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# An Improvement in Centroid Point Method for Ranking of Fuzzy Numbers

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**Abstract**

**Introduction:** In many applications, ranking of fuzzy numbers is an important component of the decision process. Many authors have investigated the use of fuzzy sets in ranking alternatives and they have studied different methods of raking fuzzy sets. Particularly, the ranking of fuzzy numbers. In a paper by Cheng [A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95 (1998) 307-317], a centroid-based distance method was suggested for ranking fuzzy numbers, both normal and non-normal. The method utilizes the Euclidean distances from the origin to the centroid point of each fuzzy numbers to compare and rank the fuzzy numbers. It is found that the mentioned method could not rank fuzzy numbers correctly. For example, it cannot rank fuzzy numbers when they have the same centroid point. Some other researches such as Chu and Tsao's , Wang and Lee and Deng et al. tried to overcome the shortcoming of the inconsistency of Cheng's method but their methods still have drawback.

**Aim:** In this paper, we want to indicate these problems of Cheng's distance, Chu and Tsao's area and Wang and Lee's revised method and then we will propose an improvement method, which can avoid these problems for ranking fuzzy numbers.

**Materials and Methods:** In point of our view, every fuzzy number may lead to zero, positive or negative real number. To overcome with this subject, we first introduce the sign function. By connecting the sign function, corrected centroid point formulas and Cheng's distance method, the improvement method will be presented.

**Results:** The proposed method includes all situations of fuzzy numbers to rank them correctly. Therefore, it improves the distance method of Cheng as well.

**Conclusion:** In order to overcomes the shortcoming in Cheng's distance method, Chu and Tsao's formulae and new revised method by Wang and Lee, a sign function was introduced and composed an improvement strategy of Cheng's distance. In this work those method are considered, which utilized the centroid points. The improved method can effectively rank various fuzzy numbers and their images. Thus, the method is superior to Cheng' distance, Chu and Tsao's area and Wang and Lee's revised method.

***Keywords***: Ranking, Normal fuzzy numbers, Non-normal fuzzy numbers, Centroid point

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## Introduction

In many applications, ranking of fuzzy numbers is an important component of the decision process. Many authors have investigated the use of fuzzy sets in ranking alternatives and they have studied different methods of raking fuzzy sets. Particularly, the ranking of fuzzy numbers.[1-5] In a paper by Cheng,[6] a centroid-based distance method was suggested for ranking fuzzy numbers. The method utilizes the Euclidean distances from the origin to the centroid point of each fuzzy numbers to compare and rank the fuzzy numbers. Chu and Tsao[7] found that the distance method could not rank fuzzy numbers correctly if they are negative and therefore, suggested using the area between centroid point and the origin to rank fuzzy numbers. Deng et al. [8] utilized the centroid point of a fuzzy number and presented a new area method to rank fuzzy numbers with the radius of gyration (ROG) points to overcome the drawback of the Cheng's distance method and Tsao's area method when some fuzzy numbers have the same centroid point. However, ROG method cannot rank negative fuzzy numbers. Abbasbandy and Asady[4] found that Tsao's area method could sometimes lead to counterintuitive ranking and hence suggested a sign distance. More recently, Wang et al.[9] pointed out that the centroid point formulas for fuzzy numbers provided by Cheng [6] are incorrect and have led to some misapplication such as by Chu and Tsao,[6] Pan and Yeh [10, 11] and Deng et al..[8] They presented the correct centroid formulae for fuzzy numbers and justified them from the viewpoint of analytical geometry. Nevertheless, the main problem, about ranking fuzzy numbers by above methods, which used the centroid point, is reminded. In 2008 Wang and Lee [12] revised Chu and Tsao's method and suggested a new approach for ranking fuzzy numbers based on Chu and Tsao's method in away to similar original point. However, there is a shortcoming in some situations. It will be illustrated in example 4.2. In the present paper, we discuss the problem of above methods. In other words, Cheng's method has still drawback, i.e., it cannot rank fuzzy numbers in some situations. To overcome the shortcoming in mentioned methods, we improve Cheng based-distance method by introducing a sign function.

The rest of the paper is organized as follows. Section 2 contains the basic definitions and notations use in the remaining parts of the paper. In Section 3, we introduce our idea to improve Cheng's method for ranking fuzzy numbers. Section 4 demonstrates by several numerical examples the fact that the centroid-based method, the area method and Wang and Lee revised method can significantly alter the result of the ranking procedure and lead to a wrong ranking order. The paper is concluded in Section 5.

## Materials and Methods

In this section, we introduce the basic concepts of fuzzy numbers and the centroid points of a fuzzy number. Then we briefly review the centroid point presented by cheng [6] and Chu and Tsao. [7] In addition the corrected formulae by Wang et al.[9]

A fuzzy number is a convex fuzzy subset like of the real line is completely defined by its membership function. Let $A$ be a fuzzy number, whose membership function $f_A(x)$ can generally be defined as [13]

$$f_A(x) = \begin{cases} f_A^L(x) & a \le x \le b, \\ \omega & b \le x \le c, \\ f_A^R(x) & c \le x \le d, \\ 0 & othrwise, \end{cases} \tag{1}$$

where $0<\omega\leq 1$ is a constant, $f_A^L(x):[a,b]\to[0,\omega]$ and $f_A^R(x):[c,d]\to[0,\omega]$ are two strictly monotonic and continuous mapping from R to closed interval $[0,\omega]$. If ω=1, then $A$ is normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by $A=(a,b,c,d;\omega)$ or $A=(a,b,c,d)$ if ω=1. In Particular, when $b=c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A=(a,b,d;\omega)$ or $A=(a,b,d)$ if ω=1. So triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonic and continuous functions, their inverse functions exist and should be continuous and strictly monotonic. Let $g_A^L:[0,\omega]\to[a,b]$ and $g_A^R:[0,\omega]\to[c,d]$ be the inverse functions of $f_A^L(x)$ and $f_A^R(x)$, respectively. Then $g_A^L(y)$ and $g_A^R(y)$ should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^\omega g_A^L(y)dy$ and $\int_0^\omega g_A^R(y)dy$ should exist. In the case of trapezoidal fuzzy number, the inverse function $g_A^L(y)$ and $g_A^R(y)$ can be analytically expressed as

$$\begin{aligned} g_A^L(y) &= \quad a+(b-a)y/\omega \quad 0\leq y\leq\omega \\ g_a^R(y) &= \quad d-(d-c)y/\omega \quad 0\leq y\geq\omega \end{aligned} \tag{2}$$

In order to determine the centroid point $(x_0(A),y_0(A))$ of a fuzzy number $A$, Cheng[6] provided the following centroid formulae

$$\begin{aligned} x_0 &= \frac{\int_a^b xf_A^L(x)dx+\int_b^c xdx+\int_c^d xf_A^R(x)dx}{\int_a^b f_A^L(x)dx+\int_b^c dx+\int_c^d f_A^R(x)dx} \\ y_0 &= \frac{\omega\left[\int_0^1 yg_A^L(y)dy+\int_0^1 yg_A^R(y)dy\right]}{\int_0^1 g_A^L(y)dy+\int_0^1 g_A^R(y)dy} \end{aligned} \tag{3}$$

Normal fuzzy numbers can be seen as special cases of non-normal fuzzy numbers with ω=1.

In Chu and Tsao,[7] the centroid point formulae were given as

$$\begin{aligned} x_0 &= \frac{\int_a^b xf_A^L(x)dx+\int_b^c xdx+\int_c^d xf_A^R(x)dx}{\int_a^b f_A^L(x)dx+\int_b^c dx+\int_c^d f_A^R(x)dx} \\ y_0 &= \frac{\left[\int_0^\omega yg_A^L(y)dy+\int_0^\omega yg_A^R(y)dy\right]}{\int_0^\omega g_A^L(y)dy+\int_0^\omega g_A^R(y)dy}. \end{aligned} \tag{4}$$

Pan and Yeh [10, 11] also adopted the above formulae. However, it is found that the formulas (3) and (4) are incorrect despite the fact that formulas of $x_0$ in relations (3) and (4) are consistent with Yager's ranking index [14, 15] $F(A)=\int_{-\infty}^{+\infty} g(x) f_A(x)dx / \int_{-\infty}^{+\infty} f_A(x)dx$ with the weighted function $g(x)=x$ and ω=1 and the same as Murakami et al.'s ranking index [16] for normal fuzzy sets $Z_0(A)=\int_{-\infty}^{+\infty} x f_A(x)dx / \int_{-\infty}^{+\infty} f_A(x)dx.$

Wang et al. [9] found from the point of view of analytical geometry and showed the corrected centroid point as follows:

$$x_0 = \frac{\int_a^b x f_A^L(x)dx + \int_b^c x dx + \int_c^d x f_A^R(x)dx}{\int_a^b f_A^L(x)dx + \int_b^c dx + \int_c^d f_A^R(x)dx}$$

$$y_0 = \frac{\left[\int_0^\omega y g_A^R(y)dy - \int_0^\omega y g_A^L(y)dy\right]}{\int_0^\omega g_A^R(y)dy - \int_0^\omega g_A^L(y)dy}. \tag{5}$$

For non-normal trapezoidal fuzzy number $A=(a,b,d;\omega)$ formulas (5) lead to following results respectively.

$$x_0 = \frac{1}{3}\left[a+b+c+d-\frac{dc-ab}{(d+c)-(a+b)}\right]$$

$$y_0 = \frac{\omega}{3}\left[1+\frac{c-b}{(d+c)-(a+b)}\right]. \tag{6}$$

Since non-normal triangular fuzzy numbers are special cases of normal trapezoidal fuzzy numbers with $b=c$, formulas (6) can be simplified as

$$x_0 = \frac{1}{3}\left[a+b+d\right]$$

$$y_0 = \frac{\omega}{3}. \tag{7}$$

Obviously, for normal trapezoidal (triangular) fuzzy numbers ω=1, the formulas of $y_0$ in relations (2.6) and (2.7) can be simplified as

$$y_0(A) = \frac{1}{3}\left[1+\frac{c-b}{(d+c)-(a+b)}\right]$$

and

$$y_0(A) = \frac{1}{3},$$

respectively.

In this case, normal triangular fuzzy numbers can be compared or ranked directly in terms of their centroid coordinates on horizontal axis. For more detail, we refer the reader to.[9]

Cheng [6] formulated his idea as follows:

$$R(A) = \sqrt{x_0(A)^2 + y_0(A)^2} \tag{8}$$

Chu and Tsao's[7] computed the area between the centroid and original points to rank fuzzy numbers as:

$$S(A) = x_0(A).y_0(A) \tag{9}$$

Wang and Lee[12] in 2008 proposed a revised method for ranking fuzzy numbers. The revised method is based on the Chu and Tsao's method.[7] In short they ranked the fuzzy numbers based on their $x_0$ 's values if they are different. In the case that they are equal, they further compare their $y_0$ 's values to form their ranks.

Further, for two fuzzy numbers $A$ and $B$ if $y_0(A) \geq y_0(B)$ based on $x_0(A) = x_0(B)$, then $A \geq B$.

To emphasis, we note that in this work trapezoidal fuzzy numbers are considered.

**An Improvement in Cheng's Distance-Based Method**

As we already mentioned, in 2006 Wang et al. [9] pointed out that the centroid point formulas for fuzzy numbers provided by Cheng [6] are incorrect. In this section, we decide to utilize Chen's distance method with corrected formulae and connect it to a sign function, which will be introduced to improve the mentioned method.

Since in Cheng's method none of fuzzy number is respected to zero. Therefore, it would be a drawback. In point of our view, every fuzzy number may lead to zero, positive or negative real number. To overcome with this subject, we first introduce the sign function as follows:

**Definition1.** Let E stands the set of non-normal fuzzy numbers, ω be a constant provided that 0<ω≤1 and $\gamma : E \to \{-\omega, 0, \omega\}$ be a function that is defined as:

$$\forall A \in E : \gamma(A) = sign\left[\int_0^{\omega} \left(g_A^L(x) + g_A^R(x)\right) dx\right],$$

i.e.

$$\gamma(A) = \begin{cases} 1 & \int_0^{\omega} \left(g_A^L(x) + g_A^R(x)\right) dx, \\ 0 & \int_0^{\omega} \left(g_A^L(x) + g_A^R(x)\right) dx, \\ -1 & \int_0^{\omega} \left(g_A^L(x) + g_A^R(x)\right) dx. \end{cases} \tag{10}$$

It is clear for normal fuzzy numbers ω=1.

**Remark 1** If inf (supp (A)) $\geq 0$ then $\gamma(A)=1$.
**Remark 2** If sup (supp (A)) $<0$ then $\gamma(A)=-1$.
**Remark 3** If $A=(a,b,d;\omega)$ be a symmetric trapezoidal fuzzy number such that $b+c=0$ then $\gamma(A)=0$.

Consider fuzzy number $A$, its centroid point is denoted as $(x_0(A), y_0(A))$, where $x_0(A)$ and $y_0(A)$ can be calculated by corrected centroid point (6).
By connecting the sign function and relations (8), the improved method denoted by I $IR(.)$ instead of $R(.)$ (Cheng [6] proposed), it will be presented as:

$$IR(A)=\gamma(A)R(A).$$

In other words

$$IR(A)=\gamma(A)\sqrt{x_0(A)^2+y_0(A)^2} \tag{11}$$

Here, the improved method $IR(A)$ is utilized to rank fuzzy numbers. Therefore, for any two fuzzy numbers $A_1$ and $A_2$,

1. $IR(A_1)>IR(A_2)$ if and only if $A_1 \succ A_2$.
2. $IR(A_1)<IR(A_2)$ if and only if $A_1 \prec A_2$.
3. $IR(A_1)=IR(A_2)$ if and only if $A_1 \approx A_2$.

Then we formulate the order $\geq$ and $\leq$ as $A_1 \geq A_2$. if and only if $A_1 \succ A_2$ or $A_1 \approx A_2$, $A_1 \leq A_2$ if and only if $A_1 \prec A_2$ or $A_1 \approx A_2$.
We consider the following reasonable properties for the ordering approaches, see [16].

1. For an arbitrary finite subset $\Gamma$ of $E$ and $A_1 \in \Gamma$, $A_1 \geq A_1$.
2. For an arbitrary finite subset $\Gamma$ of $E$ and $(A_1, A_2) \in \Gamma^2, A_1 \geq A_2$ and $A_2 \geq A_1$, we should have $A_1 \approx A_2$.
3. For an arbitrary finite subset $\Gamma$ of $E$ and $(A_1, A_2, A_3) \in \Gamma^3, A_1 \geq A_2$ and $A_2 \geq A_3$, we should have $A_1 \geq A_3$.
4. For an arbitrary finite subset $\Gamma$ of $E$ and $(A_1, A_2) \in \Gamma^2$, inf{supp($A_1$)}> sup{supp($A_2$), we should have $A_1 \geq A_2$.
5. For an arbitrary finite subset $\Gamma$ of $E$ and $(A_1, A_2) \in \Gamma^2$, inf{supp($A_1$)}> sup{supp($A_2$)}, we should have $A_1 \succ A_2$.
6. Let $A_1, A_2, A_1+A_3$ and $A_2+A_3$ be elements of $E$. If $A_1 \geq A_2$, then $A_1+A_3 \geq A_2+A_3$.

**Remark 4** The function $IR(.)$ has the properties i, ii, iii,…, vi.

**Numerical Examples**
**Example 1** Consider two fuzzy numbers $A=(-0.03,0,0.03)$ and, $B=(0,0.0484,0.0968)$ see Fig.1.

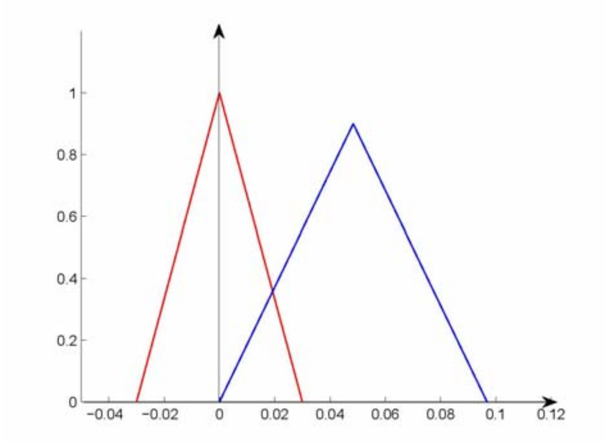


Fig. 1- Fuzzy numbers A= (-0.03, 0, 0.03) and B= (0, 0.0484, 0.0968, 0.9)

Intuitively the ranking order is $A \prec B$. Table 1 shows the results obtained by improvement centroid method, Chu and Tsao's formula, revised centroid and Cheng's method, respectively. It is very clear that Cheng's method lead to an incorrect ranking order $A \succ B$,, which is contrary to the ranking order $A \prec B$ obtained by using our proposed method. Moreover, some other methods that utilize the centroid points. This shows the fact Cheng's method can lead to wrong ranking orders.

Table 1- Comparative results of Example 1

| Fuzzy Numbers | Centroid Point $x_0, y_0$ | Improvement Centroid $IR(A) = \gamma(.)\sqrt{x_0^2 + y_0^2}$ | Chu and Tsao's Method $S(A) = x_0 . y_0$ | Wang and Lee's Revised $x_0$ | Cheng's Distance $R(A) = \sqrt{x_0^2 + y_0}$ |
|---|---|---|---|---|---|
| A<br>B | 0.0000,0.3333<br>0.0484,0.3000 | 0.0000<br>0.3039 | 0.0000<br>0.0145 | 0.0000<br>0.0484 | 0.3333<br>0.3039 |
| Results | | $A \prec B$ | $A \prec B$ | $A \prec B$ | $A \succ B$ |

**Example 2** Consider two fuzzy numbers $A = (-\frac{5}{2}, -\frac{1}{2}, 0, 1)$ and $B = (-\frac{9}{4}, -\frac{1}{4}, \frac{3}{4})$, which are demonstrated in Fig.2.

Intuitively, both the numbers are negative and the ranking order is $A \prec B$. However, by Cheng's method, the ranking order is $A \succ B$, which unreasonable result is. On the other hand the images of these to fuzzy numbers are $-A = (-1, 0, \frac{1}{2}, \frac{5}{2})$ and $-B = (-\frac{3}{4}, \frac{1}{4}, \frac{9}{4})$, respectively. By our method producing ranking order $-A \succ -B$. Clearly, similar to Chu and Tesao's area method [7] as you see in Table 2. On the contrary, the result of Cheng's distance [6] and revised centroid [12] is $-A \prec -B$, which is unreasonable. The proposed method can also overcome the shortcoming of Wang and Lee's method[12] in ranking fuzzy numbers and their images.

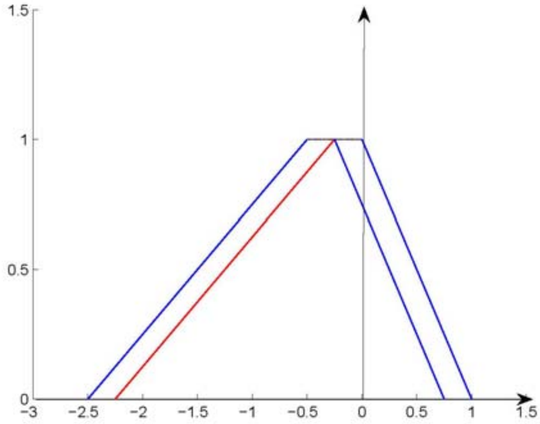


Fig. 2- Fuzzy numbers A= (-5/2,-1/2, 0, 1) and B= (-9/4,-1/4, 3/4).

Table 2- Comparative results of Example2

| Fuzzy Numbers | Centroid Point $x_0, y_0$ | Improvement Centroid $IR(A)=\gamma(.)\sqrt{x_0^2+y_0^2}$ | Chu and Tsao's Method $S(A)=x_0.y_0$ | Wang and Lee's Revised $x_0$ | Cheng's Distance $R(A)=\sqrt{x_0^2+y_0^2}$ |
|---|---|---|---|---|---|
| A<br>B | -.5625,.3750<br>-.5833,.3333 | -0.6760<br>-0.6719 | -0.2109<br>-0.1944 | -0.5625<br>-0.5833 | 0.6760<br>0.6719 |
| Results | | $A \prec B$ | $A \prec B$ | $A \prec B$ | $A \succ B$ |
| -A<br>-B | .5625,.3750<br>.5833,.3333 | -0.6760<br>-0.6719 | 0.2109<br>0.1944 | 0.2109<br>0.1944 | 0.6760<br>0.6719 |
| Results | | $-A \succ -B$ | $-A \succ -B$ | $-A \prec -B$ | $-A \succ -B$ |

**Example 3** Consider three triangular fuzzy numbers $A=(-3,0,3), B=(-3,-2,5)$ and $C=(-3,1,2)$ (See Fig. 3), whose membership functions are respectively defined as

$$f_A(x)=\begin{cases} \frac{1}{3}(x+3) & -3 \le x \prec 0, \\ 1 & x=0, \\ -\frac{1}{3} & 0 \le x \prec 3, \\ 0 & othrwise, \end{cases}$$

$$f_B(x) = \begin{cases} x+3 & -3 \le x \prec -2, \\ 1 & x = -2, \\ -\frac{1}{7}(x-5) & -2 \prec x \le 5, \\ 0 & othrwise, \end{cases}$$

$$f_C(x) = \begin{cases} \frac{1}{4}(x+3) & -3 \le x \prec 1 \\ 1 & x = 0, \\ -x+2 & 1 \le x \prec 2, \\ 0 & othrwise, \end{cases}$$

The corrected centroid point formulas (5) for mentioned numbers yield the same results, i.e., $x_0(A) = 0$ and $y_0(A) = \frac{1}{3}$. Then by using Cheng's centroid - based distance method $R(A) = R(B) = R(C) = \frac{1}{3}$. On the other hand by applying Tsao's method $S(A) = S(B) = S(C) = 0$. Therefore, both the distance method and area method producing the ranking order $A \approx B \approx C$. It shows the fact that the mentioned methods can lead to wrong ranking order. According to formulae (3.10) $\lambda(A) = 0, \gamma(B) = -1$ and $\gamma(C) = +1$. Also from the formulae (11) the results will be obtained as

$IR(A) = 0, IR(B) = -\frac{1}{3}$ and $IR(C) = \frac{1}{3}$.

Hence, the ranking order is $A \prec B \prec C$. Therefore, using our improved method can efficiently deal with the fuzzy ranking problems when different generalized fuzzy numbers have the same centroid point.

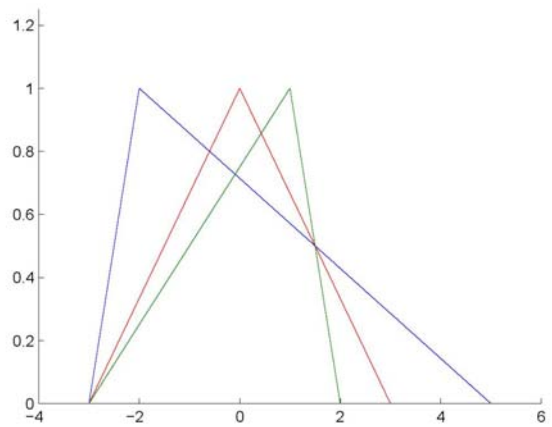


Fig. 3- Fuzzy numbers A=(-3,0,3), B=(-3,2,5) and C=(-3,1,2).

**Example 4** The two triangular fuzzy numbers $A = (-1,0,1.1), B = (-1.1,0.1,0.9)$ shown in Figure 4 are ranked by our method. It will be obtain $IR(A) = 0, IR(B) = 0.4714$. To compare with other method we refer the reader to Table 3.

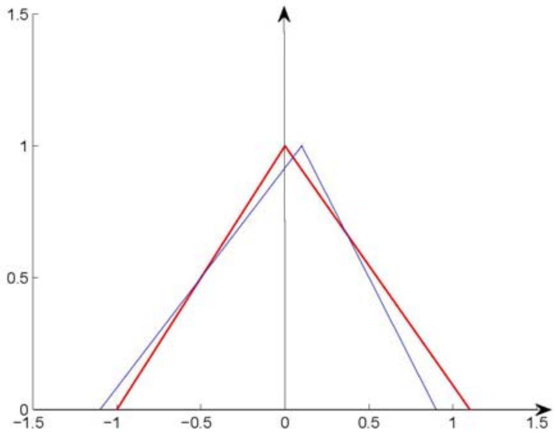


Fig. 4- Fuzzy numbers A= (-1, 0, 1.1) and B= (-1.1, 0.1, 0.9).

Table 3- Comparative results of Example 4

| Fuzzy Numbers | Modified Centroid Point | Sign Distance P=1 | Sign Distance P=2 | Chu and Tsao | Cheng's Distance |
|---|---|---|---|---|---|
| A<br>B | 0.4714<br>0.0000 | 0.0500<br>0.0000 | 0.8583<br>0.8206 | 0.1111<br>0.1111 | 0.4714<br>0.4714 |
| Results | $A \succ B$ | $A \succ B$ | $A \succ B$ | $A \approx B$ | $A \approx B$ |

**Conclusion:** In this paper, we improved Cheng distance method and presented an improvement method. The improvement method overcomes the shortcoming in Cheng's distance method, Chu and Tsao's formulae and new revised method by Wang and Lee. In this work those method are considered, which utilized the centroid point. The improved method can effectively rank various fuzzy numbers and their images. Thus, the method is superior to Cheng' distance, Chu and Tsao' area and Wang and Lee's revised method. The proposed method includes all situations of mentioned method, so it improves the distance method of Cheng as well.

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# بهبودی در روش مرکز ثقل جهت رتبه بندی اعداد فازی

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## چکیده

**مقدمه:** در بسیاری از مسائل کاربردی رتبه بندی اعداد فازی یکی از مولفه های مهم مراحل تصمیم گیری است. لذا این موضوع از اهمیت ویژه ای برخوردار است. محققین زیادی در این زمینه تحقیقات وسیعی انجام داده و به روش های متفاوتی دست یافته اند. چنگ در یکی از مقاله های خود (روشی جهت رتبه بندی اعداد فازی با استفاده ازروش فاصله، مجموعه های فازی و سیستم ها، جلد ۹۵، ۱۹۹۸) روشی به نام "فاصله ی مرکز ثقل" را جهت رتبه بندی اعداد فازی نرمال و غیر نرمال پیشنهاد داد. در روش وی از فاصله اقلیدسی بین مبدا مختصات و مرکز ثقل استفاده شده است. پس از مدتی معلوم شد که روش فاصله مرکزثقل چنگ دارای نقص می باشد و قادر به رتبه بندی همه اعداد فازی نیست. بعنوان مثال آن دسته از اعدادی که دارای مرکز ثقل یکسان هستند. محققین دیگری بر آن شدند که نقص روش چنگ را برطرف کنند و به این ترتیب بر اساس مختصات مرکزثقل روشهای دیگری را ارائه شد.

**هدف:** در این مقاله ضمن مطرح کردن اشکالات روش هایی که بر اساس مختصات مرکز ثقل ارائه شده ، روش بهبود یافته ای بر اساس روش فاصله چنگ پیشنهاد داده بطوریکه روش بهبود یافته پیشنهادی قادر به رتبه بندی اعداد گوناگون فازی باشد و مشکلات روش های قبلی را نداشته باشد.

**روش بررسی:** از نقطه نظر ما هر عدد فازی ممکن است صفر، مثبت یا منفی باشد. جهت فائق آمدن بر مشکلات موجود در روش هایی که بر اساس مرکز ثقل هستند، ابتدا تابعی به نام تابع علامت معرفی می کنیم. با استفاده ازتابع علامت، مختصات مرکز ثقل تصحیح یافته و روش فاصله چنگ، روش بهبود یافته ای بر این مبنا ساخته می شود.

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**نتایج**: روش پیشنهادی قادر به رتبه بندی کلیه اعداد فازی در هر وضعیتی می باشد. لذا بدین ترتیب روش فاصله چنگ به خوبی بهبود یافته است.

**نتیجه گیری**: با استفاده از یک تابع علامت، ترکیب آن با روش فاصله چنگ و مختصات مرکز ثقل تصحیح یافته می توان به روش مناسبی دست یافت که در واقع بهبود یافته روش چنگ است و قادر به رتبه بندی کلیه اعداد فازی در هر وضعیت میباشد و مشکلات روش های پیشین را ندارد.

***واژه‌های کلیدی***: رتبه بندی، اعداد فازی نرمال، اعداد فازی غیر نرمال، مرکز ثقل