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Solving fuzzy differential inclusions using the LU-representation of fuzzy numbers

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Solving fuzzy differential inclusions using the LU-representation of fuzzy numbers

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Abstract

Introduction: In this paper, the solution of fuzzy differential inclusions (FDIs) with Lower-Upper Representation is established.

Aim: Finding an approximate solution of fuzzy differential inclusions.

Materials and Methods: Instead of a precise fuzzy number as an initial value, we have just some information in some α-cuts of it, which shows the value of initial value at the endpoints of α-cuts and also the slopes of the fuzzy number's membership function in the mentioned points.

Results: Using the intervals formed with the α-cut of initial value we apply the Euler method and solve the FDIs to find the same information for the solution of the given problem. Also to determine the slopes of the estimated fuzzy solution again we use the Euler method and solve the fuzzy differential equations (FDEs). Finally with an specific interpolation polynomial we obtain the approximated solution.

Conclusion: The method presented in this paper is based on the LU-representation of fuzzy numbers with N=1. We find the solution set just in two α-cuts and then the approximate solution will introduce via interpolation polynomials.

Keywords: Fuzzy differential equations, Fuzzy differential inclusions, LU-representation, Fuzzy number, Fuzzy calculus

Introduction

Fuzzy differential equations are a natural way to model dynamical systems under possibilistic uncertainty. Kandel and Bayatt[1] introduced the concept of FDEs. Also, in modelling real-world phenomena, fuzzy initial value problems (FIVPs) appear naturally, as in the works of Diamond[2] and Abbasbandy[3,4] and other authors[5-9], which are always a fuzzy model adequate for some real world phenomena not a fuzzified version of a crisp problem.

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Using Hukuhara derivative[10] significant problems arise, because the solution of FDEs have quite different properties from the solution of crisp differential equations, lacking observed properties of physical systems such as stability, periodicity and bifurcation. Hülermeier[11] overcome this undesirable property by interpreting an FDE as a family of differential inclusions. The main shortcoming of using FDIs is that we do not have a derivative of a fuzzy number valued function, and so, the numerical solution of an FDE is difficult to be obtained.

In general, the arithmetic operations on fuzzy numbers can be approached either by the direct use of the membership function (by Zadeh extension principle) or by the equivalent use of the α – cut representation which is introduced in. [12] Recently Stefanini and et al. [13] introduced a parametric representation named LU-representation that produces a subspace (the complete space) of the fuzzy numbers and the results of fuzzy calculus can be parameterized by the same formes. Also LU-representation can be useful at least at a computational level, as we can approximate the FDEs by a finite set of ordinary (nonfuzzy) differential equations (ODEs) with a possibly controllable precision, by increasing eventually the number of the α – cut decomposition.

In this paper, we introduce an approximate solution of fuzzy differential inclusions with LU-representation. Fuzzy initial value is just given in some α -cuts, which define the value of initial value at the endpoints of α -cuts and also its slopes in the mentioned points. Appling the Euler method for the FDIs we find the same information for the solution of the given problem and the slopes of the estimated fuzzy solution. Finally, the approximate solution is introduced with a specific interpolation polynomial.

The organization of the paper is as follows. In Section 2, we define fuzzy number and FIVP. In Section 3, the numerical method for FDIs with initial value LU-representation are discussed. The proposed algorithm is illustrated by solving some examples in Section 4.

Materials and Methods

Notations and preliminaries

First we recall some definitions concerning fuzzy numbers. Let $\tilde{A} \in E^1$ (the class of fuzzy sets on the real line).

Definition 2.1. We write $\tilde{A}(x)$, a number in $[0,1]$, for the membership function of $\tilde{A}$ evaluated at x. An α-cut of $\tilde{A}$ written $\tilde{A}_\alpha$ is defined as $\{x \mid \tilde{A}(x) \geq \alpha\}$, for $0 < \alpha \leq 1$. We separately define $\tilde{A}_0$ as the closure of the union of all the $\tilde{A}_\alpha$, $0 < \alpha \leq 1$.

The parametric form of a fuzzy number can be defined as follows. According to the representation theorem for fuzzy numbers or intervals (see[12]), we use α – cut setting to define a fuzzy number or interval.

Definition 2.2. [13] A fuzzy number (or interval) u is completely determined by any pair $u = (u^-, u^+)$ of functions $u^\pm : [0,1] \to \mathbb{R}$, defining the end-points of the α-cuts, satisfying the three conditions:

(i) $u^- : \alpha \to u^-_\alpha \in \mathbb{R}$ is a bounded monotonic increasing (nondecreasing) left-continuous function $\forall \alpha \in]0,1]$ and right-continuous for $\alpha = 0$;

(ii) $u^+ : \alpha \to u^+_\alpha \in \mathbb{R}$ is a bounded monotonic decreasing (nonincreasing) left-continuous function $\forall \alpha \in]0,1]$ and right continuous for $\alpha = 0$;

(iii) $u^-_\alpha \leq u^+_\alpha \quad \forall \alpha \in [0,1]$.

If $u^-_1 < u^+_1$ we have a fuzzy interval and if $u^-_1 = u^+_1$ we have a fuzzy number; for simplicity we refer to fuzzy numbers as intervals.

We will then consider fuzzy numbers of normal and upper semicontinuous form also we assume that the support $[u_0^-, u_0^+]$ of u is compact (closed and bounded). The notation $u_\alpha = [u_\alpha^-, u_\alpha^+]$, $\alpha \in [0,1]$ denotes explicitly the α-cuts of u.

A new method to represent a fuzzy number is introduced in[13] that stores the value of the fuzzy number in N α – cuts and also the slopes of them.

Definition 2.3. (LU-fuzzy numbers) [13] An LU-representation of a fuzzy number on a decomposition $0 = \alpha_0 \le \alpha_1 \le \cdots \le \alpha_N = 1$ can be written as $u = (\alpha_i; u_i^-, \delta u_i^-, u_i^+, \delta u_i^+)_{i=0,1,\dots,N}$, or $u = (u_i^-, \delta u_i^-, u_i^+, \delta u_i^+)_{i=0,1,\dots,N}$ with the data

$$u_0^- \le u_1^- \le \cdots \le u_N^- \le u_N^+ \le \cdots \le u_0^+,$$

and the slopes denoted by δu (the slope associated to the element u) that

$$\delta u_i^- \ge 0, \delta u_i^+ \le 0.$$

With $N = 1$ (without internal points) and $\alpha_0 = 0, \alpha_1 = 1$ in this simple case u can be represented by a vector of 8 elements

$$u_{LU} = (u_0^-, \delta u_0^-, u_0^+, \delta u_0^+; u_1^-, \delta u_1^-, u_1^+, \delta u_1^+). \qquad (1)$$

In[13] it is shown that a parametric representation of a fuzzy number (The LU-representation of a fuzzy number) has the advantage of allowing flexible and easy to control shapes of the fuzzy number and can be used directly to obtain error-controlled approximations of the fuzzy calculus in terms of a finite set of parameters.

Consider the FIVP,

$$\begin{aligned} u'(t) &= f(t, u(t)), \\ u(0) &= u^{(0)} \in E, \end{aligned} \qquad (2)$$

where $f : [0, T] \times \mathrm{P} \to E$. In[11] it is suggested a different formulation of (2) based on a family of differential inclusions at each α – level, $0 \le \alpha \le 1$,

$$\begin{aligned} u'(t) &\in F_\alpha(t, u(t)), \\ u(0) &\in u_\alpha^{(0)}, \end{aligned} \qquad (3)$$

where $F_\alpha(t, u(t)) : \mathrm{P} \times \mathrm{P} \to K_c^1$ (K_c^1 is the nonempty compact convex subset of P) is the α-cut of $f(t, u(t))$. The idea is that the set of all solutions of (3) would be the α – level set of all solutions of (2). For more details see[14].

We use a metric in E by the relation

$$D(u, v) = \sup_{0 \le \alpha \le 1} d(u_\alpha, v_\alpha), \qquad (4)$$

where d is the Hausdorff metric for nonempty compact subsets of P (see[2]).

Results and Discution

Main results

When the initial value is in the form of LU-representation, we have an $4N + 4$ element vector. Using the intervals

$$[u_i^{(0)-}, u_i^{(0)+}], \quad i = 0, 1, \cdots, N. \qquad (5)$$

there is an FDI. For FDIs we use the Euler scheme which uses a finite difference scheme together with suitable selection procedures resulting in a sequence of approximate values

$y_0, y_1, \cdots, y_M$, on a uniform grid $0 = t_0 \leq t_1 \leq \cdots \leq t_M = T$ with step size $h = t_j - t_{j-1} = (T - t_0)/M$, for $j = 1, \cdots, M$,

$$u_{\alpha_i}^{(0)} \in [u_i^{(0)-}, u_i^{(0)+}]$$

$$u_{\alpha_i}^{(t_{j+1})} \in \bigcup_{s \in u_{\alpha_i}^{(t_j)}} \left(s + \frac{h}{2} F(t_j, u_{\alpha_i}^{(t_j)})\right) \tag{6}$$

where $\alpha_i = \frac{i}{N}$, $(i = 0, 1, \cdots, N)$. And $u_{\alpha_i}^{(t_j)} = y_j$, ($y_0$ is the initial value and other y_js will be obtained via Relation (6)). However Heun method could be used instead of Euler method, see[4] for more details.

Now we consider the first order unidimensional FDIs

$$\begin{aligned} &\frac{d}{dt} u(t) \in F_\alpha(t, u(t)), \\ &u(0) = u^{(0)}, \end{aligned} \tag{7}$$

where $F : [0, T] \times \mathrm{P} \to \mathrm{E}$ and $u^{(0)} = (u_i^{(0)-}, \delta u_i^{(0)-}, u_i^{(0)+}, \delta u_i^{(0)+})_{i=0,1,\ldots,N}$ is the LU-representation of fuzzy initial value.

We can calculate $u(t)$ by solving FDIs with Euler scheme for $i = 0, 1, \cdots, N$

$$\begin{aligned} &\frac{d}{dt} u(t) \in F_\alpha(t, u(t)), \\ &u(0) \in [u_i^{(0)-}, u_i^{(0)+}]. \end{aligned} \tag{8}$$

In practice we let $N = 1$ and discrete $[u_0^{(t_j)-}, u_0^{(t_j)+}]$ and $[u_1^{(t_j)-}, u_1^{(t_j)+}]$ to apply Euler method. Finally by taking minimum and maximum of the mentioned solutions we find $[u_0^{(t_{j+1})-}, u_0^{(t_{j+1})+}]$ and $[u_1^{(t_{j+1})-}, u_1^{(t_{j+1})+}]$, respectively.

To determine the corresponding slopes we will add to (8) the following $2N + 2$ ODEs for boundary points in each $\alpha-$cut

$$\begin{aligned} &\frac{d}{dt} \delta u_i^-(t) = \delta f(t, u(t))_i^- \quad , \quad \delta u_i^-(0) = \delta u_i^{(0)-}, \\ &\frac{d}{dt} \delta u_i^+(t) = \delta f(t, u(t))_i^+ \quad , \quad \delta u_i^+(0) = \delta u_i^{(0)+}, \end{aligned} \tag{9}$$

where the $\delta f(t, u(t))_i^{\pm}$ are defined in[13]. The ODEs in (9) will be solved by Euler method.[15]

Lemma 3.1. Let us consider a sequence of numbers $\{W_n\}_{n=0}^K$, satisfying

$$|W_{n+1}| \leq A |W_n| + B, \quad 0 \leq n \leq K - 1,$$

for some given positive constants A and B. Then,

$$|W_n| \leq A^n |W_0| + B \frac{A^n - 1}{A - 1}, \quad 0 \leq n \leq K.$$

Proof. see.

Theorem 3.1. Let $F \in C^1(\mathrm{P})$ in (3) be a compact convex valued mapping satisfying the Lipschitz condition in x with Lipschitz constant $L > 0$ and let u_α be a solution of (3). Then, $\lim_{h \to 0} Y_K(\alpha) = u_\alpha(T)$, for any $\alpha \in [0,1]$.

Proof. see[2].

Theorem 3.1 and Lemma 3.1 prove the convergence of the Euler method, so they also prove the convergence of our method which is on the base of Euler method.

Here we introduce an interpolation polynomial that will be used to find the approximate solution. Delbourgo and Gregory in[16] introduce a (2.2)-rational monotonic spline that has the following form

$$p(\alpha) = \begin{cases} \dfrac{P(\alpha)}{Q(\alpha)} & u_1 \neq u_0, \\ u_0 & u_1 = u_0, \end{cases} \tag{10}$$

where

$$P(\alpha) = (u_1 - u_0)u_1\alpha^2 + (u_0 d_1 + u_1 d_0)\alpha(1-\alpha) + (u_1 - u_0)u_0(1-\alpha)^2,$$

$$Q(\alpha) = (u_1 - u_0)\alpha^2 + (d_0 + d_1)\alpha(1-\alpha) + (u_1 - u_0)(1-\alpha)^2.$$

Without any additional parameters, the function above satisfies the Hermite interpolation conditions at the points $\alpha = 0$ and $\alpha = 1$ i.e. ($N = 1$)

$$p(0) = u_0,\ p'(0) = d_0,\ p(1) = u_1,\ p'(1) = d_1,$$

We find the approximate solution by using Relation (10) in the last step.

Theorem 3.2. Let $u_0 \in E^1$ and Ω be an open subset of $\mathrm{P} \times \mathrm{P}$ containing $\{0\} \times Supp(u_0)$. Suppose that $f : \Omega \to E^1$ is upper semicontinuous and the boundedness assumption holds, for all $u(0) \in Supp(u_0)$ and the inclusions

$$u'(t) \in F(t,u), \quad u(0) \in Supp(u_0).$$

Then, the attainable sets of the family of inclusions (3) are the level sets of a fuzzy number and their solution sets are the level sets of a fuzzy number too.

Proof. Associated with FDE (2) we can consider the deterministic differential equation (DDE):

$$u'(t) = g(t,u(t)), \quad u(0) = c, \tag{11}$$

where $u'(t)$ is the crisp derivative of a function $u : [0,T] \to \mathrm{P}$ and

$$[f(t,u)]^\alpha = g(t,[u]^\alpha).$$

By the hypotheses we have that if g is continuous, then f is continuous. Thus, since problem (11) has a unique solution $u(t,c)$ and it is continuous on Ω, $u(t,.) : \Omega \to R$ is well defined and it is continuous for each $t \in [0,T]$ fixed. Then, by (2), $u(t,.) : F(\Omega) \to F(R)$ is a continuous function and it is well defined. Therefore there exists a unique solution of the form $U(t) = u(t,U_0)$ for the FDE(3) and $[U(t)]^\alpha = [u(t,U_0)]^\alpha = u(t,[U_0]^\alpha), \forall \alpha \in [0,1]$. Therefore, given that $\alpha \in [0,1]$, we have

$$[U(t)]^\alpha = u(t,[U_0]^\alpha) = \{u(t,c) \mid c \in [U_0]^\alpha\}. \tag{12}$$

On the other hand, the α-levels of the attainable set for problem (9) are given by

$$A(t) = \{u(t,c) \mid c \in [X_0]^\alpha\}. \qquad (13)$$

From (12) and (13) follows that the attainable sets of the family of inclusions (3) are the level sets of a fuzzy number. The proof of the second assertion is similar.

By Theorem 3.2 we conclude that the final solution which is obtained via this method, will be a fuzzy number.

Examples

The following examples will be solved via an algorithm which uses the LU-representation of fuzzy numbers and Euler method. To find approximate values $y_0, y_1, \cdots, y_M$, on a uniform grid $0 = t_0 \le t_1 \le \cdots \le t_M = T$ with step size $h = t_j - t_{j-1} = (T - t_0)/M$, for $j = 1, \cdots, M$, the following algorithm introduces a n approximate solution.

Step1. Find the LU-representation of the initial value with relation (1).

Step2. For α=0 and α=1 in Relation (6), we have $u_0^{(0)} \in [u_0^{(0)-}, u_0^{(0)+}]$ and $u_1^{(0)} \in [u_1^{(0)-}, u_1^{(0)+}]$, then, find the value of the solution in the grid points by:

$$u_0^{(t_{j+1})} \in \bigcup_{s \in u_0^{(t_j)}} \left(s + \frac{h}{2} F(t_j, u_0^{(t_j)})\right)$$

and

$$u_1^{(t_{j+1})} \in \bigcup_{s \in u_1^{(t_j)}} \left(s + \frac{h}{2} F(t_j, u_1^{(t_j)})\right)$$

Step3. For α=0 and α=1 in Relation (9), solve the following ODEs:

$$\frac{d}{dt}\delta u_0^-(t) = \delta f(t, u(t))_0^- \quad , \quad \delta u_0^-(0) = \delta u_0^{(0)-},$$
$$\frac{d}{dt}\delta u_0^+(t) = \delta f(t, u(t))_0^- \quad , \quad \delta u_0^-(0) = \delta u_0^{(0)-},$$

and

$$\frac{d}{dt}\delta u_1^-(t) = \delta f(t, u(t))_1^- \quad , \quad \delta u_1^-(0) = \delta u_1^{(0)-},$$
$$\frac{d}{dt}\delta u_1^+(t) = \delta f(t, u(t))_1^+ \quad , \quad \delta u_1^+(0) = \delta u_1^{(0)+},$$

to find the slopes at the end points of α-cuts.

Step4. By the following polynomials find the approximate solution in t_M, $p_L(\alpha) = \dfrac{P_L(\alpha)}{Q_L(\alpha)}$ and

$p_U(\alpha) = \dfrac{P_U(\alpha)}{Q_U(\alpha)}$ where

$$P_L(\alpha) = (u_1^- - u_0^-)u_1^-\alpha^2 + (u_0^-\delta u_1^- + u_1^-\delta u_0^-)\alpha(1-\alpha) + (u_1^- - u_0^-)u_0^-(1-\alpha)^2,$$
$$Q_L(\alpha) = (u_1^- - u_0^-)\alpha^2 + (\delta u_1^- + \delta u_0^-)\alpha(1-\alpha) + (u_1^- - u_0^-)(1-\alpha)^2,$$
$$P_U(\alpha) = (u_1^+ - u_0^+)u_1^+\alpha^2 + (u_0^+\delta u_1^+ + u_1^+\delta u_0^+)\alpha(1-\alpha) + (u_1^+ - u_0^+)u_0^+(1-\alpha)^2,$$
$$Q_U(\alpha) = (u_1^+ - u_0^+)\alpha^2 + (\delta u_1^+ + \delta u_0^+)\alpha(1-\alpha) + (u_1^+ - u_0^+)(1-\alpha)^2.$$

The same interpolation can be made for any grid points, however the main idea is finding the solution in t_M.

Example 4.1.[17] Consider the following FDI in E^1

$$u'(t)\in u_\alpha,\ t\in[0,1]$$
$$u_\alpha^{(0)}=[0.8+0.25\alpha, 1.125-0.125\alpha],\ \alpha\in[0,1] \qquad (14)$$

The LU-fuzzy representation for initial value with $N=1$

$$u_{LU}^{(0)}=(0.75, 0.25, 1.125, -0.125, 1, 0.25, 1, -0.125).$$

The one independent equation of (14) is sufficient to be solved with (6), i.e.

$$\overline{u}'_\alpha(t)=\overline{u}_\alpha,\ t\in[0,1]$$
$$\overline{u}_\alpha^{(0)}\in[0.75+0.25\alpha, 1.125-0.125\alpha],\ \alpha\in[0,1]$$

Where $\overline{u}'_\alpha(t)\in u'_\alpha(t)$, $\overline{u}_\alpha\in u_\alpha$ and $\overline{u}_\alpha^{(0)}\in u_\alpha^{(0)}$. The exact solution $u_\alpha(t)\in[0.75+0.125\alpha, 1.125-0.125\alpha]e^t$ can be obtained analytically. Also with $N=1$, four ODEs for boundary points in each $\alpha-$cut of (9) are

$$\delta u_0^{'-}(t)=\delta u_0^-(t) \quad \delta u_0^-(0)=0.25,$$
$$\delta u_0^{'+}(t)=\delta u_0^-(t) \quad \delta u_0^+(0)=-0.125,$$
$$\delta u_1^{'-}(t)=\delta u_1^-(t) \quad \delta u_1^-(0)=0.25,$$
$$\delta u_1^{'+}(t)=\delta u_1^+(t) \quad \delta u_1^+(0)=-0.125,$$

where the Euler method with $h=0.02$ is used and the Hausdorff distance of the approximate solution is 0.0099.

$$\delta u_0^-(t)=0.25e^t,\ \delta u_0^{'-}(t)=-0.125e^t$$
$$\delta u_1^{'-}(t)=0.25e^t,\ \delta u_1^{'+}(t)=-0.125e^t.$$

We compare the exact and approximate solutions at $t=1$ in Figure 1.

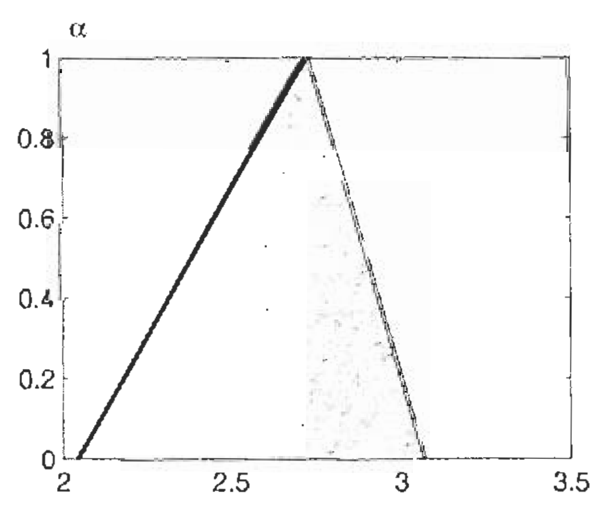

Fig. 1- Exact Solution and Approximate Solution.

Example 4.2[17]. Consider the following FDI in E^1

$$u'(t) \in -u_\alpha,\ t \in [0,1]$$
$$u_\alpha^{(0)} = [\alpha - 1, 1 - \alpha],\ \alpha \in [0,1].$$

With $N = 1$ ($\alpha = 0, 1$) the LU-representation of initial value is

$$u_{LU}^{(0)} = (-1,1,1,-1;0.1,0,-1).$$

Then we have

$$\overline{u}'_\alpha(t) = -\overline{u}_\alpha,\ t \in [0,1]$$
$$\overline{u}_\alpha^{(0)} \in [\alpha - 1, 1 - \alpha],\ \alpha \in [0,1],$$

where the exact solution is $\overline{u}_\alpha(t) \in [\alpha - 1, 1 - \alpha]e^{-t}$ also in this case with $N = 1$ the four ODEs for boundary points in each α – cut of (9) are

$$\begin{aligned}
&\delta u'^{-}_0(t) = -\delta u^{+}_0(t) \quad \delta u_0^{(0)-} = 1,\\
&\delta u'^{+}_0(t) = -\delta u^{-}_0(t) \quad \delta u_0^{(0)+} = -1,\\
&\delta u'^{-}_1(t) = -\delta u^{+}_1(t) \quad \delta u_1^{(0)-} = 1,\\
&\delta u'^{+}_1(t) = -\delta u^{-}_1(t) \quad \delta u_1^{(0)+} = -1,
\end{aligned}$$

where the Euler method with $h = 0.001$ is used and the Hausdorff distance of the approximate solution is 0.0794.

We compare the exact and approximate solutions at $t = 1$ in Figure 2.

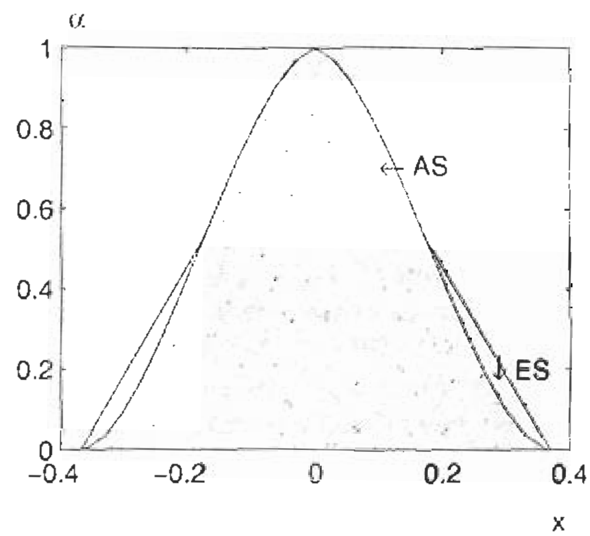

Fig. 2- Exact Solution and Approximation Solution.

Tabel1- The following table shows the Hausdorff distance of Example 1 and Example 2 for some given.

h	0.2	0.1	0.02	0.01	0.001
Hausdorff distance in Example 1	0.08	0.04	0.0099	0.005	0.0004985
Hausdorff distance in Example 2	0.3226	0.2003	0.1025	0.0904	0.0794

In [۲], the Euler method was used to solve the Example 4.2 when a parametric problem was solved, but in this paper we omit the parameter α and we solve that problem just for $\alpha=0$ and $\alpha=1$. This shows that the calculations will be so much less than the number of calculations which were needed in [۲]. Also after using interpolating polynomials the solution is so close to the solution of [۲]. In the other hand, through finding the solution the number of variables will be less and so, the less amount of memory will be needed.

Conclusion

The present paper introduces a new method for solving FDIs by using LU-representation and dividing the problem into one FDI and another FDE.

Instead of solving the FDIs with common methods as an application we propose to use LU-representation with N = ۱ to use less memory to save the information and also less computational effort. Using (2,2)-rational monotonic spline (10) we can approximate the solution of the given FDIs. For the further research we propose the problem of design and implementation of FDIs with initial conditions or parameters belonging to the space E^n of upper semicontinuous compactly supported convex fuzzy numbers.

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