_k$:

$$k \in \mathcal{K} \quad \mapsto \quad \boldsymbol{\delta}_k \in \mathcal{R}^{NG} \tag{8}$$

All entries of $\boldsymbol{\delta}_k$ are zero except those ones, which correspond to the Gauss points contained within the removed column, which are set equal to one (full damage).

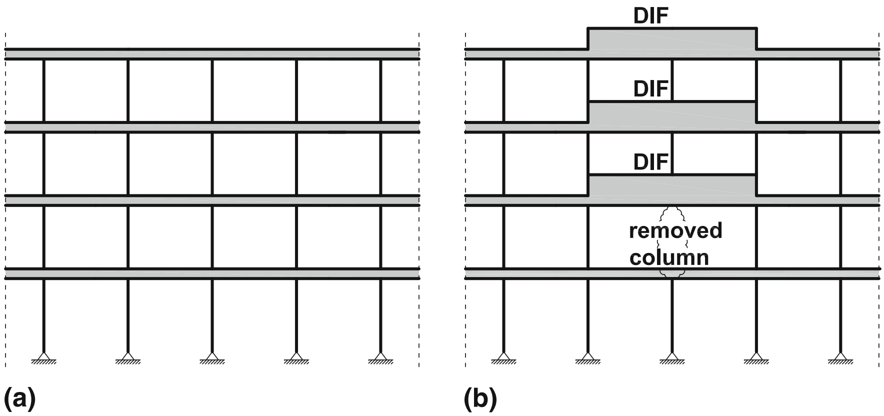

Fig. 1 Frame loading scheme: **a** intact frame, **b** column loss at second floor

DoD regulates the loadings $\boldsymbol{\phi}$ to be considered in disproportionate collapse analysis in the following way. Firstly, four basic load combinations are defined: $\boldsymbol{\phi}_B^{(i)}$, $i \in \mathcal{L}$ with $\mathcal{L} = \{1, 2, 3, 4\}$. These loadings have a global character for the building without any consideration of column removal, being distinct combinations of dead, live and snow loads. Assuming that the k-column is removed, each basic combination $\boldsymbol{\phi}_B^{(i)}$, $i \in \mathcal{L}$ generates the respective actual load cases

$$\boldsymbol{\phi}^{(i)}(k) = \boldsymbol{\phi}_B^{(i)} + (\Omega_N(k) - 1)\, \Delta\boldsymbol{\phi}_B^{(i)}(k) \tag{9}$$

where $\Omega_N(k)$ is a dynamic increase factor (DIF) to capture dynamic amplification phenomena. $\Delta\boldsymbol{\phi}_B^{(i)}(k)$ is that part of $\boldsymbol{\phi}_B^{(i)}$, corresponding to the neighboring floor areas above the removed kth column. This local overloading scheme is shown in Fig. 1.

Now, setting for the shake of brevity:

$$\alpha_{\text{LMT}}(i, k) = \alpha^*_{\text{LMT}}(\boldsymbol{\phi}^{(i)}(k), \boldsymbol{\delta}_k) \tag{10}$$

the survivability requirement of DoD can be cast in the form:

$$\min_{k \in \mathcal{K}} \min_{i \in \mathcal{L}} \alpha_{\text{LMT}}(i, k) \geq 1 \tag{11}$$

and we can define the collapse load robustness:

$$r_{\text{LMT}}(i, k) = \alpha_{\text{LMT}}(i, k) \,/\, \alpha_{\text{LMT}}(i, 0) \tag{12}$$

where:

$$\alpha_{\text{LMT}}(i, 0) = \alpha^*_{\text{LMT}}(\boldsymbol{\phi}_B^{(i)}, 0) \tag{13}$$

3 An LP model for the American steel section yield criteria

In spatial frame analysis, the local stress vector $\mathbf{s}_j$ contains the axial/shearing forces and the twisting/bending moments:

$$\mathbf{s}_j = (N, V_y, V_z, M_t, M_y, M_z)^T$$

and let the respective individual intact plastic capacities N_{pl}, $V_{pl.y}$, $V_{pl.z}$, $M_{pl.t}$, $M_{pl.y}$ and $M_{pl.z}$ be collected in the diagonal entries of a local 6×6 diagonal matrix $\mathbf{N}_j$. This way, the dimensionless local stress equals $\mathbf{N}_j^{-1}\mathbf{s}_j$.

Local stress $\mathbf{s}_j$ must belong to set $\mathcal{S}_j$. Analytically, the components of $\mathbf{s}_j$ have to satisfy the local yield criteria, modified by damage, which comprise the individual bounds posed by the individual capacities and the plastic interaction conditions. In this work, $N - M_y - M_z$ interactions are considered, which incorporate the respective individual capacity bounds.

The interactive part $\mathbf{y}_j$ of the dimensionless local stress vector has to obey the damage-modified plastic interaction yield criteria of the AISC (see [18]):

$$\begin{aligned}\mathbf{y}_j \in \mathcal{F}_j\,(\delta_j) \;&= \{(m_y, m_z, n) \in \mathbb{R}^3 \;:\; |n| + (8/9)(|m_y| + |m_z|) \leq 1 - \delta_j, \\ &\quad 0.5|n| + |m_y| + |m_z| \leq 1 - \delta_j\}\end{aligned} \tag{14}$$

The non-interactive part $\mathbf{z}_j$ has to satisfy the bounds posed by the individual plastic capacities:

$$\mathbf{z}_j \in \mathcal{C}_j\,(\delta_j) \;= \{(v_y, v_z, m_t) \in \mathbb{R}^3 \;:\; |v_y| \leq 1 - \delta_j, \quad |v_z| \leq 1 - \delta_j, |m_t| \leq 1 - \delta_j\} \tag{15}$$

Equations (14) and (15), defining the damage-modified yield criterion set $\mathcal{S}_j$ (see Fig. 2), are to be used with the corresponding partitioning of $\mathbf{s}_j$:

$$\mathbf{s}_j = \mathbf{N}_j(\mathbf{P}_y\mathbf{y}_j + \mathbf{P}_z\mathbf{z}_j) \tag{16}$$

where the permutation matrices $\mathbf{P}_y$, $\mathbf{P}_z$ are given by:

$$\mathbf{P}_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{P}_z = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

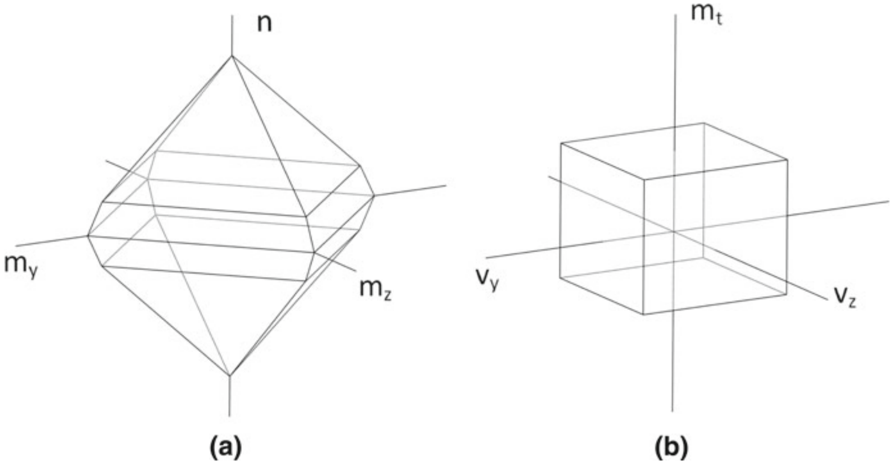

Fig. 2 AISC yield criterion set: **a** interactive, **b** non-interactive part

This way, the limit analysis problem of the damaged steel frame reads:

$$
\begin{aligned}
P_{\mathrm{LMT}}(\boldsymbol{\delta}, \boldsymbol{\phi}) \quad & \max \alpha, \\
& \sum^{NG} \mathbf{H}_j\, \mathbf{s}_j = \alpha \boldsymbol{\phi}, \\
& \quad \mathbf{s}_j = \mathbf{N}_j (\mathbf{P}_y \mathbf{y}_j + \mathbf{P}_z \mathbf{z}_j), \\
& \quad \mathbf{y}_j \in \mathcal{F}_j\,(\delta_j), \qquad \mathbf{z}_j \in \mathcal{C}_j\,(\delta_j), \\
& \quad j = 1, \ldots, NG.
\end{aligned} \tag{17}
$$

4 Numerical example

4.1 Example data

The example presented in this section is a simplified version of the "Structural Steel example" analyzed in the Appendix E of DoD; all the information regarding the geometry and the structural data of the system can be found in [6]. It is a four-story, nine-bay steel lateral moment frame and all the connections are improved welded unreinforced flange connections. For the purposes of the collapse analysis, 12 separate column losses were considered, namely 4 (1 in each floor) in each of the column grid lines A, C and E (see Fig. 3a).

The structure is modeled as a two-dimensional frame, comprising 726 nodes. Connections to foundation are considered pinned, therefore the free degrees of freedom amount to $NU = 4{,}356$. The FEM mesh contains 356 standard three-node Timoshenko beam-column isoparametric elements with 2 Gauss points per element [26]. The total number of Gauss points amounts $NG = 752$.

A respective, simple linear FEM code (cf. [23]) has been used, which provides the equilibrium matrices $\mathbf{H}_j$, not available via commercial FEM software. This code has provided the necessary data to be used by the mathematical programming software

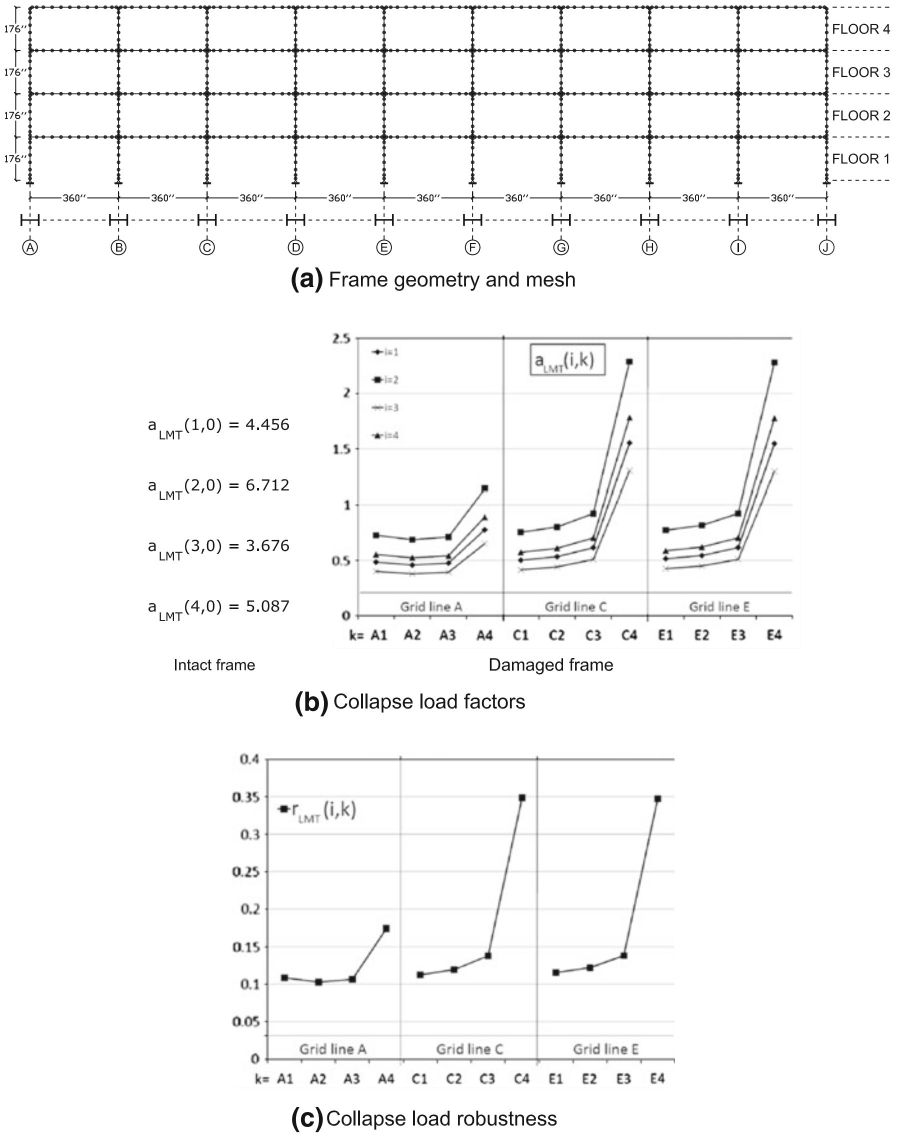

Fig. 3 Example frame: **a** geometry, **b** safety factors, **c** robustness

MOSEK [27]. Linking has been realized via a simple code, written in MATLAB [28], which provides the possibility of changing the plastic interaction criteria.

4.2 Results

Computational results are shown in Fig. 3b, c concerning all the different loadings i and removed columns k, where, e.g. column loss C2 means column removal at second floor of grid line C. The dramatic influence of the column loss to the structure's response is evident: the values of $\alpha_{\mathrm{LMT}}(i, k)$ are significantly lower than the values of safety factor for the intact case. In the worst case, the limit analysis safety factor is downgraded by a factor of approximately 10 after the column removal.

A first remark concerns the critical floor position: for all loadings and grid lines, when the column removal happens in the first floor, the impact on the structure is greater and becomes milder as the column removal happens in higher floors. Moreover, although the pattern of the results for the different grid lines is almost the same, edge grid line A remains the most critical grid line of all. The same conclusion was achieved by the results of Kim and Kim [14] and Foley et al. [15] who have concluded that the corner column of a steel frame is the most important for the structure.

The loadings prescribed by the DoD are all gravity loadings and the four load combinations are essentially affine. This property is reflected to the fact, that the values of collapse load robustness $r_{\mathrm{LMT}}(i, k)$ coincide to the third digit for the same column, i.e. they are practically independent of the loading i. Therefore, in Fig. 3c, the robustness results of the analyses are presented which are unique for each column removal. Again, grid line A corresponds to the lowest values of robustness, while floor 1 remains the worst case.

Interestingly, we have observed the same load affinity influence in the kinematical collapse patterns (plastic hinge formations) not shown in Fig. 3: they are independent on the loading case i. Generally speaking, it seems that collapse robustness is essentially independent from the loading for practically affine loading patterns. In other words, collapse robustness appears as an inherent structural system property in the aforementioned restricted sense.

A final remark concerns the comparison between the two inner grid lines B and E. By virtue of the inherent properties of plastic limit analysis and due to the regularity of the building, the influence of the neighboring edge grid line A is vanishing and, consequently, the results are almost identical.

4.3 Comments

The main practical goal of disproportionate collapse analysis consists in the re-design of the structural system of the building in order to assure its survivability against unforeseen local events. Design amelioration (re-design) is always gradual and the mathematical optimization model, presented in this work, can be used in a first series of analyses concerning ductile failures, where a second cycle will contain detailed step-by-step nonlinear analyses in order to capture phenomena not addressed in limit analysis as second-order effects, ductility considerations, etc.

Within this framework, it is mention worthy that the critical load combination can be determined either by computing the intact safety factors $\alpha_{\mathrm{LMT}}(i, 0)$ or, even more, by inspection due to the essential affinity of the load combinations. Consequently, re-design could be performed for all the respective column losses in the first floor. Remarkably, re-design can be formulated also as a mathematical programming problem.

The last topic is not addressed in the present work but some remarks seem mention worthy. Essentially, the re-design problem retains the form of problem Eq. (17) with the following modifications: the plastic capacity matrices $\mathbf{N}_j$ become now a part of the unknowns, since they are to be selected from a list of available sections. The objective function of the re-design problem has the form $\sum f_j$ where each f_j is a linear function of the respective $\mathbf{N}_j$. Finally, the safety factor α must be subjected to the additional constraint $\alpha \geq 1$. Evidently, the part of problem Eq. (17) connecting $\mathbf{s}_j$ with $\mathbf{N}_j$ becomes now a nonlinear equation, rendering the problem to be a non-convex one.

From the mechanical point of view, special attention deserves the assessment of the ductility requirements. Strictly speaking, standard limit analysis methods are valid only for unlimited ductility. In practice, the ductility demands (plastic deformations) are part of the solution of the dual optimization problem and they are automatically provided, if a primal-dual optimization algorithm is applied. Consequently, they can be a posteriori checked against given ductility thresholds. Since steel is a ductile material, ductility limitations are generally not considered as a serious overburden for the application of limit analysis for steel buildings.

Concerning the numerical implementation cost, our computational model leads to a series of typical linear or nonlinear optimization problems and respective well known facts apply. For example, on a Dell workstation with dual quad core Xeon at 3.0 GHz and 16 GB RAM, all individual LPs involved in the example required <2 s using the default settings of MOSEK.

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