October 22, 1993

Parallel image segmentation using a Hopfield neural network with annealing schedule for neural gains

Yungsik Kim
Sarah A. Rajala, *North Carolina State University*

Parallel image segmentation using a Hopfield neural network with annealing schedule for neural gains

Yungsik Kim*
Sarah A. Rajala

*Multimedia Application Section,
Human Interface Division,
Electronics and Telecommunications Research Institute,
Taejon, Korea
Email: kys@sabsal.etri.re.kr
Telephone: (42)860-6109

Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, NC 27695-7911, USA
Email: sar@eos.ncsu.edu
Telephone: (919)515-5114

ABSTRACT

Neural network architectures have been proposed as new computer architectures and a Hopfield neural network has been shown to find good solutions very fast in solving complex optimization problems. It should be noted, however, that a Hopfield neural network with fixed neural gains only guarantees to find local optimum solutions, not the global optimum solution. Image segmentation, like other engineering problems, can be formalized as an optimization problem and implemented using neural network architectures if an appropriate optimization function is defined. To achieve a good image segmentation, the global or the nearly global optimum solutions of the appropriate optimization function need to be found. In this paper, we propose a new neural network architecture for image segmentaton, "an annealed Hopfield neural network", which incorporates an annealing schedule for the neural gains. We implemented image segmentation using this annealed Hopfield neural network with an optimization function proposed by Blake and Zisserman and achieved good image segmentation in detecting horizontal and vertical boundaries. Later, we proposed an extended optimization function to achieve better performance on detecting sharp corners and diagonally-oriented boundaries. Finally, simulation results on synthetic and real images are shown and compared with general-purpose mean field annealing technique.

1. INTRODUCTION

Hopfield neural network[1,2,3] composed of neurons with graded responses has been shown to solve complex optimization problems such as a traveling salesman problem (TSP) fast once a proper optimization function has been chosen. Because Hopfield neurons are natural in that they resemble real biological neurons which have graded, continuous outputs as a function of their inputs, they can be implemented by real physical devices such as operational amplifiers. It should be noted, however,

0-8194-1369-0/93/$6.00

that a Hopfield neural network with the fixed neural gains can solve optimization problems fast but it only guarantees convergence to a local minimum nearest to the initial point, not to the global minimum solution. In some applications such as an associated memory (AM), fast convergence to the nearest local minimum would be desirable, but not in other optimization problems where the global minimum values need to be found.

Image segmentation, like other engineering problems, can be formalized as an optimization problem and the good image segmentation can be achieved by finding the global or the nearly global optimum solutions to the appropriate optimization function. Performing image segmentation by solving the optimization problem consists of two major parts:

1. Finding the proper optimization function whose global minimum value represents the desired segmented image.
2. Choosing the relaxation method to find the global solution from the proper optimization function as efficiently as possible.

The weak continuity constraints algorithm proposed by Blake and Zisserman[9] is used to define a proper optimization function for the image segmentation. The weak continuity constraints algorithm detects the discontinuities in an image composed of piecewise constant regions with step edges. The power of weak continuity constraints lies in its stability to detect discontinuities and localize them accurately and stably in the presence of substantial noise. We first use the weak continuity constraints algorithm as a proper optimization function for image segmentation and later adopt their approach for achieving improved image segmentation results.

Once the proper optimization function is defined, the next step is to find the global or the nearly global optimum of the optimization function. Relaxation algorithms such as stochastic simulated annealing (SSA)[6] and deterministic mean field annealing (MFA)[4,5] have been used to obtain the global or the nearly global optimum solutions to such problems. Stochastic simulated annealing (SSA) is guaranteed to find the global minimum under certain conditions, but it converges very slowly. Mean field annealing[4] can reach the global or the nearly global optimum solutions faster than stochastic simulated annealing, while retaining many of the advantages of stochastic simulated annealing. Furthermore, Van Den Bout *et al.*[8] showed that under certain conditions, a Hopfield neural network and mean field annealing are equivalent if the slope of the sigmoid is increased over the running of the algorithm. As mentioned previously, a Hopfield neural network with fixed neural gains only converges to the local minimum, not to the global minimum solution. In this paper, we incorporate an annealing schedule with a Hopfield neural network and adopt the weak continuity constraints algorithm to find a more optimal segmentation of images. We achieved good image segmentation results using the annealing schedule for the neural gains with the extended weak continuity constrains algorithm. Furthermore, this annealed Hopfield neural network is expected to solve other kinds of optimization problems in which a fast approach to finding the global or the nearly global minimum is desired. A potential application of this annealed Hopfield neural network is segmentation-based image coding.

2. IMAGE SEGMENTATION USING AN ANNEALED HOPFIELD NEURAL NETWORK

2.1 Image segmentation procedure

The procedure for image segmentation using an annealed Hopfield neural network is as follows:

1. Convert an image segmentation problem to an optimization problem by defining an proper optimization function. The desired segmented image should be the minimum of the optimization function.
2. Construct a Hopfield neural network which corresponds to the defined optimization function and determine network parameters such as the inter-neuron connection strengths between image neurons, the external inputs to image neurons, and the inputs to the edge neurons.
3. Simulate the network with an annealing schedule for the neural gains and interpret the converged output as the segmented image.

2.2 Proper optimization function for image segmentation

Blake and Zisserman[9] proposed the following optimization function for image reconstruction problems.

$$E_{bz} = D + S + P \tag{1}$$

where

$$\begin{aligned} D &= \sum_{i,j}(f_{i,j} - d_{i,j})^2 \\ &= \sum_{i,j}(\sum_{m} o_{i,j,m} - d_{i,j})^2 \end{aligned} \tag{2}$$

$$\begin{aligned} S &= b^2 \sum_{i,j}[(f_{i,j+1} - f_{i,j})^2(1 - v_{i,j}) + (f_{i+1,j} - f_{i,j})^2(1 - h_{i,j})] \\ &= b^2 \sum_{i,j}[\{\sum_{m}(o_{i,j+1,m} - o_{i,j,m})\}^2(1 - v_{i,j}) \\ &\quad +\{\sum_{m}(o_{i+1,j,m} - o_{i,j,m})\}^2(1 - h_{i,j})] \end{aligned} \tag{3}$$

$$P = \alpha \sum_{i,j}(v_{i,j} + h_{i,j}) \tag{4}$$

D is a measure of faithfulness of the reconstructed pixel value $f_{i,j}$ to the input pixel value $d_{i,j}$. S is a measure of how severely the reconstructed image f is deformed, and P is the penalty function that is levied for the step edges in the reconstruction.

In the above equations, $f_{i,j}$ is the gray level of the (i,j)th pixel in the output image, $d_{i,j}$ is the gray level of the (i,j)th pixel in the input image, $o_{i,j,m}$ is the output of the (i,j,m)th neuron ((i,j) defines the pixel location and m defines the gray level), $v_{i,j}$ is the (i,j)th value of vertical-edge neurons, $h_{i,j}$ is the (i,j)th value of horizontal-edge neurons, and b and α are constants. b controls the smoothness of the reconstructed image, and α controls the number of the edges in the reconstructed image.

Since $f_{i,j}$ is a gray level value and $o_{i,j,m}$ is an output of the neuron which has the value 0 or 1 at convergence, $f_{i,j}$ can be represented by summing the outputs of L neurons, where L is the maximum gray level value in the image, i.e., $L = 255$ in an 8-bit monochrome image. That is,

$$f_{i,j} = \sum_{m=0}^{L-1} o_{i,j,m} \tag{5}$$

By minimizing $E_{bz} = D + S + P$, the reconstructed image has smoothing regions which are close to original regions in gray level and has step edges between reconstructed regions.

We believe that the same optimization function can be used for image segmentation by adjusting weighting of each term. For image segmentation it is important to have more emphasis on S. Using a larger b forces the segmented regions to be smoother than appropriate for image reconstruction, and guarantees that the average gray levels are close to the actual gray levels in the regions.

2.3 Connections between and inputs to the neurons

In a Hopfield neural network, the optimization function E_{hop} which guarantees convergence to stable states is,

$$E_{hop} = -\frac{1}{2}\sum_{i,j,m}\sum_{k,l,n} A_{(i,j,m)(k,l,n)} o_{i,j,m} o_{k,l,n} - \sum_{i,j,m} e_{i,j,m} o_{i,j,m} - \sum_{i,j}[V_{i,j} v_{i,j} + H_{i,j} h_{i,j}] \tag{6}$$

where $o_{i,j,m}$ is the output of the (i,j,m)th image neuron, $A_{(i,j,m)(k,l,n)}$ represents the connection strength between the input of the (i,j,m)th image neuron and the output of the (k,l,n)th image neuron, and $e_{i,j,m}$ is the external input applied to the (i,j,m)th image neuron. $V_{i,j}$ and $H_{i,j}$ are the inputs to the (i,j)th vertical-edge neuron and to the (i,j)th horizontal-edge neuron, respectively.

By comparing E_{bz} in Eq. 1 and E_{hop} in Eq. 6, we can determine the image neuron connection strength $A_{(i,j,m)(k,l,n)}$, the external input to each image neuron, $e_{i,j,m}$, and inputs to edge neurons, $V_{i,j}$, $H_{i,j}$. Substituting in for D, S, and P, Eq. 1 becomes

$$\begin{aligned} E_{bz} &= \sum_{i,j}(\sum_{m} o_{i,j,m} - d_{i,j})^2 \\ &+ b^2 \sum_{i,j}[\{\sum_{m}(o_{i,j+1,m} - o_{i,j,m})\}^2(1 - v_{i,j}) + \{\sum_{m}(o_{i+1,j,m} - o_{i,j,m})\}^2(1 - h_{i,j})] \\ &+ \alpha \sum_{i,j}(v_{i,j} + h_{i,j}) \end{aligned}$$

As a result, parameters are

$$
\begin{aligned}
A_{(i,j,m)(i,j,n)} &= -2 - 2b^2\{(1 - v_{i,j-1}) + (1 - h_{i-1,j}) + (1 - v_{i,j}) + (1 - h_{i,j})\} \\
A_{(i,j,m)(i,j-1,n)} &= 2b^2(1 - v_{i,j-1}) \\
A_{(i,j,m)(i-1,j,n)} &= 2b^2(1 - h_{i-1,j}) \\
A_{(i,j,m)(i,j+1,n)} &= 2b^2(1 - v_{i,j}) \\
A_{(i,j,m)(i+1,j,n)} &= 2b^2(1 - h_{i,j}) \\
e_{i,j,m} &= 2d_{i,j} \\
V_{i,j} &= b^2\{\sum_{m=0}^{L-1}(o_{i,j+1,m} - o_{i,j,m})\}^2 - \alpha \\
H_{i,j} &= b^2\{\sum_{m=1}^{L-1}(o_{i+1,j,m} - o_{i,j,m})\}^2 - \alpha
\end{aligned}
\tag{7}
$$

where $n = 0,1,\ldots,L\text{-}1$ for each (i, j).

2.4 Initial outputs of the neurons

The initial output value of the each neuron in the network is set to $0.5 \pm \Delta$ where Δ is a small uniform random variable. This output value comes from statistical mechanics where the mean value of each particle on a regular lattice is about 0.5 at a high temperature if a particle can only move between 0 and 1.

2.5 Annealing schedule for the neural gains

The annealing schedule for the neural gains needs to be determined, such as initial gains, final gains, and the rate for increasing the gains at each iteration. The initial gain λ_{init} for each neuron is set to a small value, $\frac{1}{5000}$, to see whether the network segments the images successfully. The final gain does not need to be determined. Instead, the network stops iterating when there are no output changes. From the simulations, the rate for increasing the gain, η, at each iteration worked well in the range between $\frac{1}{.99}$ and $\frac{1}{.90}$. At each iteration all of the image neurons and the edge neurons in the network are updated.

3. SIMULATION RESULTS ON SYNTHETIC IMAGES

We simulated the proposed annealed Hopfield neural network with the 32×32 8-bit monochrome synthetic images, I^1_{clean}, I^2_{clean}. The synthetic test images contain the step edges of a height of 7 in gray level. I^1_{clean} is noise-free and contains the vertical and horizontal boundaries only and I^2_{clean} is noise-free and contains diagonal boundaries as well. Test images I^1_{noise} and I^2_{noise} are made by arbitrarily adding the white Gaussian noise to the noise-free images. The signal to noise ratio (SNR)

for I^1_{noise} and I^2_{noise} is 2 where SNR is defined as the ratio of step edge height to the standard deviation of the white Gaussian noise.

For test image I^1_{noise}, boundaries were detected correctly and the gray level for each region was reasonably smooth at SNR = 2. The simulation result for the image I^1_{noise} is shown in Fig. 1. For test image I^2_{noise}, the segmentation result is not as good as for I^1_{noise}. As can be seen in Fig. 2, sharp edges were not detected correctly in the segmented image. The reason for this is that as corner pixels are affected more by the background pixels than by the pixels on the triangular object, the sharp corners are more likely to be rounded or disappeared. In the next section, the extended optimization function is to be proposed to improve the performance on detecting the sharp corners and the diagonal boundaries.

4. IMPROVEMENT ON THE TRIANGULAR PATTERN

In order to better handle diagonally-oriented edges, the optimization function E_{bz} proposed by Blake and Zisserman[9] is extended. The new extended optimization function E_{ext} is given by:

$$\begin{aligned} E_{ext} &= E_{bz} + N \\ &= D + S + P + N \end{aligned} \tag{8}$$

where

$$\begin{aligned} D &= \sum_{i,j}(f_{i,j} - d_{i,j})^2 \\ &= \sum_{i,j}(\sum_m o_{i,j,m} - d_{i,j})^2 \end{aligned} \tag{9}$$

$$\begin{aligned} S &= b^2 \sum_{i,j}[(f_{i,j+1} - f_{i,j})^2(1 - v_{i,j}) + (f_{i+1,j} - f_{i,j})^2(1 - h_{i,j})] \\ &= b^2 \sum_{i,j}[\{\sum_m (o_{i,j+1,m} - o_{i,j,m})\}^2(1 - v_{i,j}) \\ &\quad +\{\sum_m (o_{i+1,j,m} - o_{i,j,m})\}^2(1 - h_{i,j})] \end{aligned} \tag{10}$$

$$P = \alpha \sum_{i,j}(v_{i,j} + h_{i,j}) \tag{11}$$

$$N = \tau \sum_{i,j}(v_{i,j} + h_{i,j+1} + v_{i+1,j} + h_{i,j})^2(v_{i,j} + h_{i,j+1} + v_{i+1,j} + h_{i,j} - 2)^2 \tag{12}$$

The D, S, and P terms are the same as in the previous optimization function E_{bz}. The N term is new. The N term includes the local interactions between the edge neurons and is added to improve the detection of closed boundaries and thus, help to detect sharp corners and diagonally-oriented boundaries better. The simulation result for test image I^2_{noise} with the extended optimization function E_{ext} and comparison with the previous simulation result with E_{bz} are shown in Fig. 3, where SNR = 2. As can be seen in Fig. 3, the segmentation result with E_{ext} was improved on detecting sharp corners and diagonally-oriented boundaries.

5. SIMULATION RESULTS ON REAL IMAGES

We appled our algorithm to several real images and compared the results with a general-purpose mean field annealing (GMFA) technique. We utilized two different real images – (1) 150 × 150 8-bit monochrome cartoon image, I_{org}^{car} and (2) 200 × 175 8-bit monochrome brain image, I_{org}^{brain}. The original cartoon image, I_{org}^{car}, was digitally recorded using a noisy camera and the original brain image, I_{org}^{brain} is the magnetic resonance image (MRI) scan taken at the Bowman Gray School of Medicine. The original cartoon image I_{org}^{car} and the original brain image I_{org}^{brain} contain noises themselves. The magnitudes of noises contained in the original images are small, thus we arbitrarily add the white Gaussian noise to them to estimate the performance of our algorithm under the reasonable noise levels. The standard deviation of the Gaussian noise added to the original cartoon image I_{org}^{car} is 7 and the standard deviation added to the original brain image I_{org}^{brain} is 10. For the noise-added cartoon image, I_{noise}^{car}, the segmentation result was good as can be seen in Fig. 4. The segmented image maintains the boundaries very well and also maintains the smoothness within the regions. The segmentation result for the noise-added brain image, I_{noise}^{brain}, is good as well. An annealed Hopfield neural network with the extended optimization function E_{ext} even detects the small and thin regions. The simulation result for the brain image, I_{noise}^{brain} is shown in Fig. 5. The segmentaton results using general-purpose mean field annealing (GMFA) technique are shown in Fig. 4 and in Fig. 5. As can be seen in Fig. 4 and in Fig. 5, the detected boundaries are sharper and the segmented regions are smoother using an annealed Hopfield neural network than using a general-purpose mean field annealing technique.

6. CONCLUSION

We proposed a new algorithm for image segmentation using an annealed Hopfield neural network with the proper optimization function. We incorporated an annealing schedule for the neural gains in the Hopfield neural network and used the optimization function proposed by Blake and Zisserman[9]. With this, we achieved good image segmentation for the images which include the vertical and horizontal boundaries only. However, the performance on the sharp corners and diagonally-oriented boundaries was not as good. Then, we proposed the new optimization function E_{ext} for improvement on detecting sharp corners and diagonally-oriented boundaries. With the extended optimization function E_{ext}, an annealed Hopfield neural network performs better on detecting triangular objects. We demonstrated the power of the proposed algorithm through the simulation results on real images, the cartoon image and the brain image. We also showed that the annealed Hopfield neural network with the extended optimization function E_{ext} performs better than a general-purpose mean field annealing (GMFA) technique for maintaining sharp edges and smooth regions in the segmented images. As a Hopfield neural network can be implemented by physical devices such as operational amplifiers, it can be used in real-time optimization applications such as segmentation-based image coding.

7. REFERENCES

1. J. J. Hopfield, "Neurons with Graded Response have Collective Computational Properties like Those of Two-state Neurons," *Proc. Natl. Acad. Sci. USA*, vol. 81, pp. 3088-3092, May, 1984.

2. J. J. Hopfield and D. W. Tank, "Neural Computation of Decisions in Optimization Problems," *Biol. Cybern.* , vol. 52, pp. 141-152, 1985.

3. J. J. Hopfield and D. W. Tank, "Computing with Neural Circuits: A Model", *Science*, vol. 233, pp. 625-633, Aug 8, 1986.

4. H. P. Hariyannaiah, G. L. Bilbro, and W. E. Snyder, "Restoration of Piecewise-constant Images by Mean-field Annealing," *Journal of Optical Society of America A* , vol. 6, no. 12, pp. 1901-1912, December, 1989.

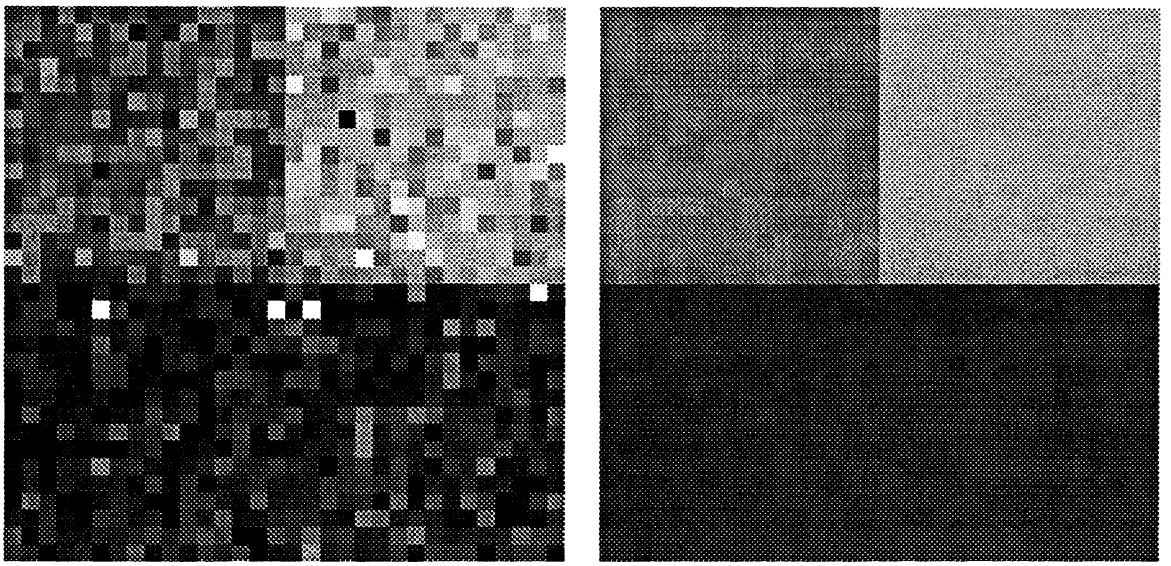
5. G. Bilbro, W. Snyder, and R. Mann, "Mean Field Approximation Minimizes Relative Entropy," *Journal of Optical Society of America A* , 8(2), February, 1991.

6. S. Geman and D. Geman, "Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images," *IEEE Transactions on PAMI*, vol. PAMI-6, no.6, pp. 721-741, November, 1984.

7. S. Kirkpatrick, C. D. Gelatt, Jr., M. P. Vecchi, "Optimization by Simulated Annealing, " *Science*, vol. 220, No. 4598, pp. 671-680, May 13, 1983.

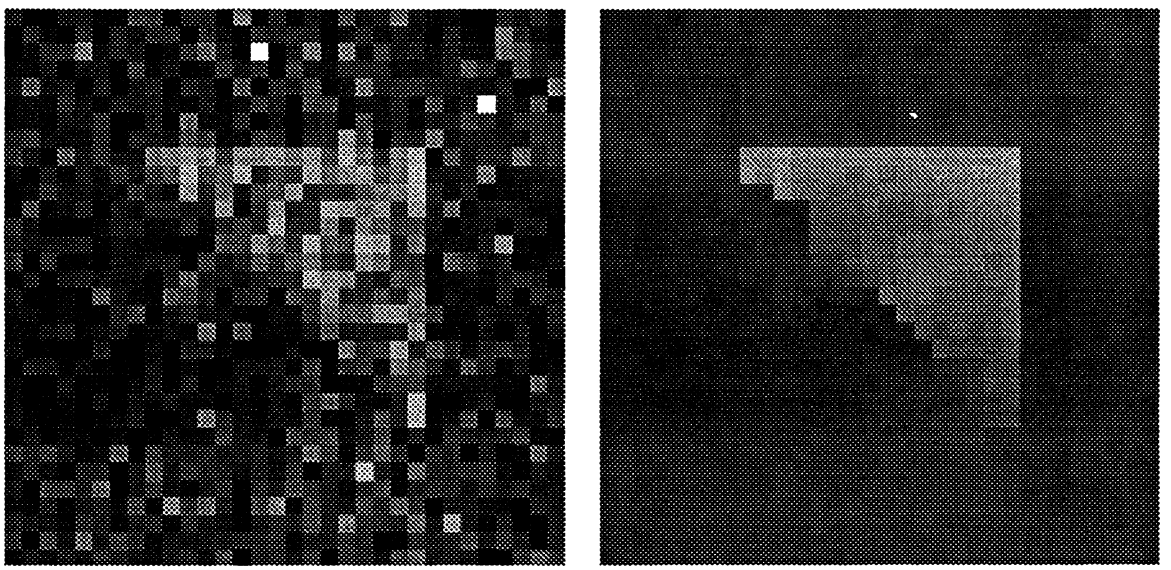
8. D. E. Van Den Bout, T. K. Miller, III., "Graph Partitioning Using Annealed Neural Networks, " *IEEE Transctions on Neural Networks* , vol. 1, no. 2, pp. 192-203, June, 1990.

9. A. Blake and A. Zisserman, *Visual Reconstruction*, The MIT Press, Cambridge, MA, 1987.



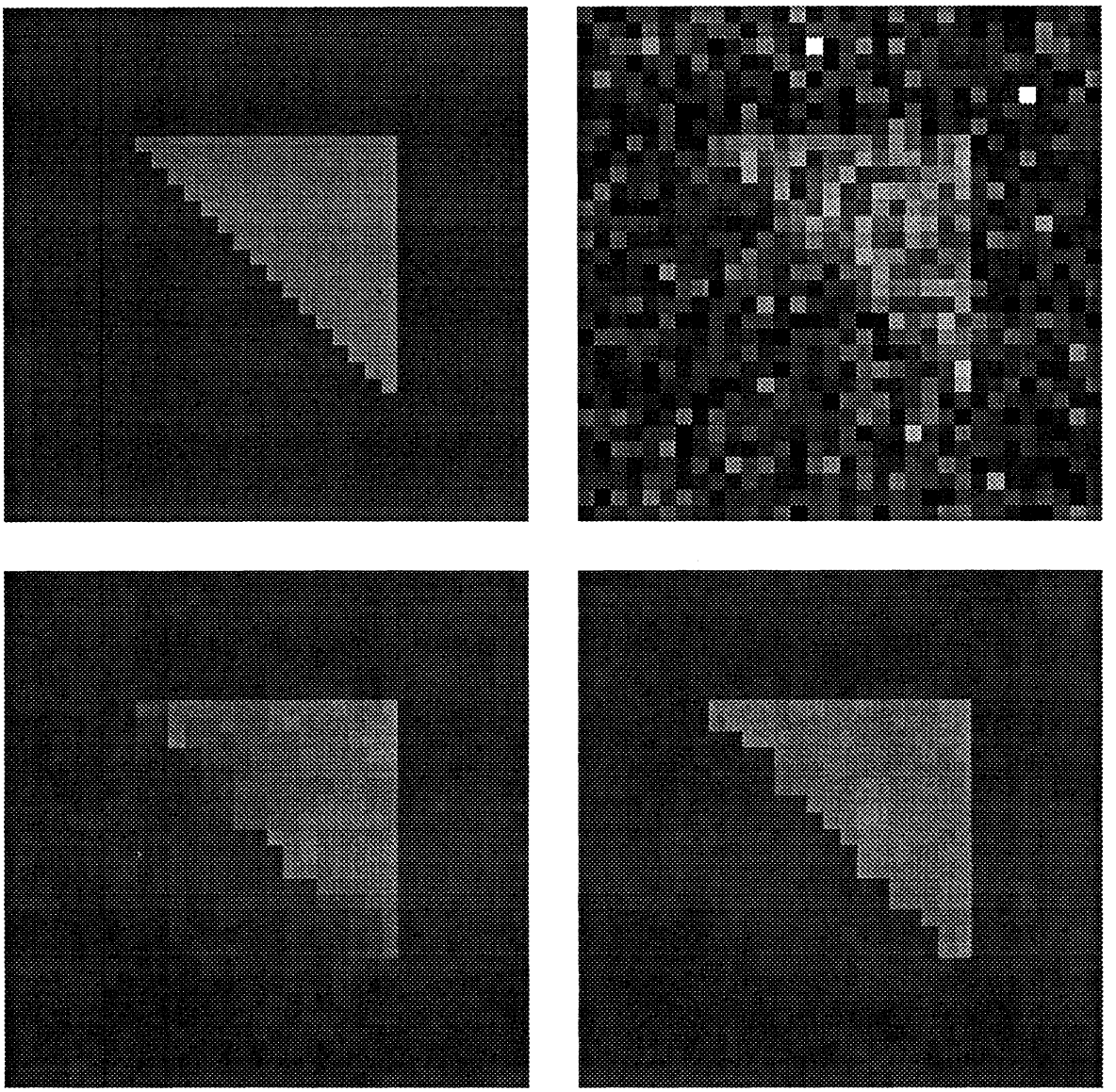
(a) (b)

Figure 1: (a) Noisy input image I^1_{noise} with $\sigma = 3.5$. SNR = 2.0. (b) Segmented output image using an annealed Hopfield neural network with $E_{bz} = D + S + P$.



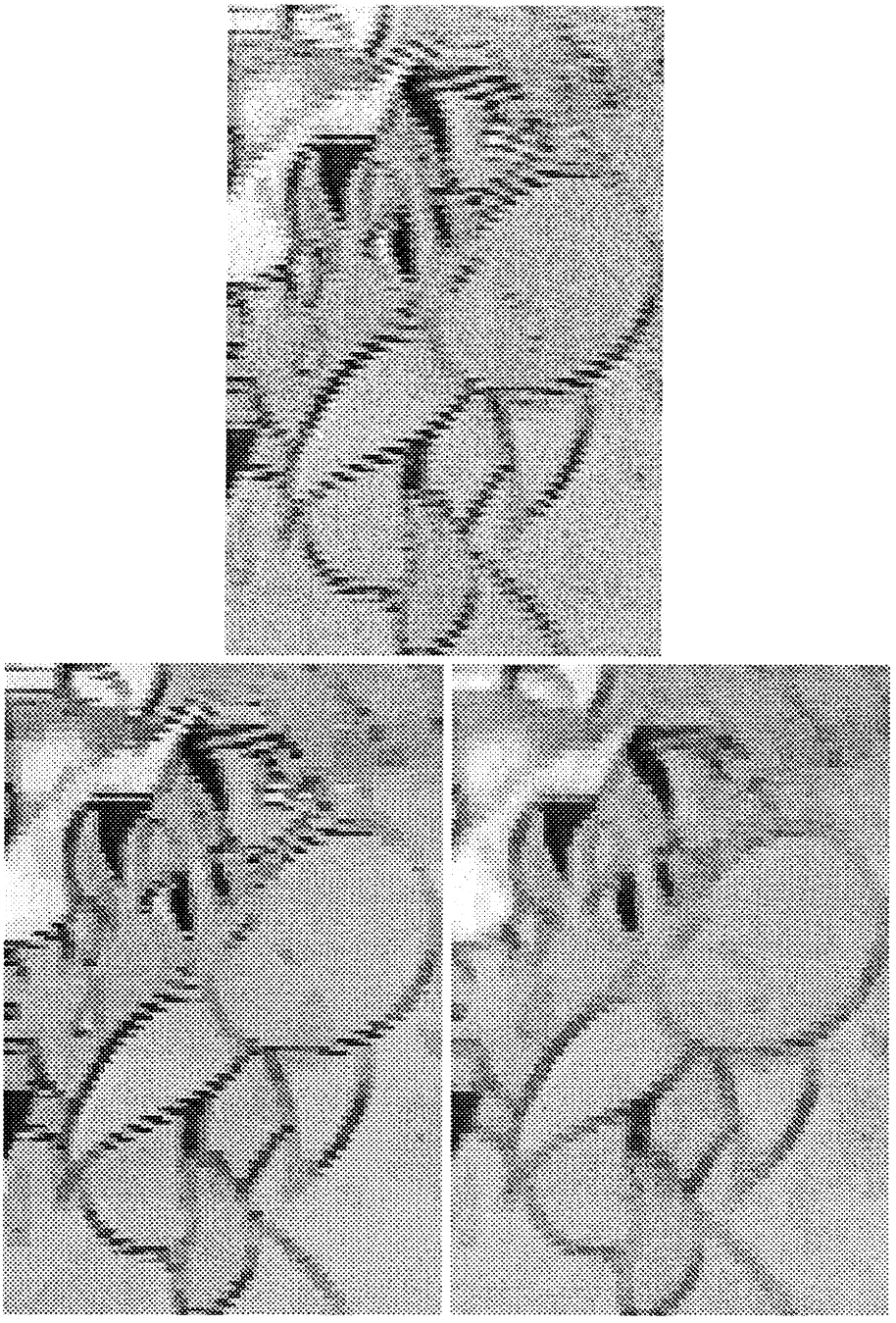
(a) (b)

Figure 2: (a) Noisy input image I^2_{noise} with $\sigma = 3.5$. SNR = 2.0. (b) Segmented output image using an annealed Hopfield neural network with $E_{bz} = D + S + P$.



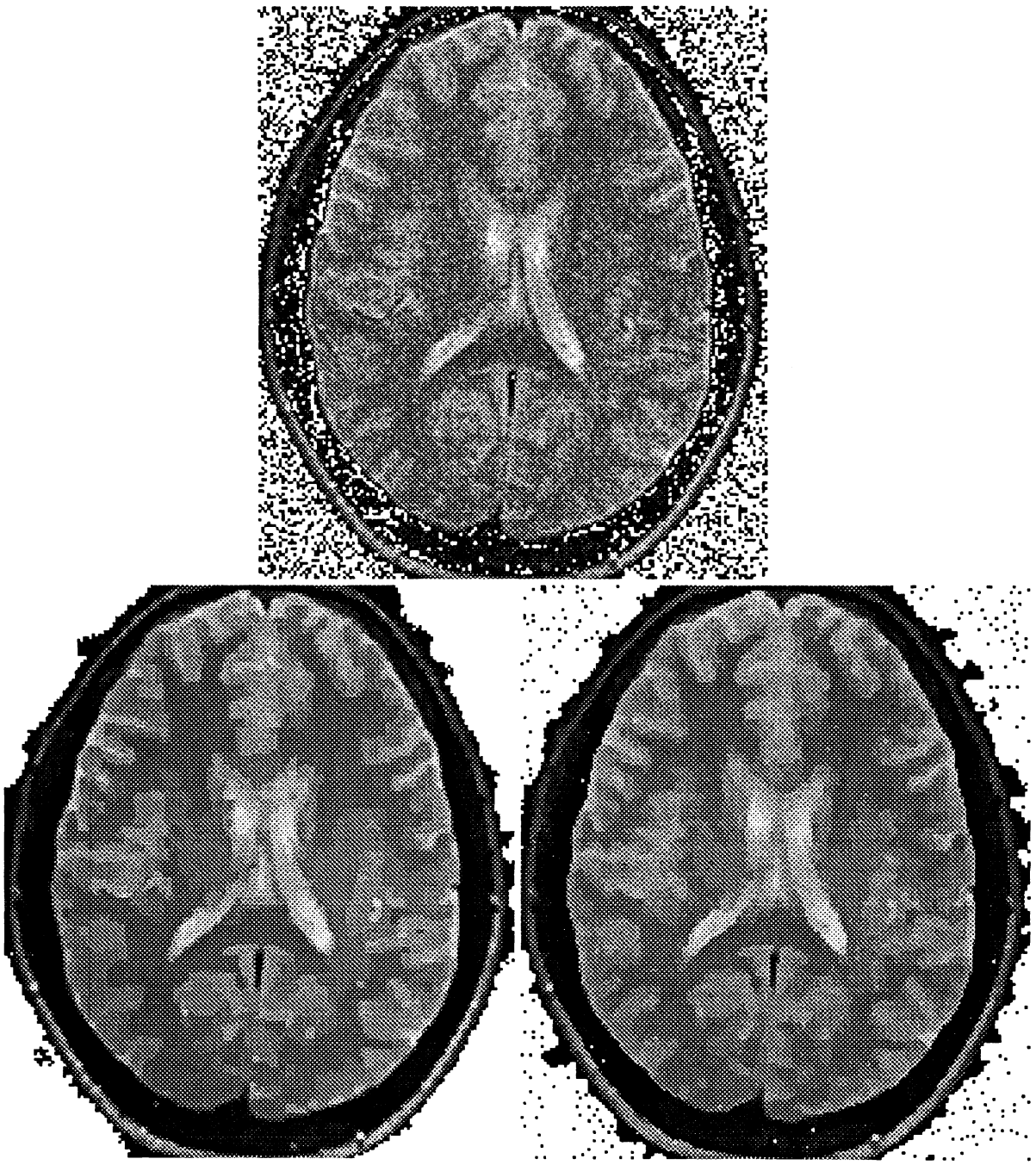
(a)	(b)
(c)	(d)

Figure 3: (a) Noise-free image I^2_{clean}. (b) Noisy input image I^2_{noise}. (c) Segmented output image using an annealed Hopfield neural network with $E_{bz} = D + S + P$. (d) Segmented output image using an annealed Hopfield neural network with $E_{ext} = D + S + P + N$. The standard deviation of the noisy input image I^2_{noise} is 3.5 and SNR = 2.0



(a)	
(b)	(c)

Figure 4: (a) Noisy input image I^{car}_{noise}. σ =7 has been added to the original cartoon image I^{car}_{org}. (b) Segmented output image using an annealed Hopfield neural network with $E_{ext} = D + S + P + N$. (c) Segmented output image using a general-purpose mean field annealing technique.



(a)	
(b)	(c)

Figure 5: (a) Noisy input image I_{noise}^{brain}. The standard deviation of the noise added is $\sigma = 10$. (b) Segmented output image using an annealed Hopfield neural network with $E_{ext} = D + S + P + N$. (c) Segmented output image using a general-purpose mean field annealing technique.