Chaos and Exchange Rates

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Chaos: Definition

At the international conference on chaos held by the Royal Society in London in 1986, chaos was defined as apparently stochastic behavior occurring in a deterministic system (Stewart (1997), p. 12). Chaotic complex dynamics are generated by nonlinearities that are present in models or in actual data. Here complexity means that observation of a given dynamic behavior does not help one to learn more about its underlying structure. Systems that exhibit complex behavior which can be described by simple, nonlinear equations are thus said to be chaotic.

The Butterfly Effect

The most important property revealing the characteristics of chaos is extreme sensitivity to initial conditions. If an infinitesimal change is made to the initial conditions of a chaotic time series, the corresponding change iterated through the system until time $t$ will grow exponentially with $t$. This is known as "the butterfly effect", where it is said that the beating of a butterfly's wings in one place can cause a great storm far away. The statistic commonly used to test for sensitivity to initial conditions is the largest Lyapunov exponent, which measures the rate at which information is lost from a system. Sensitivity to initial conditions and therefore evidence of chaos is obtained as long as the largest Lyapunov exponent is positive.
Testing for Chaos

Together with the largest Lyapunov exponent, other major tests for detecting chaos are the correlation dimension and the capacity dimension, which evaluate the complexity of a system. A dimension of less than about 5 gives further evidence of chaos. Other tests are entropy and BDS statistic. Entropy is a measure of disorder in data; its reciprocal gives roughly the time over which meaningful prediction of chaotic series is possible. The BDS statistic is a measure of deviation of the data from pure randomness. A description of these chaotic statistics is given by, for instance, Sprott and Rowlands (1995).

The Recipe for Chaos

The "recipe" for chaos in nonlinear systems seems to stem from the interaction between two conflicting tendencies: "stretching" and "folding". As time goes by, nearby points in a time series are torn apart. But although points close together move apart, some points far apart move close together. The expansion causes points that start off close together to evolve differently. At first, the difference grows regularly. But once the two points have moved far enough apart, no longer must one mimic the behavior of the other. The series becomes completely aperiodic in that it never repeats itself. This recipe is discussed in Stewart ((1997), Chapter 8).

Putting it another way, in the neighborhood of a system equilibrium a centrifugal force dominates, and further away from this point a centripetal force rules. Starting from an initial deviation a given endogenous variable perpetually fluctuates around equilibrium. As the deviations become too large, the centripetal force brings
the variable back toward equilibrium, while once back in the neighborhood of equilibrium, the centrifugal force forces it to diverge again. This sort of equilibrium attracts and repulses simultaneously and is called "strange attractor" to indicate that it is not any of the conventional equilibria as described by sinks, sources, saddles, or limit cycles.

**Strange Attractors**

Strange attractors are suggestive pictures of an equilibrium that can be plotted from a chaotic series to show some order in fake randomness. The existence of strange attractors implies that chaos does not mean totally disorderly patterns. Intervals of regular behavior can also crop up in chaotic series. Periodicity and aperiodicity can be mixed because order and chaos seem to be intimately intertwined.

To get a three-dimensional strange attractor from a chaotic time series one can use the Ruelle-Packard-Takens method (e.g. Stewart (1997), pp. 172-175). The idea is to get a picture made with the time series of three-dimensional observations built out of the original time series of one-dimensional observations by just reading successive triples. The first of these fake observations for a given variable X is, for instance, the triple \((X_{t-3}, X_{t-2}, X_{t-1})\) representing a point in three-dimensional space relative to a chosen origin. The next triple is \((X_{t-2}, X_{t-1}, X_{t})\), the subsequent one is \((X_{t-1}, X_{t}, X_{t+1})\), and so on.

**Short-Run Versus Long-Run Forecastability**

Chaos implies that long-term forecasting is futile, although short-term forecastability and controllability are possible
since there is a deterministic structure underlying the data, if only we know what it is. Long-run predictability would be possible if one had infinite precision at the starting point of a chaotic series as well as in all steps of iteration; but that is a practical impossibility.

As an illustration, if a given equation were the "true" nonlinear equation modeling exchange rate behavior, it could be used to predict where the exchange rate will go in the long run, given the starting points and parameter values. But that could only be done if the initial points and parameter values were given to infinite precision. It would also be necessary to have their entire decimal expansion, all the way out to infinity, not just, say, the first billion digits. These would be irrelevant anyway after the billionth iteration. One would need all of this, which is impossible in practical terms. This example draws on Stewart ((1997), p. 113).

Ten-digit precision, say, could allow for knowing the exchange rate path in the very short run in the example above. Thus, there is both bad and good news. The bad news addresses the long-run unpredictability. The possibility of short-run forecastability and controllability is very good news, however.

Examples of comprehensive non-technical introductions to chaos theory are Stewart (1997) and Gleick (1987), whereas Acheson (1997) and Devaney (1989) give technical introductions.

Chaos in Economics

Most work searching for chaotic dynamics in economic times series show few connections between empirical
results and economic theories (LeBaron (1994), pp. 397, 402). Studies in economic and financial data find little or no evidence of chaos, although they turn up a surprising number of unexplained nonlinear structures. Short and noisy macroeconomic times series lessen the likelihood of directly seeing chaos. Longer and cleaner financial series show no convincing evidence of chaos, although results often show strong evidence of nonlinear dependence (LeBaron (1994), pp. 397-400). Brief recent surveys of the literature on chaos in macroeconomics and finance are LeBaron (1994) and Scheinkman (1994). A former short survey focusing on chaotic economic models is given by Kelsey (1988).

Nonlinearity is a necessary but not sufficient condition for chaos to occur. Both the strong evidence concerning nonlinearity and the thin evidence of chaos suggest a puzzle that needs to be solved by redirecting research toward paying greater attention to economic theory together with conducting new empirical tests emphasizing how "noise" is processed (LeBaron (1994), p. 398). As far as the prototypical models of the exchange rate are concerned, it is suggested that foreign exchange intervention is an important source of noise interfering with the detection of chaos (Da Silva (2000); (2001)), so that a possible underlying chaotic nature of the exchange rate may be blurred by or interconnected with policymaking.

**Chaos in Exchange Rate Models**

Chaotic paths of the exchange rate are usually generated in simple models without too much structure. A purpose of these models is to argue that "if such very simple models
can demonstrate chaos, the phenomenon must surely be a possibility in much more complex systems" (Ellis (1994), p. 195). Economics-textbook examples of chaos can be produced if the rule determining exchange rate behavior is given by a logistic equation (e.g. Rivera-Batiz and Rivera-Batiz (1994), pp. 570-571; also Savit (1992)). The logistic equation (or Verhulst rule) is quite a simple equation capable of generating chaos that is usually presented in introductions to chaos theory.

Since the point that "simple economic models can generate chaos" does not need to be repeated continually, the natural subsequent step is to explore chaos using the relationships suggested by economic theory. A line of research sees in charting a source of nonlinearity leading to chaos in the context of standard sticky-price exchange-rate models (De Grauwe and Vansanten (1990); De Grauwe and Dewachter (1990); De Grauwe and Vansanten (1991); De Grauwe and Dewachter (1992); and De Grauwe, Dewachter, and Embrechts (1993)). Massive foreign exchange intervention is shown to remove chaos in these models (Da Silva (2000)). What is more, Da Silva (2001) shows that a chaotic nominal exchange rate is still possible within the framework of the Obstfeld and Rogoff ((1995); (1996), Chapter 10) model, which features monopolistic competition and short-run sticky nominal goods prices and is an attempt to update the traditional sticky-price model with microfoundations.

**Chaos and Currency Speculation**

Apparently, speculators in the foreign exchange market mostly prefer backward-looking rules rather than relying on the fundamentals of models to make forecasts.
However, there are few strong theoretical arguments to challenge the proposition that destabilizing speculators must lose money (Friedman (1953)). An exception is provided by Hart and Kreps (1986), who give an example of how rational speculation can make money and be destabilizing. But there is increasing evidence against Friedman's standpoint.

Notwithstanding, models such as those discussed in the previous section argue that charting is associated with the possibility of chaos in the foreign exchange market. Since chaos allows for predictability in the shorter term, profits from speculation based on technical analysis can be earned from accurate short-run predictability. If speculators know this fact, they will use charts intending to produce chaos. They thus have a built-in incentive to use charts to make backward-looking forecasts. Consequently, charting may be fully justified by self-fulfilling beliefs, and trading in the foreign exchange market is self-generating.

But short-run predictability would imply a great deal of money to be made forecasting the exchange rate. Nonetheless it is unlikely that such a perfect predictable structure exists in practice. Forecasts of high quality over the shortest horizon are not feasible for financial time series (LeBaron (1994), p. 401).

If there were such money making opportunities, they would be exploited and cause the exchange rate to move in such a manner as to eliminate any profits. There is not too much money to be made forecasting financial series also because (1) the actual implementation of a trading rule may involve unforeseen costs, and prices taken from recorded data sets may not actually be prices at which
trading is feasible; and (2) technical trading rules involve exposure to extensive risks; the larger the expected returns, the higher the probability of the strategy losing a considerable amount of money (LeBaron (1994), p. 401).

As a consequence, chaotic exchange-rate models can be "unrealistic" if no constraint is imposed on huge profits coming from short-run predictability. As in Da Silva ((2000); (2001)), the presence of a central bank is needed to introduce anti-chartist action in these models and play a role in lowering the profits of technical traders.

**Empirical Work on Chaos in Exchange Rates**

The tests for chaos based on algorithms that calculate the Lyapunov exponents and measures of dimension (such as the correlation dimension) are only expected to work if (1) the number of available data is large enough, say, 10000 to 40000 datapoints in natural sciences (for the computation of the correlation dimension, in particular, Ramsey and Yuan (1989) find that a data set of 5000 gives a lower bound; and even with as many as 2000 datapoints, they show a bias in favor of finding chaos even where there is none); (2) the data carry a low level of noise; and (3) the data carry an appropriate amount of aggregation. On the one hand, highly aggregate data could wash out the intrinsic nonlinearities that can a priori be responsible for chaos, but on the other, the data should have some aggregation if they are expected to be "the result of small number of degrees of freedom interconnected by nonlinear relations that could give rise to chaos" (Vassilicos (1990), p. 1); in other words, no aggregation at all is also likely to wash out chaos from the start.
Vassilicos (1990) uses tick-by-tick Deutsche mark/US dollar data advertised on Reuters FXFX page. The data set spans "only" a week, running from Sunday 9 April 1989 to Saturday 15 April 1989, giving 20408 datapoints. Ask prices are considered, instead of an average between both ask and bid prices; thus, the slightest fluctuations are captured, and there is no aggregation. Only the correlation dimension is calculated for the whole data set, as well as for three subsets. Vassilicos finds that the variations of the last two decimal digits of the DM/$ rate are not caused by low dimensional chaos.

Tata and Vassilicos (1991) use the same data set as in Vassilicos (1990) to calculate the largest Lyapunov exponent of the DM/$ rate and find that its value is not significantly larger than zero, which gives further evidence for the absence of chaos in high frequency data. To compute the largest Lyapunov exponent they use the algorithm of Wolf, Swift, Swinney, and Vastano (1985).

Following the methodology of Vassilicos (1990), Tata (1991) uses tick-by-tick ask prices of the Swiss franc/US dollar rate announced on Reuters FXFX page. The data set covers the period from Sunday 9 April 1989 to Saturday 29 April 1989, giving an available amount of 32200 datapoints. In addition to the correlation dimension, he computes the BDS statistic, compares the calculated correlation dimensions with pseudo-random numbers, and carries out a shuffle diagnostic test. Tata finds correlation dimensions that are similar to Vassilicos's and the other tests corroborate this result; he thus concludes that chaos is absent in the data.
Tata (1991) reports another study by Kugler and Lenz (1990) that analyzes 572 datapoints of exchange rates of the Deutsche mark, Swiss franc, French franc, and yen against the US dollar. They claim to have found significant nonlinearities for all cases. Although this conclusion is shared in a number of subsequent papers by other authors, the study of Kugler and Lenz shows the shortcoming of using small data sets.

The studies of Vassilicos (1990), Tata and Vassilicos (1991), and Tata (1991) are not without their problems. The data set they take is large enough, but it carries no aggregation of any kind, and this could wash out chaos from the start, as observed above. The cleanliness of their data is also questionable. Pentecost ((1993), p. 189) observes that micro-market structure dependencies can introduce noise even into the tick-by-tick data used by them. Pentecost remarks that "(t)ick-by-tick data capture bid-ask bounces and other dependencies which are caused by the micro-market structure, such as the sequential execution of limit orders on the books of the specialist as the market structure moves through those limit prices". He suggests increasing the sampling interval to average out these "artificial dependencies". The use of intra-daily data is also criticized by Guillaume, Dacorogna, Dav, Müller, Olsen, and Pictet (1994) for bringing with it the problem of excessive noise and market microstructure.

De Grauwe, Dewachter, and Embrechts ((1993), Chapter 7) use daily exchange rates of the Deutsche mark, the British pound, and the yen against the US dollar obtained from Reuters for the period running from 4 January 1971 to 30 December 1990. Raw exchange rates are transformed into daily log-returns, since the raw data are
likely to be nonstationary. They take four subsets, namely 1971-1972 (fixed exchange rates period), 1973-1981, 1982-1990, and 1973-1990 (entire floating period). They compute correlation dimensions for every subset and conclude that, although the DM/$ rate shows no evidence of chaos, there is a possibility that the phenomenon is present in the £/$ and ¥/$ rates. In particular, they find that, for the entire floating period (1973-1990), the £/$ rate is chaotic and the ¥/$ rate is "possibly chaotic". The 1973-1981 period also reveals a chaotic signal for the £/$ rate. The ¥/$ rate also appears to be chaotic for the fixed rates period (1971-1972), although this result is likely to be biased due to the small amount of datapoints.

The results of De Grauwe, Dewachter, and Embrechts (1993), Chapter 7) cannot be viewed as conclusive, however, because finding a low correlation dimension alone is not enough for concluding that chaos is present (Hsieh (1993)); computation of positive largest Lyapunov exponents is further required. Commenting on the results by those authors, Brooks (1995a), pp. 15-16) is still more sceptical: "(i)t is perhaps likely that these low dimension estimates are the product of some nonlinear stochastic (nonchaotic) data generating mechanism".

De Grauwe, Dewachter, and Embrechts (1993), Chapter 8) also test for the presence of nonlinearities using six daily, weekly and monthly exchange-rate returns, namely DM/$, ¥/$, £/$, £/DM, £/¥, and ¥/DM. The first three series are closing rates from 1 January 1971 to 30 December 1990, while the remaining three series are calculated using triangular arbitrage. They find that at daily and weekly frequencies, the presence of nonlinear structures for any of the exchange rates cannot be
discarded. However, at the monthly frequency, nonlinearity is absent in the £/¥ rate.

Brooks (1995a) uses a sample of over twenty years of daily mid-price spot exchange rates of a set of ten currencies against the British pound. The currencies are: the Austrian schilling, the Canadian dollar, the Danish krone, the French franc, the Deustche mark, the Hong Kong dollar, the Italian lira, the yen, the Swiss franc, and the US dollar. The data are taken from Datastream using Datachannel, running from 2 January 1974 to 1 July 1994. As usual, possibly nonstationary raw exchange rates are transformed into daily log-returns.

Brooks (1995a) checks for the presence of unity-root nonstationarity and finds that both the raw data and daily returns are indeed strongly I(1), although daily returns are clearly stationary. He computes the correlation dimension and the largest Lyapunov exponent, employs a surrogate data test, and compares his results with those of known chaotic systems (the logistic map, the Henon map, and the Lorenz attractor) as well as with those of a deterministic but nonchaotic map (the sine wave) and with those of a series of pure standard Gaussian noise. To calculate the largest Lyapunov exponent, Brooks uses not only the algorithm of Wolf et al. (1985) but also a more recent technique developed by Dechert and Gencay (1992), which is more powerful in the presence of noise. Indeed, results of estimation using the algorithm of Wolf et al. are likely to be highly sensitive to noise (Brock and Sayers (1988)). This is particularly problematic in economic data where noise is more prevalent.
Brooks (1995a) finds results for correlation dimension estimation that are somewhat inconclusive. In some cases, the correlation dimension stabilizes as the embedding dimension increases. This suggests the presence of chaos, although saturation of the correlation dimension is a necessary but not sufficient condition for the existence of an attractor, as already observed. However, the results of Lyapunov exponent estimation using the algorithms of Wolf et al. (1985) and Dechert and Gencay (1992) show no evidence of chaos in the pound exchange rates. In all cases, the largest Lyapunov exponent is calculated to be negative. Brooks concludes that evidence suggests the presence of some sort of nonlinear determinism in the data, but excludes the stronger possibility of deterministic chaos.

In other study, Brooks (1995b) uses the same data set as described above and also finds "irrefutable evidence of nonlinearity" in many of the series. A result almost established in literature is that if exchange rates do follow a random walk, the overwhelming evidence is that daily rates are not linearly independent of past changes. In particular, there is substantial departure from independent and identical distribution (IID). What is more, there is indeed strong nonlinear dependence. A previous study that helped such a point of view to prevail is the one by Hsieh (1989), who finds strong nonlinear dependence in daily closing bid prices of five currencies against the US dollar, namely the British pound, the Canadian dollar, the Deutsche mark, the yen, and the Swiss franc, using a total of 2510 observations from 2 January 1974 to 30 December 1983.
Once strong evidence of nonlinearities is found in most studies of exchange rates, the remaining problem is to characterize the type of nonlinearity. The first step is to distinguish between nonlinear deterministic chaos and nonlinear stochastic structures. The second step is to assess which kind of chaos, if any, is involved. The "chaos" allowing for accurate short-run forecastability is, in fact, low-dimensional "regular" chaos. That is the type of chaos addressed by the empirical studies discussed above. As seen, evidence of nonlinearity is consistent with the existence of low-dimensional "regular" chaos, although extra corroboration is needed to detect its presence. There are, however, other kinds of chaos for which there are no tests available so far: low-dimensional "irregular" chaos and high-dimensional chaos. Yet, as Brock, Hsieh, and LeBaron ((1991), p. 134) remark, "there is no practical difference between high-dimensional chaos or low-dimensional "irregular" chaos on the one hand and true randomness on the other hand". Finally, the third step is to turn attention to the possibility that the nonlinear structures underlying actual exchange rates are of a stochastic nature. Although the class of nonlinear stochastic models is extremely large, two classes encompass all models discussed in the time series literature, namely mean-nonlinearity and variance-nonlinearity (Brock, Hsieh, and LeBaron (1991), p. 135). ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are examples of variance-nonlinearity. Evidence suggests that nonlinearities are likely to enter through variances rather than through means (Brock, Hsieh, and LeBaron (1991), p. 143), if real-
world exchange rates are better described by ARCH and GARCH models.

Whether exchange rates are really chaotic may not be of great importance for traders, because they can adjust their forecasts according to the apparent predictability coming from the fact that there is strong evidence of a nonlinear structure in exchange rates (LeBaron (1994), p. 400). However, variations of exchange-rate returns are hard to forecast, although the magnitudes of these variations are indeed predictable. "Volatility persistence" thus presents a departure from pure randomness. But variations of exchange-rate returns also seem to depend on an estimate of recent volatility: predictability appears to be higher during periods of lower volatility (LeBaron (1994), p. 400). Another intriguing result which is still more upbeat about the possibility of finding low-dimensional chaos is that forecastability using such nonlinear processes is not uniform across its range of movement. There may be periods when forecasts are very good, and periods in which forecasting is almost impossible (LeBaron (1994), p. 400). Since this matches with a property of chaotic systems, the fact that such nonlinear processes can also be chaotic is a real possibility. After all, it is puzzling that "(f)or financial series there may be an extremely wide gap between successful nonlinear forecasting, and actual identification of chaotic dynamics" (LeBaron (1994), p. 401).

The fact that direct evidence for chaos is weak cannot be considered unexpected. As LeBaron ((1994), p. 397) remarks "(i)n hindsight most of these results should have been expected to some extent". An apparent reason is that "traders' ability to perceive complex patterns and trade
against them reduces the strength of these patterns" (LeBaron (1994), p. 398). Notwithstanding, in the light of the discussion presented in this article, a conclusion emerging from the use of actual exchange-rates data is that chaos "is still a very open question for economic research" (LeBaron (1994), p. 398).

References


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