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Optimal Sequential Search and Optimal Consumption-Leisure Choice

Sergey Malakhov

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Sergey Malakhov

Ph.D., Applied Economics,

Pierre Mendes France University,

Grenoble, France

To F&F

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Abstract

This paper describes the static model of consumption-leisure choice in which both the maximization of utility and the maximization of precautionary savings for sequential purchases are constrained by the equality of the marginal costs of search to their marginal benefits. The marginal rate of substitution of leisure for consumption appears in the specific form, which updates the Cobb-Douglas utility function and provides search behavior with the optimal stopping rule. This rule bridges the gap between the neoclassical explanation of search behavior and the search-satisficing concept. The original arrangements of polar models of behavior, the overconsumption effect, the Veblen effect, and Protestant work ethics complete the presentation of the model.

JEL Classification: D01, D11, D83, D91.

Introduction

During the past decades, consumer search has become one of the most dynamic themes in modern economic thought (Adams (1997), Benabou and Gartner (1993), Diamond (1971, 1982), Fishman (1992), Grewal and Marmorstein (1994), Lach (2002), Pratt et al. (1979), Reinsdorf (1994), Rothschild (1974), Salop and Stiglitz (1982), Stahl (1989), Stigler (1961), Stiglitz (1979a, 1979b)). Baye et al. (2006) categorize writings on consumer search behavior into three clusters: search-theoretic models, clearinghouse models, and bounded rationality models of price dispersion, which result in a hierarchy of subgroups, such as the fixed sample approach, sequential search analysis and asymmetric consumer models. Although the consumer search literature attempts to cover all attributes of the market environment, it produces some methodological disequilibrium. Usually, researchers focus on sellers' heterogeneity and their behavior rather than on consumers' propensity to search and their shopping preferences. This may be because many aspects of search behavior are addressed in other streams of economic

thought. The relationship between uncertainty, search, and savings has been thoroughly explored in works on the buffer stocks saving (Carroll (1992, 2001a, 2001b), Carroll and Samwick (1998), Deaton (1977), Ludvigson and Michaelidas (2001)). The economics of taxation has examined consumption inequality and the allocation of time (Stiglitz (1982), Kleven (2004)). Studies in the allocation of time theory (Becker (1965)) present detailed analyses of shopping activity (Aguiar and Hurst (2005, 2008)). Most of these studies accentuate the dynamic nature of search behavior, especially in their attempts to describe optimal sequential search rules. Although the process of sequential searching does not necessarily extend beyond one static shopping tour, there have been few attempts to integrate search theory with neoclassical utility theory (Manning (1997)). Therefore, this paper attempts to restore methodological equilibrium by presenting a model in which both the maximization of the utility of consumption-leisure choice and the maximization of saving for purchases are constrained by the equality of the marginal costs of searching to its marginal benefits.

This paper is organized as follows. The first section presents the monetary model of savings or reserve for purchases. The second section describes the corresponding consumption-leisure utility maximization model. Section III highlights different search tactics with regard to leisure complements and leisure substitutes. The fourth section examines changes in the marginal rate of substitution of consumption for leisure with regard to the time horizon of consumption-leisure choice. Section V considers the phenomenon of relative satiation, produced by the marginal rate of substitution of consumption for leisure during the search process. The sixth section describes the decision-making process with regard to the search-satisficing concept. Section VII is devoted to the famous anomaly in economic behavior of the "leisure model". This section describes how the momentous reduction of labor time transformed consumption into a quasi-complement to leisure and how this change in the consumption-leisure relationship produced the Veblen effect. In contrast, the "common model" of behavior, as demonstrated in Section VIII, exhibits major attributes of the Protestant work ethic. The brief conclusion presents the most promising extensions to the methodological approach presented here.

1. Reserve for Future Purchases

Suppose that the general relationship between the benefits and costs of a search is given by

$R(S) = wL(S) - QP(S)$, where

$wL(S)$ – labor income wL, diminishing during the search S $(\partial L/\partial S < 0)$,

QP(S) – expenditures on fixed or pre-allocated quantity ($\partial P/\partial S<0$),

R(S) – reserve (saving) for daily expenses and for purchases.

Saving for purchases occupies an important niche in the hierarchy of financial needs (Xiao & Noring (1994)). It represents a simple form of a precautionary motive because *"in reality purchases are made sequentially, some frequently and some infrequently, so that while the purchaser has exact up-to-date information about the prices of the goods actually being bought, he does not have accurate knowledge of the prices of other goods"* (Deaton (1977, p.899)).

The search for a cheaper price continues until the marginal benefit equals the value of the marginal costs, or

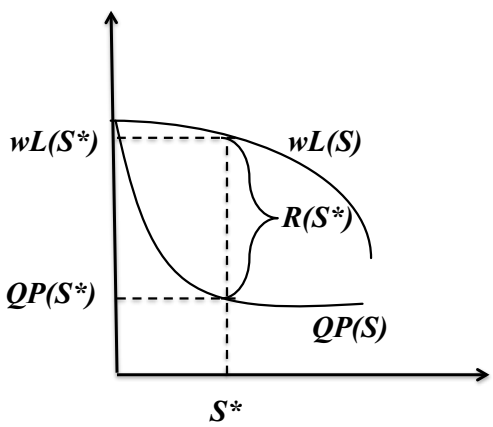
$$Q\frac{\partial P}{\partial S} = w\frac{\partial L}{\partial S} \quad (1)$$

When the consumer concludes the search, he maximizes the reserve for purchases (Fig.1):

$$w\frac{\partial L}{\partial S*} = Q\frac{\partial P}{\partial S*} \Rightarrow \frac{\partial R}{\partial S*} = 0 \, where \, R = R(S) = wL(S) - QP(S) \quad (2)$$

This is a real maximum. A diminishing marginal utility of labor leaves fewer productive hours for the search and gives us $\partial^2 L/\partial S^2<0$. However, the marginal utility of the search itself is also diminishing, or $Q\partial^2 P/\partial S^2>0$. Finally, $\partial^2 R/\partial S^2=w\partial^2 L/\partial S^2 - Q\partial^2 P/\partial S^2<0$.

Fig.1.



Sometimes, the quantity demanded is not equal to the quantity purchased.[1] When we decide to increase consumption, the expenditure curve shifts upward and becomes steeper. We should increase the absolute value of our propensity to search $|\partial L/\partial S|$ to restore the equation (1). Due to $\partial^2 L/\partial S^2<0$, we should continue the search until its marginal costs again equal its marginal benefits. However, in this case, we decrease the reserve for other purchases.

If the monetary reserve for purchases is created by a psychological precautionary motive, the consumer does not need a liquidity constraint because "*the precautionary saving motive essentially induces self-imposed reluctance to borrow (or to borrow too much)*" (Carroll (2001a, p.32)). Even if we borrow and the reserve becomes negative, we nevertheless equalize the marginal benefit of the search with its marginal costs, this time to minimize the deficit.

2. Consumption – Leisure Choice

The maximization of the reserve for purchases seems to be a necessary but not sufficient condition for the decision to conclude the search. However, we can derive the necessary condition from the analysis of the marginal utilities of consumption and leisure.

Due to $\partial^2 P/\partial S^2 >0,$ the search process decreases not only the price P itself but also the absolute value $|\partial P/\partial S|$. If the search decreases leisure time H, and if it might increase the quantity purchased Q, we can formulate a simple optimal stopping rule:

$$dU(Q(|\partial P/\partial S|),H(|\partial P/\partial S|)) = d|\partial P/\partial S|(MU_Q\frac{\partial Q}{\partial|\partial P/\partial S|}+MU_H\frac{\partial H}{\partial|\partial P/\partial S|});$$

$$\frac{dU(Q(|\partial P/\partial S|),H(|\partial P/\partial S|))}{d|\partial P/\partial S|} = MU_Q(\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{\partial H}{\partial|\partial P/\partial S|}MRS_{(H\ for\ Q)});$$

$$\frac{dU(Q(|\partial P/\partial S|),H(|\partial P/\partial S|))}{d|\partial P/\partial S|} = \frac{\partial H}{\partial|\partial P/\partial S|}MU_Q(\frac{dQ(|\partial P/\partial S|)}{dH(|\partial P/\partial S|)}+MRS_{(H\ for\ Q)});\ where$$

$$\frac{\partial H}{\partial|\partial P/\partial S|}>0; MU_Q>0; \frac{dQ(|\partial P/\partial S|)}{dH(|\partial P/\partial S|)}<0; MRS_{(H\ for\ Q)}>0.$$

$$\frac{dU(Q(|\partial P/\partial S|),H(|\partial P/\partial S|))}{d|\partial P/\partial S|}<0\ when\ (\frac{dQ(|\partial P/\partial S|)}{dH(|\partial P/\partial S|)}+MRS_{(H\ for\ Q)})<0 \qquad (3)$$

[1] See, for example, McCafferty (1977) "Excess Demand, Search, and Price Dynamics," American Economic

We see that ***the individual concludes the search for a cheaper price when an absolute value of the expected substitution rate*** $dQ_{|\partial P/\partial S|}/d\,H_{|\partial P/\partial S|}$ ***of additional searching equals the MRS (H for Q).***

Let us determine the *MRS (H for Q)* with regard to the monetary reserve for purchases. Indeed, when the consumer has no liquidity constraint, he can optimize the consumption-leisure choice *(Q,H)* with respect to the equality of the marginal values of the search. We can re-write the equation (1) to obtain the following constraint:

$$w = Q\frac{\partial P/\partial S}{\partial L/\partial S} \quad (4)$$

Therefore, the Lagrangian expression takes the following form:

$$\Lambda = U(Q,H) + \lambda(w - Q\frac{\partial P/\partial S}{\partial L/\partial S}) \quad (5)$$

When we derive the *MRS (H for Q)*, we can see that equation (1) offers the possibility to present this value in two interrelated forms, a physical form and a monetary form (Appendix A):

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (6)$$

Both of these forms include the value $\partial^2 L/\partial S\partial H$. If we denote

$$\frac{\partial L}{\partial S} = \frac{\partial L}{\partial S}(H) = \frac{H-24}{24} \quad (7)$$

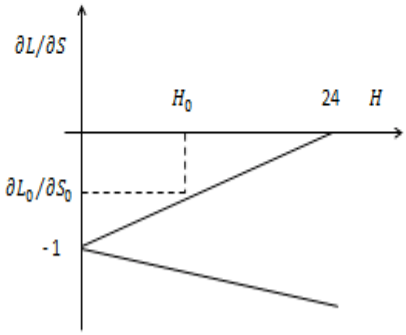
we get $\partial^2 L/\partial S\partial H = 1/24 = 1/T$, where the value T represents the time horizon of the consumption-leisure choice.[2] Therefore, the *MRS (H for Q)* provides the consumption-leisure choice with an explicit time horizon. However, if we differentiate this explicit time horizon T with respect to search time S, we get

$$\partial L/\partial S + \partial H/\partial S + 1 = 0. \quad (8)$$

(The equation (8) provides a very interesting extension to the equation (7). Indeed, when $\partial L/\partial S < -1$, the value H becomes "negative". However, in real life, where leisure time is positive, the value $\partial L/\partial S < -1$ describes an important decrease in labor time in favor of both search and leisure (Fig.2):

[2] If $dH(S) = -\,dS\times(H/T)$, then $\partial H/\partial S = -\,(H/T)$, and due to $\partial L/S + \partial H/\partial S + 1 = 0 \Rightarrow \partial L/\partial S = (H - T)/T$

Fig.2



The decision to definitely cut labor time to increase both search and leisure changes the value $\partial^2 L/\partial S\partial H$ to negative. Moreover, if we differentiate consumption Q with respect to leisure time H directly, we get

$$Q = w\frac{\partial L/\partial S}{\partial P/\partial S};\ \frac{\partial Q}{\partial H} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (9)$$

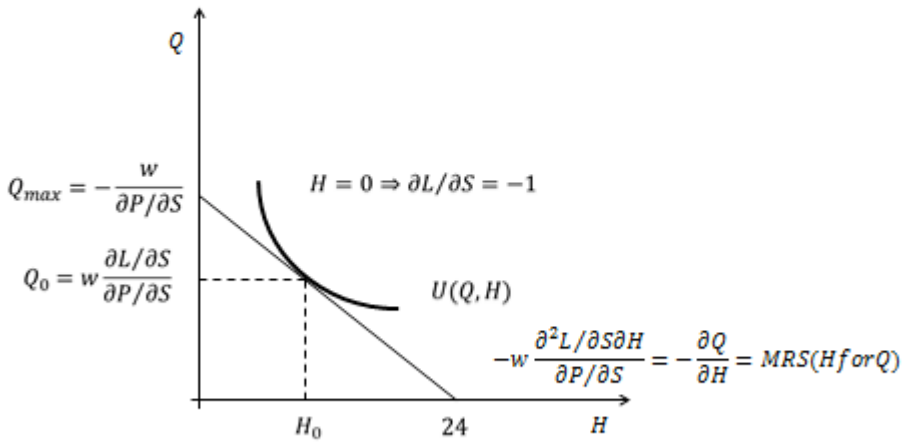
This means that when $\partial^2 L/\partial S\partial S<0$, the value $\partial Q/\partial H$ becomes positive. We can specify this particular case as the "leisure model" of behavior. Section VII presents the transformation of the "leisure model" into the Veblen effect.)

The combination of (1) and (7) results in the following form:

$$Q_0 = w\frac{\partial L_0/\partial S_0}{\partial P_0/\partial S_0} = \frac{w}{\partial P_0/\partial S_0}\frac{H_0 - 24}{24} \quad (10))$$

Now, we can present a graphical illustration of the consumption-leisure choice (Fig.3):

Fig.3



The reserve maximization model comes close to the buffer-stock concept of saving behavior. Indeed, if *MRS (H for Q)* approaches the *w/P* ratio, the consumer spends all disposable income on current consumption. However, the uncertainty of present and future price dispersions activates the search process and creates a reserve for purchases. Moreover, *"The expected savings from given search will be greater, the greater the dispersion of prices"* (Stigler (1961, p.215)).

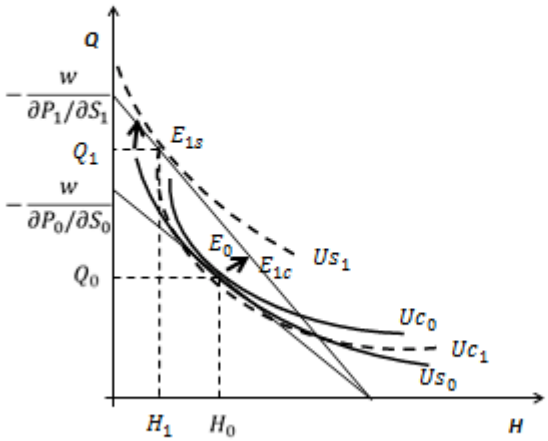
Both the monetary form and the physical form yield the same solution for the *MRS (H for Q)*. However, the physical form sometimes is more useful, because due to $\partial L/\partial S+\partial H/\partial S+1=0$ rule it can generate the following utility function (Appendix B):

$$U(Q;H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S} \qquad (11)$$

3. Leisure Complements and Leisure Substitutes

The optimal stopping rule highlights the importance of the substitution effect between consumption and leisure. The following graphical illustration facilitates understanding of the possible outcomes of an additional search for a cheaper price of leisure complements and leisure substitutes (Fig.4):

Fig.4



An individual starts from point E_0, where he has already maximized both a reserve for purchases and the consumption-leisure utility. If his preferences result in the flat indifference curve U_{s0} and if he is ready to cut the reserve for other purchases, he can continue searching until the point E_{1s}, where he increases the utility level from U_{s0} to U_{s1}. Therefore, the increase in consumption

outweighs the decrease in leisure time, and the decrease in absolute value $|\partial P/\partial S|$ increases the utility level:

$$(\frac{dQ(|\partial P/\partial S|)}{dH(|\partial P/\partial S|)} + MRS(H\ for\ Q)) < 0 \Rightarrow \frac{dU(Q(|\partial P/\partial S|), H(|\partial P/\partial S|))}{d|\partial P/\partial S|} < 0 \quad (12)$$

.

However, if his preferences produce the nearly L-shaped indifference curve U_{c0}, the consumption pattern $(Q_1;H_1)$ at point E_{1s} is not interesting to the consumer because it corresponds to a lower utility level U_{c1} with regard to the initial utility level U_{c0}; the additional search decreases the utility level:

$$(\frac{dQ(|\partial P/\partial S|)}{dH(|\partial P/\partial S|)} + MRS(H\ for\ Q)) > 0 \Rightarrow \frac{dU(Q(|\partial P/\partial S|), H(|\partial P/\partial S|))}{d|\partial P/\partial S|} > 0 \quad (13)$$

.

This simple example illustrates the common-sense idea that people are willing to search more for leisure substitutes than for leisure complements. Here we must take the opportunity to apply the results of the analysis of household labor supplies and commodity demands, presented by Blundell and Walker:

"Services and transport are strong substitutes for male leisure, whereas clothing, food, energy and our definition of durables tend to be complements to male leisure. As might be expected, these goods do not necessarily have the same relationships with female leisure. Services tend to be complementary to female leisure, clothing is a substitute and energy tends to be a compliment" (Blundell and Walker (1982), p.361).

4. Time Horizon and Reasons for Overconsumption

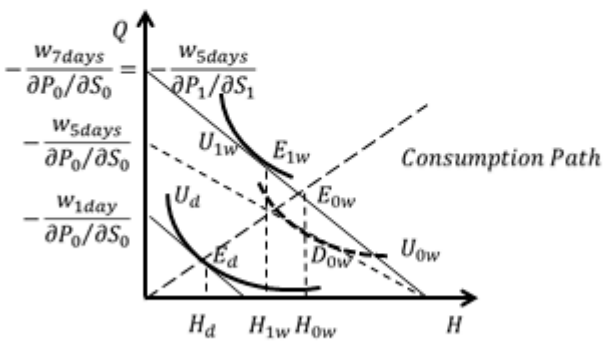
When we consider the expansion path of the consumption-leisure choice, we can see that changes in the wage rate and time horizon are usually not proportional, and shifts of constraint are not parallel. Let us consider, for example, the shift from a daily wage rate to a weekly consumption pattern.[3]

[3] Although this example is very simple, it has historical roots in times when the labor supply in developed economies was limited to 40-hour weeks and when supermarkets' networks emerged (S.M.).

Weekends decrease the labor income of a day carpenter. To compensate for this deficit, he should search for additional non-labor income; that is, he should search for a more interesting local market with a lower price level.

When a day laborer plans his weekly consumption pattern on the basis of his daily wage rate, he inevitably moves from daily equilibrium E_d not to equilibrium E_{0w} but to disequilibrium D_{0w}, which corresponds to a weekly allocation of time between labor, leisure, and search (Fig.5):

Fig.5



The disequilibrium point D_{0w} lies below the consumer's habitual consumption path. In addition, the *MRS (H for Q)* at this point is less than the *MRS (H for Q)* at his daily equilibrium E_d. However, the lower value of the *MRS* gives him the opportunity to search more. On Saturday morning, the carpenter definitely cuts his leisure time from H_{0w} to H_{1w}, and he goes to the supermarket. If he returns to his individual *MRS (H for Q),* he automatically increases his consumption well above the equilibrium level E_{0w}. The carpenter finds himself at a new equilibrium E_{1w} at utility level U_{1w}.

The following set of equations (14,15) illustrates the graphical interpretation of the overconsumption effect under this kind of uncertainty:

$$MRS(HforQ) = -\frac{w_{7days}}{\partial P_0/\partial S_0}\partial^2 L/\partial S\partial H = -\frac{w_{5days}}{\partial P_1/\partial S_1}\partial^2 L/\partial S\partial H \quad (14)$$

$$MRS(HforQ) = -\frac{Q_{E0w}}{\partial L_0/\partial S_0}\partial^2 L/\partial S\partial H = -\frac{Q_{E1w}}{\partial L_1/\partial S_1}\partial^2 L/\partial S\partial H \quad (15)\cdot$$

Additional searching decreases the absolute value $|\partial P_0/\partial S_0|$ until $|\partial P_1/\partial S_1|$. At the same time, additional searching increases the absolute value of the propensity to search from $|\partial L_0/\partial S_0|$ to

$|\partial L_1/\partial S_1|$. Therefore, when the consumer returns to his habitual *MRS (H for Q)*, he raises his weekly consumption from Q_{E0w} to Q_{E1w}.

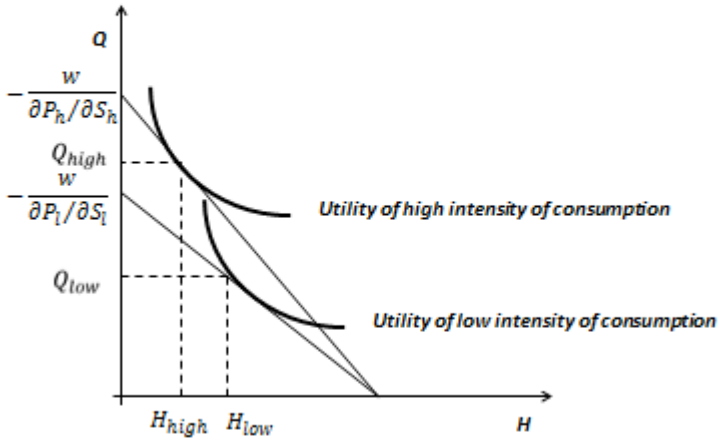
When a change in the time horizon decreases the expected value $w \times \partial^2 L/\partial S \partial H$, the need to compensate for the expected loss in labor income stimulates a search for additional non-labor income and for a lower absolute value $|\partial P/\partial S|$. If additional searching results in the return to the initial *MRS (H for Q),* the overconsumption effect is produced.

5. Relative Satiation and Different Intensities of Consumption

When an individual decides to conclude a search, he takes into consideration alternative uses of time: either working slightly more to increase the chances of receiving a bonus or enjoying his leisure. The latter feeling is more common. However, it automatically determines the value $\partial H/\partial S$, which, due to the $\partial L/\partial S + \partial H/\partial S + 1 = 0$ rule, automatically determines the value of the propensity to search $\partial L/\partial S$, if the individual has the opportunity to receive a bonus for extra hours or decides to spend extra hours in the office to improve his skills.

The value of the absolute propensity to search $|\partial L/\partial S|$ exposes the willingness to substitute labor for searching. However, "*Some individuals are more averse to work than others*" (Stiglitz (1982), p.233)). Therefore, we can expect that the absolute propensity to search differs among individuals. At the same time, the value of the absolute propensity to search determines the physical form of the *MRS (H for Q)* (Appendix B). Therefore, when individuals make different trade-offs between consumption and leisure and choose different consumption patterns, they reveal different propensities to search. The equation (1) shows that different propensities to search $\partial L/\partial S$ correspond to different search tactics and different values $\partial P/\partial S$. However, different values $\partial P/\partial S$ produce different non-labor income, and *"individuals with different abilities will make different choices of (C,Y) pairs, since they have different indifference curves"* (Stiglitz (1982) p.218) (Fig.6):

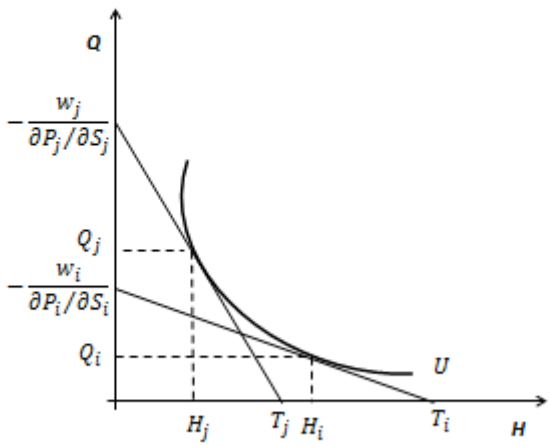
Fig.6



6. Easterlin Paradox, Search-Satisficing Concept, and Economic Decision Making

When we take the time horizon as a constant value, we can see that individuals with different abilities to consume have different search tactics. They choose different sufficient levels of consumption, which correspond to different indifference curves. However, if we take the time horizon as a variable value, we can theoretically construct an indifference curve that joins individuals with different wage rates and different propensities to search and who make purchases on different local markets (Fig. 7):

Fig.7



Indeed, the development of the reserve maximization model contributes to the analysis of the problem of the relativity of utility and to the resolution of the so-called “Easterlin paradox” (de la Croix (1998), Easterlin (1974), Ferrer-i-Carbonell (2005), van de Stadt et al. (1985)). However, this analysis seems to be methodologically overly long because it requires the willpower to understand the relative nature of utility itself.

The discussion of the procedural approach and the search-satisficing concept seems to be much shorter. Simon wrote,

"In an optimizing model, the correct point of termination is found by equating the marginal cost of search with the (expected) marginal improvement in the set of alternatives. In a satisficing model, search terminates when the best offer exceeds an aspiration level that itself adjusts gradually to the value of the offers received so far" (Simon (1978, p.10)).

The reserve maximization model redirects our attention from the marginal values of search to the value of *MRS (H for Q)*. The equalization of marginal values in the decision-making process becomes a problem of secondary importance. The focal point in decision-making changes from the marginal values of search to the concept of the marginal rate of substitution.

In fact, the optimal search represents a search for a price level that corresponds, at the given wage and time horizon, to a physical trade-off between consumption and leisure. (We can expose the decision-making process more informally if we take the value of the wage rate as W per period and the value of $\partial^2 L/\partial S \partial H=1$):

$$-\frac{dQ}{dH}=\frac{MU_H}{MU_Q}=-\frac{w}{\partial P/\partial S}\partial L^2/\partial S\partial H=-W\frac{\Delta S}{\Delta P} \quad (16)$$

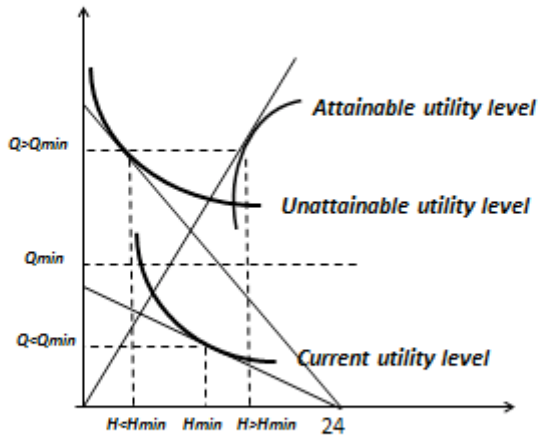
In reality, consumers do not calculate the marginal costs and marginal benefits of search. If they enter the market and find prices higher than the reservation level (the actual *MRS (H for Q)* less its target level), they simply spend search time S=ΔS to get ΔP, which produces a ***satisficing as well as optimal*** combination of consumption and leisure. This satisficing level should correspond to their *MRS (H for Q)* because otherwise ***they feel frustrated*** (Malakhov (2011)).

Of course, an adjustment takes place. When consumers quickly find a price slightly higher than the reservation level, they can accept it because the short search time, with respect to the lower ΔP value, can produce the optimal $\Delta S/\Delta P$ ratio (the *QP(S)* curve unexpectedly becomes flatter). Contrarily, when consumers unexpectedly and quickly find the reservation price itself (the *MRS (H for Q)* is still below its optimal level) and additional search presumes a real journey, consumers remember the place, make a short tour around to assure themselves of their choice and then return. This tour can increase search time ΔS to the optimal $\Delta S/\Delta P$ ratio.

7. "Leisure Model" of Behavior and the Veblen Effect

If we return to the "leisure model" of behavior, we see that the value $\partial L/\partial S<-1$ makes the relationship between search and leisure positive $(\partial H/\partial S>0)$ (Fig.8):

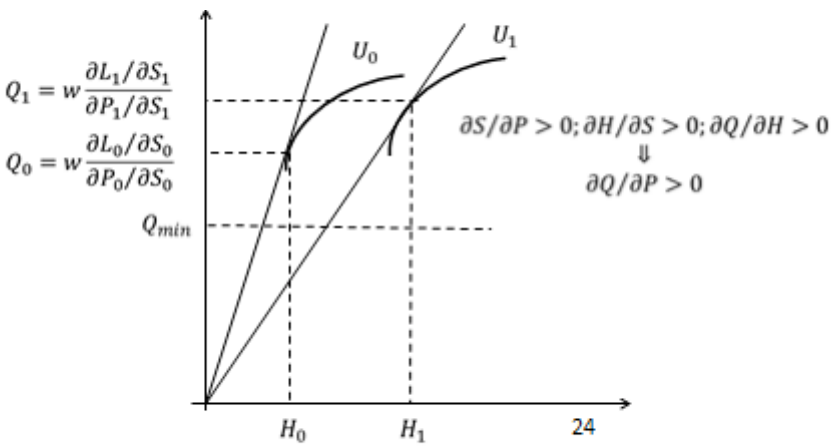
Fig.8



At the same time, the value $\partial L/\partial S<-1$ gives us $\partial^2 L/\partial S\partial H<0$. However, when the value $\partial^2 L/\partial S\partial H$ becomes negative, the equation (9) results in a positive relationship $\partial Q/\partial H$. The budget constraint changes its slope, and the indifference curve changes its shape. This occurs when the aspiration or minimum level of consumption is unattainable because an individual must give up the physical and/or psychological minimum of leisure time *Hmin*.

We can find examples of the "leisure model" of behavior not only in developed economies but also in underdeveloped economies. The most interesting attribute of the "leisure model" is the rationalization of the Veblen effect. We can see that the ***Veblen effect is rational for the "leisure model" of behavior*** (Fig.9):

Fig.9



Price growth shifts *QP(S)* curves upward, and the new price level stimulates searching. The relationship between the price and time of the search becomes positive *(∂S/∂P>0).*[4] Let us imagine a Hollywood star who begins to search at a new price level. If she increases her search time, she automatically increases her leisure time *(∂H/∂S>0).* To "load" this additional leisure, she increases consumption *(∂Q/∂H>0).* As a result, price growth, followed by an increase in leisure as well as in consumption, raises the utility level *(∂Q/∂P>0;* $U_1>U_0$*).* A Hollywood star may feel happier when she increases her consumption at the new price level. Indeed, conspicuous consumption and conspicuous leisure complement one another (Veblen (1984)).

8. Protestant Work Ethics

Returning to the "common model" of behavior, we can find an example that contrasts with the Veblen effect, when individuals decrease consumption to avoid ostentation (Lea et al. (1987)). The counter-Veblen effect guides the analysis of the reserve maximization model to an important conclusion. If we interpret the ***low absolute propensity to search*** as a manifestation of ***high willingness to work***, the key equation (1) of the reserve maximization model tells us that high willingness to work corresponds to a low level of consumption.

If we convert the key equation of the model (1) into elasticity form, we get

$$w\frac{\partial L}{\partial S} = Q\frac{\partial P}{\partial S} \Rightarrow \frac{PQ}{wL} = \frac{e_{l,s}}{e_{p,s}} \quad (17).$$ [5]

We can see that for a given relative price reduction $e_{p,s}$, the relative propensity to search $e_{l,s}$ corresponds to the average propensity to consume. Therefore, the equation (17) represents the algebraic illustration of the major attributes of Protestant ethics: hard work, the propensity to save, and modesty in consumption (Furnham (1990), Weber (1976)). The reserve maximization model transforms these considerations into the following form: a low relative propensity to search corresponds to a low average propensity to consume (Malakhov (2003).

Conclusion

[4] Here, we should keep in mind both income and substitution effects. The discussion about price dispersion and inflation (see, for example, Reinsdorf (1994)) reveals the complexity of this problem. However, recent studies (see, for example, McKenzie and Shargorodsky (2011)) highlight the validity of the *∂S/∂P>0* assumption.

[5] When we try to check the equation (17) we should keep in mind that objects of search&shopping in developed economies represent usually not more than a quarter of total expenditures. The most part of expenditures are fixed and, therefore paid from the reserve for purchases.

The model presented here suggests many interesting reflections in different fields of economic science. The analysis of intertemporal decision making based on interest rate is beyond the scope of this paper. Here, we can only note the fact that the *MRS(H for Q)* ratio consists of three variables, which are time dependent. The simple mathematical logic says that we cannot eliminate the factor of time from the *MRS (H for Q)* ratio. Therefore, the concept of the *MRS (H for Q)* receives additional confirmation of its time dependency. This consideration becomes crucial when we consider product lifecycle (i.e., the lifecycle of durables). The search for a big-ticket item is followed by the absolute value of the future time horizon and the discounted value of future wages. In this way, the time horizon reinforces the substitution effect between consumption and leisure and decreases the *MRS (H for Q)*.

The present paper considers the wage rate to be a constant value. If we proceed to the analysis of the income elasticity of search with regard to the income elasticity of consumption itself, we can easily explain some "anomalies" of individual labor supply decisions, such as women's higher willingness to substitute leisure for labor and the "irrational shirking" in underdeveloped economies.

Taken together, the analyses of intertemporal decision-making and the wage rate elasticity of search can contribute to the understanding of the effects of individual labor supply with regard to precautionary savings, such as an increase in labor supply when future wages are uncertain.

The relationship between searching and home production also requires additional attention. We can describe some home activities in a form of the *QP(S)* function, where the search for a cheaper price corresponds to the "production of useful commodities" (for example, cooking). Moreover, this approach can integrate the analysis of the reserve for purchases with the utility maximization rules of the attribute model, in which consumers' preferences can result in specific marginal rates of the substitution of leisure to consumption as well as in specific price decisions.

Appendix A.

Setting the Lagrangian expression $\Lambda = U(Q,H) + \lambda(w - Q\frac{\partial P/\partial S}{\partial L/\partial S}.)$, the first-order conditions for a maximum are

$$\frac{\partial \Lambda}{\partial Q} = \frac{\partial U}{\partial Q} - \lambda \frac{\partial P/\partial S}{\partial L/\partial S} = 0; \quad \frac{\partial \Lambda}{\partial H} = \frac{\partial U}{\partial H} - \lambda Q \partial \frac{\partial P/\partial S}{\partial L/\partial S} \Big/ \partial H = 0 .$$

Trying to determine the marginal rate of substitution of leisure for consumption, we get[6]

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=\frac{\partial Q\dfrac{\partial P/\partial S}{\partial L/\partial S}\Big/\partial H}{\dfrac{\partial P/\partial S}{\partial L/\partial S}}=-Q\frac{\partial P/\partial S\times\partial^2 L/\partial S\partial H}{\dfrac{\partial P/\partial S}{\partial L/\partial S}(\partial L/\partial S)^2}=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$Q\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S}\Rightarrow\frac{Q}{\partial L/\partial S}=\frac{w}{\partial P/\partial S}\Rightarrow$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (18)$$

Appendix B.

The physical form of the *MRS (H for Q)* results in the following equation:

$$MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{Q\times T}{T(H-T)}=\frac{Q}{L+S} \quad (19)$$

The relationship between the intensity of consumption and the *MRS (H for Q)* is provided by the implicit psychological value of the propensity to search *∂L/∂S*. We can identify this relationship in the following form:

$$MRS(HforQ)=-\frac{\partial Q}{\partial H}=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-e_{\partial L/\partial S,H}\frac{Q}{H} \quad (14) \quad (20).$$

Then, we come to

$$e_{Q,H}=e_{\partial L/\partial S,H}=\frac{1}{T}\frac{T}{H-T}H=-\frac{H}{S+L} \quad (21)$$

The leisure elasticity of consumption in the "common model" of behavior equals the relative allocation of time.

Now we can present the *MRS (H for Q*) with regard to the elasticity of substitution between leisure and consumption:

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[6] We take the value *∂P/∂S* as a given by the price dispersion of local market. If we presuppose that an individual can always adjust price reduction to a pre-allocated quantity *(∂P/∂S=∂P/∂S(Q))* and to target leisure time *(∂P/∂S=∂P/∂S(H))*, consumption and leisure become perfect complements. The model implies that consumers can choose a market with certain price dispersion, but they are still price-takers there—now, *price-reduction takers*.

$$MRS(HforQ) = -\frac{\partial Q}{\partial H} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$\frac{\partial Q}{\partial H} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = \frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = \frac{QT}{(H-T)}\frac{1}{T} = \frac{Q}{H}\frac{H}{(H-T)} = \frac{Q}{H}\frac{(H-T+T)}{(H-T)}; \quad (22)$$

$$\frac{\partial Q}{\partial H} = \frac{Q}{H}(1+\frac{T}{H-T}) = \frac{Q}{H}(1+\frac{1}{\partial L/\partial S}) = \frac{Q}{H}(\frac{\partial L/\partial S+1}{\partial L/\partial S});$$

$$if\ \partial L/\partial S = -\alpha \Rightarrow \frac{\partial Q}{\partial H} = -\frac{Q}{H}(\frac{1-\alpha}{\alpha})$$

$$MRS(HforQ) = (\frac{1-\alpha}{\alpha})\frac{Q}{H} = (\frac{-\partial H/\partial S}{-\partial L/\partial S})\frac{Q}{H}$$

We can get the same result for the following Cobb-Douglas utility function:

$$U(Q,H)=Q^{-\partial L/\partial S}H^{\partial H/\partial S}.$$

If we follow the $\partial L/\partial S+\partial H/\partial S+1=0$ rule, the elasticity of substitution between leisure and consumption is $\sigma=1$.

If we apply the equation (7) to the trends in time allocation (Aguiar and Hurst (2007), McFarlane and Lindsey (2007)), we can get the examples of utility functions of the American and Canadian households during last decades.

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