Production and financial linkages in inter-firm networks: structural variety, risk-sharing and resilience

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Production and financial linkages in inter-firm networks: structural variety, risk-sharing and resilience

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Abstract The paper analyzes how (production and financial) inter-firm networks can affect firms’ default probabilities and observed default rates. A simple theoretical model of shock transfer is built to investigate some stylized facts on how firm-idiosyncratic shocks are allocated in the network, and how this allocation changes firm default probabilities. The model shows that the network works as a perfect “risk-pooling” mechanism, when it is both strongly connected and symmetric. But the “risk-sharing” does not necessarily reduce default rates, unless the shock firms face is lower on average than their financial capacity. Conceived as cases of symmetric inter-firm networks,
industrial districts might have a comparative disadvantage in front of heavy crises.

**Keywords** Firm clusters · Industrial districts · Interlinking transactions · Resilience · Systemic risk

**JEL Classification** R11 · R12 · G20

1 Introduction

The world is now experiencing the economic tail of the sub-prime financial crisis. The idiosyncrasy of this crisis (e.g. Shiller 2008; Reinhart and Rogoff 2008, 2009) and the different resilience exhibited by countries (e.g. Frenkel and Rapetti 2009) have already been investigated. Quite interestingly, this investigation has led to a certain “revitalization” of some past interpretations of financial crises, which seem to fit the current one as well as more recent theories, although with important amendments: the Minsky approach to asset bubbles and crises is just an example (e.g. Dymski 2010; Arestis and Singh 2010; Eggertsson and Krugman 2011).

On the contrary, at the best of our knowledge, the resilience of the different models of production organization to the crisis has not yet received attention. Nonetheless, this is a further test for the alleged superiority of the “flexible specialization” model of production (e.g. Storper and Christopherson 1987; Hirst and Zeitlin 1989; Storper 1995; Herrigel 1996; Le Heron 2009), and of industrial districts in particular (e.g. Harrison 1992; Guerrieri et al. 2003). From an evolutionary perspective to urban and regional studies (Ter Wal and Boschma 2011), the current crisis brings to the front the selection mechanisms that macroeconomic fluctuations entail at the meso-level, depending on the features of local production systems.

In trying to fill this gap, the paper analyzes how inter-firm (production and financial) networks affect the firm’s resilience to financial shocks. In particular, it focuses on the risk of default entailed by firm-specific credit constraints and investigates: (i) how the allocation of risk depends on the structure of such networks; and (ii) how this allocation changes firms’ default probabilities.

The remainder of the paper is organized as follows. In Section 2, we review and combine the theoretical literature on network models of risk-sharing, contagion and financial stability, with the empirical literature on the different typologies of inter-firm networks. In Section 3, we build up a simple stylized model to analyze the transfer mechanisms of firm’s idiosyncratic financial shocks in inter-firm networks. Section 4 presents the main results of the model. Section 5 discusses their empirical implications and concludes.
2 Background literature and stylized facts

The scope and speed of diffusion of the recent financial crisis have stimulated the analysis of the conditions under which financial contagion can actually arise. This literature dates back at least to Diamond and Dybvig (1983), who refer to phenomena of “bank run” and self-fulfilling panic in the banking system. Drawing on them, Allen and Gale (2000) analyze how interbank lending brings about domino-like effects, which could increase the risk of collapse of the whole financial system, that is “systemic risk”. Iori et al. (2006), Nier et al. (2007), Gallegati et al. (2008) and Battiston et al. (2009) have recently re-examined this issue and found a possible trade-off between the mutual insurance of financial institutions and the systemic risk.

Out of the financial literature, the conditions for domino effects (here called “cascading failures”) to occur and produce “global cascades” have been studied by Watts (2002), Motter and Lai (2002) and Whitney (2009). Along the same research stream, diffusion and contagion in networks have been investigated by, among the others, Pastor-Satorras and Vespignani (2001, 2002), Dodds and Watts (2005) and López-Pintado (2008). Finally, recent economic studies have analyzed the efficient and stable configurations of “risk-sharing networks”, i.e. networks the links of which guarantee the nodes bilateral mutual insurance (Bramoullé and Kranton 2007a, b; Fafchamps and Gubert 2007; Bloch et al. 2008).

In spite of its consistency, this literature has not yet been applied to investigate the way in which inter-firm networks affect the resilience of firms to external shocks. This is unfortunate, as a number of empirical and theoretical studies have addressed inter-firm relationships in clusters and their actual structures. Taxonomies of them (e.g. constellations, hub-and-spokes, satellite platforms and different kinds of industrial districts) have been put forward by Markusen (1996), Paniccia (1998) and Carbonara (2002). Their evolutionary patterns have been studied in an industry life-cycle perspective by Carbonara et al. (2002) and Albino et al. (2006, 2007)—for supply chains in industrial districts—and by Ter Wal and Boschma (2011)—in terms of co-evolutionary processes of industries, firms and networks in clusters. Finally, the structure of ownership and non-ownership ties in industrial districts, their evolution over time and the presence of business groups in industrial districts have been investigated by Brioschi et al. (2002, 2004).

These inter-firm networks are mainly made up of production linkages between different typologies of firms (e.g. final producers vs. subcontractors) with heterogeneous capabilities.¹ These production linkages become

¹Firms look for networking also in other spheres, such as innovation. For an analysis of the networks of R&D collaborations see, for instance, Orsenigo et al. (2001), Goyal and Moraga-Gonzalez (2001) and Goyal and Joshi (2003).
extremely important in the aftermath of crises that expose firms to demand declines and credit restrictions from formal banking institutions.2

Indeed, inter-firm production relationships usually entail inter-firm credit relationships. On the one side, firms can obtain credit from subcontractors through payments delays, i.e. trade credit, which requires different contractual power between the parties (Peterson and Rajan 1997). On the other side, the supplier may obtain credit from the buyer on the basis of an underlying commercial transaction, possibly by discounting the refunding from the relative payment.

While the nature of trade-credit has been largely investigated, that of the latter deserves more attention. As Dei Ottati (1994) argues, this is a kind of credit which a final firm might want to “interlink” with the underlying subcontracting relationship with the supplier, in order to allow it to deliver what is required, according to certain technical specifications. Unlike trade-credit, which is somehow indirect, the “interlinking of credit and subcontracting” is actually a direct credit, as the client firm actually provides the supplier with financial resources, usually before the underlying production transaction occurs, and in order to make it occur should the provider face some financial difficulties. In a sense, the outcome of the subcontracting contract, to which the parties mutually commit, is the collateral of such a particular kind of credit.

On the one hand, because of its peculiar nature, this interlinking requires a minimum level of cooperation and mutual trust between firms. On the other, it helps reducing the emergence of opportunistic behaviors and thus raises the level of social capital in the socio-economic cluster. It is not by chance that this kind of relationship has been detected for the first time in the investigation of the industrial district of Prato (Florence, Italy) (Dei Ottati 1994). In general, it may be seen in those contexts where spatial proximity, face-to-face contacts, long-lasting relationships and in-depth social and cultural closeness play an important role (Cainelli 2008).

As is well-known, the Italian manufacturing system is an emblematic example of the coexistence of all these features. Accordingly, although to a certain extent idiosyncratic, it could be taken as a good empirical test to support the relevance of our theoretical arguments. On the one hand, most of the Italian manufacturing activities are concentrated within local systems of small and medium sized firms and industrial districts. In 2001, the 199 Italian industrial districts, identified according to the National Statistical Office’s (Istat)

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2Alessandrini et al. (2008, 2009) and Alessandrini and Zazzaro (2009) suggest that local banking systems, affecting information asymmetries between lenders and borrowers at the local level, can reduce firms’ financing constraints. As a matter of fact, physical proximity, involving long-lasting relationships and in-depth cultural affinity, allows local banks to collect a greater amount of “soft” information on local borrowers, thus increasing the quality of screening and monitoring. Nonetheless, since bank decision centers have been concentrated over the last decade in a few places, the “functional” distance between banks and local production systems has increased, thus counter-balancing the positive effects of local closeness. Their findings show that these negative effects prevail over the positive ones due to “operational” distance, making firms’ financial constraints actually more binding.
definition, accounted for about 38% of the total value added, 44% of the total employment and 46% of the total manufacturing exports of the country (ISTAT 2005). The relevance of these local production systems increases if their production specialization is considered. For example, the textile districts accounted for about 58% of the Italian manufacturing employment, while those operating in the footwear industry for 61% (Cainelli and Zoboli 2004). On the other hand, in Italian local production systems, subcontracting and trade credit are very pervasive, too. As for sub-contracting, in presenting the results of a survey conducted by the Bank of Italy on the Italian industrial districts, Omiccioli (2000) shows that 32.3% of the surveyed firms were actually sub-contractors, 43.5% of which were with respect to final producers as such, and the rest with respect to final firms which were in turn subcontractors. According to the same study, the incidence of sub-contracting firms is higher among small firms (about 35% of the total) and in those sectors in which Italian industrial districts are typically specialized, such as textiles (40.2%) and mechanics (40.2%). As far as trade credit is concerned, with reference to the same Bank of Italy study, Cocozza (2000) shows that about 27% of Italian manufacturing firms in the survey (both subcontractors and final producers) used trade credit as one of the main sources of external financing. Omiccioli (2000) supports these findings and qualifies them by noticing that final firms are generally larger than subcontractors and that, for the latter, the incidence of trade credit out of total sales is as much as 7.3%. Moreover, trade credit relationships are found to be particularly widespread among firms operating in such district-like sectors as textiles (9.2% of the total sales) and mechanics (8.1%).

Quite interestingly, the intertwining of these production and credit relationships, both empirically documented, turns out to be crucial in periods of financial crisis such as the current one. As is shown in a recent report by the Bank of Italy about the effects of the international crisis on the Italian economy (Bugamelli et al. 2009), first of all, final producers have been transferring to their subcontractors part of their non-diversifiable risk due to their credit constraints. In so doing, subcontracting relationships have allowed final firms to benefit from a greater production and financial flexibility, thus mitigating the effects of the crisis itself. On the other hand, a number of final producers have financed some of their suppliers through “factoring” operations, in order to allow them to continue in their production activities. As shown by the interviews of the report, the need to protect these long-lasting supplier relationships was the main reason for these strategic choices.

In general, the empirical evidence about firm reaction to the crisis is quite rich and complex.3 The underlying mechanisms are as usual difficult to disentangle, unless by simplifying the picture, an exercise to which the following model is dedicated.

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3 The 2009 Innobarometer survey (Kanerva and Hollanders 2009), although limited to innovation, is a significant example of this richness.
3 Model

Let us consider a network of \( n \) firms. Assume these firms are linked through production relationships only, in which one firm acts as supplier (\( S \)) of intermediate commodities or labor services for another final producer (\( F \)).\(^4\) On the basis of the arguments developed in Section 2, these production relationships could entail two possible credit relationships between \( S \) and \( F \): (i) trade credit, that is, the credit granted by \( S \) to \( F \) via payments delays, the extent of which depends on the relative contractual power of the parties; and (ii) the credit to the subcontractor, which is granted by \( F \) to \( S \) in an interlinking of subcontracting and credit, as a means to reduce opportunistic behaviors and sustain long-term relationships (Fig. 1a).

These credit channels are very important. They can act as possible transfer mechanisms, between \( S \) and \( F \), of the shocks which could hit them. In particular, trade credit can allow \( F \) to transfer part of its own shock to \( S \). The credit granted to the subcontractor, as well as the pre-existence of a credit relationship, can instead enable \( S \) to transfer part of its shock to \( F \) (Fig. 1b).

The shock which could hit the firms—in a way we will clarify below—is assumed to be exogenous. With respect to the issue at stake in this paper, it can be thought of as a credit shortage, originating from bank downturns and collapses—external to the model—which make \( F \) and \( S \) suffer from financial constraints in operating their businesses. However, providing it has repercussions on the financial conditions of \( S \) and \( F \), the shock could be of any kind, such as a macroeconomic or an industrial one.

The working of the transfer mechanisms crucially depends on the network structure. In order to study this effect, assume that each firm \( i \) of the \( n \) in the network is hit by an external shock \( x_{i0} \). We then represent the transfer mechanisms of these idiosyncratic shocks by a weighted directed network \( \Gamma \), where the valued directed edge from firm/node \( i \) to firm/node \( j \) (\( \delta_{ij} \)) measures the share of the total idiosyncratic shock of \( i \) that \( i \) can transfer to \( j \) (\( \sum_{j=1}^{n} \delta_{ij} = 1 \)) (Fig. 2).

\(^4\) Although at the price of a certain lack of realism, the model is kept in its simplest benchmark version, in order better to show its functioning and potentiality.
With this assumption, the shock experienced by firm $i$ after one round ($x_{i1}$) is simply equal to:

$$x_{i1} = \sum_j \delta_{ij} x_{j0}$$

and:

$$(x_{11} \ldots x_{n1}) = (x_{10} \ldots x_{n0}) \begin{pmatrix} \delta_{11} & \ldots & \delta_{1n} \\ \vdots & \ddots & \vdots \\ \delta_{n1} & \ldots & \delta_{nn} \end{pmatrix}$$

or, in matrix form:

$$x'_1 = x'_0 T$$  \hspace{1cm} (1)$$

where $T$ is the adjacency matrix of the network $\Gamma$, and $x'_1$ the row vector of firm-specific shocks after one round of interaction.

It follows that:

$$x'_t = x'_{t-1} T = x'_0 T^t$$  \hspace{1cm} (2)$$

where $x_t$ is the allocation vector after $t$ rounds of interactions, assuming that the network $\Gamma$ stays constant throughout the process.

If the process converges in the limit, so that, by further increasing the rounds of interactions, the allocation vector does not change:

$$\hat{x}' = x'_0 \left( \lim_{t \to \infty} T^t \right) = x'_0 \left( \lim_{t \to \infty} T'^t \right) T$$  \hspace{1cm} (3)$$

we can retain such vector $\hat{x}$ as the vector of the equilibrium allocation of the idiosyncratic shocks. It is a function of the initial allocation $x_0$, given the adjacency matrix $T$: $\hat{x}(x_0; T)$.
Just to give an example, in a simple transfer network made up of three firms—1, 2, 3—with the following structure (Fig. 3a):

\[
T = \begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}
\]

we have:

\[
T^{25} \approx T^{26} = \ldots = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{pmatrix}
\]

So the system soon converges to the equilibrium and thus:

\[
\hat{x}' = x_0' \left( \lim_{t \to \infty} T^t \right) = (x_{10}, x_{20}, x_{30}) \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & \frac{1}{6} \\
\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{pmatrix} = \left( \frac{1}{2}, \frac{1}{3}, \frac{1}{6} \right) \sum_{i=1}^{3} x_{i0}.
\]

As for the initial exogenous shock \(x_0\), we model it as a random vector, which generic element \(x_{i0}\) is made up of a common trend (\(\mu\)) and an idiosyncratic random component (\(\epsilon_i\)):

\[
x_{i0} = \mu + \epsilon_i
\]

\(\text{5This idiosyncratic component can capture the individual differences in the experienced shock or in the buffer level of the internal absorption of the shock.}\)
Moreover, each node/firm $i$ is characterized by a given threshold $\theta_i$, which represents its resistance to external shocks.$^6$

Assuming that $\hat{x}$ exists and is in fact unique, the default condition for firm $i$ can be stated as follows:

$$\hat{x}_i(x_0; T) > \theta_i.$$  (5)

The reference to the equilibrium allocation of the shocks in the default condition greatly simplifies the analysis. However, the higher (lower) the speed of convergence of the system to the equilibrium, the more (less) reasonable is such assumption. We address this issue in Section 4.2.3.

### 4 Results

Of the simple model above, we first search for the limit distribution of the idiosyncratic shocks in the network. Provided that it exists and is indeed unique, we then investigate its impact on firms’ default probabilities—that is, the probability that the shock overcomes the firm’s financial capacity (i.e. threshold)—and on expected default rates—i.e. the number of defaulted firms over the total number of firms in the network—induced by the network of shock transmissions formalized by $T$.

#### 4.1 Shock transfer and risk distribution

As far as the analysis of the limit distribution of the idiosyncratic shocks in the network is concerned, it is important to note that, in spite of the fact that, for what concerns the shock transfer, our model is not probabilistic, $T$ is formally a right stochastic matrix. Hence, in order to study the allocation of shocks in equilibrium, we can use a number of useful results from the theory of finite Markov chains.$^7$

First of all, following the standard definitions, we define the network $\Gamma$ (and the related matrix $T$) as strongly connected if each node can reach every other by a directed path, i.e. a sequence of distinct nodes $i_1, i_2, \ldots, i_K$ such that $T_{i_k, i_{k+1}} > 0$, for each $k \in \{1, 2, \ldots, K\}$.

---

$^6$Such parameter can be conceived as a resistance threshold to unexpected losses. As such, it is not simply a threshold to the loss distribution. If the shock was somehow expected or if the firm was usually operating in a high volatile environment, the firm would tend to accumulate resources to better resist to the possible losses.

$^7$For a textbook treatment of Markov chains, see Karlin and Taylor (1975, 1981) and the references therein. Iterated matrices have been used also in studies on the convergence of beliefs in networks (DeGroot 1974; DeMarzo et al. 2003; Golub and Jackson 2010), prestige and status (Bonacich 1987), and in strategic games for networks with neighbors’ influence (Ballester et al. 2006).
Let us say that the network $\Gamma$ (and the related matrix $T$) is aperiodic if the greatest common divisor of the lengths of its directed cycles is 1, where a directed cycle is a directed path joining a node to itself, and the length of the cycle is the number of distinct nodes in the path.\footnote{Strictly speaking, a cycle is not a path because the starting (and ending) node appears twice. However, apart from this minor inconsistency, the definition is correct and is made here for convenience.}

We can therefore state our first proposition.

**Proposition 1** If the inter-firm network $\Gamma$ is strongly connected and aperiodic, the system always reaches an equilibrium in which each firm bears a definite proportion $(s_i)$ of the sum of all the idiosyncratic shocks $(\sum_i x_{i0})$:

\[
\hat{x}' = s' \left( \sum_i x_{i0} \right)
\]

where $s'$ is the left eigenvector of $T$ corresponding to eigenvalue 1 the entries of which sum to 1: $s'(I - T) = 0, \sum_i s_i = 1$.

Let us note that the condition of aperiodicity for the network is rather weak, and can be assumed as almost always satisfied in the present framework. Indeed, a sufficient condition for the network to be aperiodic is that there is at least one loop ($\delta_{ii} > 0$ for some $i$), that is, at least one firm which is not able to transfer all the experienced shock in each round.

A formal proof of the proposition is provided in the Appendix. Here we instead discuss the simple three-firm example in Section 3 (Fig. 3a). In that example, although firms 2 and 3 are able to transfer all the one-round shock to the others, the system soon converges to the equilibrium and such equilibrium entails the following redistribution of the sum of all the shocks: $(1, 2, 1, 6)$. Thus, when firms are hit by the same initial shock, the system of relations is detrimental (beneficial) for firm 1 (3).\footnote{So, for instance, if $x_0' = (100, 100, 100)$, we have $\hat{x}_0' = (150, 100, 50)$.}

As we will say in the discussion, this is what could be expected when a generic cluster of firms—that is, a cluster in which all the firms are either directly or indirectly connected among them, but with no need of reciprocity—is hit by a shock. Its distribution in the network has, for the firms that constitute it, ambiguous effects. In other words, in absence of qualified forms of network relationships—such as, for example, those occurring in an industrial district—being part of a network will not necessarily reduce the severity of the shock the firms face, although that might be possible. In the light of this result, the expectations about the destiny of small and medium enterprises based in non-firmly embedded networks in front of the crisis are at most misty.

Therefore, in general, the proportion of the sum of all the shocks that, in equilibrium, accrues to each firm is not the same. On the contrary, given a
symmetric network, i.e. a network whose adjacency matrix is symmetric ($\delta_{ij} = \delta_{ji}$ for each $i, j$), the following holds:

**Proposition 2** If the network is strongly connected, aperiodic and symmetric, the risk distribution is egalitarian, i.e. each firm gets in equilibrium a common shock amounting to an average of all the shocks:

$$\hat{x}_i = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}, \quad \forall i \in N.$$ 

A formal proof of the statement is given in the Appendix. Here we just note that, apart from the strong connectivity and aperiodicity, the condition for this “egalitarian” distribution to occur is not that all the linkages are equal, but only that they are perfectly reciprocated, as the example of Fig. 4 illustrates. In this simple three-firm network, the equilibrium values are: $s_1 = s_2 = s_3 = \frac{1}{3}$, in spite of the fact that the structure of relations of the three firms is strongly different.¹⁰

As we better argue in Section 5, the symmetry condition could be retained proper of a network configuration toward which industrial districts would tend “asymptotically”. It can be understood by thinking that the higher the level of social capital in a local production system, that is, the more the system resembles an industrial district—in the characterization given of it by the famous Becattini’s tradition (Pyke et al. 1990)—the more the interlinking of credit and subcontracting is used to re-balance the contractual power of the parties in the supplier-user relationships (Dei Ottati 1994), the more the correspondent network turns out to be symmetric.

In general, when the symmetry condition does not hold, the risk distribution is not egalitarian and a different portion of the total shock accrues to firms ($s_i \neq s_j$). Still, industrial districts are such that, being part of them tends to align firms as far as the supported shock is concerned. In other words, in industrial districts, the shock distribution is more “democratic” than in other networks, even when it is not perfectly egalitarian (such as in a symmetric network). As we will see, this is not necessarily a good thing for district firms.

What is important is that the portion of the supported shock depends on the overall structure of relations. An example is provided in Fig. 3b, where, despite the symmetry in the reciprocal relationship, 1 and 2 get different parts of the total shock because of the differences in their relation with 3: $(s_1, s_2, s_3) = (.3, .6, .1)$.

¹⁰Indeed, at a first glance, firm 1 might look in a better position than 3, because it is able to transfer a much greater portion of its initial shock to the others (0.9 against 0.1 of firm 3).
The final network structure that we consider is that of a network which is not strongly connected, where one or more nodes/firms may have zero outdegree or indegree: in economic terms, firms with no forward or backward linkages, respectively.

It is self-evident that nodes with zero indegree and a positive outdegree turn out to be shock releasers: firms which, in the limit, are always able to transfer their shock to the others completely. This is the case shown in Fig. 5a, where firm 1 transfers all of its shock to the others. As we will say in the discussion, this might be thought as the case of strongly hierarchical networks, where the suppliers do not have any contractual power (in the limit position) over the client firm, which dominates them.

On the other hand, nodes with zero outdegree and a positive indegree turn out to be shock absorbers. In other words, these are firms that sustain the shock of the others without being able to transfer theirs. This is the case of firms 2, 3 and 4 in the previous example, as well as of firm 3 in Fig. 5b.

4.2 Risk distribution and default probabilities

By affecting the actual allocation of idiosyncratic shocks, inter-firm networks can change firms’ default probabilities and, via this, observed default rates.

In this section, we investigate only the two most paradigmatic cases, namely, (i) the default probability of a supplier in a subcontracting relation without interlinking credit (Fig. 6); and (ii) the default rate of firms in a symmetric and strongly connected network. These cases are quite extreme, and this

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11 In the example, each supplier (firms 2–4) faces its idiosyncratic shock plus a fraction (1/3) of the buyer’s shock.
necessarily lead to a loss of generality. They can, nonetheless, deliver clear-cut results and, therefore, be a useful starting point in the analysis of more complex situations. To this end, we also provide some insights for the case of strongly and asymmetric networks.

The section ends with a discussion on the speed of convergence, which is of utmost importance. In fact, given our analysis of default probabilities at the equilibrium, the lower the speed at which the system reaches the equilibrium, the more unrealistic is our operational suggestion to analyze default probabilities at the equilibrium allocation.

4.2.1 Default probabilities of suppliers in supplier-buyer relations

Given the idiosyncratic shock:

\[ x_i = \mu + \epsilon_i \]

and assuming that \( \epsilon_i \sim (0, \sigma^2) \), the shock faced by the supplier is:

\[ \hat{x}_S = x_{S0} + \delta_{FS} x_{F0} \sim ((1 + \delta_{FS}) \mu, (1 + \delta^2_{FS}) \sigma^2). \]

Thus, the shock experienced by the supplier (\( \hat{x}_S \)) is higher on average and it is also more volatile than its initial shock (\( x_{S0} \)). Clearly, this increases its default probability with respect to the one of an isolated firm facing the shock \( x_i \) (Fig. 7).

The economic correspondent of this result is pretty intuitive. The suppliers of a hierarchical network, with a pivotal client firms—in the following, we will refer as an example to the case of the Fiat automobile value chain—suffer twice as much the consequence of the shock than if they were isolated, not
only because of the higher scale of the shock, but also because of its lower predictability.

4.2.2 Default rates in strongly connected networks

In case of firms in a symmetric and strongly connected network, from Proposition 2 it follows that $\hat{x}_i = \hat{x}_j = \bar{x}_0$, for each $i, j$. Still assuming that $\epsilon_i \sim (0, \sigma^2)$, this implies:12

$$\hat{x}_i = \bar{x}_0 \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Thus, given that each firm faces in equilibrium an average of all the shocks, its volatility is reduced. This leads us to state two further propositions.

**Proposition 3** Assume that the idiosyncratic components of the shocks are independently distributed and the number of firms in the network sufficiently large. Then, the default probability of a firm in a symmetric and connected network is higher (lower) than that of the same firm in isolation when the expected value of the random shock is higher (lower) than its threshold.

A formal proof of this proposition is in the Appendix. In intuitive terms, Fig. 8 shows the case of a normally distributed shock for a firm $i$ with a certain

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12Let us note that the assumption of an equal distribution of $\epsilon$ is not strictly needed for any of the results and even the assumption of equality in variance can be relaxed. Indeed, by using the Lindeberg-Lévy central limit theorem, one can show that, if $\epsilon_i \sim (0, \sigma^2_i)$, then:

$$\hat{x}_i = \bar{x}_0 \sim N\left(\mu, \frac{\sigma^2_i}{n}\right)$$

with $\sigma^2 = \sum_{i=1}^n \sigma_i^2 / n$ as long as the Lindeberg condition holds, that is, $\sigma^2$ is not dominated by any single term.
threshold $\theta_i$. The shadow area under the curves measure the probability that the shock is lower than the firms’ threshold, that is, the probability of survival for the firm. As clearly emerges from the Figure, this area for $x_i$—the random shock faced by the firm in isolation—is higher (lower) than the corresponding area for $\hat{x}_i$—the random shock faced by the firm in the network—when the average of the shock ($\mu$) is lower (higher) than the firm’s threshold ($\theta_i$).

As we will emphasize in the discussion, this is one of the most interesting results of the paper. A district firm, or a firm in a district-like environment, is not necessarily safer than an isolated one. It depends on the severity of the (financial) crisis firms face. Should we able to conclude that the current crisis is indeed a very large one, with respect to others, firms in industrial districts are expected to have a comparative disadvantage.

Moving from the individual firm to the network of firms, if the threshold is heterogeneous across firms, but drawn from a common distribution, the following holds:

**Proposition 4** Assume the idiosyncratic components of the shocks are independently distributed, and so are the threshold values ($\theta_i$), and in addition the number of firms sufficiently large. Then, the average default rate in a symmetric and connected network is higher (lower) than the one for isolated firms if $\Pr(\theta_i < \mu) > \Pr(\theta_i - x_{i0} < 0)$ ($\Pr(\theta_i < \mu) < \Pr(\theta_i - x_{i0} < 0)$).

For the simpler case of a homogeneous threshold, the previous proposition reduces to the following one:

**Proposition 5** Assume the idiosyncratic components of the shocks are independently distributed, the threshold value is homogeneous across firms, and the number of firms sufficiently large. Then, the average default rate in a symmetric and connected network is higher (lower) than the one for isolated firms if the expected value of the random shock is higher (lower) than the common threshold.

Finally, let us consider the case of strongly connected asymmetric networks, in which the allocation of the total shock is not equal across firms. As it depends on the overall structure of bilateral relations, no easy generalization can be made. In general, if $s_i$ is the fraction of the total shock accruing to firm $i$,

---

13In reality, there could be a relation between the firm size, the value of $\theta$ and the network structure. So, for instance, in strongly hierarchical networks, more central firms are usually bigger and therefore likely associated with higher thresholds. However, the assumption of a homogeneous threshold across firms seems to be less unrealistic in the case of strongly connected-(nearly) symmetric networks, because such inter-firms networks are usually Marshallian districts where differences in size tend to be small.
i will face in equilibrium a shock which is asymptotically normally distributed (by the central limit theorem) with expected value:

\[
E[\hat{x}_i] = \mathbb{E}
\left[
\sum_i s_i x_i^0 \right] = s_i \mu
\]

and variance:

\[
\text{Var}[\hat{x}_i] = \text{Var}
\left[
\sum_i s_i x_i^0 \right] = s_i^2 n \sigma^2
\]

Hence, if \( s_i < 1/n \), the firm in the network have to face a shock which is less volatile and lower on average than the one it faced in isolation. If instead \( 1/n < s_i < 1/\sqrt{n} \), the firm’s shock in the network is higher on average but still less volatile. Finally, when \( 1/\sqrt{n} < s_i \leq 1 \), the firm’s shock is both higher on average and more volatile.

In any case, asymptotically, the following proposition holds (proved in the Appendix):

**Proposition 6** Assume the idiosyncratic components of the shocks are independently distributed and the number of firms in the network sufficiently large. Then, the default probability of a firm in a strongly connected asymmetric network is higher (lower) than that of the same firm in isolation if the expected value of the random shock is higher (lower) than \( \theta_i \).

In the case of a heterarchical network, therefore, but without the symmetric ties of an industrial districts, the results in terms of default crucially depend on the size of the shock. In particular, we need to take into account the share of the overall shock the firm gets in equilibrium and whether this share actually makes the expected value of the shock exceed the firm’s threshold.

At the level of the overall network, what matters is the correlation between the equilibrium shares, as determined by the network, and the firms’ thresholds. So, for instance, if the firms that in equilibrium get the higher shares of the overall shocks are those with the lowest thresholds, the default rate exhibited by these networks can be relatively high in the case of low shocks, but comparatively lower in case of strong common shocks. Indeed, the system of relations makes the weakest firms, which would have died anyway, take a larger share of the total shock.

**4.2.3 Default analysis and speed of convergence**

Our default analysis strongly relies on the operational device to work out default probabilities at the limiting distribution of shocks. In fact, the slower the rate at which the system converges to the equilibrium, the less realistic is our assumption.

Hence, understanding the relationship between the structure of the firms’ network and the speed of convergence is crucial. In formal terms, the question
amounts to calculate how long it does take the Markov matrix $T$ to approach its limit.

This is a well known issue, on which there is in fact a large literature. As reported by Golub and Jackson (2010), the convergence time is proportional to $1/\log(|\lambda_2(T)|)$, where $\lambda_2(T)$ stands for the second largest eigenvalue of $T$. Therefore, a second eigenvalue close to 1 implies a very low speed of convergence.

As for the relationship between this mathematical condition and its insights for our model, a useful perspective is the one provided by the approach based on measuring bottlenecks (Diaconis and Stroock 1991). The basic idea is that if there are pieces of the network connected only by narrow linkages, the convergence is slow.

5 Discussion and final remarks

The results of the model suggest a number of interesting interpretations, related to the background literature and stylized facts we reviewed.

First of all, the network capacity of working as a “system” in financial terms—in which individual firms exchange their idiosyncratic shock for a certain portion of the total shock of the network—crucially depends on the structure of the network itself. In particular, the strong connectivity of the network is crucial. Should some or even only one of the firms be “isolated” from the twofold transfer mechanism we have described, the network would lose its system properties.

This can be considered in the case of clusters, in which firms are linked through subcontracting relationships but with little socio-economic embeddedness. In these chains of “atomistic” producer-user relationships, the client firms exploit their larger market power to transfer, via trade credit, their risk to the subcontractors themselves, which thus get subject to financial default exclusively and/or earlier than the former.

This result can be used to interpret what is happening, for example, in the supply-chain network of Fiat automobile in Italy (Abatecola 2009). Here, the small subcontractors of components are actually providing the producer with a remarkable margin of flexibility both in production and financial terms. The FIAT contractual power and the absence of a district-like environment for the supply-chain are crucial for this to occur.

While strongly connected networks are able to work as financial systems, on the other hand, their capacity to translate the idiosyncratic risk of each firm into an average of the risk of all the firms in the cluster is not guaranteed. In order to have such “egalitarian” risk-pooling, the inter-firm bilateral relationships need to be perfectly reciprocated. With benefit of hindsight, we could say that the district atmosphere the network in which it is embedded must be such as, to compensate exactly, or tend to compensate, the asymmetries in contractual powers which emerge from user-supplier differences in size. Quite interestingly, this result is consistent with the trade-off local studies
find between contractual opportunistic behaviors, on the one side, and social capital, on the other. For example, this has been shown to be the case of the footwear district of San Mauro Pascoli in the Italian region of Emilia-Romagna (Brioschi et al. 2004).

Interesting implications can be drawn also in terms of default probability, that is, of the actual capacity of the network firms to bear a financial risk such as the current one. In the case of non strongly connected networks, when “isolated firms” are present, those firms which act as pure absorbers have been shown to be two times in trouble: not only because they end up receiving a shock larger on average than the one would have accrued in isolation, but also because such a shock encapsulates the variability of that faced by the other firms.

Definitely more interesting is the result for the district network, where trade credit and interlinking of credit and subcontracting coexist. District firms have been shown to be more resilient than isolated ones only under two important conditions: in the case of symmetric relationships, and providing the average shock is lower than the threshold of the firm itself. Conversely, belonging to the district could even increase the default probability of the firm.

This is possibly the most important result of the model. Indeed, it seems to show that the industrial district model, while enabling firms to share the risk of a moderate shock, and to be actually more resilient in “normal” conditions, does not help and is actually disadvantageous in front of “heavy” financial crisis (such as possibly the current one).14 Quite interestingly, this result is invariant with respect to the actual structure of the relationships in the district: “canonically” or not Paniccia (1998) does not make any difference for its financial behavior. Indeed, recent data seem to show that the crisis have had a major impact on more traditional districts, such as those in textiles and footwear, no matter their actual structure of the network. See, for instance, the recent report on the textiles district of Carpi (R&I2011).

If, in strongly connected networks, the twofold credit relationships we have envisaged are asymmetric, the implications of the model becomes more blurred, as they depend on the ratio between the share of the overall shock firms get (in equilibrium) and the firms’ threshold. Still, the insight is that, in this case, the networked firms actually split into two groups: the “winners”, so to say, which are able to transfer to the others part of both the average and the variance of their shock, and the “losers”, whose default rate increases both because of a higher and a more variable shock. This is another interesting result, which recovers the relevance of the structure of local production systems in evaluating their resilience to the crisis: indeed, such a structure turns

14 Under a different perspective, the same result points to the production specialization of the districts, making more (less) fragile those which are specialized in sectors more (less) exposed to international competition: the different destiny of the ceramic tales district of Sassuolo and of the mechanical one of Bologna in Italy, for example, can also be read in this terms.
out to be more important than the bilateral relationships on which local studies usually focus.

The results of this paper contribute to the evolutionary analysis of the dynamics of local production systems and of industrial districts at least in two respects. First of all, we extend to financial linkages the array of factors which intervene in the adjustment processes that clusters of firms experience in front of external shocks (Boschma and Lambooy 2002). As we said, rather than on the peculiarities of local banks and local credit, which have already received attention in urban and regional studies (e.g. Ughetto 2009; Alessandrini and Zazzaro 2009), these financial linkages depend on the network structure and on the mutual coordination mechanisms (in particular, on the “interlinking of subcontracting and credit”) that social capital and institutions allow local firms to undertake. The second contribution concerns the extension of the manifold co-evolutionary processes through which regional dynamics go. Indeed, in addition to a life-cycle perspective—which has also received a certain attention (e.g. Neffke et al. 2011; Albino et al. 2007)—the local interlinking of industrial dynamics, evolution of firm capabilities and industry-wide knowledge (Ter Wal and Boschma 2011) should also consider a business-cycle perspective, in which the “shocks” we have addressed in this paper are of crucial importance.

We think the main value added of the paper is that the stylized model we propose is analytically tractable and can deliver very striking predictions. However, as usual, such tractability comes at a price. In particular, we make the implicit assumption that the dynamics of propagation of the shocks do not significantly differ. In fact, the shock transfer mechanisms implied by the different credit channels linking together suppliers and final producers can exhibit very different dynamics. Moreover, we assume that the structure of the network stays constant all along the process. In fact, the network structure will probably change along the process of propagation as the result of the firm’s strategies aimed at minimizing the default probabilities. In order to analyze these issues, one probably needs to give up the analytical tractability and build an agent-based model relying on simulations. This is the next step, and we think that the results of this paper can prove useful in such a step.

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15 In this respect, it seems plausible that the slower the propagation, the higher the probability that such changes occur.
Appendix

Proof of Proposition 1 When $T$ is strongly connected, it is a standard result of the theory of Markov chains that aperiodicity is a necessary and sufficient condition for $T$ to be convergent (e.g. Kemeny and Snell 1960). Moreover, when this happens, $T$ is also primitive, i.e. $T^t$ has only strictly positive entries for some $t \geq 1$ (e.g. Perkins 1961), and there is a unique (up to scale) left eigenvector $s$ of $T$, corresponding to the unit eigenvalue, such that for any $v$:

$$\lim_{t \to \infty} T^t v = s' v. $$

Since $T$ is convergent, $S \equiv \lim_{t \to \infty} T^t$ exists and hence:

$$ST = \lim_{t \to \infty} T^t T = \lim_{t \to \infty} T^t = S$$

where each row of $S$ is equal to $s'$.

It follows that:

$$\hat{x}' = x'_0 \left( \lim_{t \to \infty} T^t \right) = x'_0 S = x'_0 \begin{pmatrix} s' \\ \vdots \\ s' \end{pmatrix} = s' \left( \sum_i x_{i0} \right).$$

\[\square\]

Proof of Proposition 2 A symmetric network implies $T = T'$ and therefore:

$$S' = \left( \lim_{t \to \infty} T' \right)' = \lim_{t \to \infty} T' = S$$

i.e. $S$ must be symmetric too ($s_{ij} = s_{ji}$). As in $S$ by definition $s_{ji} = s_{ii}$, the symmetry implies $s_{ii} = s_{ij}$.

Moreover, since the sum by column of each row is one, it follows that:

$$\sum_{j=1}^{n} s_{ij} = n s_{ii} = 1$$

for each $i$ and all the elements of $S$ are equal to $1/n$. Hence:

$$\hat{x}_i = x'_0 \begin{pmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{pmatrix} = \frac{1}{n} \sum_i x_{i0} = \bar{x}$$

for each $i \in N$. \[\square\]

Proof of Proposition 3 Given that, in a connected-symmetric network $\hat{x}_i = \bar{x}_0$ and this variable is asymptotically normally distributed with variance $\sigma^2/n$ and mean $\mu$, when $n$ gets larger it converges in probability toward $\mu$. Therefore, we have:

$$\lim_{n \to \infty} \Pr(\hat{x}_i > \theta_i) = \begin{cases} 1 & \text{if } \theta_i > \mu \\ 0 & \text{if } \theta_i < \mu. \end{cases}$$
Production and financial linkages in inter-firm networks

By contrast, since $\sigma^2 > 0$, there is always a $\epsilon > 0$ such that $\epsilon < \Pr(x_{i0} > \theta_i) < 1 - \epsilon$ and this probability is so strictly bound between 0 and 1.

**Proof of Proposition 4** Assuming that $\theta_i$ are identically and independently distributed, and so are $x_{i0}$, the number of firm defaults follows a binomial distribution with expected value $n \Pr(\theta_i - x_{i0} < 0)$. The expected value of the default rate is, therefore, simply $\Pr(\theta_i - x_{i0} < 0)$.

For firms in a symmetric-connected network, the expected value of the binomial is instead: $n \Pr(\theta_i - \bar{x} < 0)$, with an expected default rate equals to $\Pr(\theta_i - \bar{x} < 0)$.

Given that $\bar{x} \xrightarrow{P} \mu$ we have that:

$$\lim_{n \to \infty} \Pr(\theta_i - \bar{x} < 0) = \Pr(\theta_i < \mu)$$

Hence, the expected default rate of firms when the number of firms gets large is higher (lower) in isolation than in a symmetric-connected network if $\Pr(\theta_i - x_{i0} < 0) > \Pr(\theta_i < \mu)$ ($\Pr(\theta_i - x_{i0} < 0) < \Pr(\theta_i < \mu)$).

**Proof of Proposition 5** For a common threshold ($\theta_i = \theta$), we have:

$$\lim_{n \to \infty} \Pr(\bar{x} < \theta) = \begin{cases} 1 & \text{if } \theta > \mu \\ 0 & \text{if } \theta < \mu \end{cases}$$

while $\Pr(x_{i0} < \theta)$ remains strictly bound between 0 and 1.

**Proof of Proposition 6** The probability of default in a strongly connected asymmetric network for firm $i$ is:

$$\Pr(\hat{x}_i > \theta_i) = \Pr\left(\sum_{i} x_{i0} > \theta_i\right) = \Pr\left(\frac{\sum_{i} x_{i0}}{s_i n} > \theta_i\right) = \Pr\left(\bar{x} > \frac{\theta_i}{s_i n}\right)$$

$\bar{x}$ converges in probability toward $\mu$, therefore we have that $\Pr(\hat{x}_i > \theta_i)$ tends to 1 if $\bar{x} > \frac{\theta_i}{s_i n}$ and 0 if instead $\bar{x} < \frac{\theta_i}{s_i n}$.

By contrast, since $\sigma^2 > 0$, there is always a $\epsilon > 0$ such that $\epsilon < \Pr(x_{i0} > \theta_i) < 1 - \epsilon$ and the probability in this case is strictly bound between 0 and 1.

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