Application of detuned laser beatwave for generation of few-cycle electromagnetic pulses

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Abstract. An approach to compressing high-power laser beams in plasmas via coherent Raman sideband generation is described. The technique requires two beams: a pump and a probe detuned by a near-resonant frequency $\Omega < \omega_p$. The two laser beams drive a high-amplitude electron plasma wave (EPW) which modifies the refractive index of plasma so as to produce a periodic phase modulation of the incident laser with the laser beat period $\tau_b = 2\pi/\Omega$. After propagation through plasma, the original laser beam breaks into a train of chirped beatnotes (each of duration $\tau_b$).

The chirp is positive (the longer-wavelength sidebands are advanced in time) when $\Omega < \omega_p$ and negative otherwise. Finite group velocity dispersion (GVD) of radiation in plasma can compress the positively chirped beatnotes to a few-laser-cycle duration thus creating in plasma a sequence of sharp electromagnetic spikes separated in time by $\tau_b$. Driven EPW strongly couples the laser sidebands and thus reduces the effect of GVD. Compression of the chirped beatnotes can be implemented in a separate plasma of higher density, where the laser sidebands become uncoupled.

INTRODUCTION

For nearly two decades the chirped-pulse amplification (CPA) was a dominating trend in the technology of generating the ultra-short high-power laser pulses [1]. However, CPA technique meets its natural limit because of technological difficulties in manufacturing the large-size compressor gratings and high susceptibility of these gratings to thermal damage. This is the basic challenge for the development of high-repetition-rate petawatt laser systems. Fortunately, a fully ionized rarefied plasma $[\omega_p \ll \omega_0$, where $\omega_p = (4\pi e^2 n_0/m_e)^{1/2}$ is an electron Langmuir frequency, $\omega_0$ is a fundamental laser frequency, $n_0$ is an electron density of plasma, $-|e|$ is an electron charge, and $m_e$ is an electron mass at rest] can itself be used as a medium for compression and amplification of electromagnetic (EM) wave packets at intensities much higher than allowed in the conventional CPA setups.

In this report, we show that a train of ultrashort radiation spikes can be created in plasmas by means of coherent generation of a Raman cascade. The electron plasma wave driven by the ponderomotive beat wave of a two-frequency laser modifies the refractive index of plasma so as to produce the cascade of EM sidebands shifted by integer multiples of the beat frequency $\Omega$ [2]. When the beat wave is detuned off resonance with the natural weakly relativistic plasma mode ($\Omega \neq \omega_p$, $|\Omega - \omega_p| < \omega_p$), the phase modulation of the pump becomes periodic with a beat period $\tau_b = 2\pi/\Omega$. Within each beat period the longer-wavelength sidebands are advanced in time if $\Omega < \omega_p$ (positive chirp), and retarded if $\Omega > \omega_p$ (negative chirp). The group velocity dispersion (GVD) of...
EM radiation in plasmas can compress the positively chirped laser beatnotes to a few-laser-cycle duration thus creating in plasma a train of sharp EM spikes separated in time by the beat period \( \tau_b \).

We show that strong coupling of Raman sidebands via the driven electron plasma wave (DEPW) can substantially reduce the effect of GVD. In this case, the train of few-cycle EM spikes can be created in two stages which employ two separate plasmas. In the first plasma that we call modulator the near-resonant excitation of the plasma wave occurs, and the laser field becomes phase modulated with the beat period \( \tau_b \) while its amplitude modification due to the GVD is minimal. In the second, much denser, plasma called compressor no resonance occurs between the laser beat wave and natural plasma mode \( \omega_{p(c)} \gg \Omega \), and the laser sidebands become uncoupled. The compressor plasma remains almost unperturbed as the chirped laser beam propagates through it, and its high GVD shrinks of the positively chirped beatnotes. Similar concept of generating the train of EM spikes in molecular gases via generation of Raman cascade was proposed in Ref. [4]. If compared with that scheme, our approach is advantageous as it is viable at laser intensities much higher than the gas ionization threshold.

**BASIC EQUATIONS**

The beat wave of the two-component laser, \( E_0(\omega_0, k_0) \) and \( E_1(\omega_0 + \Omega, k_0 + k_\Omega) \) [now and later, \( \Omega \approx \omega_p, k_\Omega = \Omega/v_g, v_g = c^2 k_0/\omega_0 \approx c, c k_0 = \omega_0(1-d)/2 \), and \( d = (\omega_p/\omega_0)^2 \equiv n_0/n_c \ll 1 \), where \( n_c = m_e \omega_0^2/(4\pi e^2) \) is a critical plasma density for the fundamental laser component] drives in plasma the electron density perturbation which, in turn, produces the cascade of Raman sidebands (1D in space),

\[
a(z, \xi) = \text{Re} \sum_n a_n(z, \xi) \exp(-i \omega_n \xi/v_g),
\]

where \( \omega_n = \omega_0 + n\Omega \) with \( n \) integer, \( a_n = eE_n/(m_e \omega_0 c), \) \( v_g |\partial a_n/\partial \xi| \ll \omega_0 |a_n| \), and \( \xi/v_g = t - z/v_g \) is a retarded time. In the weakly relativistic limit, \( |a_n| < 1 \), the amplitudes \( a_n \) obey the set of equations

\[
\frac{2i}{k_0} \frac{\partial a_n}{\partial z} - \frac{1}{d} \left( \frac{\omega_0}{\omega_n} \right)^2 \omega_n \approx \frac{1}{2} \left( \frac{\omega_0}{\omega_n} \right) (N_e a_{n-1} + N_e^* a_{n+1}).
\]

Second term in the left-hand side describes the GVD of the EM satellites in plasma, and the right-hand side (RHS) corresponds to the nonlinear coupling of sidebands via the normalized amplitude of weakly nonlinear DEPW, \( (n_e - n_0)/n_c \equiv \text{Re}[N_e(z, \xi) \exp(i k \Omega \xi)], \) \( |\partial N_e/\partial \xi| \ll k \Omega |N_e| \). Temporal profiles \( a_0(\xi) \) and \( a_1(\xi) \) of the incident laser beams are given at the plasma boundary \( z = 0 \):

\[
a_n(0, \xi) = a_\sigma(\xi) \delta_{\sigma n}, \ \sigma = 0, 1
\]

(here, \( \delta_{\sigma m} \) is the Kroneker delta). The amplitude \( N_e \) obeys the equation

\[
[\partial/\partial \xi - i(\delta \omega_1 + \delta \omega_{nl})/c] N_e = -i(k \Omega d/4) \rho (z, \xi)
\]
EM CASCADING IN PLASMA WITH ZERO GVD

We derive here the qualitative scaling laws for the EM cascading under assumption of negligible GVD, which simplifies the set (2) as follows:

\[ \frac{\partial a_n}{\partial z} \approx -i(k_0/4)(N_e a_{n-1} + N_e^* a_{n+1}) \]  

(5)

The integral \( \rho(z, \xi) \equiv \sum a_n a_{n-1}^* \) of the set (5) makes the RHS of Eq. (4) \( z \)-independent.

In the case of exact resonance (\( \delta \omega = 0 \)), the DEPW is close to a weakly relativistic natural mode of plasma. Its amplitude, for \( \rho = \rho_0 \) = const, is expressed through the elliptic integral of the first kind [2]. The RFC eventually destroys the resonance between the DEPW and the laser beat wave and thus limits the amplitude of density perturbation by \( |N_e|_{\text{max}} = 2d(2|\rho_0|/3)^{1/3} \), which corresponds to the maximal RFC \( \delta \omega_{nl}^{\text{max}} \approx (\omega_p/4)(|\rho_0|^2/2)^{1/3} \). The amplitude of DEPW oscillates with time around \( |N_e| = |N_e|_{\text{max}}/2 \) [2], and the steady-state phase modulation of pump at a given \( z \) is impossible. Besides, in a 2D geometry, the transverse variation of pump, and, hence, of the RFC, produces the radial inhomogeneity of the DEPW phase leading eventually to a transverse wavebreaking [5]. The undesirable relativistic effects can be eliminated by the choice of the non-resonant, or detuned, beat wave (\( |\delta \omega| \gg |\delta \omega_{nl}^{\text{max}}| \)) which fixes the amplitude and phase of the DEPW [numerical example from Appendix gives a sufficient frequency mismatch \( |\delta \omega| \geq 3 |\delta \omega_{nl}^{\text{max}}| \)]. The amplitude of linear DEPW \( N_e(\xi) = -i(k_\Omega d/4) \int_0^\xi \rho(\tau) \exp[i(\delta \omega(\tau - \xi)/v_g)] d\tau \) enters the solution of Eq. (5),

\[ a_n(z, \xi) = \sum_{\sigma=0,1} a_\sigma(\xi) \exp[i(n-\sigma)(\psi + \pi)]J_{n-\sigma}(2W), \]  

(6)

where \( J_n(x) \) is the Bessel function of the first kind, and \( \psi(z, \xi) \) and \( W(z, \xi) \) are the phase and modulus of the generating function \( W(z, \xi) \equiv We^{i\psi} = i(k_0/4) \int_0^z N_e(z', \xi) dz' \).

If the driver amplitude varies slowly on the scale \( \delta \omega_f^{-1} \), the electron density perturbation follows adiabatically the ponderomotive force: \( N_e(\xi) \approx |\rho(\xi)d/4| \Omega/\delta \omega_f \) (in the example from Appendix, \( v_g |\partial \rho/\partial \xi| \leq 0.05 |\delta \omega_f \rho| \) is sufficient). In this case, \( \psi = \pi/2 \) and \( W(z, \xi) = N_e(\xi)k_0z/4 \) (\( \rho \) is taken real), and the sum (1) gives a train of phase modulated beatnotes,

\[ a(z, \xi) = \sum_{\sigma=0,1} a_\sigma(\xi) \cos\{\omega_\sigma \xi/v_g + [N_e(\xi)k_0z/2]\cos(k_\Omega \xi)\}. \]  

(7)

As \( |\partial N_e/\partial \xi| \ll k_\Omega |N_e| \), the phase modulation is nearly periodic in time with the beat period \( \tau_b \), and effective number of sidebands, \( 2M \approx N_e k_0 z_M \propto |n_e - n_0| \lambda_0 z_M \), is excited in plasma of the length \( z \approx z_M \).
COMPRESSION OF LASER BEATNOTES IN PLASMA

According to Eq. (7), the wavelength grows in time within each laser beatnote for $\delta \omega_{l} < 0$. In this case, due to the natural GVD of radiation in plasma, the shorter-wavelength sidebands catch up the longer-wavelength ones thus building up the field amplitude near the center of each beatnote. So, in common, the phase modulation proceeds concurrently with the beatnote compression. We evaluate the effect of GVD perturbatively, $a_{n}(z) = a_{n}^{(0)}(z) + d^{2}a_{n}^{(1)}(z) + O(d^{4})$, with $a_{n}^{(0)}$ given by Eq. (6). Equation (2) gives an order relation, $i\partial a_{n}^{(1)}/\partial z \sim n^{2}k_{0}a_{n}^{(0)}/2$, which evaluates the correction to the sideband amplitudes at $z = z_{M}$, $a_{n}(z_{M}) \sim (1 - i\varphi_{n}\mathcal{A}_{n,M}^{(0)})a_{n}^{(0)}$, where $\varphi_{n} = (n^{2}/2)d^{2}k_{0}z_{M}$.

Given $M$ sidebands at $z = 0$ and plasma density artificially set constant ($N_{e} = 0$, so that the sidebands are uncoupled), the $n$th sideband ($|n| \leq M$) acquires the phase shift $\varphi_{n}$ upon propagating over the distance $z_{M}$. If $\varphi_{n} > \pi$, the GVD in unperturbed plasma is not negligible. When the sidebands are coupled via the DEPW, the phase shift is corrected by the factor $\mathcal{A}_{n,M}$. If $|\mathcal{A}_{n,M}\varphi_{n}| < \pi$ while $\varphi_{n} > \pi$ for the same density of plasma, the GVD in the system of coupled sidebands is suppressed, and the phase modulation of the pump proceeds as considered in the previous section. The scaling for the reduction of GVD in the system of $M$ coupled sidebands follows from Eq. (8). Equation (6) gives the largest sideband amplitudes for $|n| \approx M$. Estimates $J_{M}(M) \sim (\sqrt{2}/\pi)M^{-1/3}$, and $\int_{0}^{M}J_{M}(\tau)d\tau \sim O(1)$ at $M \to +\infty$, give the phase shift correction for the utmost satellites $|\mathcal{A}_{M,M}| \sim (\pi/\sqrt{2})M^{-2/3}$. In effect, the GVD is reduced for $M > 3$ ($|\mathcal{A}_{M,M}| < 1$). Effective suppression of GVD occurs in the range $0$ parameters

$$\left(N_{e}\tilde{z}\right)^{4/3} < \sqrt{2}/(\tilde{z}d^{2}) < (\sqrt{2}/\pi)(N_{e}\tilde{z})^{2}$$

(here, $\tilde{z} \equiv k_{0}z/2$) that follows from the condition $|\mathcal{A}_{M,M}\varphi_{M}| < \pi < \varphi_{M}$. Fixed $\tilde{z}$, the processes of phase modulation and compression become equally important if the plasma density is high enough, $d > N_{e}^{-2/3}\tilde{z}^{-7/6}$.

In plasma with nonzero GVD, $\rho$ varies along $z$, and the analytical results from the previous section are only approximately valid. Assuming the adiabatic approach to hold (that is, $v_{k}[\partial \rho/\partial \xi] \ll |\delta \omega_{l}\rho|$), we substitute $N_{e} \approx [\rho(z, \xi)d/4](\Omega/\delta \omega_{l})$ into Eqs. (2) and solve the resulting set of nonlinear coupled equations numerically with the boundary conditions (3). In the two following numerical examples only the normalized plasma density $d$ is altered. We set $\delta \omega_{l} = -3\delta \omega_{nl}^{\text{max}}$, assume $a_{0} = a_{1} = \text{const}$, and fix the number of sidebands, $M = 8$ and the normalized plasma density perturbation $N_{e} = 0.5 \times 10^{-4}$; hence, the modulator length is $z_{8} \approx 0.5 \times 10^{5}\lambda_{0}$ [that is, $z_{8} \approx 4$ cm at $\lambda_{0} = 0.8 \mu$m; such long plasma can be created either in a plasma guiding structure (plasma channel) or by increasing the radius of laser focal spot up to $r_{0} \approx 70 \mu$m to match the interaction length $z_{M}$ with twice the Rayleigh length $z_{R} = \pi r_{0}^{2}/\lambda_{0}$].

Figure 1 illustrates the compression of a positively chirped laser beatnote in two stages. The modulator density $d = 0.5 \times 10^{-3}$ corresponds to the lower border of the
range (9), and laser amplitudes \( a_{0,1} \approx 0.2 \) and frequency mismatch \( \delta \omega_t \approx -0.1 \omega_p \) are quite high. The spectrum of chirped laser [see Fig. 1(a)] is symmetric in the case of zero GVD [see Eq. (6)], whereas nonzero GVD brings some asymmetry. The beatnote profile just slightly shrinks in the modulator, so further compression is made in the plasma of higher density, \( \omega_{p(c)} \gg \Omega \), which is assumed to be unperturbed by the laser beams. The laser temporal profile at the compressor output is \( a(z_M + z_c, \xi) = \Re \sum_n a_n(z_M) \exp \{-i(\omega_n/v_g)\{\xi + z_c[1 - (v_g/c)\epsilon_n]\}\} \), where \( \epsilon_n \equiv 1 - \omega_{p(c)}^2/\omega_n^2 \), and \( a_n(z_M) \) are the spectral components of the chirped laser [bars in Fig. 1(a)]. The utmost higher- and lower-frequency components of laser are separated in time by roughly \( \tau_b/2 \) at the input of compressor. The maximum compression is expected at a distance \( z_c \), when they meet at the center of the beatnote. It takes time \( z_c/c \), that is, \( \Delta v_g(z_c/c) \approx c\tau_b/4 \). Assuming \( \Delta v_g \approx M\Omega (\partial v_g/\partial \omega)\omega_0 \) and knowing that \( k_0(\partial v_g/\partial \omega)\omega_0 \approx (3/2) [\omega_{p(c)}/\omega_0]^2 \), we find \( z_c \approx (k_0M)^{-1} [\omega_0/\omega_{p(c)}]^2 (\omega_0/\Omega)^2 \). Plasma in the compressor is by a factor of 25 denser than in the modulator, so that \( z_c \approx 0.065z_8 \approx 2.7 \) mm. These short dense plasmas may be created by ablation of microcapillaries [6]. Fig. 1(b) shows roughly eight-cycle spike which has approximately eight times higher intensity than the beatnote at the input of compressor.

Figure 2 shows that doubling the plasma density \( d = 10^{-3} \) results in the concurrent phase modulation and compression of the beatnote. Here, laser amplitudes and beatwave detuning are quite low: \( a_{0,1} \approx 0.071 \) and \( \delta \omega_t \approx -0.025\omega_p \). The output spectra reveal considerable modification due to the GVD [compare the bars in Figs. 2(a) and 1(a)]. The beatnote compression and amplification are almost the same as in the two-stage compressor scheme discussed above. As in the GVD-dominated regime the second compressor stage is unnecessary, it is obviously preferable for experimental implementation.

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**FIGURE 1.** The two-stage compression of a chirped laser beatnote in plasma. Modulator plasma density is \( d = 0.5 \times 10^{-3} \) (i.e., \( n_0 = 8.75 \times 10^{17} \) cm\(^{-3} \)). Plot (a) shows the laser spectrum (bars and stairs stand for the modulator with and without GVD, respectively). Dash-dotted line in the plot (b) presents the initial beatnote amplitude (\( z = 0 \)). Dashed line shows the envelope of the chirped beatnote at \( z = z_8 \). This chirped beatnote is compressed in a higher-density plasma with \( \omega_{p(c)} = 5\omega_p \) after propagation over the distance \( z_c = 0.07z_8 \) (solid curve).
FIGURE 2. Single-stage plasma pulse compressor: modulation and compression occur concurrently. Plasma density is \( n = 10^{-3} \ (n_0 = 3.5 \cdot 10^{18} \text{ cm}^{-3}) \). Plot (a): bars and stairs stand for the plasma with and without GVD, respectively. Plot (b): the temporal profile of the compressed beatnote (solid line), compression rate being roughly the same as in Fig. 1(b). Dashed line: the beatnote amplitude at \( z = 0 \).

CONCLUSIONS

We have demonstrated a novel approach to laser pulse compression in the plasma via coherent Raman cascading. The laser beat wave non-resonant with a natural plasma mode drives the DEPW which produces a phase modulation of the pump laser with the beat period. Large number of laser sidebands (more than ten) can be excited under realistic experimental parameters. The GVD of radiation in plasma can compress the positively chirped laser beatnotes and thus create a train of few-laser-cycle EM spikes (single-stage compressor). Strong coupling of laser sidebands via DEPW can reduce the effect of GVD; in this case, compression can be made in a separate plasma of high density (two-stage compressor). We conjecture that this technique could be used for compressing petawatt laser pulses.

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APPENDIX: EVOLUTION OF THE EPW AMPLITUDE DRIVEN BY STEADILY GROWING LASER BEAT WAVE

We give here an example of DEPW that creates a desirable periodic phase modulation of the input laser. We take a an exponentially growing flat-top laser beat wave, \( \rho(\xi) = \rho_0 \left[ \exp(\Delta \omega \xi/v_g) - 1 \right] \left[ \exp(\Delta \omega T) - 1 \right]^{-1} \) for \( 0 \leq \xi/v_g \leq T \) and \( \rho(\xi) = \rho_0 \equiv a_0 a_1 = \text{const} \) for \( \xi/v_g \leq T \). Here \( 1/\Delta \omega \) is a characteristic growth time of the beat wave at a given \( z \).

Omitted the RFC, Eq. (4) admits the solution

\[
N_e(\xi) \approx \frac{\rho_0 \Omega d/4}{\delta \omega_l + i \Delta \omega} \left[ 1 - \frac{i \Delta \omega}{\delta \omega_l} \left( e^{i(\delta \omega_l/v_g)(\xi-L)} - 1 \right) \right]
\]
FIGURE 3. Temporal evolution of DEPW in the case of adiabatically slow ($\Delta \omega \ll |\delta \omega|$) excitation: solid line — numerical solution of full Eq. (4), dash-dotted line — analytic linear solution (10), dashed line — solution of Eq. (4) with $\delta \omega_l$ omitted. The beat wave parameters are $\rho_0 = 0.05$, $\Delta \omega = 5.85 \cdot 10^{-3} \omega_p$, $\Delta \omega T \approx 3$. Where nonzero, $\delta \omega = -3 \delta \omega_{nl}^{\max} = -0.117 \omega_p$, so that $|\Delta \omega/\delta \omega| = 0.05$. The large frequency detuning completely suppresses the relativistic effect: the slowly growing non-resonant beat wave excites almost steady DEPW with small variations of amplitude near the equilibrium value $|n_e/n_0 - 1| = (\rho_0/4)|\Omega/\delta \omega|$. That holds for $\xi/v_g \geq T$ and $\Delta \omega T > 1$. We distinguish the regimes of (a) rapid ($\Delta \omega \gg |\delta \omega|$) and (b) slow ($\Delta \omega \ll |\delta \omega|$) beat wave growth. In the case (a), $N_e \approx -i(\rho_0 d/2)(\Omega/\delta \omega) \sin \psi \exp(i\psi)$ [with $\psi = (\delta \omega/2v_g)(\xi - L)$], which, plugged into Eqs. (6) and (1), give the double-periodic phase modulation of laser field with close periods ($\tau_b$ and $\tau_p = 2\pi/\omega_p$),

$$a \approx \sum_{\sigma=0,1} a_{\sigma} \cos \left\{ \frac{\omega_\sigma \xi}{v_g} + \frac{|N_e|_{\text{max}} k_0 z}{2} \left[ \cos(k\Omega \xi) - \cos \left( k_p \xi - \frac{\delta \omega_l}{v_g} L \right) \right] \right\} \quad (11)$$

[here, $|N_e|_{\text{max}} = (\rho_0 d/2)(\Omega/\delta \omega_l)$]. The lower- and higher-frequency field components are not separated in time within one beatnote, which gives no chance for its compression into a single spike in a high-GVD medium. In the case (b), $a(z, \xi)$ is given by Eq. (7) with constant $N_e \approx (\rho_0 d/4)(\Omega/\delta \omega_l)$. Then, the phase modulation of laser occurs with the single beat period $\tau_b$, and the regime (b) appears to be useful for the pulse compression purposes. Figure 3 illustrates the DEPW evolution in this case.

REFERENCES