Summer June, 2001

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ABSTRACT

Suppression of a large-angle stimulated Raman scattering (LA-SRS) of a short modulated (two-frequency) laser pulse in a transparent plasma in the presence of a linear long-wavelength electron plasma wave (LW EPW) having relativistic phase velocity is considered under the conditions of weak and strong coupling. The laser spectrum includes two components with a frequency shift equal to the frequency of the LW EPW. The mutual influence of different spectral components of a laser on the SRS under a given angle in the presence of the LW EPW is examined.

1 INTRODUCTION

The stimulated Raman scattering through large angles of laser radiation in plasmas[1–3] is known to have a significant effect on a propagation of a short (< 1 ps) intense laser pulse in plasmas,[4,5] and, hence, on operation of various schemes of laser-plasma particle accelerators.[6] In order to optimize these schemes, one must clarify the conditions under which the detrimental pulse errosion due to the LA-SRS could be minimized.

We have shown previously[7–9] that certain nonlinear processes of a laser pulse evolution in a rarefied plasma can suppress the backward and near-backward SRS. Namely, when an amplitude modulation of a pulse occurs with a spatial period close to $\lambda_p = 2\pi c/\omega_{pe}$, where $\omega_{pe} = \left(4\pi e^2 n_0/m_e\right)^{1/2}$ is an electron plasma frequency corresponding to the electron background density n_0 (which is the case under the conditions of resonant self-modulation[10] of a pulse), the resonant suppression of the LA SRS of higher-frequency spectral components of the modulated pulse occurs.[8] We noted[7,8] that the self-modulation of a pulse is accompanied by the excitation of a large amplitude long-wavelength electron plasma wave with a phase velocity close to the group velocity of a pulse, hence, to ignore the influence of the LW EPW on the scattering process would be incorrect in many cases. We found[9] that the features of the LA-SRS of a monochromatic pulse (we denote its frequency ω_0, $\omega_0 \gg \omega_{pe}$) in the presence of the LW EPW whose phase velocity coincides with a group velocity of a laser radiation are follows:

- The Stokes band of the weakly coupled SRS [$a_1 = eE_1/(m_e\omega_0 c) \ll (\omega_{pe}/\omega_0)^{1/2}$, where a_1 is a normalized amplitude of the laser electric field] undergoes a suppression when a normalized amplitude of a density perturbation $N_{LW} \equiv \delta n_{LW}/n_0$ in a LW EPW becomes of order or exceeding the ratio ω_{pe}/ω_0; under the same condition the anti-Stokes branch appears.

Laser Optics 2000: Ultrafast Optics and Superstrong Laser Fields, Alexander A. Andreev, Vladimir E. Yashin, Editors, Proceedings of SPIE Vol. 4352 (2001) © 2001 SPIE · 0277-786X/01/$15.00

- For the limit of strong coupling $[(\omega_{pe}/\omega_0)^{1/2} < a_1 < 1]$, the density perturbation amplitude must be as large as $N_{LW} > (a_1\omega_{pe}/\omega_0)^{2/3} \gg \omega_{pe}/\omega_0$ to approximately halve the maximum increment of the LA SRS.

The suppression of the LA-SRS in the presence of the LW EPW is caused by the multi-wave nature of the process. In an unperturbed plasma, the weakly coupled LA-SRS is a three-wave resonant process, in which a pump EM wave $(\omega_0, \mathbf{k}_0)$ decays into a scattered EM wave $(\omega_0 - \omega_{pe},\ \mathbf{k}_s)$ and a plasma natural mode $(\omega_{pe},\ \mathbf{k}_e)$, where $\mathbf{k}_e = \mathbf{k}_0 - \mathbf{k}_s$. In a plasma perturbed by a LW EPW the phase modulation of scattering Langmuir waves[9,11] occurs. Then, the complex spectrum of scattering plasma waves $(\omega_{pe} + n\omega_{LW}, \mathbf{k}_e + n\mathbf{k}_{LW})$[11] participates in the scattering process. The sidebands (which are not natural plasma modes) are shifted to integer multiples of a frequency $\omega_{LW} \approx \omega_{pe}$ and wavenumber $k_{LW} \ll k_e$ of the LW EPW, and can exist only at the expense of the original natural Raman mode $(\omega_{pe}, \mathbf{k}_e)$, whose energy can be completely exhausted for $N_{LW} \sim \omega_{pe}/\omega_0$.[11] Hence, in the presence of a LW EPW, the transfer of the energy of the unstable plasma oscillations to a great number of satellites, which are not natural modes, dramatically decreases the growth rate of the instability.[9]

The results reported in the present paper substantially supplement the earlier investigations.[7–9] We have examined the spectral features of the LA-SRS of a modulated (two-frequency) laser pulse in a rarefied plasma in the presence of a given linear LW EPW with a phase velocity close (not necessarily equal) to the phase velocity of a laser pulse. In order to describe the LA-SRS of a laser pulse under the conditions of resonant self-modulation, the two-frequency pulse is considered with a frequency difference equal to the frequency of the LW EPW, which is close to the electron plasma frequency. Since the frequency difference of the laser spectral components (pump waves) is close to ω_{pe}, resonant suppression of the SRS of the higher-frequency pulse component[7,8] does not occur, because such effect necessitates the frequency difference of the pump waves close to $2\omega_{pe}$. In the presence of a given linear LW EPW, the convective amplification of the unstable modes in the frame of reference co-moving with the pulse is studied in the approximations of weak and strong coupling. In the regime of weak coupling, the dispersion analysis of the instability has shown that the Stokes components of scattered radiation from spectral components of the pulse are both suppressed provided $N_{LW} > \omega_{pe}/\omega_0$. The latter condition fulfilled, the anti-Stokes bands from both components of a pulse appear in the spectrum of scattered light (however, the anti-Stokes increments remain small if compared with the increments of the instability in a non-perturbed plasma). We have found that the LA-SRS of a higher-frequency pump wave undergoes more severe suppression in the presence of the LW EPW than the scattering of the lower-frequency one (it is notable that the increment of the latter even increases until $N_{LW} \leq 0.5\,\omega_{pe}/\omega_0$, and for larger N_{LW} the decrease occurs). The phenomenon of suppression is found to be almost independent on the detuning $\Delta\omega_{LW} \equiv \omega_{LW} - \omega_{pe}$ of a frequency of the LW EPW, which can occur in the process of the resonant self-modulation (in such case, $|\Delta\omega_{LW}| \ll \omega_{pe}$[12]). Under the conditions of strong coupling, when the scattering process is nonresonant in character, and its spectrum is much wider than the electron plasma frequency,[1,2] the instability experiences suppression for $N_{LW} > (a_1\omega_{pe}/\omega_0)^{2/3} \gg \omega_{pe}/\omega_0$ rather than for $N_{LW} \sim \omega_{pe}/\omega_0$.

2 BASIC EQUATIONS

To describe the LA SRS of a laser radiation in rarefied plasmas we represent a high-frequency (HF) electric field in a plasma as a sum of two components with close frequencies

$$\mathbf{a}(\mathbf{r}, t) = \frac{1}{2}\left\{\mathbf{a}_0(\mathbf{r}, t)e^{-i\omega_0 t + ik_0 z} + \mathbf{a}_s(\mathbf{r}, t)e^{-i\omega_s t + i(\mathbf{k}_s, \mathbf{r})}\right\} + c.c., \tag{1}$$

where the dimensionless amplitudes of a laser pulse $\mathbf{a}_0 = e\mathbf{E}_0/(m_e\omega_0 c)$ and scattered radiation $\mathbf{a}_s = e\mathbf{E}_s/(m_e\omega_s c)$ are assumed to vary slowly in time and space on the scales $\omega_{0(s)}^{-1}$ and $k_{0(s)}^{-1}$, respectively. In the present report we consider the nonrelativistic electron motion in the electromagnetic field, so $|\mathbf{a}_{0(s)}| \ll 1$. Both incident and scattered radiation obey the dispersion relation for electromagnetic (EM) waves in plasmas $\omega_{0(s)}^2 = \left(k_{0(s)}c\right)^2 + \omega_{pe}^2$. Hence, in the case of strongly rarefied plasma $(\omega_{0(s)} \gg \omega_{pe})$, one can let, without loss of generality, $\omega_0 = \omega_s$ and

$|\mathbf{k}_s| \equiv k_s = k_0$ thus including possible deviation of a frequency and wave number of scattered radiation from ω_0 and k_0 in the spatio-temporal dependence of the envelope $\mathbf{a}_s(\mathbf{r}, t)$. Hence, the wave vector of the scattered field is determined as $\mathbf{k}_s = (\mathbf{k}_{s\perp}, k_0 \cos\alpha)$, where $|\mathbf{k}_{s\perp}| \equiv k_{s\perp} = k_0 \sin\alpha$, and $\alpha \leq \pi$ is a scattering angle reckoned from the direction of a pulse motion. Since now, we consider the case of the linearly polarized laser light and analyze the SRS through a given angle in the plane orthogonal to the plane of the laser polarization ($\mathbf{k}_{s\perp} \perp \mathbf{a}_0$), where the coefficient of amplification of waves achieves a maximum.[5,9] Ponderomotive force at the beat frequency of incident and scattered EM waves excites an electron density perturbation with a chatacteristic wave vector $\mathbf{k}_e = \mathbf{e}_z k_0 - \mathbf{k}_s$ [so that $k_{ez} = 2k_0 \sin^2(\alpha/2)$ and $k_e \equiv |\mathbf{k}_e| = 2k_0 \sin(\alpha/2)$], which is responsible for the stimulated Raman scattering through an angle α:

$$\delta\tilde{n}_s(\mathbf{r}, t) = \frac{1}{2}\, \delta n_s(\mathbf{r}, t) e^{i(\mathbf{k}_e, \mathbf{r})} + c.c. \tag{2}$$

Assuming that the scattering angle is not small [$\sin(\alpha/2) \sim 1$], we suppose that the envelope of the scattering plasma waves $\delta n_s(\mathbf{r}, t)$ is also slowly varying on the laser time and space period. Apart from the short-wavelength plasma waves, which participate in the scattering process, the short laser pulse can create a long-wavelength electron plasma density perturbations.[10,13] These long-wavelength electron plasma waves possess a relativistic phase velocity close to the group velocity of a laser pulse, and can be utilized in various schemes of plasma-based particle accelerators.[6] The electron density perturbations related with such waves may have an amplitude of order tens per cent of the background electron density.[14]

We will describe the SRS of a laser radiation under a given angle in a time scale short compared to the ion plasma period $\tau_i = 2\pi/\omega_{pi}$ [where $\omega_{pi} = (4\pi e^2 n_0/m_i)$ is an ion plasma frequency], using nonrelativistic hydrodynamic equations for a cold electron fluid with the immobile ion background and the Maxwell equations for scattered radiation. Apart from the scattering plasma waves (2), we account for the presence of a LW EPW in a plasma. We arrive at a pair of linear coupled equations for the amplitudes $\mathbf{a}_s$ and $N_s \equiv \delta n_s/n_0$, in which the electron quiver velocity $\tilde{\mathbf{v}}_{LW}(\mathbf{r}, t)$ of the LW EPW enters ($\tilde{\mathbf{v}}_{\mathrm{LW}}$ known, corresponding long-wavelength electron density perturbation $\delta\tilde{n}_{\mathrm{LW}}(\mathbf{r}, t)$ can be retrieved using the continuity equation):

$$i\left(\frac{\partial}{\partial t} + (\mathbf{v}_g, \nabla)\right)\mathbf{a}_s = \frac{\omega_{pe}^2}{4\omega_0}\mathbf{a}_0 N_s^*, \tag{3}$$

$$\left(\left(\frac{\partial}{\partial t} + (\tilde{\mathbf{v}}_{LW}, \nabla) - i(\mathbf{k}_e, \tilde{\mathbf{v}}_{LW})\right)^2 + \omega_{pe}^2\right) N_s^* = -\frac{1}{2}(k_e c)^2 (\mathbf{a}_0^*, \mathbf{a}_s). \tag{4}$$

In Eq. (3), $\mathbf{v}_g = c^2\mathbf{k}_s/\omega_0$ (as the limit of strongly rarefied plasma is under consideration, we set $|\mathbf{v}_g| = c$). The functions $\mathbf{a}_0(\mathbf{r}, t)$ and $\tilde{\mathbf{v}}_{LW}(\mathbf{r}, t)$ may be determined self-consistently, or be given in a certain form. In the latter case, Eqs (3) and (4) form the basis of a linear theory of the LA SRS in plasmas. Eq. (4) is linear in the amplitudes of the decay waves, and nonlinear in $\tilde{\mathbf{v}}_{LW}$. Generally, the LW EPW might be nonlinear wave, and $\tilde{\mathbf{v}}_{LW}$ may be represented as a row expansion in harmonics of ω_{LW} and k_{LW} whose amplitudes are expressed through powers of a normalized amplitude of the density perturbation $N_{LW} = \delta n_{LW}/n_0$ in a linear wave[11,15] [The connection between N_{LW} and $\tilde{\mathbf{v}}_{LW}$ in a linear wave is given by Eq. (9)]. In the present report we presume the LW EPW to be the linear wave. This is possible when the effect of the LW EPW nonlinearity on the scattering process is negligible, which necessitates the following limitation of the amplitude of the density perturbation[9]

$$N_{LW} < \left(\frac{\omega_{pe}}{\omega_0}\right)^{1/2}. \tag{5}$$

Throughout the present report, we meet the condition (5) and omit the term $(\tilde{\mathbf{v}}_{LW}, \nabla)$ in the second-order operator of Eq. (4).[9] Further, for the sake of simplicity, we exclude the variable coefficient $\tilde{\mathbf{v}}_{LW}$ from Eq. (4) by means of a unitary replacement

$$\{\mathbf{a}_s, N_s^*\} = \left\{\hat{\mathbf{a}}_s, \hat{N}_s^*\right\}\exp\left(-i\tilde{\Psi}\right), \tag{6}$$

$$\tilde{\Psi} = -\int_0^t (\mathbf{k}_e, \tilde{\mathbf{v}}_{LW})\, d\tau,$$

which accounts for the phase modulation of the decay waves in the presence of the LW EPW. Since the transformation (6) preserves the absolute value, the instability onset may be studied in terms of the amplitudes $\left\{\hat{\mathbf{a}}_s, \hat{N}_s^*\right\}$. Instead z and t, it is convenient to use variables $\xi = z - ct$ and $\eta = t$ of the frame of reference co-moving with the pulse ($\mathbf{r}_\perp$ remains the same in both frames). When the change of variables is made, the equations for the amplitudes of the modulated waves read

$$i\left(\frac{\partial}{\partial\eta} + (\tilde{\mathbf{v}}_g, \tilde{\nabla})\right)\hat{\mathbf{a}}_s + \hat{\mathbf{a}}_s\left(\frac{\partial\tilde{\Psi}}{\partial\eta} + (\tilde{\mathbf{v}}_g, \tilde{\nabla})\tilde{\Psi}\right) = \frac{\omega_{pe}^2}{4\omega_0}\mathbf{a}_0\hat{N}_s^*, \tag{7}$$

$$\left(\left(\frac{\partial}{\partial\eta} - c\frac{\partial}{\partial\xi}\right)^2 + \omega_{pe}^2\right)\hat{N}_s^* = -\frac{1}{2}(k_e c)^2(\mathbf{a}_0^*, \hat{\mathbf{a}}_s). \tag{8}$$

Here, $\tilde{\mathbf{v}}_g = \left(c^2\mathbf{k}_{s\perp}/\omega_0, -2c\sin^2(\alpha/2)\right)$, $\tilde{\nabla} = (\partial/\partial\mathbf{r}_\perp, \partial/\partial\xi)$.

We consider below a given shape of both laser pulse and LW EPW thus neglecting their evolution on time scales of interest. The LW EPW is given to be one-dimensional, linear and free, with the electron quiver velocity

$$\tilde{\mathbf{v}}_{LW} = \mathbf{e}_z v_{ph_{LW}} N_{LW}\cos\phi, \tag{9}$$

$$\phi = k_{LW}z - \omega_{LW}t + \varphi_0, \tag{10}$$

where $\varphi_0 = \text{const}$ is a constant phase shift of the LW EPW, and $v_{ph_{LW}} = k_{LW}/\omega_{LW} \approx c$ is its phase velocity. We account for the possible deviation of a wave number and frequency of the LW EPW from the plasma wave number $k_p = \omega_{pe}/c$ and plasma frequency,[12] so that in Eq. (10) $k_{LW} \equiv k_p + \delta k_{LW}$, and $\omega_{LW} \equiv \omega_{pe} + \delta\omega_{LW}$. In order to make a dispersion analysis of the SRS instability of the laser pulse subjected to the resonant self-modulation, we consider a laser pulse which consists of two spectral components (pump waves) shifted to the frequency and wave number of the LW EPW:

$$a_0 = a_1 + a_2 e^{i\phi}, \tag{11}$$

where a_1 and a_2 are the constant amplitudes of the pump waves in the region $-L_{pulse} < \xi < 0$ (L_{pulse} is a longitudinal pulse length) and are zero outside this interval. The laser pulse with the envelope (11) consists of the components with a carrier frequency ω_0 and blue-shifted (anti-Stokes) frequency $\omega_0 + \omega_{LW}$. The representation (11) of a laser envelope corresponds to the three-dimensional regime of the self-modulation of a laser pulse with a power which is near critical for a relativistic self-focusing[14] and describes the on-axis structure of the pump field in that 3D regime (see Fig. 2 of Ref.[14]).

The pump field with an envelope (11) represents a pair of one-dimensional quasi-plane waves. As applied to the problem of scattering through large angles, such choice of the pulse envelope can be substantiated as follows. It was shown[3] that the LA-SRS of a laser pulse, which is bounded in two dimensions, enters the regime of steady-state spatial amplification in the co-moving variables as the time $\tau_0 = \max\left\{L_{pulse}/c, L_{pulse}/[2c\sin^2(\alpha/2)]\right\}$ passes since the pulse entered the plasma. It can be shown,[3,16] that the one-dimensional regime of spatial amplification dominates in a plasma, if the transverse dimension $L_\perp$ of a pulse is large enough to satisfy the inequality

$$L_\perp/L_{pulse} \gg \cot(\alpha/2). \tag{12}$$

Throughout this report we meet the condition (12), which supports the choice of the one-dimensional geometry of a pump field. In the one-dimensional regime, we will find the increments of spatial amplification of the unstable modes in the co-moving variables. These increments determine the maximum possible amplification of the unstable waves in the co-moving frame, rather than an exact structure of the scattered EM field, which depends on both shape of the laser pulse and boundary-value conditions at the pulse side boundaries.[16]

3 DISPERSION EQUATION

We substitute (9) and (11) into Eqs. (7) and (8) and make the Fourier transformation of the envelopes $\hat{\mathbf{a}}_s$ and $\hat{N}_s^*$ with respect to the transverse spatial variable

$$\left\{\hat{\mathbf{a}}_s, \hat{N}_s^*\right\}_{(\mathbf{r}_\perp,\xi,\eta)} = \int \left\{\hat{\mathbf{a}}_s, \hat{N}_s^*\right\}_{(\mathbf{k}_\perp,\xi,\eta)} e^{i(\mathbf{k}_\perp,\mathbf{r}_\perp)}\, d\mathbf{k}_\perp, \tag{13}$$

where $\mathbf{k}_\perp$ is a real vector, and represent the Fourier transformants in the Floquet form

$$\left\{\hat{\mathbf{a}}_s, \hat{N}_s^*\right\}_{(\mathbf{k}_\perp,\xi,\eta)} = \sum_j e^{-i\omega\eta+ik_j\xi} \sum_{n=-\infty}^{n=\infty} \left\{\hat{\mathbf{a}}_s^{(n)}, \hat{N}_s^{*(n)}\right\}_{(\mathbf{k}_\perp,k_j,\omega)} e^{in\phi}, \tag{14}$$

from which the recurrent relation for the ampliudes $\hat{\mathbf{a}}_s^{(n)}$ follows:

$$-Y^{(n-1)}\hat{\mathbf{a}}_s^{(n-1)} + X^{(n)}\hat{\mathbf{a}}_s^{(n)} - Y^{(n)}\hat{\mathbf{a}}_s^{(n+1)} = 0. \tag{15}$$

The dispersion equation for the infinite set of the coupled equations (15) is expressed through continued fractions

$$X^{(n)} = \frac{\left(Y^{(n)}\right)^2}{X^{(n+1)} - \dfrac{\left(Y^{(n+1)}\right)^2}{X^{(n+2)} - \ldots}} + \frac{\left(Y^{(n-1)}\right)^2}{X^{(n-1)} - \dfrac{\left(Y^{(n-2)}\right)^2}{X^{(n-2)} - \ldots}}. \tag{16}$$

Solution to the Eq. (16) $k_j(\omega, k_\perp)$ is a complex number; its positive imaginary part is an increment of spatial amplification, which corresponds to the solution growing towards the pulse trailing boundary. The functions, which enter (16), are defined in the following way:

$$\begin{aligned}
X^{(n)} &= D_1^{(n)} + \frac{\omega_{pe}^3}{2}\left(\frac{\beta_1}{D_{2(n)}^{(+)}D_{2(n)}^{(-)}} + \frac{\beta_2}{D_{2(n-1)}^{(+)}D_{2(n-1)}^{(-)}}\right), \\
Y^{(n)} &= \frac{\omega_{pe}}{2}\left[\mu\left(\frac{v_{ph\,LW}}{c} - \cos\alpha\right) - \frac{\omega_{pe}^2\sqrt{\beta_1\beta_2}}{D_{2(n)}^{(+)}D_{2(n)}^{(-)}}\right], \\
D_1^{(n)} &= \omega + ck_j - \Delta\omega_d + n\left(\omega_{LW} + \frac{\delta\omega_{LW} - c\delta k_{LW}}{1-\cos\alpha}\cos\alpha\right), \\
D_{2(n)}^{(\pm)} &= \omega_{pe} \pm (\omega + ck_j + n\omega_{LW}),
\end{aligned}$$

where $\Delta\omega_d \equiv (ck_\perp \sin\alpha - \omega\cos\alpha)/(1-\cos\alpha) = \omega_d - \omega_0$ is a frequency shift of the scattered light detected in the lab frame, $\beta_{1(2)} = \left(a_{1(2)}^2/2\right)(\omega_0/\omega_{pe})$ are the coupling coefficients of the waves, and $\mu = (\omega_0/\omega_{pe})N_{LW}$ is a normalized amplitude of the density perturbation in a given linear LW EPW. There is a principal difference in the behavior of the instability under the conditions of weak ($\beta_{1(2)} \ll 1$) and strong ($\beta_{1(2)} \gg 1$) coupling. These limits are investigated in the two following Sections.

4 LIMIT OF WEAK COUPLING

4.1 Frequency domains of the instability

In the limit of weak coupling ($\beta_{1(2)} \ll 1$), the instability onset is due to the resonant interaction of waves, which are close to the natural modes. This regime implies that the spatial growth rate remains much smaller than the plasma wavenumber k_p. As the equations $D_1^{(n)}=0$ and $D_{2(n)}^{(\pm)}=0$ are the dispersion equations (written in the co-moving frame) for normal EM and electron plasma modes, the domains of parameters, in which the complex solutions $k_j(\Delta\omega_d)$ of Eq. (16) are to be searched for, are determined by the solutions to the following four sets of equations: (a) $D_1^{(0)}\approx 0$, $D_{2(0)}^{(+)}\approx 0$, (b) $D_1^{(0)}\approx 0$, $D_{2(0)}^{(-)}\approx 0$, (c) $D_1^{(1)}\approx 0$, $D_{2(1)}^{(+)}\approx 0$, (d) $D_1^{(1)}\approx 0$, $D_{2(1)}^{(-)}\approx 0$. Eq. (a) gives $\omega_d\approx\omega_0-\omega_{pe}$ — the Stokes domain for the "red" (ω_0) pump wave; Eq. (b) gives $\omega_d\approx\omega_0+\omega_{pe}$ — the anti-Stokes domain for the "red" pump wave; Eq. (c) gives $\omega_d\approx\omega_0+\omega_{LW}-\omega_{pe}$ — the Stokes domain for the "blue" $(\omega_0+\omega_{LW})$ pump wave; Eq. (d) gives $\omega_d\approx\omega_0+\omega_{LW}+\omega_{pe}$ — the anti-Stokes domain for the "blue" pump wave.

4.2 Limit of small μ

For $\sqrt{\beta_{1(2)}}\ll\mu\ll 1$, one can retain in the expansion (14) the fundamental terms ($n=0$) and a pair of additional terms, corresponding to $n=\pm 1$. With the account for those terms, the dispersion equation becomes

$$X^{(0)}=\frac{\left(Y^{(0)}\right)^2}{X^{(1)}}+\frac{\left(Y^{(-1)}\right)^2}{X^{(-1)}}. \tag{17}$$

While $\mu\ll 1$, there is no complex roots of (17) in the anti-Stoke regions, whereas in the Stokes domains the imaginary parts $\operatorname{Im}k\equiv\kappa(\Delta\omega_d)$ read as follows:

$$\kappa(\omega_d\approx\omega_0-\omega_{pe}) = \frac{1}{2c}\sqrt{\tilde\beta_1\omega_{pe}^2-[\omega_d-(\omega_0-\omega_{pe})]^2}, \tag{18}$$

$$\kappa(\omega_d\approx\omega_0+\delta\omega_{LW}) = \frac{1}{2c}\sqrt{\tilde\beta_2\omega_{pe}^2-[\omega_d-(\omega_0+\omega_{LW}-\omega_{pe})]^2}, \tag{19}$$

where $\tilde\beta_1=\beta_1R_1^{(+)}$, and $\tilde\beta_2=\beta_2R_2^{(-)}$. Here, $R_{1(2)}^{(\pm)}(\mu,\delta\omega_{LW},\delta k_{LW})$ are the renormalizing functions for the coupling coefficients in the presence of the LW EPW:

$$R_{1(2)}^{(\pm)}=\frac{1\pm\mu\sqrt{\dfrac{\beta_{2(1)}}{\beta_{1(2)}}}\left(\dfrac{v_{ph\,LW}}{c}-\cos\alpha\right)\left(1+\dfrac{\delta\omega_{LW}}{\omega_{pe}}+\dfrac{\delta\omega_{LW}\cos\alpha-c\delta k_{LW}\sin\alpha}{\omega_{pe}(1-\cos\alpha)}\right)}{1+\dfrac{\mu^2}{2}\left(\dfrac{v_{ph\,LW}}{c}-\cos\alpha\right)^2}. \tag{20}$$

As it follows from (20), with the increase in μ the monotonic decrease of the coupling coefficient $\tilde\beta$ occurs in case the pulse is monochromatic (when either β_1 or β_2 equals zero); the correction quadratic in μ reduces the maximum of the increment. In the case of the two-frequency pulse an asymmetry appears in the dependence on μ of the coupling coefficients $\tilde\beta_1$ and $\tilde\beta_2$. That is, on the contrary to the case of a monochromatic pulse, the coupling coefficient $\tilde\beta_1$, which corresponds to the "red" pump wave, grows with μ when $\mu\ll 1$, and in the presence of the "blue" pump wave the increment (18) increases due to the linear correction in μ. On the other hand, the increment (19) of the SRS from the "blue" pump wave decreases even faster than in the case of the monochromatic pulse; that is, the negative correction to this increment is also linear in μ rather than quadratic. Note that the angular dependence of the increments (18) and (19) emerge only in the presence of the LW EPW; in a non-perturbed plasma (i. e. in the absence of LW EPW) the increment of the spatial growth in the one-dimensional regime of the LA-SRS is independent on scattering angle.[3,9,16]

4.3 Numerical solutions of the dispersion equation for arbitrary μ

In the case of arbitrary μ restricted from above by the value $(\omega_0/\omega_{pe})^{1/2}$ [see (5)], the dispersion equation (16) is solved numerically, and its complex solutions are searched for in both Stokes and anti-Stokes regions defined above in the Subsection 4.2. Previously,[9] for the LA-SRS of a monochromatic pulse in the presence of a given linear LW EPW with a phase velocity equal c (namely, $\omega_{LW} = \omega_{pe}$, and $k_{LW} = k_p$), the suppression of a Stokes and generation of the anti-Stokes branches of the instability were established. As is already noted in the Introduction, the suppression of the instability is connected with the participation in the scattering process of new short wavelength satellites $(\omega_{pe} + n\omega_{LW}, \mathbf{k}_e + n\mathbf{k}_{LW})$, $n \neq 0$, which appear as a consequence of periodic spatio-temporal phase modulation (6) of scattering plasma waves in the presence of a LW EPW. The satellites (which are not natural plasma modes) gain the energy from the original eigenmode $(\omega_{pe}, \mathbf{k}_e)$, which is resonantly driven by the ponderomotive force at the beat frequency ($\approx \omega_{pe}$) of incident and scattered EM waves. Therefore, the energy, which in in the absence of LW EPW would be transfered to the single mode of plasma oscillations, is expended in excitation of a large number ($n \sim \mu^2 \gg 1$) of plasma modes (*not* natural modes), which causes the dramatic decrease in the increment.

We investigate first how the phenomenon of suppression is affected by the additional component in the pulse spectrum. Numerical solution to the Eq. (16) yields the dependence on μ of the spatial increments, corresponding to the Stokes and anti-Stokes bands from both spectral components (ω_0 and $\omega_0 + \omega_{LW}$) of a pulse. These dependencies corresponding to the central frequencies of both Stokes and anti-Stokes bands are shown for $v_{ph\,LW} = c$ and $\alpha = \pi$ (direct backscatter) in Fig. 1 ("red" pump wave) and Fig. 2 ("blue" pump wave). In these Figures suppression of the Stokes branches of the instability as well as generation of the anti-Stokes branches are clearly seen for $\mu > 1$. The anti-Stokes branches are, in turn, suppressed for $\mu > 2$. In the case of a two-frequency pump, the Stokes increment of the "red" pump wave increases until $\mu < 0.5$ and then drops [see Fig. 1(a)]. The Stokes increment of the "blue" pump wave experiences more severe suppression than in the case of the monochromatic pulse [compare Fig. 1(a) and 2(a) for $\beta_1 = \beta_2$]. The linear growth with μ of the "red" pump wave increment and linear decrease in the increment of the "blue" pump wave for small μ are discussed in the previous Subsection and described by formulas (18)-(20).

The self-modulation of the laser pulse can be accompanied by shifts in frequency and wave number of the LW EPW from the resonant values ω_{pe} and k_p,[12] which can also exert some influence on the scattering process. We have considered the effect of the frequency detuning $\delta\omega_{LW}$ on the increments of backward SRS ($\alpha = \pi$) of a single pump wave ($\beta_2 = 0$). The increments corresponding to the centers of the Stokes and anti-Stokes lines are shown in Fig. 3. The curves shown in Fig. 3 correspond to small frequency shifts ($\delta\omega_{LW} \ll \omega_{pe}$), as in most regimes of a pulse self-modulation these shifts are indeed small if compared to the electron plasma frequency in the rarefied plasmas. In particular, an electron plasma should be too dense ($n_0 \approx 0.2n_c$) to make the frequency shift $\delta\omega_{LW} = (\omega_{pe}/2)(\omega_{pe}/\omega_0)^3$ (which corresponds to the 1D-2 regime of the self-modulation[12]) equal the quite moderate value $\delta\omega_{LW} = 0.05\omega_{pe}$ used in the calculation of Fig. 3. As is clear from Fig. 3, the influence of a frequency shift $\delta\omega_{LW} \ll \omega_{pe}$ on the suppression of the SRS-instability does not produce significant changes in the effect of suppression.

The effect of suppression of the SRS-instability also depends on the angle between $\mathbf{k}_e$ and $\tilde{\mathbf{v}}_{LW}$ [in the geometry accepted in the present paper, this angle coincides with the scattering angle – see Eq. (4)]. Reduction of the scattering angle (then, $|\mathbf{k}_e|_{\alpha \to 0} \to 0$) makes the influence of the LW EPW on the scattering process less pronounced, as is seen in Fig. 4. Thus, the fraction of the energy losses due to the side-scattering becomes more pronounced in the presence of a LW EPW.

To conclude the discussion of the limit of weak coupling, let us consider a certain regime of the laser pulse self-modulation in which the discovered effect of suppression of the LA SRS would be observed. That is, in the known 3D regime of the self-modulation of a laser pulse with a power close to the critical power for the relativistic self-modulation,[14] the laser spectrum consists of the component with the carrier frequency and the anti-Stokes component blue-shifted to the plasma frequency [the on-axis pulse structure is described approximately by the dependence (11)]. For such regime of the self-modulation instability, the numerical modelling[14] performed for a plasma and laser parameters such that $\omega_0/\omega_{pe} = 20$, $\beta_1 = 0.256$, and $\kappa L_{pulse} \approx 13.5$ (hence, the LA SRS of such pulse is in the weakly coupled regime and is far from the nonlinear saturation) gives the maximum amplitude of the electron density perturbation $N_{LW} \approx 0.1$ {under the parameters of modelling,[14] N_{LW} remains less than $(\omega_{pe}/\omega_0)^{1/2} \approx 0.22$ [see Eq. (5)], which yet allows us to use the linear approximation (9) for the LW EPW}. The

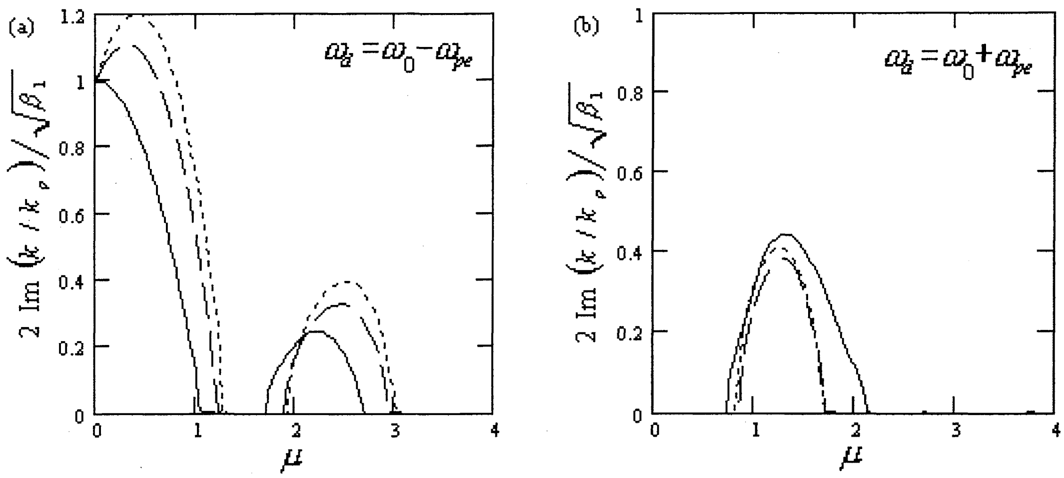

Figure 1. Maximum increments of the Stokes (a) and anti-Stokes (b) bands of backscatter ($\alpha=\pi$) of the "red" spectral component of a pulse (ω_0) versus $\mu=(\delta n_{LW}/n_0)(\omega_0/\omega_{pe})$. The amplitude of the "blue" pulse component ($\omega_0+\omega_{pe}$) is such that $\beta_2=0$ (monochromatic pulse, solid line), $\beta_2=0.5\beta_1$ (dashed line), $\beta_2=\beta_1$ (dotted line).

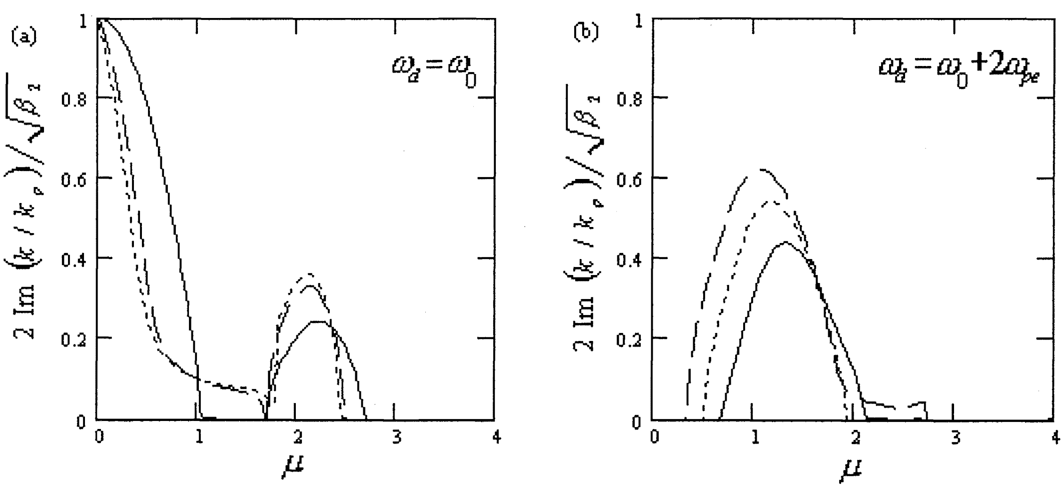

Figure 2. Maximum increments of the Stokes (a) and anti-Stokes (b) bands of backscatter ($\alpha=\pi$) of the "blue" spectral component of a pulse ($\omega_0+\omega_{pe}$) versus $\mu=(\delta n_{LW}/n_0)(\omega_0/\omega_{pe})$. The amplitude of the "red" component (ω_0) is such that $\beta_1=0$ (monochromatic pulse, solid line), $\beta_1=0.5\beta_2$ (dashed line), $\beta_1=\beta_2$ (dotted line).

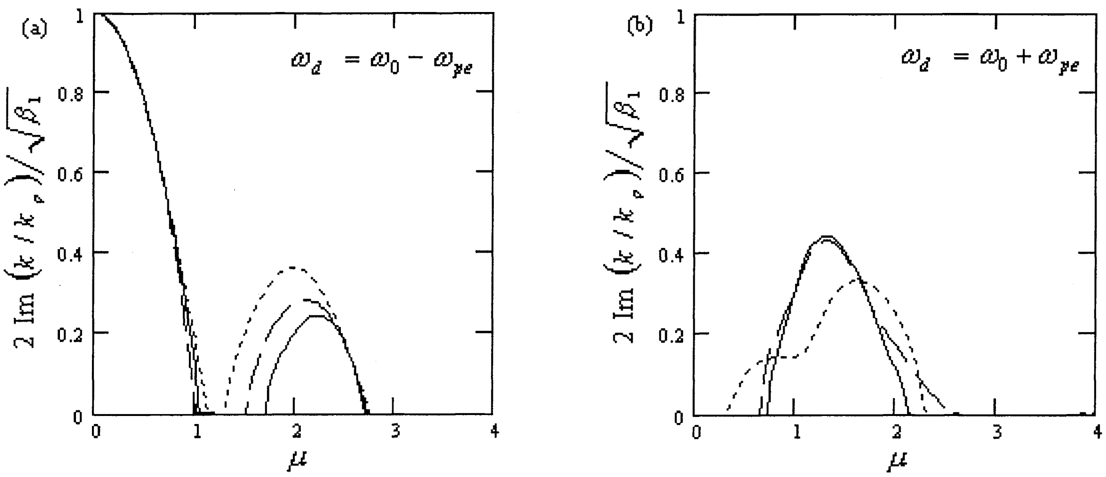

Figure 3. Maximum increments of the Stokes (a) and anti-Stokes (b) bands of backscatter ($\alpha=\pi$) of a single-frequency pulse versus $\mu=(\delta n_{LW}/n_0)(\omega_0/\omega_{pe})$. The detuning of a frequency of the LW EPW from the resonant value ω_{pe} equals zero (solid line), $0.01\omega_{pe}$ (dashed line), $0.05\omega_{pe}$ (dotted line).

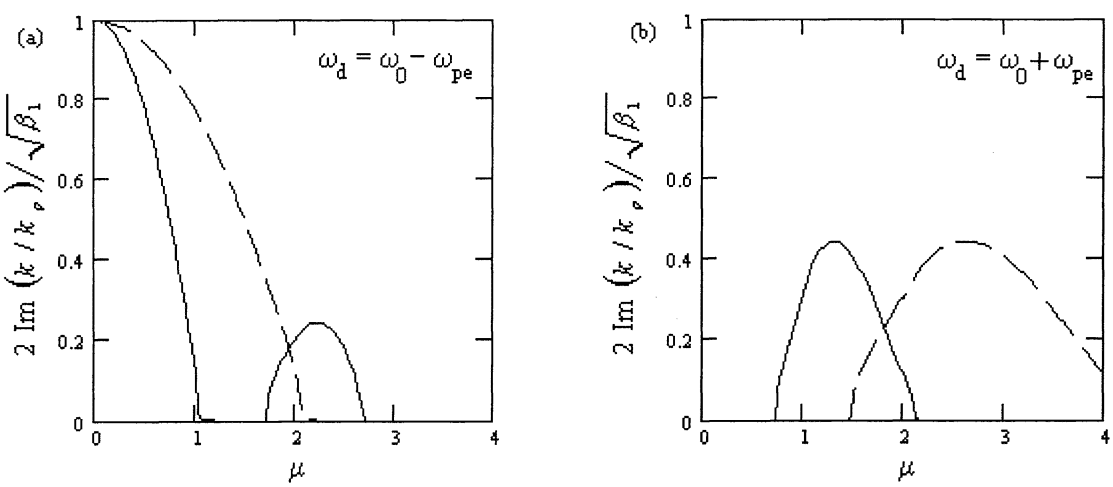

Figure 4. The influence of the reduction of the scattering angle ($\alpha=\pi$- solid line, $\alpha=\pi/2$- dashed line) on the SRS of a monochromatic pulse in the presence of the LW EPW. The normalized amplitude of the density perturbation $\mu=(\delta n_{LW}/n_0)(\omega_0/\omega_{pe})$, which corresponds to the first vanishing of the Stokes increment, grows with the decrease in the scattering angle.

maximum value of the density perturbation achieved in the considered regime of the laser pulse self-modulation yields $\mu \approx 2$, and it is evident that the suppression of the LA SRS is possible under such conditions.

Let us sum up the basic features of the LA-SRS of a two-frequency pulse in the presence of a LW EPW having a phase velocity close to the group velocity of a pulse.

- The most significant effect of the LW EPW on the scattering process in the limit of weak coupling is the suppression of the Stokes and generation of the anti-Stokes branches of the instability provided the normalized electron density perturbation N_{LW} in the LW EPW exceeds the value ω_{pe}/ω_0; in strongly rarefied plasmas $(n_0/n_c < 10^{-2})$ the suppression occurs for a quite moderate density perturbation $N_{LW} < 0.1$.
- The mutual influence of the spectral components of the pulse shifted to the frequency close to ω_{pe} leads to the asymmetry of the suppression of the SRS of different components: The maximum of the Stokes increment of the lower-frequency pump component slightly grows for $N_{LW} < 0.5\,\omega_{pe}/\omega_0$, and for larger N_{LW} the suppression follows (for $N_{LW} > \omega_{pe}/\omega_0$); the maximum of the Stokes increment of the higher-frequency pump component undergoes mononotonic decrease with N_{LW} for $\mu < 1$ and more severe suppression than in the case of the monochromatic pulse.
- The effect of suppression is almost independent on the detuning of a phase velocity of the LW EPW from the group velocity of a pulse if $\left|v_{ph\,LW} - c\right| \ll c$.
- Reducing the scattering angle makes the effect of suppression less pronounced, and the density perturbation amplitude N_{LW}, at which the Stokes increments vanish for the first time, grows.

5 LIMIT OF STRONG COUPLING

In the limit strong coupling $(\beta_{1(2)} \gg 1)$, scattering plasma waves are not natural modes. In the case of a single-frequency pulse $(\beta_2 = 0)$, the scattered radiation in a plasma without LW EPW has a wide-spread (if compared to ω_{pe}) spectrum, which is extended to the blue side: $\omega_0 - (3/2)\sqrt[3]{\beta_1}\omega_{pe} \leq \omega_d \leq \omega_0 + (1/2)\beta_1\omega_{pe}$. The maximum value of the increment $\kappa_0 = (\sqrt{3}/2)k_p(\beta_1/2)^{1/3}\left[1 - (2\beta_1^2)^{-1/3}\right] \gg k_p$,[5,9] which can be obtained from the dispersion equation $X^{(0)} = 0$, corresponds to the frequency $\omega_d = \omega_0$. For a two-frequency pulse with a frequency difference small compared to $\sqrt[3]{\beta_1}\omega_{pe}$, and close amplitudes of the spectral components, the spectral width and maximum increment can be described by the above dependencies with β_1 replaced by $\left(\sqrt{\beta_1} + \sqrt{\beta_2}\right)^2$.

When a plasma is perturbed by a small-amplitude LW EPW such that $\mu \ll \beta_{1(2)}^{1/6}$, one may obtain from Eq. (17) the correction to the increment κ_0 which accounts for the influence of the LW EPW on the strongly coupled LA-SRS. In the case of a monochromatic pulse the corrected increment reads

$$\kappa = \kappa_0 - \mu^2 \frac{3\sqrt{3}}{8} k_p \left(\frac{\beta_1}{2}\right)^{-1/3}. \tag{21}$$

Eq. (21) proves to be useful for estimation of the effect of a LW EPW on the strongly coupled LA SRS even far beyond its strict validity region. So, expression (21) predicts a decrease in $\kappa(\omega_d = \omega_0)$ when $\mu \approx \sqrt[3]{\beta_1}$, or $N_{LW} \sim (a_0\omega_{pe}/\omega_0)^{2/3}$. This prediction is confirmed in part by the results of numerical solution to the general dispersion equation (16). The increments were calculated using (16) for both single-frequency (ω_0) and two-frequency $(\omega_0 + \omega_{pe})$ pulses. The dependencies $\kappa(\omega_d = \omega_0)$ versus μ, which are drawn for a direct backscatter $(\alpha = \pi)$, are seen in Fig. 5. The curves shown in Fig. 5 demonstrate the reduction in the increment of the strongly coupled SRS for both single- and two-frequency pulse. Although the complete suppression of the SRS of a single-frequency pulse does not occur, the decline by nearly one half of the growth rate of the SRS-instability is the case, in accordance with the approximate expression (21), for $\mu \approx \sqrt[3]{\beta_1}$, or $N_{LW} \sim (a_1\omega_{pe}/\omega_0)^{2/3}$.

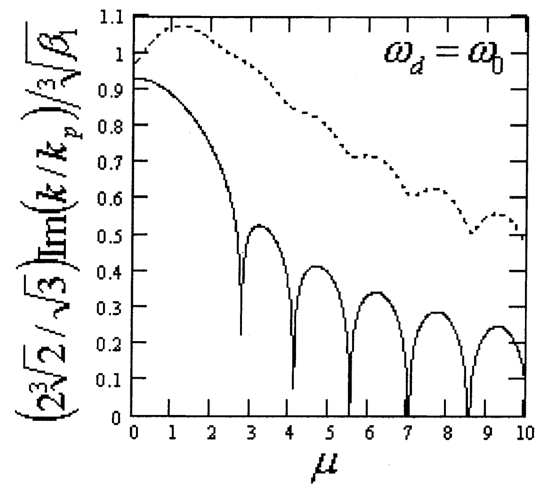

Figure 5. The spatial increment $\kappa(\omega_d = \omega_0)$ of the direct backscattering ($\alpha=\pi$) under the conditions of strong coupling, which correspond to $\beta_1=20$, $\beta_2=0$ (monochromatic pulse, solid line), and $\beta_1=\beta_2=20$ (two-frequency pulse with a frequency difference equal to ω_{pe}, dashed line), versus the normalized amplitude of the density perturbation $\mu=(\delta n_{LW}/n_0)(\omega_0/\omega_{pe})$.

To illustrate the effect of suppression of the LA SRS under the condition of strong coupling, let us consider the long laser pulse ($L_{pulse} > \lambda_p$) with a sharp leading edge ($L_{front} < 0.5\lambda_p$), which excites in a plasma a LW EPW whose amplitude is approximately equal to $N_{LW} = (a_1/2)^2$.[13] Leaving aside the fact that such a plasma wave can itself seed a laser pulse self-modulation, we consider first an influence of this wave on the LA SRS of a monochromatic laser pulse with a given rectangular envelope (this is valid on time scales larger than a pulse duration, but small enough for the development of the self-modulation, and corresponds to the early stage of the self-modulation instability). The normalized amplitude of density perturbation $N_{LW} = (a_1/2)^2$ gives $\mu = \beta_1/2$, which, under the conditions of strong coupling, is significantly larger than that is necessary to halve the maximum increment [i.e. $\mu \approx \sqrt[3]{\beta_1}$ and $N_{LW} \approx (a_1\omega_{pe}/\omega_0)^{2/3}$]. Using the dependencies plotted in Fig. 5, we find that the increment of direct backscattering $\kappa(\omega_d = \omega_0, \alpha = \pi)$ takes at $\mu = \beta_1/2 = 10$ the value equal to approximately one fifth of its value in an unperturbed plasma. Now it is clear that the strongly coupled LA SRS of a pulse with a sharp leading edge can be suppressed at the very beginning of the self-modulation process. However, the important requirement of linearity of the LW EPW (5) should be met, which limits the density perturbation amplitude $N_{LW} < (\omega_{pe}/\omega_0)^{1/2}$, and, hence, the amplitude of the laser $a_1 < 2\,(\omega_{pe}/\omega_0)^{1/4}$. In the case of more intense laser, one must account for the effect of harmonics of a nonlinear LW EPW on the scattering process along with the effect of the linear component (9) of the LW EPW excited by the front edge of the laser pulse. This can drastically modify the suppression of the scattering process. As is seen in the results of modelling[17] of the LA-SRS of a short ($L_{pulse} \sim \lambda_p$) relativistically strong ($a_1 \sim 1$) laser pulse generating a strongly nonlinear LW EPW, significant variations of a spectral shape of LA-SRS were established[17] depending on a pulse length and intensity in the parameter regions $\beta > 50$ (hence, $\mu > 25$), and $\omega_0/\omega_{pe} > 20$ rather than suppression of the instability.

6 CONCLUSION

The results reported in the present paper clearly point to the fact that in the presence of a linear LW EPW the LA-SRS of a two-frequency laser pulse acqires new specific features. That is, under the conditions of weak coupling the instability undergoes suppression when a density perturbation in the LW EPW exceeds the value, which for rarefied plasmas ($n_0/n_c \approx 10^{-2}$) represents about ten per cent of the background electron plasma density. Mutual influence of diffrent spectral components of a laser pulse shifted to approximately ω_{pe} retains the common features of the suppression effect the same as in the case of monochromatic pulse, except the somewhat asymmetry in the behavior of the increments of different pump components at $N_{LW} \ll \omega_{pe}/\omega_0$. As the scattering angle reduces, the suppression becomes less pronounced in comparison with the case of direct backscatter (thus, in the presence of the LW EPW the fraction of the energy losses due to the side-scatter increases). Besides, in the anti-Stokes ranges corresponding to each pump spectral component new branches of the instability appear (however, with an increment small in comparison to the conventional increment of the LA-SRS in a non-perturbed plasma). Under the condition of strong coupling the complete suppression of the instability is not the case; though, the significant reduction of the increment is the case for $N_{LW} \sim (a_1\omega_{pe}/\omega_0)^{2/3}$. In the case of the wake-field excitation by the sharp leading edge of a pulse [then, $N_{LW} = (a_1/2)^2$], when the condition of strong coupling fulfulled, the increment of the SRS falls approximately to one fifth of the value corresponding to an unperturbed plasma. This work was supported in part by the Russian Foundation for Basic Research under Grant no. 98-02-16263.

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