2011

The Structure of Efficient Liability Rules

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The motivation for this paper is two-fold. One purpose is to obtain a complete characterization of efficient liability rules within the framework of a model which is essentially the standard tort model with only some minor differences; but with a liability rule notion more general than the standard one. The standard model within which by and large efficiency of liability rules has been considered in the law and economics literature is essentially the one within the framework of which Brown (1973) had made his seminal contribution. The important elements of the standard model include: a two-party interaction with loss due to accident falling only on one party; information regarding costs of care and expected loss functions as part of common knowledge; legal specifications of due care levels being appropriate from the perspective of minimization of social costs; risk-neutral behaviour on the part of the two parties; among others.¹

The usual notion of a liability rule is that of a rule which determines the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of whether and to what extents the parties involved in the interaction were negligent. A party is called nonnegligent if its care level is at least equal to the due care level; otherwise it is called negligent. In particular, it makes no difference whether a party’s care level is equal to the due care or more than the due care. In this paper, however, we differentiate between taking the due care and taking more than the due care. A party is called negligent if its care level is less than the due care; exactly nonnegligent if its care level is equal to the due care; and over-nonnegligent if its care level is greater than the due care. A liability rule is defined as a function which for any nonnegligence proportions of the

¹Considerations relating to the efficiency of liability rules have occupied an important place in the law and economics literature right from its inception. The pioneering contribution by Calabresi (1961) analyzed the effect of liability rules on parties’ behaviour. In his seminal contribution Coase (1960) looked at liability rules from the point of view of their implications for social costs. The rule of negligence was analyzed by Posner (1972) from the perspective of economic efficiency. The first formal analysis of liability rules was done by Brown (1973). His main results demonstrated the efficiency of both the rule of negligence and the rule of strict liability with the defense of contributory negligence. Formal treatment of some of the most important results of the extensive literature on liability rules is contained in Landes and Posner (1987), and Shavell (1987). A necessary and sufficient condition for a liability rule to be efficient, with the notion of a liability rule defined in the usual way, is given in Jain and Singh (2002).
two parties specifies how the loss is to be apportioned between the two parties in case of accident.

An analysis of the totality of all liability rules shows that the subclass of efficient liability rules is characterized by the conjunction of two conditions, namely the requirement of non-reward for over-nonnegligence (RNO) and the condition of negligence liability (NL). The requirement of non-reward for over-nonnegligence essentially requires that if one party is exactly nonnegligent then the other party must not benefit by moving from a position of exact nonnegligence to over-nonnegligence. More precisely, what is required is that if one party is exactly nonnegligent then the liability share of the other party when he is over-nonnegligent is greater than or equal to his liability share when he is exactly nonnegligent. The condition of negligence liability requires that if one party is exactly nonnegligent and the other party is negligent then the entire loss in case of accident must be borne by the negligent party. Thus it is possible to have a liability rule which is efficient and punishes individuals for excessive care although none of the liability rules employed in practice does so. If one does not make a distinction between the due care and more than the due care, as indeed is the case as a general rule, then RNO is automatically satisfied; and consequently it follows that the efficient liability rules are characterized by condition NL alone.

The second motivation for the paper starts from the observation that although as a general rule no distinction is made in tort law between the due care and more than the due care, the practice of making no such distinction does not seem to have a basis in the efficiency requirement. From the necessity and sufficiency of the conjunction of RNO and NL for efficiency, it follows that there are efficient rules which make a distinction between the due care and more than the due care as well as those which do not; and also there are inefficient rules which make a distinction between the due care and more than the due care and which do not. Because of complete logical independence between efficiency on the one hand and making no distinction between the due care and more than the due care, it follows that efficiency alone cannot be an explanation of this tort law feature. The feature must, at least partly, have a non-efficiency explanation. This paper attempts to provide an explanation, although a tentative one, of the no distinction feature. For this purpose, a condition called monotonicity is introduced in the paper, which requires that if a party’s nonnegligence proportion goes up then his liability share must not increase,
given that there is no change in the other party’s nonnegligence proportion. As taking
greater care never results in greater expected accident loss, fairness would seem to require
that a party taking greater care should not be penalized. Therefore it can be argued that
monotonicity condition is a formalization of at least one important aspect of fairness. It
is shown in the paper that every monotonic liability rule which is efficient must be such
that it makes no distinction between the due care and more than the due care. Because
monotonicity, by itself, like efficiency, is unrelated to the no-distinction feature, in view of
the theorem linking efficient monotonic liability rules with no-distinction feature, it seems
appropriate to claim that the no-distinction feature is grounded partly in efficiency and
partly in fairness. Although in this paper only one built-in tort law feature has been an-
alyzed, from the conclusions arrived at certain general points of a methodological nature
seem to emerge which we now discuss.

The economic analysis of law in general and that of tort law in particular, both norma-
tive and positive, revolves around the core idea of economic efficiency. This, however, does
not necessarily mean that non-efficiency values are altogether irrelevant in the context of
economic analysis of law. It is of course true that if in a certain context it turns out that
there is only one rule or procedure which is efficient then giving precedence to efficiency
over all other values implies that any value which is not satisfied by the unique efficient
rule or procedure cannot be incorporated. But in general there would be more than one
rule which would be efficient in a given context; and consequently the choice of rule for
adoption from among efficient rules could be made on the basis of values other than that
of economic efficiency. From analogous considerations it follows that non-efficiency values
have some relevance for positive analysis of law as well. Even if the observed choice of
rule is efficient, efficiency can explain the observed choice only partly if the set of efficient
rules in the given context contains more than one member. For a complete explanation
it might be necessary to invoke values other than that of economic efficiency.

Thus in the context of positive analysis of law, even if it turns out that by and large
laws are efficient, a two-stage analysis might be desirable in which in the first stage the
totality of efficient rules or procedures is identified and in the second stage an explanation
of the observed rule or procedure is attempted in terms of additional values. Similarly,
even if efficiency is accorded primacy over all other values, a two-stage normative analysis
of law seems appropriate in which first the efficient rules are identified and then efficient
rules are analyzed from the perspective of other values which might be considered relevant
in the given legal context. Both kinds of exercises have their place. It is important to note
that in both positive and normative kinds of exercises the identification of the totality of
efficient rules or procedures is crucial. If the objective is to study the structure of efficient
liability rules, whether positively or normatively, one has to begin with the identification of the set of efficient rules as has been done in this paper.

The paper is divided into three sections. Section 1 contains definitions, assumptions and the framework of analysis. In section 2 the structure of efficient liability rules is discussed in terms of the two characterizing conditions of the requirement of non-reward for over-nonnegligence and the negligence liability. Among other things, the section contains formal treatment of monotonicity condition and that of the tort law feature of treating the due care and more than the due care alike. The last section contains some concluding remarks on the interpretation and implications of the analysis of this paper and on viewing of liability rules as embodiments of values. The formal proofs are relegated to the appendix.

1 Definitions and Assumptions

In this paper we consider accidents resulting from interaction of two parties within the framework of a model, which is essentially the standard tort model, first formulated by Brown (1973), with only some minor differences. The two parties will be assumed to be strangers to each other. It will be assumed that, to begin with, the entire accident loss falls on one party to be called the victim (plaintiff); the other party would be referred to as the injurer (defendant). We denote by \( c \geq 0 \) the cost of care taken by the victim; and by \( d \geq 0 \) the cost of care taken by the injurer. We assume that \( c \) and \( d \) are strictly increasing functions of levels of care of the two parties. This of course implies that \( c \) and \( d \) themselves can be taken as indices of levels of care of the victim and the injurer respectively.

Let

\[
C = \{ c \mid c \text{ is the cost of some feasible level of care which can be taken by the victim}\};
\]

and

\[
D = \{ d \mid d \text{ is the cost of some feasible level of care which can be taken by the injurer}\}.
\]

We assume \( 0 \in C \land 0 \in D \).

(A1)

Assumption (A1) merely says that taking no care is always a feasible option for both parties.

Let \( \pi \) denote the probability of occurrence of accident and \( H \geq 0 \) the loss in case of occurrence of accident. Both \( \pi \) and \( H \) will be assumed to be functions of \( c \) and \( d \);

\[\text{\textsuperscript{3}}\text{For elaboration of the standard tort model and for the rationale of some of its crucial assumptions see Brown (1973), Landes and Posner (1987), and Shavell (1987).}\]
\( \pi = \pi(c, d), H = H(c, d) \). Let \( L = \pi H \). \( L \) is thus the expected loss due to accident. We assume:

\[
(\forall c, c' \in C)(\forall d, d' \in D)[c > c' \rightarrow L(c, d) \leq L(c', d)] \land [d > d' \rightarrow L(c, d) \leq L(c, d')] .
\]

(A2)

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.\(^4\) Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; \( TSC = c + d + L(c, d) \). Let \( M = \{(c', d') \in C \times D \mid c' + d' + L(c', d') \text{ is minimum of } \{c + d + L(c, d) \mid c \in C \land d \in D\}\} \). Thus \( M \) is the set of all costs of care configurations \((c', d')\) which are total social cost minimizing. It will be assumed that:

\( C, D, \pi \) and \( H \) are such that \( M \) is nonempty. (A3)

In order to characterize a party’s level of care as negligent or otherwise a reference point (the due care level) for the party needs to be specified. Let \( c^* \) and \( d^* \), where \((c^*, d^*) \in M\), denote the due care levels of the victim and the injurer respectively. We define nonnegligence functions \( p \) and \( q \) as follows:

\[
p : C \rightarrow [0, \infty) \text{ such that}^{5}:
p(c) = \begin{cases} \frac{c}{c^*} & \text{if } c^* > 0; \\ 1 & \text{if } c^* = 0 \end{cases}
\]

\[
q : D \rightarrow [0, \infty) \text{ such that}:
q(d) = \begin{cases} \frac{d}{d^*} & \text{if } d^* > 0; \\ 1 & \text{if } d^* = 0. \end{cases}
\]

Remark 1 Instead of defining \( p(c) = 1 \) for all \( c \in C \) when \( c^* = 0 \), one could also define it in any other way subject to the following two restrictions without affecting the results of this paper:

(i) \( p(c^*) = 1 \)

(ii) \( p \) is an increasing function of \( c \), i.e., \((\forall c, c' \in C)[c > c' \rightarrow p(c) \geq p(c')]\).

Analogous remarks apply for function \( q \) when \( d^* = 0 \).

\(^4\)\( L \), in general, is not strictly decreasing in \( c \) and \( d \) because of two reasons. \( L \) may become zero for sufficiently high values of \( c \) and \( d \); then increasing them beyond these levels will not bring about any further decrease in \( L \). In some cases of complementarities between cares of the two parties, \( L \) may be such that at a particular combination of care levels of the two parties a reduction in \( L \) can be brought about only if both care levels are increased.

\(^5\)We use the standard notation to denote:

\{\( x \mid 0 \leq x \leq 1 \)\} by \([0, 1]\), \{\( x \mid 0 \leq x < 1 \)\} by \([0, 1)\), \{\( x \mid 0 < x \leq 1 \)\} by \((0, 1]\), \{\( x \mid 0 < x < 1 \)\} by \((0, 1)\), \{\( x \mid x \geq 0 \)\} by \([0, \infty)\) and \{\( x \mid x > 1 \)\} by \((1, \infty)\).
In case there is a legally binding due care level for the plaintiff, it would be taken to be identical with $c^*$ figuring in the definition of function $p$; and in case there is a legally binding due care level for the defendant, it would be taken to be identical with $d^*$ figuring in the definition of function $q$. Thus implicitly it is being assumed that the legally binding due care levels are always set appropriately from the point of view of minimizing total social costs. Knowledge of $C, D, L$ and the legally specified due care levels will be assumed to be part of common knowledge.

$p$ and $q$ would be interpreted as proportions of nonnegligence of the victim and the injurer respectively. The victim would be called negligent if $p < 1$; exactly nonnegligent if $p = 1$; and over-nonnegligent if $p > 1$. Similarly, the injurer would be called negligent if $q < 1$; exactly nonnegligent if $q = 1$; and over-nonnegligent if $q > 1$.

We now proceed to define the notion of a liability rule. The way the notion of a liability rule is defined in this paper is more general than the one employed in the law and economics literature. From a technical point of view it is desirable to define the notion of a liability rule independently of its applications. The context in which a liability rule can be applied is completely specified if in addition to $C, D, \pi$ and $H$ we also specify the configuration of due care levels $(c^*, d^*) \in M$. The set of all applications $< C, D, \pi, H, (c^*, d^*) \in M >$ satisfying (A1)-(A3) will be denoted by $\mathcal{A}$.

Formally, a liability rule is a function $f$ from $[0, \infty)^2$ to $[0, 1]^2$, $f : [0, \infty)^2 \mapsto [0, 1]^2$, such that: $f(p, q) = (x, y)$, where $x + y = 1$.\footnote{The usual law and economics literature notion of a liability rule is defined as follows: A liability rule is a function $f$ from $[0, 1]^2$ to $[0, 1]^2$, $f : [0, 1]^2 \mapsto [0, 1]^2$, such that: $f(p, q) = (x, y)$, where $x + y = 1$.}

Thus, a liability rule is a rule which specifies the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of proportions of nonnegligence of the two parties.

Let $f$ be a liability rule. Consider a particular application $< C, D, \pi, H, (c^*, d^*) \in M >$ of the rule. If accident takes place and loss of $H(c, d)$ materializes, then $xH(c, d)$ will be borne by the victim and $yH(c, d)$ by the injurer; where $(x, y) = f(p, q) = f[p(c), q(d)]$. As, to begin with, in case of occurrence of accident, the entire loss falls upon the victim, $yH(c, d)$ represents the liability payment by the injurer to the victim. The expected costs of the victim and the injurer, to be denoted by $EC_1$ and $EC_2$ respectively, therefore are $c + xL(c, d)$ and $d + yL(c, d)$ respectively. Both parties are assumed to prefer smaller
expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Let $f : [0, \infty)^2 \mapsto [0, 1]^2$ be a liability rule. $f$ is defined to be efficient for a given application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $A$ iff $(\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. In other words, a liability rule is efficient for a particular application satisfying (A1)-(A3) iff (i) every $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium is total social cost minimizing, and (ii) there exists at least one $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium. A liability rule is defined to be efficient with respect to a class of applications iff it is efficient for every application belonging to that class.

Throughout this paper we denote $f(1, 1)$ by $(x^*, y^*)$, i.e., we write $x(1, 1)$ as $x^*$ and $y(1, 1)$ as $y^*$. We also denote $L(c^*, d^*)$ by $L^*$.

### 2 The Structure of Efficient Liability Rules

#### 2.1 Characterization of Efficient Liability Rules

First we define two conditions on liability rules.

Condition of Negligence Liability (NL): A liability rule $f$ satisfies the condition of negligence liability iff $[(\forall p \in [0, 1])[f(p, 1) = (1, 0)] \land [(\forall q \in [0, 1])[f(1, q) = (0, 1)]]$.

In other words, a liability rule satisfies the condition of negligence liability iff its structure is such that (i) whenever the injurer is exactly nonnegligent and the victim is negligent, the entire loss in case of occurrence of accident is borne by the victim, and (ii) whenever the victim is exactly nonnegligent and the injurer is negligent, the entire loss in case of occurrence of accident is borne by the injurer.

Requirement of Non-Reward for Over-Nonnegligence (RNO): A liability rule $f$ satisfies the requirement of non-reward for over-nonnegligence iff $[(\forall p \in (1, \infty))[x(p, 1) \geq x(1, 1)] \land [(\forall q \in (1, \infty))[y(1, q) \geq y(1, 1)]]$.

That is to say, a liability rule satisfies the requirement of non-reward for over-nonnegligence iff its structure is such that (i) given that the injurer is exactly nonnegligent, the liability share of the victim when he is over-nonnegligent is greater than or equal to his liability.
share when he is exactly nonnegligent, and (ii) given that the victim is exactly nonneg-
ligent, the liability share of the injurer when he is over-nonnegligent is greater than or
equal to his liability share when he is exactly nonnegligent.

The most important result of this paper, stated in Theorem 1, says that efficient
liability rules are characterized by the conjunction of the two conditions defined above.
The following is the formal statement of the characterization theorem:

**Theorem 1** A liability rule \( f : [0, \infty)^2 \mapsto [0, 1]^2 \) is efficient with respect to \( \mathcal{A} \) iff it satis-
fies the requirement of non-reward for over-nonnegligence and the condition of negligence
liability.

The Theorem is proved via four propositions whose statements and proofs are given
in the Appendix. Proposition 1 establishes that if a liability rule satisfies both RNO and
NL then regardless of which application belonging to \( \mathcal{A} \) is considered, \((c^*, d^*) \in M \) consti-
tutes a Nash equilibrium. Proposition 2 establishes that regardless of which application
belonging to \( \mathcal{A} \) of a liability rule satisfying RNO and NL is considered, all \((\overline{c}, \overline{d}) \in C \times D \)
which are Nash equilibria are total social cost minimizing. Propositions 1 and 2 together
thus show that a liability rule satisfying RNO and NL is efficient for every application
belonging to \( \mathcal{A} \); and therefore establish the sufficiency of RNO and NL for efficiency with
respect to \( \mathcal{A} \). Propositions 1 and 2 together, in fact, establish more than the sufficiency
part. To establish sufficiency part we need, for every application belonging to \( \mathcal{A} \), only (i)
all \((c, d) \in C \times D \) which are Nash equilibria to be total social cost minimizing, and (ii) the
existence of a \((\overline{c}, \overline{d}) \in C \times D \) which is a Nash equilibrium. Rather than merely showing
the existence of a \((\overline{c}, \overline{d}) \in C \times D \) which is a Nash equilibrium, Proposition 1 establishes
a much stronger result; namely that the configuration of due care levels is always a Nash
equilibrium.

Proposition 3 establishes that RNO is a necessary condition for efficiency with respect
to \( \mathcal{A} \) of any liability rule; and Proposition 4 establishes that NL is a necessary condition
for efficiency with respect to \( \mathcal{A} \) of any liability rule. Propositions 3 and 4 together there-
fore establish the necessity of conjunction of RNO and NL for efficiency with respect to \( \mathcal{A} \).

**Remark 2** As a liability rule is inefficient with respect to a class of applications iff it is
inefficient for at least one application belonging to that class, it follows that a rule which
is inefficient with respect to a class of application may be efficient with respect to a proper
subclass of that class. The conjunction of NL and RNO is necessary and sufficient for
efficiency with respect to \( \mathcal{A} \). Let:

\( \mathcal{A}^0_i = \{ < C, D, \pi, H, (c^*, d^*) \in M > \in \mathcal{A} \mid c^* = 0 \} \); and
\( \mathcal{A}_0^c = \{ C, D, \pi, H, (c^*, d^*) \in M \mid \exists A \mid d^* = 0 \} \).

In other words, \( \mathcal{A}_0^c \subset A \) is that subclass of applications belonging to \( A \) such that \( c^* = 0 \); and \( \mathcal{A}_0^d \subset A \) is that subclass of applications belonging to \( A \) such that \( d^* = 0 \). If \( c^* = 0 \) then we have \((\forall c \in C)[p(c) = 1]\). Consequently, \([\forall p \in [0, 1]]\langle f(p, 1) = (1, 0) \rangle\) part of the NL condition and \([\forall p \in (1, \infty)]\langle x(p, 1) \geq x(1, 1) \rangle\) part of the RNO condition are trivially satisfied. Therefore the set of liability rules efficient with respect to \( \mathcal{A}_0^c \) is given by the conjunction of \([\forall q \in [0, 1]]\langle f(1, q) = (0, 1) \rangle\) and \([\forall q \in (1, \infty)]\langle y(1, q) \geq y(1, 1) \rangle\). Similarly, if \( d^* = 0 \) then we have \((\forall d \in D)[q(d) = 1]\); and consequently \([\forall q \in [0, 1]]\langle f(1, q) = (0, 1) \rangle\) part of NL and \([\forall q \in (1, \infty)]\langle y(1, q) \geq y(1, 1) \rangle\) part of RNO are trivially satisfied. Therefore the set of liability rules efficient with respect to \( \mathcal{A}_0^d \) is given by the conjunction of \([\forall p \in [0, 1]]\langle f(p, 1) = (1, 0) \rangle\) and \([\forall p \in (1, \infty)]\langle x(p, 1) \geq x(1, 1) \rangle\). From the above it is clear that every liability rule is efficient with respect to \( \mathcal{A}_0^c \cap \mathcal{A}_0^d \).

Both RNO and NL put restrictions on the assignment of liability shares when one party is exactly nonnegligent and the other party is not. Neither of the two conditions constrains in any way if neither party is exactly nonnegligent or if both parties are exactly nonnegligent. In particular if one party is negligent and the other party is over-nonnegligent then neither condition constrains liability assignments in any way whatsoever. Consequently it follows that it is possible for a liability rule to be efficient and at the same time exhibit rather perverse features. Consider the following example:

**Example 1** Let the liability rule \( f : [0, \infty)^2 \mapsto [0, 1]^2 \) be defined by:

\[
\begin{align*}
\text{if } p < 1 \land q < 1; & & \quad f(p, q) = \left( \frac{1}{2}, \frac{1}{2} \right) \\
\text{if } p < 1 \land q = 1; & & \quad = (1, 0) \\
\text{if } p < 1 \land q > 1; & & \quad = (0, 1) \\
\text{if } p = 1 \land q < 1; & & \quad = \left( \frac{1}{2}, \frac{1}{2} \right) \\
\text{if } p = 1 \land q = 1; & & \quad = \left( \frac{1}{2}, \frac{1}{2} \right) \\
\text{if } p = 1 \land q > 1; & & \quad = (1, 0) \\
\text{if } p > 1 \land q < 1; & & \quad = \left( \frac{1}{2}, \frac{1}{2} \right) \\
\text{if } p > 1 \land q = 1; & & \quad = \left( \frac{1}{2}, \frac{1}{2} \right) \\
\text{if } p > 1 \land q > 1. & & \quad = \left( \frac{1}{2}, \frac{1}{2} \right)
\end{align*}
\]

This liability rule is efficient as it satisfies both RNO and NL. The perverse feature of the rule lies in the fact that when one party is negligent and the other is over-nonnegligent the entire liability falls on the over-nonnegligent party.

\( ^7 \)A conditional is true if its antecedent is false.
2.2 The No-Distinction between the Due Care and More than the Due Care Requirement

One of the basic features of tort law is that for the purpose of assigning liability shares, as a general rule, it does not distinguish between the due care and more than the due care. There are two different ways in which this characteristic feature of tort law can be incorporated in the analysis of liability rules. One can either treat the no-distinction feature as a condition on liability rules or the feature can be incorporated in the definition of a liability rule itself. The no-distinction feature of tort law as a condition on liability rules can be stated as follows:

The No-Distinction between the Due Care and More than the Due Care Requirement (NDMR): A liability rule \( f \) satisfies the no-distinction between the due care and more than the due care requirement if \( \forall p, q \in [0, \infty) \) \( |p \geq 1 \rightarrow f(p, q) = f(1, q) \) \( \wedge q \geq 1 \rightarrow f(p, q) = f(p, 1) \] \].

It is immediate that any liability rule which satisfies NDMR would also satisfy RNO. Therefore, it follows, from Theorem 1, that for the subclass of liability rules satisfying NDMR a necessary and sufficient condition for efficiency is that NL holds. We formally state the result as a theorem.

**Theorem 2** Let liability rule \( f : [0, \infty)^2 \rightarrow [0, 1]^2 \) belong to the subclass of liability rules satisfying the no-distinction between the due care and more than the due care requirement. Then, a necessary and sufficient condition for \( f \) to be efficient for every possible choice of \( C, D, \pi, H \) and \( (c^*, d^*) \in M \) satisfying (A1)-(A3) is that it satisfy the condition of negligence liability.

If the no-distinction between the due care and more than the due care requirement is to be incorporated in the definition of a liability rule itself then the most appropriate way to do so seems to be to define a liability rule as a function from \([0, 1]^2\) to \([0, 1]^2\), rather than from \([0, \infty)^2\) to \([0, 1]^2\); along with the required changes in the definitions of functions \( p \) and \( q \). Under this procedure for incorporating the no-distinction between the due care and more than the due care requirement, a liability rule \( f \) would be defined by:

\[
f : [0, 1]^2 \rightarrow [0, 1]^2, \text{ where } f(p, q) = (x, y), \; x + y = 1.
\]

An application of \( f \) would consist of specification of \( C, D, \pi, H, (c^*, d^*) \in M \) satisfying (A1)-(A3); along with functions \( p \) and \( q \) defined as follows: 

\[
p : C \rightarrow [0, 1] \text{ such that: }
p(c) = \begin{cases} \frac{c}{c^*} & \text{if } c < c^*; \\ 1 & \text{if } c \geq c^*. \end{cases}
\]
$q : D \mapsto [0, 1]$ such that:

\[
q(d) = \begin{cases} 
\frac{d}{d^*} & \text{if } d < d^*; \\
1 & \text{if } d \geq d^*.
\end{cases}
\]

RNO, of course, is inapplicable in this framework. Within this framework, the efficient liability rules are characterized by condition NL.\(^8\)

The no-distinction between the due care and more than the due care requirement is logically completely independent of efficiency. There are both efficient and inefficient liability rules satisfying NDMR as well as violating it. Therefore, it follows that this very important feature of tort law cannot possibly have an explanation solely rooted in the normative criterion of economic efficiency. In what follows we argue that this feature of tort law is based partly on fairness considerations and partly on efficiency considerations. In order to argue this we first introduce the property of monotonicity.

### 2.3 Monotonic Liability Rules

As taking greater care never results in greater expected accident losses, fairness would seem to require that greater levels of care should not be associated with greater liability shares. In other words, the use of liability rules which perversely associate larger liability shares for larger proportions of nonnegligence might be thought inappropriate on grounds of fairness. A liability rule which does not perversely associate larger liability shares with larger proportions of nonnegligence would be called a monotonic liability rule. Intuitively, the monotonicity requirement seems to be quite compelling on considerations of fairness and justice. It is therefore not surprising that all liability rules used in practice satisfy the monotonicity requirement. Formally, the monotonicity condition is defined as follows:

**Monotonicity (M):** A liability rule $f$ satisfies the condition of monotonicity iff $\forall p, p', q, q' \in [0, \infty)][[p \geq p' \rightarrow x(p, q) \leq x(p', q)] \land [q \geq q' \rightarrow y(p, q) \leq y(p, q')]$.

If one considers only the subclass of monotonic liability rules, then in view of Theorem 1 it follows that all efficient monotonic liability rules satisfy the no-distinction requirement of tort law. We state this important result as a theorem. The proof is given in the Appendix.

**Theorem 3** Let liability rule $f : [0, \infty)^2 \mapsto [0, 1]^2$ belong to the subclass of liability rules satisfying the condition of monotonicity. Then, if $f$ is efficient for every possible choice

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\(^8\)For the result that efficient liability rules in this framework are characterized by condition NL, see Jain and Singh (2002).
of $C, D, \pi, H$ and $(c^*, d^*) \in M$ satisfying (A1)-(A3) then it satisfies the no-distinction between the due care and more than the due care requirement.

The monotonicity condition, like the NDMR condition, is logically completely independent of efficiency. From Theorem 1 it follows that the class of efficient liability rules includes both monotonic and non-monotonic rules; and so does the class of inefficient rules. Monotonicity by itself is unrelated to NDMR as well; as there are monotonic rules both satisfying and violating NDMR, and non-monotonic rules both satisfying and violating NDMR. The NDMR, however, seems to be, in view of Theorem 3, partly grounded in fairness and partly in efficiency.

2.4 The Requirement of No-Distinction among Levels of Care Which are less than the Due Care

Most liability rules used in practice do not distinguish among levels of care which are less than the due care. A notable exception is the rule of comparative negligence. The requirement that no distinction be made among different levels of care as long as they are all less than the due care can be formalized as follows:

The Requirement of No-Distinction among Levels of Care Which are less than the Due Care (RNDL): A liability rule $f$ satisfies the requirement of no-distinction among levels of care which are less than the due care iff $\forall p, q \in [0, \infty)\left[p < 1 \implies f(p, q) = f(0, q)\right] \land \left[q < 1 \implies f(p, q) = f(p, 0)\right]$.

It is easy to see that this condition is not only unrelated to efficiency and monotonicity (which is being interpreted as a formalization of an aspect of fairness) taken singly but also when they are considered jointly. Thus, this requirement, although analogous to NDMR, does not seem to be based on any value of a compelling nature.

The structure of liability rules which satisfy both NDMR and RNDL is extremely simple as the liability assignments depend only on whether and which parties are negligent. If both the conditions are incorporated in the definition of a liability rule itself, then a liability rule $f$ becomes a function from $\{0, 1\}^2$ to $[0, 1]^2$; $f : \{0, 1\}^2 \to [0, 1]^2$, where $f(p, q) = (x, y)$, $x + y = 1$. An application of $f$ consists of specification of $C, D, \pi, H, (c^*, d^*) \in M$ satisfying (A1)-(A3); along with functions $p$ and $q$ defined as follows:

$p : C \to \{0, 1\}$ such that:
\[ p(c) = \begin{cases} 0 & \text{if } c < c^*; \\ 1 & \text{if } c \geq c^* \end{cases} \]
\[ q : D \mapsto \{0, 1\} \text{ such that:} \]
\[ q(d) = \begin{cases} 0 & \text{if } d < d^*; \\ 1 & \text{if } d \geq d^* \end{cases} \]

3 Concluding Remarks

This paper has been concerned with the characterization of efficient liability rules on the one hand; and with the normative analysis of an important built-in feature of tort law, namely the practice of making no distinction between the due care and more than the due care, on the other. The tentative conclusion which emerged from the latter is that the no-distinction feature appears to be partly grounded in efficiency and partly in fairness. The reason for terming the conclusion as tentative is that it is based on the analysis of liability rules within the framework of the standard model which neglects the administrative costs of the legal system. If all costs are taken into account, not merely the costs of care and expected accident loss as is done in the standard tort model, then the use of monotonic liability rules may additionally be justified on efficiency considerations, as they are likely to be less costly to administer than non-monotonic liability rules.

However, it is important not to conclude on considerations of this type that if all costs are taken into account then a purely efficiency explanation can be constructed for observed rules and features. The widespread use of comparative negligence provides a counter-example. Comparative negligence is as efficient as negligence rule within the framework of the standard tort model. However, when all costs are taken into account, comparative negligence, because of higher administrative costs, is less efficient than the negligence rule. Regardless of which rule or rules are most efficient\(^9\), when all costs are taken into account, we know for certain that comparative negligence, being less efficient than the negligence rule, cannot be among the most efficient rules. The only way to explain the observed choice of comparative negligence is to invoke some normative criterion like fairness or by analyzing the problem with a different set of assumptions.

Coming back to the question of excessive care (more than the due care) from the perspective of efficiency, it is possible to think of situations where it will be a matter of indifference whether excessive care is punished or not, situations where punishment

---

\(^9\)It should be noted that there can be no presumption that the set of efficient rules, when all costs are taken into account, will be a subset of the rules which are efficient within the framework of the standard model.
for excessive care may be indicated, and situations where reward for excessive care may be indicated. Within the framework of the standard model it does not matter whether excessive care is punished or not; what is required for efficiency is that it must not be rewarded. In contrast, in the context of pure economic loss punishment for excessive care will generally be indicated. In pure economic loss cases, the loss suffered by the victim is generally greater than the social loss as the victim’s loss is partially offset by third party gain.\textsuperscript{10} In pure economic loss cases, often victims are not compensated. As the loss that the victims ultimately bear is greater than the socially relevant loss, they will be induced to take greater care than the socially optimal level. Therefore, a policy of punishment for excessive care may be indicated; merely a policy of non-reward will not be sufficient. In cases where care levels are chosen sequentially\textsuperscript{11}, it may be socially desirable, for the party choosing the care level after observing the choice of the party choosing first, to choose a level of care greater than the the due care in case the party choosing first chooses a care level less than the due care. The party choosing second, however, will not do so within the framework of the standard model. A policy of reward may be necessary for inducing excessive care. While, as seen above, there are all kinds of cases, the important point to note is that excessive care is never punished, not even when doing so may be appropriate from the perspective of efficiency; although it may be encouraged (as in last clear chance doctrine). This strongly suggests a non-efficiency rationale for it, in addition to efficiency.

From the above, one thing seems to be reasonably clear, namely the need to analyze features, particularly the built-in features, of tort law along the lines of the analysis of the no-distinction feature carried out in this paper so as to make explicit their connections with the relevant normative criteria.

As far as liability rules are concerned, their analysis can be done in two different, although related, ways. Given any property defined for liability rules one can ask the question as to which liability rules satisfy the property and which liability rules do not. Alternatively, one can consider a given liability rule and find out which properties are or are not satisfied by the given rule. In a given context if the purpose is to explain the choice of a particular liability rule it would be ideal if one could obtain a complete characterization of the liability rule in question in terms of the embodied values. The characterizing set of values would then constitute a complete explanation of the observed choice.

The normative criteria embodied in liability rules are important from several perspec-

\textsuperscript{10}On pure economic loss see Bishop (1982), Rizzo (1982) and Dari-Mattiacci (2004), among others.

tives. They are of course important from the perspective of attributes of the outcomes which result under the rules when everyone behaves rationally. At times, for various reasons, there can be deviations from the course dictated by rationality; some of the embodied criteria come into relevance by linking the deviations from rational behaviour with the outcomes. However, it is important to note that even when there is never any deviation from rational behaviour, the normative criteria which become relevant only when there are deviations from rational behaviour, nevertheless are important as law has not only a consequentialist purpose but an expressive one as well. Consider two efficient rules: one which punishes over-nonnegligence severely, and the other which is monotonic; and an application where the total social costs minimizing configuration is unique. If everyone always behaves rationally, no one will ever be over-nonnegligent. If no one is ever going to be over-nonnegligent, whether individuals are punished for over-nonnegligence or not will never come into the play. But this does not mean that it is a matter of indifference as to which liability rule is adopted. Adoption of one rule rather than another is also a declaration by the society as to which normative criteria it subscribes to.
References

Appendix

**Proposition 1** If a liability rule $f : [0, \infty)^2 \mapsto [0, 1]^2$ satisfies conditions NL and RNO then for any arbitrary choice of $C, D, \pi, H$ and $(c^*, d^*) \in M$ satisfying (A1)-(A3), $(c^*, d^*)$ is a Nash equilibrium.

*Proof:* Let liability rule $f$ satisfy conditions NL and RNO. Take any $C, D, \pi, H$ and $(c^*, d^*) \in M$ satisfying (A1)-(A3).

Suppose $(c^*, d^*)$ is not a Nash equilibrium. This implies:

\[
(\exists c' \in C) [c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^*L^*] \lor (\exists d' \in D)[d' + y[p(c^*), q(d^*)]L(c^*, d') < d^* + y^*L^*].
\]  

(1.1)

Suppose $(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^*L^*]$ holds.

\[
c' < c^* \land (1.2) \to c' + L(c', d^*) < c^* + x^*L^*, \text{ as } x[p(c'), q(d^*)] = 1 \text{ by condition NL.}
\]  

(1.2)

This is a contradiction as total social costs are minimum at $(c^*, d^*)$. Therefore we conclude:

\[
c' < c^* \to (1.2) \text{ cannot hold.} \quad (1.3)
\]

For $c' > c^*$, we have:

If $c^* = 0$ then $x[p(c'), q(d^*)] = x(1, 1) = x^*$;

If $c^* > 0$ then $x[p(c'), q(d^*)] \geq x^*$, by condition RNO.

Therefore,

\[
c' > c^* \to x[p(c'), q(d^*)] \geq x^*.
\]

Consequently,

\[
c' > c^* \land (1.2) \to c' + x^*L(c', d^*) < c^* + x^*L^*.
\]

\[ (1 - x^*)c + x^*[c' + d^* + L(c', d^*)] < (1 - x^*)c^* + x^*[c^* + d^* + L^*]
\]

\[ (1 - x^*)c' < (1 - x^*)c^*, \text{ as } TSC(c', d^*) \geq TSC(c^*, d^*). \quad (1.4)
\]

\[ (1 - x^*) > 0 \land (1.4) \to c' < c^*, \text{ which contradicts the hypothesis that } c' > c^*. \quad (1.5)
\]

\[ (1 - x^*) = 0 \land (1.4) \to 0 < 0, \text{ a contradiction.} \quad (1.6)
\]

(1.5) and (1.6) establish that (1.4) cannot hold. Therefore it follows that:

\[
c' > c^* \to (1.2) \text{ cannot hold.} \quad (1.7)
\]

(1.3) and (1.7) establish that (1.2) cannot hold. By an analogous argument one can show that $(\exists d' \in D)[d' + y[p(c^*), q(d^*)]L(c^*, d') < d^* + y^*L^*]$ cannot hold.

This establishes that $(c^*, d^*)$ is a Nash equilibrium.

**Proposition 2** If a liability rule $f : [0, \infty)^2 \mapsto [0, 1]^2$ satisfies conditions NL and RNO then for every possible choice of $C, D, \pi, H$ and $(c^*, d^*) \in M$ satisfying (A1)-(A3):

\[ (\forall (\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})] \text{ is a Nash equilibrium } \to (\overline{c}, \overline{d}) \in M]. \]
Proof: Let liability rule \( f \) satisfy conditions NL and RNO. Take any \( C, D, \pi, H \) and \((c^*, d^*) \in M \) satisfying (A1)-(A3).

Let \((\bar{c}, \bar{d})\) be a Nash equilibrium. \((\bar{c}, \bar{d})\) being a Nash equilibrium implies:

\[
(\forall c \in C)[\bar{c} + x[p(\bar{c}), q(\bar{d})] L(\bar{c}, \bar{d}) \leq c + x[p(c), q(\bar{d})] L(c, \bar{d})]
\]
and

\[
(\forall d \in D)[\bar{d} + y[p(\bar{c}), q(\bar{d})] L(\bar{c}, \bar{d}) \leq d + y[p(\bar{c}), q(d)] L(\bar{c}, d)]
\]

(2.1) and (2.2) imply respectively:

\[
\bar{c} + x[p(\bar{c}), q(\bar{d})] L(\bar{c}, \bar{d}) \leq c^* + x[p(c^*), q(\bar{d})] L(c^*, \bar{d})
\]

(2.3)

\[
\bar{d} + y[p(\bar{c}), q(\bar{d})] L(\bar{c}, \bar{d}) \leq d^* + y[p(\bar{c}), q(d^*)] L(\bar{c}, d^*)
\]

(2.4)

Adding inequalities (2.3) and (2.4) we obtain:

\[
\bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + x[p(c^*), q(\bar{d})] L(c^*, \bar{d}) + y[p(\bar{c}), q(d^*)] L(\bar{c}, d^*).
\]

(2.5)

By the definitions of functions \( p \) and \( q \); and condition RNO:

if \( \bar{c} \geq c^* \) then \( y[p(\bar{c}), q(d^*)] \leq y^* \); and

if \( \bar{d} \geq d^* \) then \( x[p(c^*), q(\bar{d})] \leq x^* \).

By condition NL:

if \( \bar{c} < c^* \) then \( y[p(\bar{c}), q(d^*)] = 0 \); and

if \( \bar{d} < d^* \) then \( x[p(c^*), q(\bar{d})] = 0 \).

Also, by (A2),

if \( \bar{c} \geq c^* \) then \( L(\bar{c}, d^*) \leq L^* \); and

if \( \bar{d} \geq d^* \) then \( L(c^*, \bar{d}) \leq L^* \).

In view of the above,

\[
\bar{c} \geq c^* \wedge \bar{d} \geq d^* \wedge (2.5) \rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + x^* L(c^*, \bar{d}) + y^* L(\bar{c}, d^*)
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + x^* L^* + y^* L^* = c^* + d^* + L^*;
\]

(2.6)

\[
\bar{c} < c^* \wedge \bar{d} \geq d^* \wedge (2.5) \rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + x[p(c^*), q(\bar{d})] L(c^*, \bar{d})
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(c^*, \bar{d})
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L^*;
\]

(2.7)

\[
\bar{c} \geq c^* \wedge \bar{d} < d^* \wedge (2.5) \rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + y[p(\bar{c}), q(d^*)] L(\bar{c}, d^*)
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L(\bar{c}, d^*)
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L^*;
\]

(2.8)

\[
\bar{c} < c^* \wedge \bar{d} < d^* \wedge (2.5) \rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + y[p(\bar{c}), q(d^*)] L(\bar{c}, d^*)
\]

\[
\rightarrow \bar{c} + \bar{d} + L(\bar{c}, \bar{d}) \leq c^* + d^* + L^*.
\]

(2.9)

(2.6)-(2.9) establish that:

\( (2.5) \rightarrow TSC(\bar{c}, \bar{d}) \leq TSC(c^*, d^*) \).

As \( TSC(c^*, d^*) \) is minimum, it follows that we must have \( TSC(\bar{c}, \bar{d}) = TSC(c^*, d^*) \).

This establishes that \((\bar{c}, \bar{d}) \in M \).

Proposition 3 If a liability rule \( f : [0, \infty)^2 \rightarrow [0, 1]^2 \) is efficient for every possible choice of \( C, D, \pi, H \) and \((c^*, d^*) \in M \) satisfying (A1)-(A3), then it satisfies condition RNO.
Proof: Let liability rule \( f \) violate condition RNO. Then:
\[ \exists p \in (1, \infty) \] \( x(p, 1) < x(1, 1) \) \( \lor \) \( \exists q \in (1, \infty) \) \( y(1, q) < y(1, 1) \).
Suppose \( \exists q \in (1, \infty) \) \( y(1, q) < y(1, 1) \) holds.
Let \( t \) be a positive number. Choose \( u \) such that:
\[ u > \frac{(q-1)t}{y(1, 1) - y(1, q)}. \]
It should be noted that \( u > 0 \).
Choose a positive number \( \mu \) such that \( \mu < (q-1)t \).
Let \( s, \epsilon \) and \( \delta \) be any positive numbers. Let \( C, D \) and \( L \) be specified as follows:
\[ C = \{0, s\}, D = \{0, t, qt\} \]
\[ L(0, 0) = s + \epsilon + t + \delta + u, L(s, 0) = t + \delta + u \]
\[ L(0, t) = s + \epsilon + u, L(s, t) = u \]
\[ L(0, qt) = s + \epsilon + u - (q-1)t + \mu, L(s, qt) = u - (q-1)t + \mu. \]
\[ \epsilon > 0, \delta > 0 \] and \( 0 < \mu < (q-1)t \) imply that \( (s, t) \) is the unique total social cost minimizing configuration, i.e., \( M = \{(s, t)\} \).
Let \( (c^*, d^*) = (s, t) \).
Now,
\[ EC_2(s, t) \]
\[ = t + y(1, 1)L(s, t) \]
\[ = t + y(1, 1)u \]
\[ EC_2(s, qt) \]
\[ = qt + y(1, q)L(s, qt) \]
\[ = qt + y(1, q)[u - (q-1)t + \mu] \]
\[ EC_2(s, t) - EC_2(s, qt) \]
\[ = t + y(1, 1)u - qt - y(1, q)[u - (q-1)t + \mu] \]
\[ = -(q-1)t + [y(1, 1) - y(1, q)]u + y(1, q)[(q-1)t - \mu] \]
As \( y(1, q)[(q-1)t - \mu] \geq 0 \), we conclude:
\[ EC_2(s, t) - EC_2(s, qt) \geq -(q-1)t + [y(1, 1) - y(1, q)]u. \]
As \( u > \frac{(q-1)t}{y(1, 1) - y(1, q)} \), it follows that:
\[ EC_2(s, t) - EC_2(s, qt) > 0. \]
This implies that the unique total social cost minimizing configuration \( (s, t) \) is not a Nash equilibrium. Consequently \( f \) is not efficient.
If \( \exists p \in (1, \infty) \) \( x(p, 1) < x(1, 1) \) holds, then by an analogous argument one can show that \( f \) is not efficient.
This establishes the proposition.

**Proposition 4** If a liability rule \( f : [0, \infty)^2 \rightarrow [0, 1]^2 \) is efficient for every possible choice of \( C, D, \pi, H \) and \( (c^*, d^*) \in M \) satisfying (A1)-(A3), then it satisfies condition NL.

Proof: Let liability rule \( f \) violate condition NL. Then:
\[ \exists p \in [0, 1) \left[ f(p, 1) \neq (1, 0) \right] \lor \exists q \in [0, 1) \left[ f(1, q) \neq (0, 1) \right]. \]

Suppose \( \exists q \in [0, 1) \left[ f(1, q) \neq (0, 1) \right] \) holds.

Let \( t > 0 \). Choose \( r \) such that \( 0 \leq y(1, q)t < r < t \). Let \( v = \frac{r}{1-q} \). Let \( u > 0 \) and \( \epsilon > 0 \).

Let \( C, D \) and \( L \) be specified as follows:

\[
C = \{0, u\}, \quad D = \{0, qv, v\} \\
L(0, 0) = u + \epsilon + t + qv, \quad L(u, 0) = t + qv \\
L(0, qv) = u + \epsilon + t, \quad L(u, qv) = t \\
L(0, v) = u + \epsilon, \quad L(u, v) = 0.
\]

\( \epsilon > 0 \) and \( t > r = (1 - q)v \) imply that \( M = \{(u, v)\} \).

Let \( (c^*, d^*) = (u, v) \).

Now,

\[
EC_2(u, v) = v \\
EC_2(u, qv) = qv + y(1, q)L(u, qv) \\
= qv + y(1, q)t \\
EC_2(u, v) = v - qv - y(1, q)t \\
= (1 - q)v - y(1, q)t \\
= v - y(1, q)t \\
> 0.
\]

This implies that the unique total social cost minimizing configuration \( (u, v) \) is not a Nash equilibrium. \( f \) is therefore not efficient.

If \( \exists p \in [0, 1) \left[ f(p, 1) \neq (1, 0) \right] \) holds, then by an analogous argument it can be shown that \( f \) is not efficient.

This establishes the proposition.

**Proof of Theorem 1:** Let liability rule \( f \) satisfy the requirement of non-reward for over-nonnegligence and the condition of negligence liability. Then by Propositions 1 and 2 \( f \) is efficient for every possible choice of \( C, D, \pi, H \) and \( (c^*, d^*) \in M \) satisfying (A1)-(A3). Proposition 3 and 4 establish that if \( f \) is efficient for every possible choice of \( C, D, \pi, H \) and \( (c^*, d^*) \in M \) satisfying (A1)-(A3), then it satisfies the requirement of non-reward for over-nonnegligence and the condition of negligence liability.

**Proof of Theorem 3:** Let \( f \) be an efficient monotonic liability rule. Therefore, \( f \) satisfies RNO and NL by Theorem 1.

\[
p \geq 1 \rightarrow x(p, q) \leq x(1, q), \text{ by condition M} \tag{1}
\]

\[
q \geq 1 \rightarrow x(1, q) \leq x(1, 1), \text{ by RNO} \tag{2}
\]

\[
q \geq 1 \rightarrow x(p, q) \geq x(p, 1), \text{ by M} \tag{3}
\]
\[ p \geq 1 \rightarrow x(p, 1) \geq x(1, 1), \text{ by RNO} \] (4)

From (1) and (2) we conclude:
\[ p \geq 1 \land q \geq 1 \rightarrow x(p, q) \leq x(1, 1) \] (5)

From (3) and (4) we conclude:
\[ p \geq 1 \land q \geq 1 \rightarrow x(p, q) \geq x(1, 1) \] (6)

(5) and (6) imply:
\[ p \geq 1 \land q \geq 1 \rightarrow x(p, q) = x(1, 1) \] (7)

In view of (1), (2) and (7) it follows that:
\[ p \geq 1 \land q \geq 1 \rightarrow x(p, q) = x(1, q) \] (8)

Now,
\[ q < 1 \rightarrow x(1, q) = 0, \text{ by NL} \] (9)

(1) and (9) imply:
\[ p \geq 1 \land q < 1 \rightarrow x(p, q) = x(1, q) \] (10)

(8) and (10) imply:
\[ p \geq 1 \rightarrow x(p, q) = x(1, q) \] (11)

By an analogous argument it follows that:
\[ q \geq 1 \rightarrow x(p, q) = x(p, 1) \] (12)

(11) and (12) establish the theorem.