Multiple Cointegrating Vectors and Structural Economic Models: An Application to the French Franc/U.S. Dollar Exchange Rate*

SELAHATTIN DIBOOGLU
Southern Illinois University
Carbondale, Illinois

WALTER ENDERS
Iowa State University
Ames, Iowa

I. Introduction

Structural models of the exchange rate have performed very poorly for the industrialized nations during the post-Bretton Woods period. The time series behavior of exchange rates seems to conform to other asset prices in that volatility is large and short-term changes seem to respond primarily to the "news." Using the Engle and Granger [16] technique, studies by Baillie and Selover [4], Baillie and McMahon [5], and Kim and Enders [21] among others, provided evidence that there are no long-run relationships between bilateral nominal exchange rates and the so-called fundamentals. More recently, papers by Baillie and Pecchenino [6] and Adams and Chadha [1] used the maximum-likelihood Johansen [18] and Johansen and Juselius [19] method to provide evidence of cointegration between exchange rates and some of the fundamentals. However, none of these papers is able to validate any of the standard models of exchange rate determination.

Our departure from the previous literature is that we develop a simple modelling strategy that is useful in the presence of *multiple* cointegrating vectors. In our view, a well specified economic model indicates the number of cointegrating vectors that exist among a set of variables. Moreover, the presence of multiple cointegrating vectors conveys valuable information that should not be wasted. We extend the suggestion of Bagliano, Favero, and Muscatelli [3] and Smith and Hagan [26] and interpret each cointegrating vector as a behavioral or as a reduced form equation from a structural model. The technique is illustrated using U.S./French exchange rate and money market data. In doing so, it is shown that it is not possible to reject a popular structural exchange rate determination model. We demonstrate that in the presence of multiple cointegrating vectors, the theory can guide us in "identifying" the behavioral equations. The exactly identified long-run relationships can be properly considered to be behavioral equations resulting from a structural model of exchange rate determination. Given that these equations represent long-run properties

*The authors would like to thank Harvey Cutler and an anonymous referee for making helpful suggestions. All remaining errors are our own.

of the data, an appropriate dynamic structure can be obtained by imposing this long-run analysis on the short-run behavior of the system. To that end we use conventional innovation accounting techniques (variance decomposition and impulse response functions) based on the correctly specified error-correction representation of the system.

In order to introduce the key variables relevant for exchange rate determination, section II reviews the two-country Dependent Economy Model as developed by Dornbusch [13; 14]. Our use of the model is designed to be illustrative in that it suggests a convenient set of forcing variables, has a simple formulation, and contains some straightforward identification restrictions. Moreover, the illustrative model is general enough to be consistent with the Dornbusch "Overshooting" Model [15] or with the Mussa Monetary Model [24]. Section III discusses our modelling strategy and shows how it can be employed in the presence of multiple cointegrating vectors. We employ the technique to estimate a structural model of the Dollar/Franc exchange rate in section IV. To preview our results, we find two distinct cointegrating vectors linking the exchange rate to money supplies, output levels, interest rates, price levels, and productivity levels. We find reasonable support for our attempt to restrict the cointegrating vectors so as to identify (1) the money market equilibrium relationship and (2) the modified Purchasing Power Parity (PPP) relationship that is consistent with the Dependent Economy Model. In section V, we estimate the appropriately specified error-correction representation (with the restricted cointegrating vectors) in order to characterize the short-run properties of the system. Using innovation accounting techniques, we show that a modest proportion of exchange rate variability is explained by the fundamentals. Conclusions, limitations of the methodology, and directions for further research are contained in section VI.

II. An Illustrative Model

Most conventional exchange rate determination models assume money market equilibrium conditions of the form:

$$m_t = k + p_t + \eta y_t - \lambda r_t; \quad \eta, \lambda > 0 \tag{1}$$

$$m_t^* = k^* + p_t^* + \eta^* y_t^* - \lambda^* r_t^*; \quad \eta^*, \lambda^* > 0, \tag{2}$$

where m is the domestic money supply; p is the domestic price level; y is domestic income; r is the domestic interest rate; t is a time subscript; (*) denotes the foreign counterpart of domestic variables; all variables except interest rates are expressed in logarithms, and k, θ, and λ are constants.

A constrained version of the money market equilibrium relationship is the conventional two-country relative version:

$$m_t - m_t^* = \alpha_0 + \alpha_1(p_t - p_t^*) + \alpha_2(y_t - y_t^*) - \alpha_3(r_t - r_t^*). \tag{3}$$

Although there is no a priori reason to expect symmetry in the behavioral coefficients, this widely used specification saves considerable degrees of freedom in a dynamic model. Moreover, the specification is compatible with the "missing money" literature if money demands shift in both countries [2]. Of course, it is possible to test the restrictions implied by equation (3).

Recent work on the demand for money suggests that equation (3) holds only as long-run

equilibrium relationship.[1] Thus, we depart somewhat from the original Dornbusch formulation in that our equation (3) is *not* intended to represent equilibrium at a point in time; rather, it represents the long-run/cointegrating relationship in the money market.

Monetary Models of the exchange rate, such as Mussa [24] usually link national price levels (i.e., p and p^*) by assuming the existence of PPP. However, the questionable performance of PPP during post-Bretton Woods period suggests the use of an alternative approach followed by the Dependent Economy Model of Dornbusch [13; 14]. Let the domestic price level be a weighted average of the prices of traded goods and non-traded goods:

$$\begin{aligned} p_t &= \theta p_t^T + (1-\theta)p_t^N \\ &= p_t^T + (1-\theta)\rho_t \end{aligned} \tag{4}$$

where

$$\rho_t \equiv p_t^N - p_t^T$$

and p^T is logarithm of the price of traded goods; p^N is logarithm of the price of non-traded goods; ρ is logarithm of the relative price of non-traded good; and θ is a share parameter. A similar relationship exists for the prices in the foreign country.

Assuming commodity arbitrage in the traded good only:

$$p_t^T = p_t^{*T} + s_t \tag{5}$$

where s is the logarithm of the domestic currency price of foreign exchange.

Combining equations (4) and (5) and assuming that the share parameters for the two countries are equal (i.e., $\theta = \theta^*$), yields:

$$s_t = p_t - p_t^* - (1-\theta)(\rho_t - \rho_t^*). \tag{6}$$

Equation (6) is the modified Purchasing Power Parity (PPP) relationship; as long as there are no relative price changes, relative PPP will hold. However, relative price shocks (i.e., changes in $\rho - \rho^*$) can cause deviations from PPP.

An important feature of the Dependent Economy Model is that the relative price of non-traded goods is determined by productivity differentials across sectors. To best understand the relationship between productivity and relative prices, consider Figure 1. The production possibility curve between tradables and non-tradables is given AB. Let the relative price of the non-traded good be given by the slope of line CD; hence, $\rho_t = OC/OD$. Note that line EF is parallel to CD. Assuming full employment, at this particular value of ρ_t, production takes place at point H. As shown in the Figure, consumption takes place at point K (on the indifference curve labeled I). The market for the non-traded good clears since domestic production equals domestic demand. The excess demand for the tradable good can be satisfied by importing $EC = KH$ units of tradables.

The depiction of a nation with a trade deficit is necessarily a short-run phenomenon; given that a nation cannot forever borrow from abroad, long-run equilibrium requires that net imports

1. For example, Boughton [9] examines the long-run properties of the demand for money in five large industrial countries (United States, Japan, Germany, France, and the United Kingdom) under the hypothesis that the long-run functions are stable but that the dynamic adjustment processes are complex. To capture this property of the money demand functions, the author suggests using cointegration techniques. The results broadly support this hypothesis. Additional recent empirical evidence on the demand for money can be found in [2].

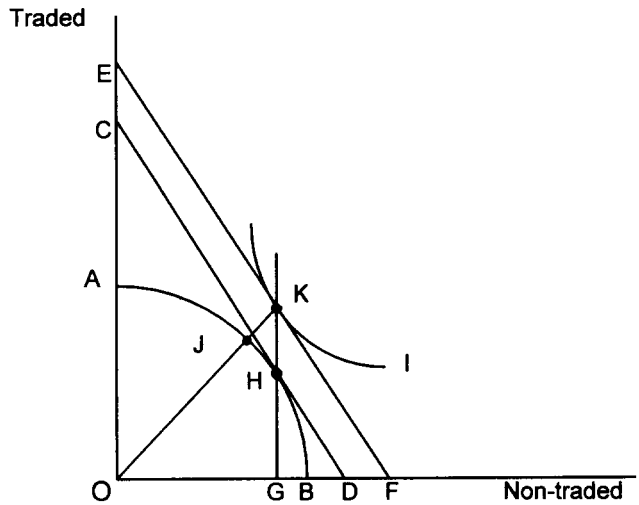

Figure 1. Equilibrium with Non-traded Goods

equal zero. The dynamic adjustment of the system is such that the wealth reduction associated with the trade deficit leads to a reduction in expenditure levels. Let the Engle curve be given by line OK. Hence, any decline in expenditures must be accompanied by a relative price change so as to preserve equilibrium in the market for non-tradables. Thus, long-run equilibrium must occur on the production possibilities curve somewhere along line segment JH. The point is that there is a unique *long-run* relative price of non-tradables for any given set of preferences and technology. As can be inferred from Figure 1, given preferences, an increase in the relative productivity of tradables will be associated with a relatively high price of the non-traded good.

Closing the model entails (i) linking domestic and foreign interest rates (usually through uncovered interest parity), (ii) the appropriate specification of the goods market clearing relationships (e.g., sticky prices versus full market clearing), and (iii) linking spot and forward exchange rates. We elect not to make any specific assumptions regarding these issues; our intent is to construct a model that is consistent with this framework. The error-correction representation can suggest the proper specifications regarding the dynamics of the model.

III. Cointegration and Structural Models: Methodological Issues

Johansen [18] and Johansen and Juselius [19] provide a unified approach to the estimation and testing of cointegrating relationships. The procedure is particularly useful since: (i) it uses maximum likelihood techniques; (ii) it can detect and estimate multiple cointegrating vectors; and (iii) it allows us to test restrictions on the cointegrating vector(s).

According to Engle and Granger [16], a single cointegrating vector has a straightforward interpretation as a long-run "equilibrium" relationship. Nevertheless, the existing literature has paid scant attention to the existence and interpretation of multiple cointegrating vectors.

The econometric problem is to properly identify each of the cointegrating vectors. The Johansen procedure necessitates imposing the same restrictions across *all* cointegrating vectors whereas appropriate identification requires the imposition of different restrictions on each vector. In order to identify a structural model we undertake the following steps:

(1) Pretest all variables to be included in the VAR for the order of integration. It is generally inappropriate to mix variables that are integrated of different orders.[2] Having selected the appropriate variables, use the Johansen procedure to obtain the number of cointegrating vectors. This is the tentative number of behavioral or reduced form relations in the model.

(2) Economic theory may suggest the existence of certain structural relationships that conform to the results in step 1). In order to identify each behavioral relation, one can impose various zero identifying restrictions by running the Johansen procedure with the appropriate variables excluded. If the remaining variables are then found to be cointegrated, the exclusion restriction suggested by the model is deemed to be appropriate. If no cointegrating vector is found, the restriction must be rejected. Additional restrictions suggested by the structural model can be tested using the cointegrating vectors with the imposed zero restrictions.

(3) The error-correction models using equilibrium errors from both the restricted and unrestricted models can be estimated.[3] Innovation accounting (variance decomposition, and impulse response functions) can be used to obtain information concerning the dynamics of the restricted and unrestricted systems. Comparing the results of the two estimations can provide additional evidence concerning the plausibility of the underlying theory.

Making specific reference to the Dependent Economy Model, we let the 7×1 column vector $\mathbf{X}_t$ consist of the variables $(m_t - m_t^*)$, a measure of relative productivity denoted by $(pr_t - pr_t^*)$, $(r_t - r_t^*)$, $(y_t - y_t^*)$, $(p_t - p_t^*)$, s_t, and a constant. If equations (3) and (6) hold as long-run cointegrating relationships, we should find at least two independent cointegrating vectors for $\mathbf{X}_t$. Given that there are two such cointegrating vectors, it should be possible to impose a set of zero restrictions such that the coefficients on $(pr_t - pr_t^*)$ is zero in one of the vectors and that the coefficients on $(m_t - m_t^*)$, $(y_t - y_t^*)$, and $(r_t - r_t^*)$ are zero in the other vector.[4] These restrictions are imposed by running the cointegration test without the corresponding variables to see if the hypothesized long run relations exist. If these criteria are met, it is possible to test specific restrictions on the other coefficients (e.g., the normalized price coefficient is unity in the PPP relation).

IV. The Unrestricted Cointegrating Relationships

As described at the bottom of Table I, our data set consists of variables typically included in the simple monetary model of exchange rate determination such as that of Dornbusch [15], Mussa [24], or Frenkel [17]. Specifically, we include relative money supplies (M1), the short-term interest rate differential, relative output levels (as measured by real GNP), the relative price level (as measured by the GNP deflators), and nominal exchange rate. In addition, we include a measure of the

2. It is also inappropriate to arbitrarily mix trending variables with integrated variables. If the system contains integrated variables as well as trending variables, an additional requirement for cointegration is that those linear combinations which cancel the unit roots also cancel the deterministic trends. For details see [10].

3. Standard innovation accounting techniques (Variance Decompositions and Impulse Response Functions) can be used to analyze the dynamics of the unrestricted system. For details on innovation accounting in a VAR framework with cointegrated variables see [22].

4. In a sense, such structural identification of the cointegrating vectors is straightforward as compared with Classical Identification. Here, it is not necessary to make assumptions as to which variables are endogenous (since the validity of the Johansen procedure does not require such assumptions). Moreover, exact identification is facilitated since each equation in the system can be estimated directly.

Table I. Augmented Dickey Fuller (ADF) Tests.

Variable	Lags	ADF t-statistic
s_t	4	−1.77
	8	−1.64
$(p_t - p_t^*)$	4	−1.62
	8	−2.02
$(m_t - m_t^*)$	4	−2.06
	8	−2.26
$(pr_t - pr_t^*)$	4	−2.42
	8	−2.14
$(y_t - y_t^*)$	4	−1.65
	8	−1.19
$(r_t - r_t^*)$	4	−2.87
	8	−2.62

Notes:

(1) Definition of the Variables: s_t, nominal, period average exchange rate (e.g., Francs per US Dollar); m_t, money supply (M1); p_t, price level as measured by the GNP or GDP deflator; r_t, short term interest rate as measured by call money rate or equivalent; y_t, real output, GNP or GDP in constant prices; pr_t, productivity rate as measured by productivity index of the manufacturing sector; an asterisk denotes foreign counterparts, and all variables except interest rates are measured in logarithms.

(2) The critical values at the 0.05, 0.025, and 0.01 significance levels are −2.89, −3.17, and −3.51, respectively.

relative productivity between tradables and non-tradables. We follow the usual practice measuring relative productivity in the tradable sector as the index of industrial employment to industrial output. This ratio for France divided by the ratio for the U.S. is our measure of international relative productivity.[5] The data is quarterly from 1971.I, (the end of the Bretton Woods period) to 1990.IV. All data are taken from the ROM-disk edition of *International Financial Statistics*.

As a first step, we investigate the stochastic properties of the individual series; we are particularly interested in the order of integration of the various series. Table I reports the results of Augmented Dickey-Fuller [11; 12] unit root tests using 4 and 8 quarter lags. It is clear from the table that a unit root cannot be rejected for all series with a possible exception of the short-run interest rate differentials.

Next we use the Johansen procedure to determine the number of cointegrating relationships for the Franc/dollar nominal exchange rate, relative money supply, relative price level, short-term interest rate differential, relative output level, and the relative productivity level. Since the data is quarterly, (and we have potential degrees of freedom problem with 8 lags) we use a lag length of 4.[6] Likelihood ratio test statistics and their critical values regarding the number of long run equilibrium relationships in the system are presented in Table II.

Using the trace (λ_{trace}) test, we can reject the null hypotheses of $r = 0$ and $r \leq 1$ (against alternatives that $r \geq 1$ and $r \geq 2$ respectively) at conventional significance levels. The calculated value of λ_{trace} for the null of $r = 2$ is barely rejected at the 95% significance level. Given that there are *2 or more* cointegrating vectors, the maximal eigenvalue ($\lambda_{\max}$) test for a null of $r = 2$

5. It is common practice to proxy relative productivity in the traded good sector to that in the non-tradable sector by manufacturing productivity alone. Thus for the French/U.S. case, $pr_t - pr_t^*$ is defined to be: French manufacturing employment/French index of industrial production all divided by the corresponding ratio for the U.S.

6. Diagnostic tests indicate that the residuals from the 4 lag model approximate white noise.

Table II. Cointegration Between the French Franc and the "Fundamentals"

	Statistics and Critical Values			
Null	λ_{tr}	$\lambda_{trace}(.95)$	λ_{max}	$\lambda_{max}(.95)$
$r = 0$	135.87	102.14	53.75	40.30
$r = 1$	82.12	76.07	28.47	34.40
$r = 2$	53.66	53.12	22.58	28.14
$r = 3$	31.07	34.91	12.90	22.00
$r = 4$	18.17	19.96	11.79	15.67
$r = 5$	6.38	9.24	6.38	9.24

Notes: Maximum lag in VAR is 4. Variables included in the VAR: s_t, $(m_t - m_t^*)$, $(p_t - p_t^*)$, $(y_t - y_t^*)$, $(r_t - r_t^*)$, $(pr_t - pr_t^*)$, and an intercept.

against the specific alternative $r = 3$ cannot be rejected at the 95% level. Hence, there is strong evidence for two cointegrating relationships in the system. Allowing for a constant as the seventh element in the cointegrating vectors, the 7×2 cointegrating matrix $\boldsymbol{\beta}$ is given by:

$$\begin{bmatrix} -2.90 & 11.72 & -1.72 & 10.96 & -2.91 & 2.71 & -2.30 \\ 0.22 & 24.24 & 0.60 & -21.21 & -11.86 & 1.21 & 0.14 \end{bmatrix}. \tag{7}$$

Normalizing each of the two cointegrating vectors with respect to the exchange rate, the two long-run equilibrium relationships are:

$$\begin{aligned} s_t &= 1.071(m_t - m_t^*) + 1.074(p_t - p_t^*) - 4.048(y_t - y_t^*) \\ &\quad + 0.635(r_t - r_t^*) - 4.330(pr_t - pr_t^*) + 0.850 \end{aligned} \tag{8}$$

$$\begin{aligned} s_t &= -0.186(m_t - m_t^*) + 9.812(p_t - p_t^*) + 17.552(y_t - y_t^*) \\ &\quad - 0.501(r_t - r_t^*) - 20.057(pr_t - pr_t^*) - 0.116. \end{aligned} \tag{9}$$

At this point, equations (8) and (9) can be interpreted as long-run reduced form relationships. Note that the first equation (the most "significant" of the two cointegrating vectors) has many of the properties of a reduced form structural model of the exchange rate. The franc price of the dollar moves nearly proportionately to the relative supply of francs, and relative increases in real French GNP and/or productivity act to appreciate the franc. The positive relationship between the interest rate differential and the franc price of the dollar is consistent with uncovered interest parity. Uncovered interest rate parity implies that the French interest rate will exceed the U.S. rate only when individuals anticipate a depreciation of the Franc. If such expectations are discounted into the current exchange rate, the current franc price of the dollar will be positively related to the interest rate differential. The second relationship is not directly interpretable. Note, however, that the exchange rate, relative prices and productivities do have signs that are consistent with the modified PPP relationship. The critical point is whether it is possible to impose (i) money market equilibrium relationship and (ii) the modified PPP relationship of the Dependent Economy Model on equations (8) and (9). The issue is whether it is statistically possible to restrict the $\boldsymbol{\beta}$ matrix such $\beta_{11} = \beta_{16} = 0$ and $\beta_{22} = \beta_{24} = \beta_{26} = 0$ and still find "reasonable" values for the other behavioral coefficients.

As shown in Table III, the estimated values of λ_{trace} and λ_{max} strongly indicate a single cointegrating vector between relative money supplies, price levels, income levels, and interest rates.

Table III. Cointegration Among Money Market Variables

	Statistics and Critical Values			
Null	λ_{trace}	$\lambda_{\text{trace}}(.95)$	$\lambda_{\max}$	$\lambda_{\max}(.95)$
$r = 0$	55.26	53.12	28.80	28.14
$r = 1$	26.46	34.91	14.04	22.00
$r = 2$	12.42	19.96	7.17	15.67
$r = 3$	5.25	9.24	5.25	9.24

Notes: Maximum lag is 6. Variables included in the VAR: $(m_t - m_t^*)$, $(p_t - p_t^*)$, $(y_t - y_t^*)$, $(r_t - r_t^*)$, and an intercept.

Table IV. Cointegration Tests of Modified-PPP

	Statistics and Critical Values			
Null	λ_{trace}	$\lambda_{\text{trace}}(.95)$	$\lambda_{\max}$	$\lambda_{\max}(.95)$
$r = 0$	54.93	34.91	35.60	22.00
$r = 1$	19.33	19.96	14.67	15.67
$r = 2$	4.66	9.24	4.66	9.24

Notes: Maximum lag is 4. Variables included in the VAR: s_t, $(p_t - p_t^*)$, $(pr_t - pr_t^*)$, and an intercept.

Moreover, we cannot reject a null of a single cointegrating vector between the exchange rate, relative price levels, and relative productivity at conventional significance levels. Appropriately normalizing these two cointegrating vectors, we find the following long-run relationships:

$$m_t - m_t^* = 1.247(p_t - p_t^*) + 3.735(y_t - y_t^*) - 0.235(r_t - r_t^*) - 0.011 \tag{10}$$

$$s_t = 2.72(p_t - p_t^*) - 4.53(pr_t - pr_t^*) - 2.27. \tag{11}$$

The money market "equilibrium" relationship is reasonably well-behaved, although the implied income elasticity of demand is rather large while the estimated price elasticity of 1.247 is "too high."[7] However, the restriction that money and prices have equal coefficients cannot be rejected (A $\chi^2_{df=1}$ test fails to reject this restriction at 12.6% significance level). When long-run money neutrality is imposed, the resulting money market equilibrium relationship is:

$$m_t - m_t^* = (p_t - p_t^*) + 3.838(y_t - y_t^*) - 0.659(r_t - r_t^*) - 0.014. \tag{12}$$

A possible explanation for the problematic coefficient magnitudes may be the imposed symmetry restrictions. Using an 8-variable system with 4 lags, we estimated an unrestricted version of equation (10). After pretesting variables for unit roots (a unit root cannot be rejected for all series with the possible exception of French interest rates), we sought to determine the number of cointegrating relationships of the form:

$$0 = \gamma_0 + \gamma_1 p_t - \gamma_1^* p_t^* + \gamma_2 y_t - \gamma_2^* y_t^* - \gamma_3 r_t + \gamma_3^* r_t^* - \gamma_4 m_t + \gamma_4^* m_t^*. \tag{10'}$$

7. However, there are questions as to what the elasticities "should be" in the studies of the demand for money. Boughton [9] found, among other things, that the traditionally accepted restrictions about long-run homogeneity with respect to the price level and unitary or less than unitary real income elasticity are questionable. Moreover, this conclusion is robust with respect to a variety of estimation strategies.

The λ_{trace} statistic indicated 4, and the $\lambda_{\max}$ statistic indicated 3 linearly independent cointegrating vectors among the 8-variable system. Given that there are four cointegrating vectors, we impose the following symmetry restrictions on the cointegrating vectors.

Null Hypothesis	Significance Level
$H_1 : \gamma_1 = \gamma_1^*$	0.037
$H_2 : \gamma_2 = \gamma_2^*$	0.000
$H_3 : \gamma_3 = \gamma_3^*$	0.010
$H_4 : \gamma_4 = \gamma_4^*$	0.084
$H_5 : \gamma_1 = \gamma_1^*;\ \gamma_2 = \gamma_2^*;\ \gamma_3 = \gamma_3^*;$ and $\gamma_4 = \gamma_4^*$	0.000
$H_5' : \gamma_1 = \gamma_1^*;\ \gamma_2 = \gamma_2^*;\ \gamma_3 = \gamma_3^*;$ and $\gamma_4 = \gamma_4^*$	0.042

The symmetry restrictions implied by H_1 through H_5 are imposed on all four significant cointegrating vectors. It is clear that most of the restrictions are rejected at conventional significance levels. Following Johansen and Juselius [20], we also imposed the overall restrictions on the first and most significant cointegrating vector; the result is given as H_5'. If imposed on the first cointegrating vector, the restrictions are marginally binding at the 5% significance level. Similarly, we estimated an unrestricted version of equation (11) and found two significant cointegrating vectors. Symmetry restrictions imposed on the two significant cointegrating vectors were rejected; however, the restrictions are not rejected if imposed on the first cointegrating vector.

The modified-PPP relationship [i.e., equation (11)] also has the "correct" signs. Relative increases in the French price level are associated with increases in the price of the dollar, whereas relative increases in French productivity are associated with decreases in the price of the dollar. The coefficient on the price level is too large; the $\chi^2_{df=1}$ test statistic that this coefficient is unity can be rejected at less than 1% significance level. Finally, a joint test that the price coefficient is unity while the productivity term is zero (strict PPP holds) yields a $\chi^2_{df=2}$ of 20.38 suggesting that the restriction is clearly rejected.[8] Although this finding is troublesome, it is consistent with MacDonald's [23] argument that different means of constructing national indices leads to non-proportional measured price movements.

Overall, the evidence is not favorable to the symmetry restrictions. Nevertheless, in order to retain a reasonable number of degrees of freedom, the dynamic model below continues to impose the symmetry restrictions on both the money market and the modified PPP relationships. However, the key point is, other than the magnitude of the coefficient on the price term in the PPP relation, we have identified two key long-run relationships that are consistent with a structural exchange rate model. In that sense, we have found a meaningful long-run relationship between the exchange rate and the "fundamentals" suggested by the Dependent Economy model.

V. The Dynamic Model

The Granger Representation Theorem indicates that a cointegrated system has an error-correction representation. Consider the following dynamic specification:

$$\Delta \mathbf{X}_t = \mathbf{A}(L)\Delta \mathbf{X}_{t-1} + \alpha\boldsymbol{\beta}'\mathbf{X}_{t-1} + \mathbf{C}Q + \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \tag{13}$$

8. Baillie and Pecchenino [6] finds stable money market equilibrium relationships for the U.K./U.S. case using cointegration methods. The study concludes that the persistent deviations from PPP is the only source for the rejection of the monetary model.

where $\mathbf{A}(L)$ is a 6×6 matrix with elements which are k-order polynomials in the lag operator L, Q is a dummy variable reflecting intervention in the market for French francs with the European Monetary System (EMS), $\mathbf{C}$ and $\boldsymbol{\mu}$ are column vectors of constants, and $\boldsymbol{\epsilon}_t$ is a column vector of disturbances denoted by ε_{1t} through ε_{6t}.[9]

Engle and Granger [16] propose a clever way to circumvent the cross-equation restrictions involved in the direct estimation of equation (13). The residuals from the cointegrating relationship(s) represent the deviations from "long-run equilibrium" in period $t-1$. The traditional methodology uses the residuals from the unconstrained cointegration vector(s) to form $\boldsymbol{\beta}'\mathbf{X}_{t-1}$. In our view, it is the residuals from the identified cointegrating vectors that should be used in the dynamic specification. The use of the restricted cointegrating vectors imposes the theoretical long-run structure on the short-run dynamics of the system. Hence, the error-correcting terms we used in estimating equation (13) are the residuals from equations (11) and (12).

The top portion of Table V reports the Variance Decomposition Analysis for four different forecasting horizons using a Choleski Decomposition. The order of the variables is that implied by the Table; money → productivity → interest rate → income → prices → exchange rate. Note that money and productivity "explain" the preponderance of their forecast error variance. After 4 quarters, income and prices explain approximately 50% of their forecast error variance. Possibly the most important result is that the exchange rate is affected by other variables in the system. The exchange rate shares a long-run equilibrium with other variables and these other variables explain approximately 20% of exchange rate forecast error variance. Although the exchange rate follows near random walk behavior we have found other variables which share common trends with the exchange rate. Notice also that the exchange rate accounts for relatively little of the forecast error variance of these other variables.

The lower portion of the table shows the contemporaneous correlation matrix of the innovations. The correlation coefficients between income and productivity innovations (.50), income and interest rate (.37), and productivity and interest rate innovations (.36) are high. The middle portion of the Table shows the Variance Decomposition at a 12 quarter forecasting horizon using the reverse ordering exchange rate → prices → income → interest rates → productivity → money. As expected from the correlation coefficients, the importance of productivity for income and for prices is diminished. Without further restrictions, it is not possible to easily disentangle productivity, interest rate and income shocks, and productivity and interest rate shocks.

The dynamic relationships among the variables can be best understood by examining the impulse response functions (we present the impulse responses for the first ordering only). In response to a typical money supply shock (Figure 2), both the price level and the price of the foreign currency increase. As predicted by the Dependent Economy Model, domestic money expansion is associated with domestic inflation and a depreciation of the domestic currency. After the third quarter the price level, but not the exchange rate, begins to fall back towards its original level. The money supply innovation appears to have "Keynesian" effects in that it is associated with what appear to be permanent effects on output. A typical money supply shock is associated with increases in income; since the changes are positive, the increases appear to be permanent. Our explanation of the findings is that the ordering of the variables is crucial in such interpretations; reversing the ordering (so that income comes before money) yields results that income shocks increase the money supply.

9. There were 5 realignments of the franc during the period covered by the study. Devaluations on 5 October 1981 (3%), 14 June 1982 (5.75%), 21 March 1983 (2.5%) and 7 April 1986 (3%). There was one revaluation on 21 July 1985 (2%). The variable Q is constructed as taking the per cent change in the quarter corresponding to the intervention and zero otherwise.

Table V. Variance Decompositions

		Innovation in					
	k	$\Delta(m-m^*)$	$\Delta(pr-pr^*)$	$\Delta(r-r^*)$	$\Delta(y-y^*)$	$\Delta(p-p^*)$	Δs
$\Delta(m-m^*)$	1	100.0	0	0	0	0	0
	4	89.1	2.2	2.5	1.4	3.1	1.7
	8	82.9	4.7	2.8	1.8	4.5	3.2
	12	82.6	4.8	2.8	1.8	4.6	3.4
$\Delta(pr-pr^*)$	1	1.9	98.1	0	0	0	0
	4	5.8	83.4	6.7	0.1	1.7	2.4
	8	6.0	79.2	7.7	0.8	2.1	4.1
	12	6.2	78.1	8.0	1.1	2.2	4.4
$\Delta(r-r^*)$	1	0.1	12.8	87.1	0	0	0
	4	2.5	21.7	61.3	6.7	1.5	6.3
	8	5.5	20.8	56.8	7.2	1.8	7.9
	12	5.7	20.5	56.1	7.5	1.9	8.4
$\Delta(y-y^*)$	1	0.0	25.5	4.2	70.3	0	0
	4	13.0	24.5	7.6	50.8	2.0	2.0
	8	13.9	25.4	7.0	44.4	1.8	7.5
	12	14.4	24.6	6.8	42.9	1.8	9.5
$\Delta(p-p^*)$	1	2.6	8.8	1.8	4.6	82.1	0
	4	10.3	27.0	3.6	5.4	49.7	4.0
	8	11.8	25.1	4.1	6.1	45.4	7.3
	12	12.1	24.6	4.2	6.1	44.5	8.6
Δs	1	1.6	1.1	1.2	0.7	1.7	93.6
	4	7.4	1.2	1.5	5.2	1.8	82.8
	8	7.2	1.8	5.2	1.7	1.7	81.6
	12	7.3	2.6	2.0	5.2	1.7	81.3

		Reverse Order					
	k	$\Delta(m-m^*)$	$\Delta(pr-pr^*)$	$\Delta(r-r^*)$	$\Delta(y-y^*)$	$\Delta(p-p^*)$	Δs
$\Delta(m-m^*)$	12	74.3	12.6	1.1	1.2	6.5	4.2
$\Delta(pr-pr^*)$	12	4.3	53.8	9.8	19.1	6.8	6.2
$\Delta(r-r^*)$	12	7.8	13.5	55.4	9.5	5.1	8.6
$\Delta(y-y^*)$	12	13.7	6.0	5.2	59.0	3.5	12.5
$\Delta(p-p^*)$	12	9.7	6.3	3.4	14.0	57.1	9.5
Δs	12	2.9	1.4	2.0	1.2	5.2	87.3

	Correlation Matrix					
	$\Delta(m-m^*)$	$\Delta(pr-pr^*)$	$\Delta(r-r^*)$	$\Delta(y-y^*)$	$\Delta(p-p^*)$	Δs
$\Delta(m-m^*)$	1.00	−0.14	−0.03	0.01	0.16	0.13
$\Delta(pr-pr^*)$		1.00	0.36	0.50	0.27	−0.12
$\Delta(r-r^*)$			1.00	0.37	0.23	−0.14
$\Delta(y-y^*)$				1.00	−0.00	−0.15
$\Delta(p-p^*)$						−0.12

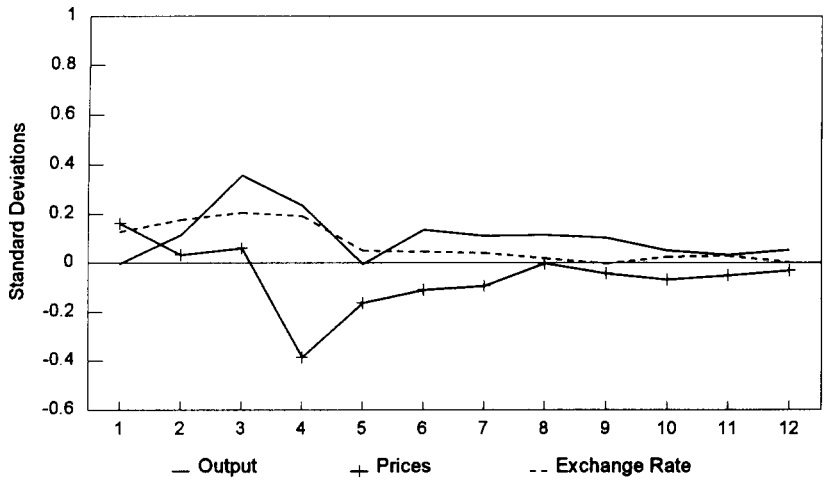

Figure 2. Responses to a Money Supply Innovation

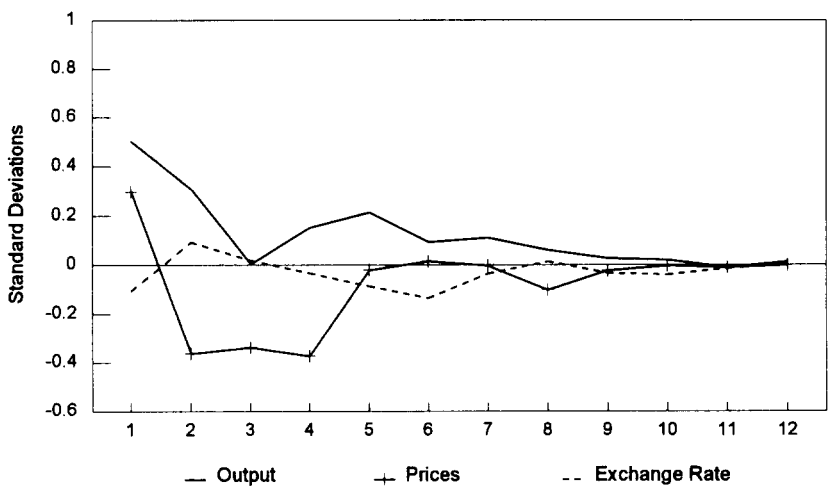

Figure 3. Responses to a Productivity Innovation

The effects of productivity innovations (see Figure 3) on income quickly die out after the first quarter. Although the contemporaneous effect on prices is positive, there is a general decline in prices after the productivity shock. Surprisingly, the productivity shock has little effect on the exchange rate. The implication is that the productivity shocks are responsible for deviations from PPP; however, it is prices which respond to productivity shocks. The responses to an interest rate shock are also quite interesting. As shown in Figure 4, after an initial increase, output decreases. The price and exchange rate respond by moving in opposite directions with no overall permanent effects.

Finally, exchange rate shocks (see Figure 5) have little price level effects (that surprisingly are negative) and the own effects die out exponentionally. This is due to the fact that the exchange rate has a sizable noise component. On the other hand, an exchange rate shock has Keynesian effects in that a currency depreciation is associated with increases in domestic output.

With two cointegrating vectors, the dimensions of α and β' in equation (13) are (6×2)

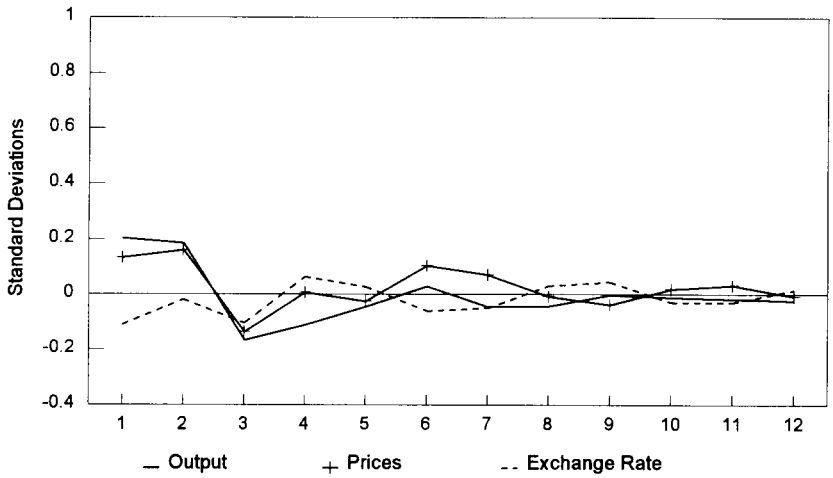

Figure 4. Responses to an Interest Rate Innovation

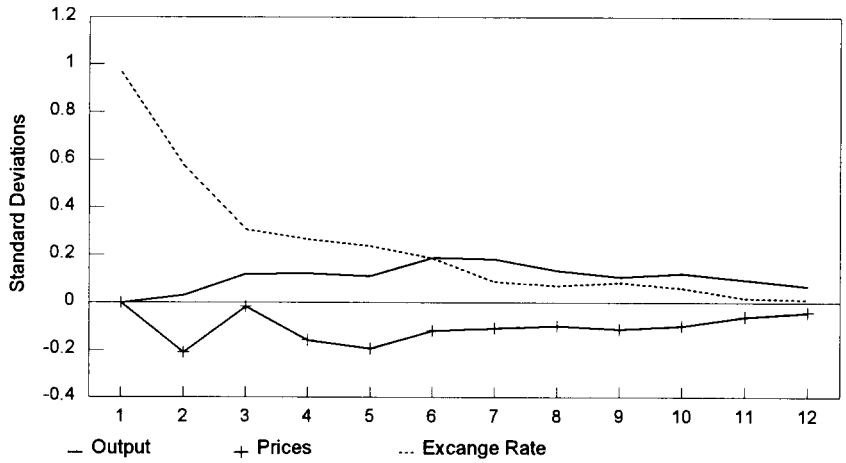

Figure 5. Responses to an Exchange Rate Innovation

and (2×6), respectively. The identified cointegrated vectors from (11) and (12) are contained in β'. The "speed of adjustment" coefficients contained in α provide an interesting insight into the short-run dynamics. The estimate of α is:

$$\alpha = \begin{bmatrix} 0.083 & 0.038 \\ 0.012 & 0.041 \\ -0.035 & -0.110 \\ -0.002 & 0.011 \\ 0.024 & -0.034 \\ 0.123 & 0.131 \end{bmatrix}.$$

As such, the contemporaneous change in the exchange rate is over 12% of any deviation from long-run money market equilibrium and over 13% of any deviation from the modified long-run PPP relation. It is noteworthy that these speed of adjustment coefficients are larger than those of the other variables in the system.

Identified Cointegrating Vectors and Structural Decompositions

The arbitrary nature of the Choleski decomposition can be alleviated by using the type of structural VAR proposed by Sims [25] and Bernanke [7]. The Sims-Bernanke procedure entails specifying a structural model of the contemporaneous relationship among the variables.

Suppose that the "true" structural error-correction model is:

$$\begin{bmatrix} 1 & v_{12} \dots v_{16} \\ v_{21} & 1 \dots v_{26} \\ \dots & \\ v_{61} & v_{26} \dots 1 \end{bmatrix} \begin{bmatrix} \Delta(m_t - m_t^*) \\ \Delta(pr_t - pr_t^*) \\ \dots \\ \Delta s_t \end{bmatrix} = V_1(L)\Delta X_{t-1} + V_2 X_{t-1} + cQ + u + \begin{bmatrix} \epsilon_{mt} \\ \epsilon_{prt} \\ \dots \\ \epsilon_{st} \end{bmatrix} \quad (14)$$

or more compactly:

$$\mathbf{V}\Delta\mathbf{X}_t = \mathbf{V}_1(L)\Delta\mathbf{X}_{t-1} + \mathbf{V}_2\mathbf{X}_{t-1} + \mathbf{c}Q + \mathbf{u} + \boldsymbol{\epsilon}_{xt}$$

where $\mathbf{X}$ is defined as above, $\mathbf{V}_1(L)$ is a 6×6 matrix with elements which are k-order polynomials in the lag operator L, Q is a dummy variable reflecting intervention in the market for French francs within the European Monetary System (EMS), $\mathbf{c}$ and $\mathbf{u}$ are column vectors of constants, and $\boldsymbol{\epsilon}_{xt}$ is a column vector of the structural innovations denoted by ε_{mt} through ε_{prt}. These structural innovations are assumed to be "pure" innovations in that they are orthogonal. It is important to note that $\mathbf{V}_2$ is a cointegrating matrix of rank 2 and the elements of $\mathbf{V}$ yield the contemporaneous relationship between the variables in the system. For example, if $v_{12} = 0$ the relative price has no contemporaneous effect on $\Delta(m_t - m_t^*)$.

Equation (13) is obtained by premultiplying (14) by $\mathbf{V}^{-1}$. Hence: $\mathbf{A}_1(L) = \mathbf{V}^{-1}\mathbf{V}_1(L)$; $\boldsymbol{\alpha\beta}' = \mathbf{V}^{-1}\mathbf{V}_2$; $\mathbf{C} = \mathbf{V}^{-1}c$; $\boldsymbol{\mu} = \mathbf{V}^{-1}\mathbf{u}$; and: $\boldsymbol{\epsilon}_t = \mathbf{V}^{-1}\boldsymbol{\epsilon}_{xt}$. As such, the residuals from (13) are composites of the pure structural innovations of the variables in the system. Sims and Bernanke propose using economic analysis to model the contemporaneous relationship among the variables by restricting the various v_{ij}. The problem is to take the estimated value of $\boldsymbol{\epsilon}_t$ and to restrict the v_{ij} so as to (1) recover $\boldsymbol{\epsilon}_{xt}$ as $\mathbf{V}^{-1}\boldsymbol{\epsilon}_t$ and (2) preserve the assumed error structure concerning the independence of the various elements of $\boldsymbol{\epsilon}_{xt}$.

Let $E\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t' = \boldsymbol{\Sigma}$ denote the variance/covariance matrix of the residuals in $\boldsymbol{\epsilon}_t$ and let $\boldsymbol{\Sigma}_x$ denote the variance/covariance matrix of the structural innovations. The relationship between $\boldsymbol{\Sigma}_x$ and $\boldsymbol{\Sigma}$ is given by:

$$\boldsymbol{\Sigma}_x = \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}'. \quad (15)$$

Given the estimated value of $\boldsymbol{\Sigma}$, the problem is to restrict $\mathbf{V}$ so as to identify $\boldsymbol{\Sigma}_x$. To solve this identification problem, simply count equations and unknowns. Since $\boldsymbol{\Sigma}$ is a symmetric 6×6 matrix, there are only 21 free elements. There are 6 elements along the principal diagonal, 5 on the first off-diagonal, 4 along the second off-diagonal, . . . , and 1 corner element for a total of 21 free elements. Given that the diagonal elements of $\mathbf{V}$ are normalized to unity, $\mathbf{V}$ contains 30 unknown values. In addition, there are the 6 unknown values of Var (ε_{mt}) through Var (ε_{prt}) for a total of 36 unknowns in the structural model. As such, the structural can be exactly identified by restricting 15 elements of $\mathbf{V}$.

Notice that a Choleski decomposition imposes exactly 15 zero restrictions on $\mathbf{V}$. The problem is that these restrictions are imposed in an ad hoc fashion. Thus, the identified cointegrating vectors can be useful in suggesting economically meaningful restrictions for $\boldsymbol{\Sigma}$. In order to highlight the use of the structural cointegrating vectors in the Sims-Bernanke decomposition consider the

Table VI. Structural Variance Decompositions

		Innovation in					
	k	$\Delta(m-m^*)$	$\Delta(pr-pr^*)$	$\Delta(r-r^*)$	$\Delta(y-y^*)$	$\Delta(p-p^*)$	Δs
$\Delta(m-m^*)$	1	96.0	0.0	0.6	0.1	3.2	0.0
	4	81.7	7.5	1.4	2.1	5.6	1.7
	8	76.9	8.8	2.1	2.4	6.7	3.2
	12	76.6	8.8	2.1	2.5	6.7	3.3
$\Delta(pr-pr^*)$	1	0.0	100.0	0.0	0.0	0.0	0.0
	4	3.3	82.3	9.2	0.3	1.9	2.4
	8	4.0	77.6	10.5	1.7	2.2	4.0
	12	4.3	76.2	10.9	2.2	2.2	4.3
$\Delta(r-r^*)$	1	0.0	0.0	100.0	0.0	0.0	0.0
	4	3.5	21.0	59.5	9.5	1.0	5.4
	8	6.4	21.0	54.1	10.5	1.3	6.6
	12	6.6	21.2	53.2	10.9	1.3	6.9
$\Delta(y-y^*)$	1	0.0	0.0	0.0	100.0	0.0	0.0
	4	13.7	8.7	9.2	68.2	0.8	2.4
	8	14.0	11.1	10.5	60.4	0.8	4.0
	12	14.2	10.9	10.9	58.5	0.9	4.3
$\Delta(p-p^*)$	1	0.0	0.0	0.0	0.0	100.0	0.0
	4	9.4	11.0	4.2	7.4	66.3	4.3
	8	10.7	10.9	5.0	8.1	57.3	7.9
	12	10.7	10.7	5.0	8.1	56.1	9.2
Δs	1	0.0	0.9	0.0	0.0	1.0	98.1
	4	3.3	1.2	1.6	3.1	2.7	88.1
	8	3.1	2.4	2.1	3.2	2.5	86.7
	12	3.1	2.7	2.3	3.3	2.5	86.1

over-identified system suggested by equations (11) and (12). Let the first and last rows of $\mathbf{V}\Delta\mathbf{X}_t$ in (14) be:

$$\Delta(m_t - m_t^*) + v_{13}\Delta(r_t - r_t^*) + v_{14}\Delta(y_t - y_t^*) + v_{15}\Delta(p_t - p_t^*) \tag{16}$$

$$v_{62}\Delta(pr_t - pr_t^*) + \Delta s_t + v_{65}\Delta(p_t - p_t^*) \tag{17}$$

and all other off-diagonal elements equal zero.

Of course, this pattern is overly restrictive; we impose 25 zero restrictions while only 15 are necessary for an exactly identified decomposition. However, our goal is to illustrate the use of the identified cointegrating vectors in the Sims-Bernanke decomposition. Given the estimate of $\mathbf{\Sigma}$ from (14) and the restrictions of $\mathbf{V}$, we find the contemporaneous relationship among the variables to be:

$$\Delta(m_t - m_t^*) - 1.37\Delta(p_t - p_t^*) - 0.133\Delta(y_t - y_t^*) + 0.126\Delta(r_t - r_t^*) \tag{16'}$$

$$-\,0.786\Delta(p_t - p_t^*) + \Delta s_t + 0.225\Delta(pr_t - pr_t^*). \tag{17'}$$

Table VI reports the variance decompositions using this structural decomposition. Note that the 12-step ahead forecast error decompositions are very similar to those in the top portion of Table V. The two main exceptions concern the effects of structural productivity shocks; the role of productivity innovations is diminished in the structural decomposition. In Table VI, the structur-

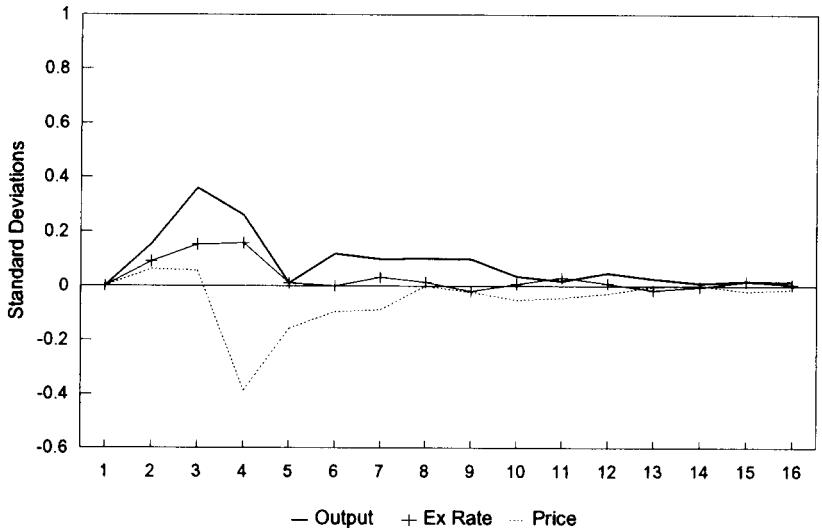

Figure 6. Responses to a Money Innovation

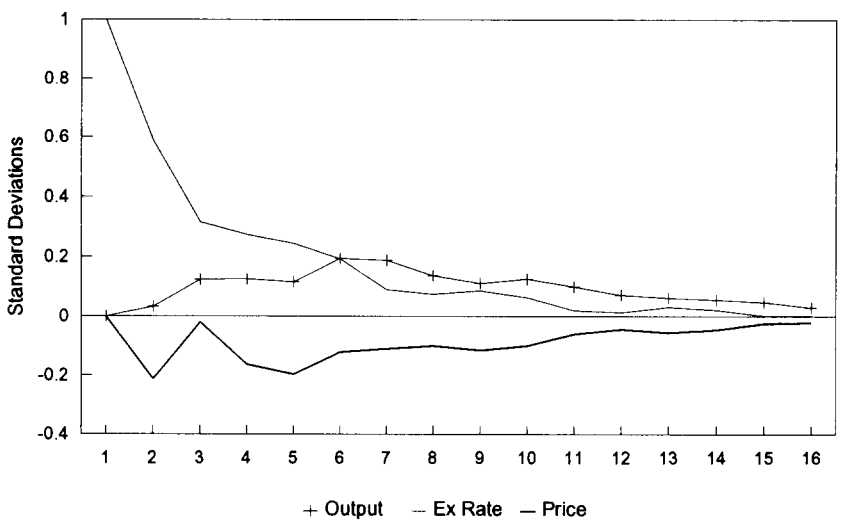

Figure 7. Responses to an Exchange Rate Innovation

ally decomposed productivity shocks explain 10.7% of the 12-step ahead forecast error variance of prices. Using the Choleski decomposition in the top portion of Table V, the corresponding proportion is 24.6%. Also, in Table VI, productivity innovations explain 10.9% of the 12-step ahead forecast error variance of income. Using the Choleski decomposition in the top portion of Table V, the corresponding proportion is also 24.6%. Since the contemporaneous relationship among the innovations is highly restrictive, the 1-step ahead and 4-step ahead forecast error variances are quite different from those in Table V.

Figure 6 shows some of the impulse responses of the system to a money supply innovation. It is remarkable that the income, price, and exchange rate impulse responses are so similar to those shown in Figure 2. Comparing Figures 5 and 7 suggests that the effects of exchange rate

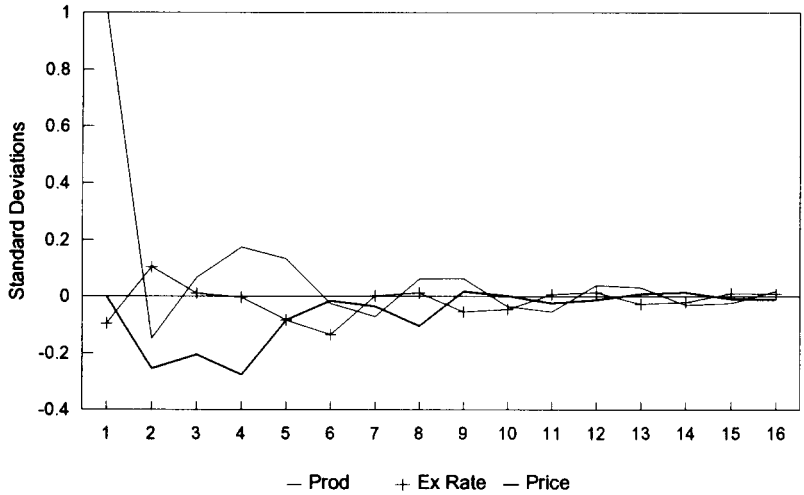

Figure 8. Responses to a Productivity Innovation

innovations are invariant to the decomposition. In contrast, the alternative decomposition implies a very different response of the prices to a productivity shock. As shown in Figure 8, prices exhibit a sustained decline. In the lower portion of Figure 3, prices initially increase in response to a positive shock.

Since **V** is overidentified, it is possible to relax some of the restrictions to obtain other plausible decompositions. For example, it is possible to let productivity have a contemporaneous effect on output. Yet, this takes us too far afield from our goal of illustrating the use of the identified cointegrating vectors in a Sims-Bernanke decomposition.

VI. Concluding Discussion

The major aim of the paper is to provide a reasonable interpretation for the existence of multiple cointegrating vectors among a given set of variables. Each of the individual cointegrating vectors can be viewed as a behavioral or reduced form equation resulting from a structural model. Given this natural interpretation, structural identification can be quite straightforward since it is not necessary to specify the set of endogenous versus exogenous variables. Exact identification of the behavioral equations can be facilitated by the imposition of zero restrictions on the cointegrating vector(s). Moreover, it is possible to test these restrictions.

We illustrated the procedure using the set of variables suggested by the Dependent Economy Model of exchange rate determination. Bilateral data for France and the U.S. revealed two independent cointegrating among the exchange rate, relative money supplies, relative price levels, relative income levels, interest rate differential, and relative productivity levels. We were able to impose zero restrictions on the two cointegrating relationships; the restricted vectors can be reasonably interpreted as a money market equilibrium equation and the modified-PPP equation. We have found that the hypothesized structural relationships of the Dependent Economy Model exist as long-run equilibrium relationships.

Interpreting the restricted cointegration vectors as long-run behavioral relationships has important implications for the error-correction representation of the model. The resulting impulse

response functions have the property that all variables necessarily return to levels consistent with the long-run behavior posited by a structural model. Using the French/U.S. data, the restricted dynamics appears to be consistent with a "Keynesian" model. In response to a "typical" domestic money shock, the relative price level, output level, productivity level, and price of foreign currency all tend to increase. Possibly the most important result of the dynamic analysis is that the exchange rate shares some common trends with other variables in the system, and these variables account for a reasonable proportion of the forecast error variance of the exchange rate.

One problem with standard impulse response analysis is that there is no unique way to obtain orthogonalized innovations. Our dynamic response functions are conditional on the ordering in a Choleski decomposition. However, the willingness to interpret the cointegrating vectors as behavioral equations can be helpful in obtaining the necessary restrictions to conduct a Sims-Bernanke decomposition. Moreover, it is possible incorporate the decomposition technique developed by Blanchard and Quah [8] into the dynamic analysis. The added restrictions can be that some variables have no long-run effects on other variables.

References

1. Adams, Charles and Bankim Chadha, "Structural Models of The Dollar." *IMF Staff Papers*, September 1991, 525–59.

2. Baba, Yoshihisa, David F. Hendry and Ross M. Starr, "The Demand for M1 in the U.S.A., 1960–1988." *Review of Economic Studies*, January 1992, 25–61.

3. Bagliano, Fabio C., Carlo A. Favero and Vito A. Muscatelli. "Cointegration and Simultaneous Models: An Application to the Italian Money Demand." Paper presented at the International Workshop on large-scale economic and financial applications, University of Urbino, May 1991.

4. Baillie, Richard T. and David D. Selover, "Cointegration and Models of Exchange Rate Determination." *International Journal of Forecasting*, January 1987, 43–51.

5. Baillie, Richard T. and Patrick McMahon. *The Foreign Exchange Market: Theory and Econometric Evidence*. Cambridge: Cambridge University Press, 1989.

6. Baillie, Richard T. and Rowena A. Pecchenino, "The Search for Equilibrium Relationships in International Finance: The Case of the Monetary Model." *Journal of International Money and Finance*, December 1991, 582–93.

7. Bernanke, Ben, "Alternative Explanations of the Money-Income Correlation." *Carnegie-Rochester Conference Series on Public Policy*, 1986, 49–100.

8. Blanchard, Oliver and Danny Quah, "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review*, September 1989, 655–73.

9. Boughton, James M., "Long-run Money Demand in Large Industrial Countries." *IMF Staff Papers*, March 1991, 1–32.

10. Campbell, John Y. and Pierre Perron, "Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots." *NBER Macroeconomics Annual*, 1991, 141–201.

11. Dickey, David A. and Wayne A. Fuller, "Distribution of the Estimates for Autoregressive time Series With a Unit Root." *Journal of the American Statistical Association*, June 1979, 427–31.

12. ———, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica*, July 1981, 1057–72.

13. Dornbusch, Rudiger, "Devaluation, Money, and Nontraded Goods." *American Economic Review*, December 1973, 871–80.

14. ———, "Real and Monetary Aspects of the Effects of Exchange Rate Changes." In *National Monetary Policies and the International Financial System*, edited by R. Z. Aliber. Chicago: University of Chicago Press, 1974.

15. ———, "Expectations and Exchange Rate Dynamics." *Journal of Political Economy*, December 1976, 1161–76.

16. Engle, Robert F. and Clive W. J. Granger, "Cointegration and Error-Correction: Representation, Estimation, and Testing." *Econometrica*, March 1987, 251–76.

17. Frenkel, Jacob A., "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence." *Scandinavian Journal of Economics*, May 1976, 200–24.

18. Johansen, Soren, "Statistical Analysis of Cointegration Vectors." *Journal of Economic Dynamics and Control*, June–September 1988, 231–54.

19. ——— and Katerina Juselius, "Maximum Likelihood Estimation and Inference on Cointegration with Application to the Demand For Money." *Oxford Bulletin of Economics and Statistics*, May 1990, 169–09.

20. ———, "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and the UIP for UK." *Journal of Econometrics*, July 1992, 211–44.

21. Kim, Jin-Ock and Walter Enders, "Real and Monetary Causes of Real Exchange Rate Movements in the Pacific Rim." *Southern Economic Journal*, April 1991, 1061–70.

22. Lutkepohl, Helmut and Hans-Eggert Reimers, "Impulse Response Analysis of Cointegrated Systems." *Journal of Economic Dynamics and Control*, January 1992, 53–78.

23. MacDonald, Ronald, "Purchasing Power Parity: Is It For Real." *Review of Economics and Statistics*, November 1993, 690–95.

24. Mussa, Michael, "The Exchange Rate, The Balance of Payments, and Monetary and Fiscal Policy under a Regime of Controlled Floating." *Scandinavian Journal of Economics*, May 1976, 229–48.

25. Sims, Christopher, "Are Forecasting Models Usable for Policy Analysis?" Federal Reserve Bank of Minneapolis *Quarterly Review*, Winter 1986, 3–16.

26. Smith, Jeremy and Jim Hagan. "Multivariate Cointegration and Error Correction Models: An Application to Manufacturing Activity in Australia." Working Paper No. 223. Australian National University, March 1991.