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All-Optical Control of Nonlinear Self-Focusing in Plasmas Using Non-Resonantly Driven Plasma Wave

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Abstract. Excitation of plasma density perturbations by an initially bi-color laser pulse helps to control nonlinear refraction in the plasma and enables various types of laser self-guiding. In this report we consider a setup that not only makes possible the transport of laser energy over cm-long relatively dense plasmas ($n_0 = 10^{18}$ cm^{-3}) but also transforms the pulse into the unique format inaccessible to the conventional amplification techniques (relativistically intense periodic trains of few-cycle spikes). This well focusable pulse train is a novel light source interesting for ultra-fast high-field science applications. The opposite case of suppression of nonlinear self-focusing and dynamical self-guiding of an over-critical multi-frequency pulse is proposed for the proof-of-principle experimental study.

Keywords: Laser self-focusing, self-guiding, electromagnetic cascading, radiation pulse train
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INTRODUCTION

Over almost 40 years, interaction of multi-color radiation beams with underdense plasmas [$\omega_0 \gg \omega_{pe}$, where ω_0 is the laser carrier frequency, $\omega_{pe} = (4\pi e^2 n_0/m_e)^{1/2}$ is the electron plasma frequency, m_e is the electron rest mass, n_0 is the background electron density, and e is electron charge] has been a topic of keen interest in the context of enhanced plasma heating [1], electron acceleration [2, 3, 4] and all-optical control over the nonlinear focusing and dynamical self-guiding [5, 6, 7, 8]. Recent research shows that the nonlinear refraction induced by the periodic index grating (electron density perturbation) excited by a bi-color laser pulse with a non-resonant difference frequency, $\Omega \neq \omega_{pe}$, can either enhance or suppress the nonlinear focusing [2, 8]. Cooperation of the relativistic self-focusing [9] and plasma wave-induced focusing in the case $\Omega \leq \omega_{pe}$ results in a feedback loop between the three-dimensional (3-D) plasma wave excitation and laser focusing, and, eventually, in a longitudinal breakup of a long radiation beam into a train of short beat notes guided by the buckets of the plasma wave [4, 10]. However, in the strictly resonant case, $\Omega = \omega_{pe}$, the resonant nature of the interaction results in an excitation of the relativistic plasma wake which rapidly depletes the laser beam. On the other hand, high amplitude of the electron plasma wave close to the wavebreaking limit results in the electron self-injection and acceleration up to a hundred-MeV energy [11] (although with a Maxwellian energy spectrum). With $\Omega < \omega_{pe}$, the feedback loop is reduced, and the laser self-focusing is not catastrophic; the plasma wake remains non-broken even in the vicinity of nonlinear focus [5, 6]. Simultaneously, the laser beam witnesses periodic index modulation at all times which results in periodic phase modulation. In spectral terms, it translates into an electromagnetic cascade — generation of frequency satellites shifted to integer multiples of the difference frequency with respect to the carrier frequency ω_0. Within each radiation beat note, red-shifted components are advanced in time, and the blue-shifted are delayed; due to the group velocity dispersion (GVD) in plasma, they pick up at the center of the beat note, thus producing a few-laser-cycle radiation spike [5]. Thus, nonlinear refraction due to plasma wave excitation transforms the beat wave of an initially bi-color long laser beam into the train (hair-comb) of few-cycle multi-terawatt pulses. In the next section we propose a numerical experiment with parameters accessible for the existing laser facilities which shows that the radiation pulse train can be self-guided by the combination of plasma wave-induced guiding and longitudinal compression of beat notes over cm-length, low-density plasmas (Compressor) with minimal energy losses. The beat notes are finally compressed to 3 laser cycles, whereas their intensity remains mildly relativistic, $I \sim 10^{18}$ W/cm^2. A final boost in intensity (beyond 10^{19} W/cm^2) is achieved in a thin ($\sim$ few mm) and dense plasma (the Lens) [12]. Relativistic self-focusing in the dense plasma enforces the self-focusing of the radiation pulse train whereas the sub-10 fs duration of beat notes is preserved. The resulting train of ultrarelativistic pulses may have applications in laser-solid interactions, such as experiments on monoenergetic proton acceleration [13] and symmetric hole boring in inertial confinement fusion (ICF) [14].

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In the opposite case of an over-detuned laser beat wave, $\Omega > \omega_{pe}$, the index grating is *de*-focusing, and the feedback loop between laser beam focusing and plasma wave excitation appears to be broken. As a result, there is an opportunity for guiding the over-critical beams [7, 8] {with total power exceeding the critical power for relativistic self-focusing, $P_{\rm cr} = 16.2(\omega_0/\omega_{pe})^2$ GW [9]}. We report here a numerical experiment on the plasma wave-assisted self-guiding for the parameters of a bi-color laser developed by the group of Prof. M. C. Downer in The University of Texas [15]. It is shown that splitting 10% of energy of a long ($\tau_L \sim 200$ fs) 1 J laser pulse into a frequency shifted same-size beam and overlapping the beams in a plasma indeed helps to avoid the relativistic collapse and propagate the beams without depletion over a half-centimeter in a high-density plasma ($n_0 \sim 10^{19}$ cm^{-3}). Presence of the near-resonant electron density perturbation together with the frequency chirp of the pulses suppresses the resonant Raman instabilities (Raman side-scatter) [16, 17]. A proof-of-principle laboratory experiment is presently under preparation.

TRAIN OF RELATIVISTICALLY INTENSE FEW-CYCLE RADIATION PULSES: NOVEL OPTICAL SOURCE BASED ON NON-RESONANT AMPLITUDE SELF-MODULATION AND ELECTROMAGNETIC CASCADING IN PLASMAS

The idea of single-stage radiation beam compressor using electromagnetic cascading was proposed in Refs. [5, 6]. The initially bi-color laser beam with a difference frequency $\Omega < \omega_{pe}$ propagates in the underdense plasma and drives an electron density perturbation, which acts as a periodic focusing channel [2, 8]. Feedback between the enhancement of focusing and increase of the plasma wave amplitude results in a resonant self-modulation instability [3]. Simultaneously, the laser beam witnesses the periodic index perturbation (plasma wave) at all times, which results in a periodic frequency modulation (chirp); in spectral terms this means generation of the radiation cascade — ensemble of frequency components $\omega_0 \pm m\Omega$, where m is integer. When $\Omega < \omega_{pe}$, the lower-frequency ("red") components are advanced in time with respect to the higher-frequency ("blue") ones within each chirped radiation beat note. Due to the difference in group velocities, the spectral components eventually pick up in the center of the beat note giving rise to the radiation spike. If the cascading broadens the laser spectrum as much as $\Delta\omega \sim \omega_0$, the beat notes may be compressed to a single laser cycle. We have found [5, 6] that as soon as the self-modulation instability saturates, the multi-frequency laser beam enters the stage of dynamic self-guiding. Each beat note rides inside its plasma wave bucket which partly suppresses the diffraction [4, 10]. At the same time, continuous temporal compression partly compensates intensity reduction due to the diffraction. As a result, the intensity on axis remains high, but the radiation beam transforms into a periodic (with the beat period $\tau_b = 2\pi/\Omega$) sequence of sharp, few-fs long and tens of microns wide almost flat radiation beamlets. Although such a light source (train of few-fs relativistically intense pulses with THz repetition rate) is unique, and certainly cannot be reproduced using conventional laser technology, to make it interesting for the high-field science applications (particle acceleration [13], stable hole boring in ICF [14] *etc.*) one has to focus this train of ultrashort, broad-bandwidth beat notes to high intensity. This may be not be possible with conventional optics because of high dispersion of the glass and aberrations of mirrors.

Here we propose to employ the nonlinear focusing properties of plasmas, and make the entire setup (Compressor and Lens) plasma-based. The low-density long Compressor plasma (where the laser beam increases its bandwidth, and the beat notes get compressed) is followed by a short dense slab where the relativistic focusing nonlinearity [9] imparts into the beam a converging phase front and makes it focus. Unfortunately, as the following simulation shows, the idea of thin lens so nicely working for the narrow-bandwidth beams [12, 18], does not work here, and the focusing should occur inside the Lens plasma slab. However, the intensity boost by more than an order of magnitude appears to be possible without compromising the short duration of beat notes.

We make the simulation using a fully relativistic PIC code WAKE. The code uses a quasi-paraxial solver for radiation beam propagation, which calculates precisely the beam group velocity and beam depletion due to the wake excitation in plasma. Simulation runs in the "moving window" using the variables $r = \sqrt{x^2+y^2}$ (3-D cylindrically symmetric geometry), z (propagation variable), and $\xi = z - ct$ ("co-moving coordinate"). Plasma response to the time-averaged (over laser period) ponderomotive force is calculated in the quasi-static approximation, which enormously speeds-up particle pushing. These approximations are capable of correctly describing the near-forward Raman scattering. Although validity of the ponderomotive approximation may be compromised as soon as the beat note length approaches single-cycle, we checked that a mildly relativistic ($a \sim 1$), 2–3 cycle-long pulse propagation is described quite precisely. Therefore, we adjusted the simulation parameters so as to avoid too strong temporal compression.

The physical setup is as follows. A pair of fully overlapped 9 TW pulses with full width at half-maximum in

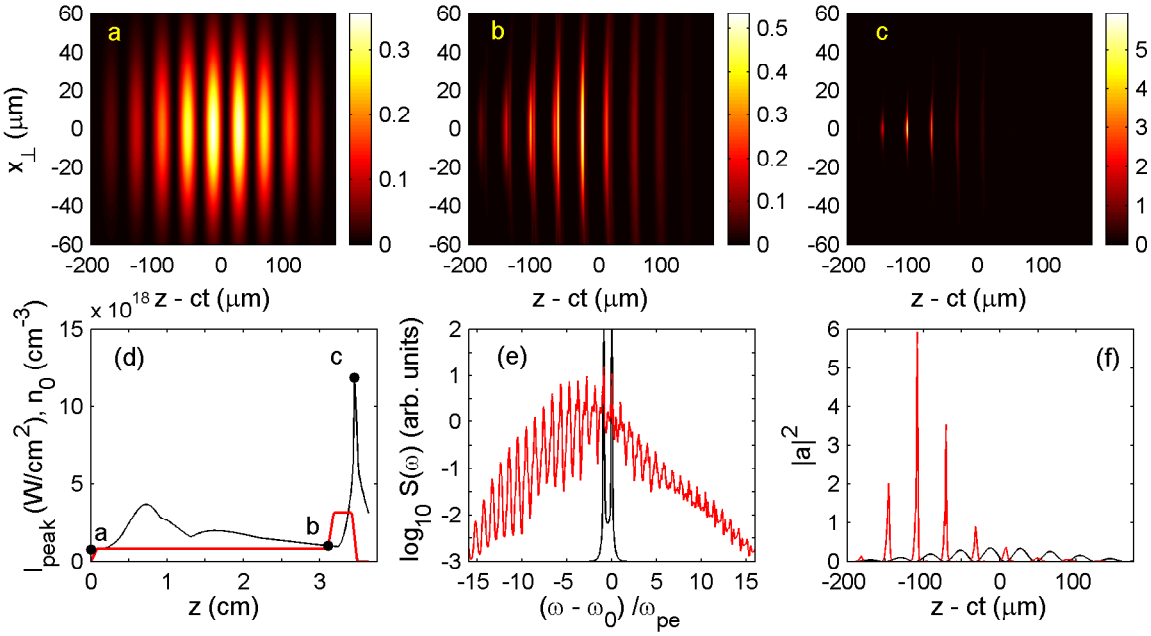

FIGURE 1. (Color online). Normalized laser intensity $|a(z-ct,r)|^2$ (a) at the plasma entrance ($z=0$); (b) after the Compressor ($z=3.1$ cm); and (c) after the nonlinear lens ($z=3.45$ cm). These points are indicated on the curve of peak intensity in panel (d) (black curve); thick red/gray line shows the piecewise-flat electron density profile. A long, low-density plasma serves as a modulator and compressor, and a short and dense one as a nonlinear lens. Laser spectrum at the exit of plasma [panel (e)] indicates the presence of red-shifted electromagnetic cascade, which is the signature of near-resonant amplitude self-modulation. The Compressor generates a train of 3-cycle pulses of weakly relativistic intensity which are focused by the nonlinear plasma lens to the intensity 1.2×10^{19} W/cm^2. Axial lineout of laser intensity at the plasma entrance (black) and exit (red) is shown in panel (f). Combination of the Compressor and nonlinear lens makes it possible to achieve a factor of 17 enhancement in intensity.

intensity $\tau_L = 660$ fs, same spot size $r_0 = 56.7$ μm, and central wavelengths $\lambda_0 = 0.8$ μm, and $\lambda_1 = 0.8165$ μm propagate in the Compressor plasma of density $n_C = 8.8\times10^{17}$ cm^{-3}. For this density, the difference frequency $\Omega = \omega_0 - \omega_1 = 0.9\omega_{pe}$. The Compressor plasma length is $L_C = 3.15$ cm [$\approx 2.5Z_R$, where $Z_R = (\pi/\lambda_0)r_0^2$ is the Rayleigh length for the carrier-frequency beam]. Such a long homogeneous plasma can be created in a gas cell or with the help of a slit nozzle. The Compressor is followed by a 3 mm-long, four times denser slab, the Lens (it may be a separate compartment of a gas cell held at a higher pressure or an embedded short gas jet). The background density profile is shown in Fig. 1(d). For the Compressor density, the normalized beam sizes are $\omega_{pe}\tau_L \approx 34.6$, $k_p r_0 = 10$ (where $k_p = \omega_{pe}/c$), and $P_{\rm cr} \approx 32$ TW. According to Ref. [8], the same-size beams self-focus if their power satisfies the inequality $P/P_{\rm cr} > [1-(\Omega/\omega_{pe})^2]/[3-(\Omega/\omega_{pe})^2] \approx 0.14$. This condition is satisfied with a vast margin, as for each 9 TW beam $P/P_{\rm cr} \approx 0.28$. As a consequence, the beams self-focus, and their intensity [as seen in Fig. 1(d)] increases by factor 4.75 after the first half of the Rayleigh length ($z \approx 0.65$ cm). At this point, as the earlier study [6] has shown, spectral broadening is not significant, and the laser evolution is totally dominated by the transverse effect of plasma wave-induced focusing. Later on, a cascade develops, the laser spectrum broadens, and the beat notes compression begins. Notably, close examination of the frequency spectrum 1(e) shows that the frequency mismatch between neighboring modes can be either Ω or ω_{pe}, which is an indication of the resonant plasma response and, hence, of the resonant self-modulation. Another indication of the self-modulation process is the spectral satellite generation predominantly in the red part of the spectrum, which is typical for Raman-type instabilities. Nevertheless, the Raman processes do not disrupt the beat note compression. Figure 1(c) shows the result of laser beam evolution in the Compressor. After $z = 3.1$ cm of propagation, initially smooth beam transforms into a sequence of five 3-cycle pulses with a time delay $\Delta T \approx 140$ fs. At this point, 75% of the laser energy remains in the box.

The compressed beat notes have intensities in the range 0.5–1 $\times 10^{18}$ W/cm^2, and, hence, instantaneous power 15–30 TW. This low intensity beam can be useful for the self-guided plasma beat wave acceleration [6] which, however, still requires external injection and is unlikely to produce mono-energetic beams. The laser-solid interactions (ion acceleration, ICF) require at least an order of magnitude higher intensity. Thus, the multi-spike beam exiting the

Compressor should be focused. Importantly, Fig. 1(b) shows no sign of transverse beam perturbations, which means that the beat wave driven plasma wave has suppressed the sideward Raman scattering almost completely [17]. Hence, it is not surprising that the beam passing through the Lens appears to be very focusable; Figs. 1(c) and (f) show the factor of 12 intensity increase with respect to that at the exit of Compressor. Examination of the beat note lengths at the exit of the Lens [Fig. 1(f)] does not show further compression. Thus, the beat note length at the exit of Compressor is almost transform limited, so we used the Compressor plasma with maximal efficiency. Importantly, after all manipulations in plasmas, the beam preserves 65% of its energy. The remaining energy, 7.8 J, is certainly interesting for laser-solid interaction experiments.

DYNAMICAL GUIDING OF OVERCRITICAL RADIATION BEAMS: PROPOSED EXPERIMENT

Standard theory of stationary nonlinear (combined relativistic and ponderomotive) self-focusing [19] predicts that a single overcritical beam ($W_{\rm tot} > 1$, where $W_{\rm tot} = P_{\rm tot}/P_{\rm cr}$ is the ratio of the total power to the critical power for the relativistic self-focusing [9]), experiences a catastrophic focusing until a complete electron evacuation and/or higher-order nonlinearities stop the collapse. Simultaneously, the longitudinal beam breakup and enhanced plasma wave generation deplete the pulse and limit the length of its propagation. The experimental question is: having the output power from the amplifier and plasma density fixed, can we prevent the collapse-like dynamics of the radiation beam by purely optical means? The answer is positive; based on the earlier developed weakly relativistic theory, one has to split a fraction of energy,

$$W_1 \geq (W_{\rm tot} - 1)[(\Omega/\omega_{pe})^2 - 1], \tag{1}$$

into a geometrically equivalent (same spot size, same duration) frequency shifted (by $\Omega > \omega_{pe}$) pulse, and optically mix the two in plasma [7, 8]. The de-focusing index grating (with a period $\tau_b = 2\pi/\Omega$) suppresses the self-focusing of the two-beam system, and turns the collapse of the monochromatic beam into a mild de-focusing of the bi-color beam; dynamical guiding follows. Notably, the power remaining in the main beam may still be over-critical: $W_0 = W_{\rm tot} - W_1 > 1$. We explore the possible experimental verification of this scenario using the parameters of a bi-color laser recently developed by the group of Prof. M. C. Downer in The University of Texas [15]. In this setup, the main amplifier delivers a 1 J pulse with the central wavelength $\lambda_0 = 0.8$ μm, and bandwidth-limited full width at half-maximum in intensity $\tau_{L0} = 30$ fs. 10% of the main pulse energy is split from the main pulse and sent through a Raman shifter and amplifier; the Raman-shifted wavelength is $\lambda_1 = 0.873$ μm. The difference frequency is thus $\Omega \approx 1.97 \times 10^{14}$ s^{-1}, which corresponds to the resonant density $n_{\rm res} = 1.22 \times 10^{19}$ cm^{-3} (at which $\Omega = \omega_{pe}$). For this range of densities, the pulse length is about one plasma period, $\tau_p = 2\pi/\omega_{pe} = 32$ fs, and $W_{\rm tot} = 12.9$. To effectively use the de-focusing properties of the plasma beat wave, the laser pulse should be stretched in time to several plasma periods (the bandwidth is to be preserved at the expense of the frequency chirp), so that the main beam is still slightly overcritical. The latter is important to ensure the balance between the relativistic mass effects and plasma wave-induced nonlinear refraction; apparently, both weak and strong pulses should be chirped in the same direction, so that $\Omega = $ const.

For the numerical experiment performed using the code WAKE we select the following parameters. The main 1 J pulse is split in the proportion 9/1. The pulses with the central wavelengths $\lambda_0 = 0.8$ μm and $\lambda_1 = 0.873$ μm are chirped and stretched to the duration $\tau_L = 7\tau_{L0} = 210$ fs. Focal spot [beam radius at $\exp(-2)$ of peak intensity] is $r_0 = 25$ μm. Plasma length is $L_p = 2Z_R = 5$ mm. The electron density $n_0 = 7.8 \times 10^{18}$ cm^{-3} corresponds to $\Omega = 1.25\omega_{pe}$, $\tau_L = 33/\omega_{pe} = 41.35/\Omega$, $W_{\rm tot} = 1.234$, $W_1 = 0.1234$, and $W_0 = 1.11$. The normalized peak intensity of the combined bi-color pulse is $a_0^2 + a_1^2 = 0.2$, where $a_0 = 0.434$, and $a_1 = 0.152$. Characteristic amplitude of the driven plasma wave [8] is $\delta N_\Omega = a_0 a_1 (\Omega^2/2)/(\Omega^2 - \omega_{pe}^2) \approx 0.1$.

Figure 2 compares propagation of the stretched (chirped) single-frequency pulse ($W = W_{\rm tot} = 1.234$) (top row) with that of the initially bi-color pulse (bottom row). Comparison of panels (a) and (e), and (c) and (g) shows that the initially bi-color pulse self-focuses much weaker than the monochromatic one and maintains the large focal spot almost constant starting from $z = 0.5$ mm. And the increase in the peak intensity appears to be more than 5 times less than in the monochromatic case. As a consequence, the energy transfer from the multi-frequency pulse to the plasma wake is insignificant, which reduces drastically the energy losses (from 22.5% in the monochromatic case to 5.5% in the "bi-color" case). In the case of initially bi-color pulse, the radiation spectrum displays a set of frequency sidebands shifted to integer multiples of Ω (an electromagnetic cascade), whereas the initially monochromatic (chirped) pulse shows only the red-shift of central frequency by $\approx \omega_{pe}/2$ and no Raman sidebands. The absence of Raman sidebands $\omega_0 \pm$

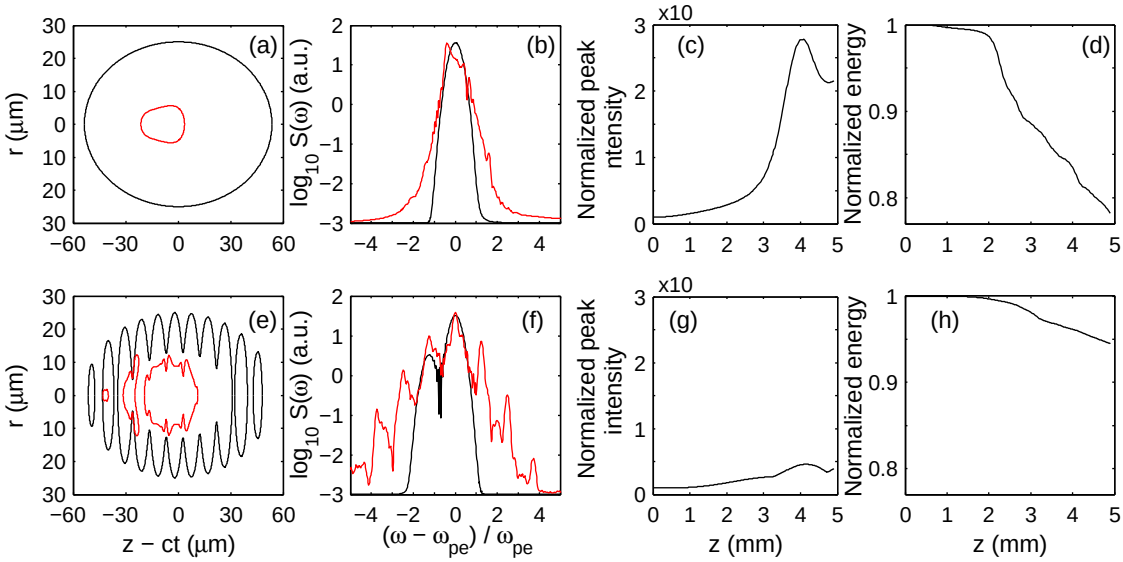

FIGURE 2. (Color online). Suppression of the nonlinear focusing by the frequency upshifted electron plasma wave. Top row: chirped initially monochromatic pulse (initial bandwidth corresponds to the transform-limited bandwidth of a 30 fs pulse; pulse propagates to the right). Bottom row: initially bi-color pulse with the difference frequency $\Omega = 1.25\omega_{pe}$, energy partition between the components 9/1, and the same chirp of both spectral components. Panels (a) and (e): iso-contours of intensity at $\exp(-2)$ of the peak; black corresponds to $z = 0$, red/gray — to $z = 5$ mm ($2Z_R$). Panels (b) and (f): frequency spectra at $z = 0$ (black) and $z = 5$ mm (red/gray). Bandwidth of initially single-color beam remains the same, and no Raman cascade develops. Red shift of central frequency by $\approx 0.5\omega_{pe}$ occurs after 5 mm propagation. Electromagnetic cascade develops in the case of initially bi-color pulse; spectral components $\omega_n = \omega_0 \pm n\Omega$, $n = 1,2,3$ are clearly seen. There are no Raman satellites $\omega_0 \pm n\omega_{pe}$ in the radiation spectrum. Panels (c) and (g): normalized peak intensity of the beam. Intensity increase in the case of bi-color pulse is reduced by a factor > 5 against the monochromatic case. Panels (d) and (h): normalized energy in the simulation box. Absence of collapse and low plasma wave amplitude in the bi-color case explains low depletion rate of the dynamically guided multi-color beam.

$n\omega_{pe}$ in either case can be explained by the effect of frequency chirp which introduces the initial frequency bandwidth of the pulse ($\Delta\omega \approx \omega_{pe}$) well exceeding the bandwidth of the near-forward stimulated Raman scattering [16], $\Delta\omega_{FSRS} \geq \omega_{pe}(a_0/4)\sqrt{\omega_{pe}/\omega_0} \approx 0.03\omega_{pe}$. Thus, energy loss due to the stimulated side-scatter and the threat of transverse beam breakup appear to be drastically reduced.

In conclusion, a numerical experiment with realistic parameters shows that all-optical methods of manipulation with self-guiding can result in a stable propagation of the overcritical, long ($\omega_{pe}\tau_L >> 1$) laser beams in high-density ($n_0 \sim 10^{19}$ cm^{-3}) plasmas. Suppression of the relativistic self-focusing and Raman-type instabilities avoids both longitudinal and transverse beam breakup and achieves robust transport of energy over cm-scale length in the experimental conditions available at existing facilities [15].

CONCLUSION

Nonlinear refraction due to plasma wave excitation transforms a beat wave of an initially bi-color pulse into a train (hair-comb) of few-cycle pulses. When the difference frequency of two initial pulses is less than the electron Langmuir frequency, the non-resonantly driven 3D electron density perturbation acts as a periodic focusing channel that drastically reduces the self-focusing threshold. A feedback loop between the plasma wave excitation and laser focusing results in the amplitude self-modulation instability, which is sensitive to the plasma parameters and thus can be controlled in order to preserve the laser beam quality. After the stabilization of self-modulation, electromagnetic cascading and local index gradients inside the plasma wave buckets increase the laser bandwidth. Subsequent self-consistent compression of individual beat notes due to the effect of group velocity dispersion produces a periodic sequence of few-cycle length radiation spikes guided by the plasma wave buckets. This unique light beam cannot be obtained through the conventional chirped-pulse amplification. The radiation pulse train is well focusable to high

intensity (possibly in higher-density plasma). This, combined with the large total beam energy, makes it interesting for particle accelerator and fast ignition applications. The recent surge of interest in using radiation pulse trains for all-optical manipulation of laser-solid interactions makes this novel light source especially valuable. The opposite situation with suppression of nonlinear focusing and beat note compression is also possible; it requires the difference frequency exceed ω_{pe}. The periodic de-focusing index grating can result in the stable propagation of the long over-critical radiation beam over a few Rayleigh lengths; this scenario is ready for experimental verification using an available bi-color laser source.

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