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# Newton's method for solving a system of dual fuzzy nonlinear equations

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**Abstract**

In this paper, we propose a numerical solution for a system of dual fuzzy nonlinear equations by Newton's method. The fuzzy quantities are presented in parametric form. Some numerical illustrations are given to show the efficiency of algorithm.

**Keywords:** Newton's method, Fuzzy parametric form, Dual fuzzy nonlinear equations.



## 1 Introduction

Solving fuzzy equations and system of fuzzy equations has long been a problem in fuzzy set theory and many works have been done on these topics; see [5, 4, 13]. In [4], using classical methods based on the extension principle, the authors investigated solutions to linear and quadratic equations when the coefficients were real or complex fuzzy numbers and concluded that too often these equations do not have a solution. This result prompted the authors of [4] to investigate other solutions to fuzzy equations. In [5] they gave new solution concept and showed that the fuzzy quadratic equation, where the coefficients are all real fuzzy numbers, always has a solution as a real or complex fuzzy number. In [1], the authors investigated a model for solving a fuzzy nonlinear

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equation as $F(\overline{X}) = \overline{C}$, whose all parameters are fuzzy numbers. Then Abbasbandy and Ezzati [2], by using the parametric form of fuzzy numbers [11], investigated the following fuzzy nonlinear system

$$\begin{cases} F(\overline{X}, \overline{Y}) = \overline{C}, \\ G(\overline{X}, \overline{Y}) = \overline{D}, \end{cases}$$

and according to [1], they replaced the original system by crisp system.

According to this fact that system of simultaneous nonlinear equations play a major role in various areas such as mathematics, statistics, engineering and social sciences, therefore we need to develop numerical methods to find the roots of these systems. The plan for the rest of this paper is as follows. Section 2 formalized the notation and formal expression of the concepts with which we deal. In section 3, we will introduce Newton's method for solving a system of dual fuzzy nonlinear equations. In section 4, some numerical examples will be presented to show the efficiency of algorithm. Finally, section 5 draws the conclusive remarks.

## 2 Preliminaries

**Definition 2.1** *A fuzzy number is a mapping $\overline{U} : R \to I = [0, 1]$ with the following properties, see [9, 11]:*

1. *$\overline{U}$ is upper semi-continuous,*
2. *$\overline{U}(x) = 0$ outside some interval $[c, d]$,*
3. *There are real numbers $a, b$ such that $c \leq a \leq b \leq d$ and*
   - *3.1 $\overline{U}(x)$ is monotonic increasing on $[c, a]$,*
   - *3.2 $\overline{U}(x)$ is monotonic decreasing on $[b, d]$,*
   - *3.3 $\overline{U}(x) = 1, \ a \leq x \leq b$.*

The set of all these fuzzy numbers is denoted by $E^1$. An equivalent parametric is also given in [11] as follows.

**Definition 2.2** *A fuzzy number $\overline{U}$ in parametric form is a pair $(\overline{U}_1, \overline{U}_2)$ of functions $\overline{U}_1(r), \overline{U}_2(r),\ 0 \le r \le 1$, which satisfy the following requirements:*

1. *$\overline{U}_1(r)$ is a bounded monotonic increasing left continuous function,*

2. *$\overline{U}_2(r)$ is a bounded monotonic decreasing left continuous function,*

3. *$\overline{U}_1(r) \le \overline{U}_2(r),\ 0 \le r \le 1$.*

A popular type of fuzzy number is the set of trapezoidal fuzzy number $\overline{U} = (x_0/x_1/y_0/y_1)$ with interval defuzzifier $[x_1, y_0]$ where the membership function is

$$\overline{U}(x) = \begin{cases} \dfrac{x - x_0}{x_1 - x_0} & x_0 \le x \le x_1, \\[2ex] 1 & x \in [x_1, y_0], \\[2ex] \dfrac{y_1 - x}{y_1 - y_0} & y_0 \le x \le y_1, \\[2ex] 0 & Otherwise. \end{cases}$$

Its parametric form is

$$\overline{U}_1(r) = (x_1 - x_0)r + x_0, \qquad \overline{U}_2(r) = y_1 - (y_1 - y_0)r.$$

If $x_1 = y_0$ then $\overline{U} = (x_0/x_1/y_1)$ is called triangular fuzzy number. Note that the triangular fuzzy numbers are special cases of $L - L$ fuzzy numbers, see [12].

Let $TF(R)$ be the set of all trapezoidal fuzzy numbers. The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows.

For arbitrary $\overline{U} = (\overline{U}_1(r), \overline{U}_2(r))$, $\overline{V} = (\overline{V}_1(r), \overline{V}_2(r))$ and $k > 0$ we define addition $(\overline{U} + \overline{V})$ and multiplication by scalar $k$ as

$$(\overline{U} + \overline{V})_1(r) = \overline{U}_1(r) + \overline{V}_1(r), \quad (\overline{U} + \overline{V})_2(r) = \overline{U}_2(r) + \overline{V}_2(r), \tag{1}$$

$$(k\overline{U})_1(r) = k\overline{U}_1(r), \quad (k\overline{U})_2(r) = k\overline{U}_2(r). \tag{2}$$

## 3 The Newton's method

Usually, there is no inverse element for an arbitrary fuzzy number $x \in E^1$, i.e. there exists no element $y \in E^1$ such that $x + y = 0$. Actually, for all non-crisp fuzzy number $x \in E^1$ we have $x + (-x) \neq 0$. Therefore the fuzzy system

$$\begin{cases} P(\overline{X}, \overline{Y}) = Q(\overline{X}, \overline{Y}) + \overline{C}, \\ R(\overline{X}, \overline{Y}) = S(\overline{X}, \overline{Y}) + \overline{D}, \end{cases} \tag{3}$$

cannot be equivalently replaced by the fuzzy system

$$\begin{cases} P(\overline{X}, \overline{Y}) - Q(\overline{X}, \overline{Y}) = \overline{C}, \\ R(\overline{X}, \overline{Y}) - S(\overline{X}, \overline{Y}) = \overline{D}. \end{cases}$$

In the sequel, we will call the fuzzy nonlinear system (3), a dual fuzzy nonlinear system where all parameters are fuzzy numbers.

Now our aim is to obtain a solution for fuzzy nonlinear system 3. The parametric form of this system, for all $r$ belong $[0, 1]$, is as follows:

$$\begin{cases} P_1(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) = Q_1(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) + \overline{C}_1(r), \\ P_2(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) = Q_2(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) + \overline{C}_2(r), \\ R_1(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) = S_1(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) + \overline{D}_1(r), \\ R_2(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) = S_1(\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2, r) + \overline{D}_2(r). \end{cases} \tag{4}$$

The solution of 4, for any $0 \leq r \leq 1$, is called analytical solution of 3.

Suppose that $\overline{X} = (\alpha_1, \alpha_2)$ and $\overline{Y} = (\beta_1, \beta_2)$ are the solutions of 4, i.e. for all $r$ belong

$[0, 1]$

$$\begin{cases} P_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = Q_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) + \overline{C}_1(r), \\ P_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = Q_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) + \overline{C}_2(r), \\ R_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = S_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) + \overline{D}_1(r), \\ R_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = S_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) + \overline{D}_2(r). \end{cases}$$

Hence we suppose functions $F_1, F_2, G_1$ and $G_2$ are defined as follows:

$$\begin{cases} F_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = P_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) - Q_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r), \\ F_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = P_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) - Q_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r), \\ G_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = R_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) - S_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r), \\ G_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = R_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) - S_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r). \end{cases}$$

Consequently

$$\begin{cases} F_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = \overline{C}_1(r), \\ F_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = \overline{C}_2(r), \\ G_1(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = \overline{D}_1(r), \\ G_2(\alpha_1, \alpha_2, \beta_1, \beta_2, r) = \overline{D}_2(r). \end{cases}$$

Therefore, if $\overline{X_0} = (\overline{X_{01}}, \overline{X_{02}})$ and $\overline{Y_0} = (\overline{Y_{01}}, \overline{Y_{02}})$ are approximation solutions for this system, then for all $r$ belong to $[0, 1]$, there are $h_i(r), k_i(r)$; for $i = 1, 2$, such that

$$\begin{cases} \alpha_1(r) = \overline{X_{01}}(r) + h_1(r), \\ \alpha_2(r) = \overline{X_{02}}(r) + k_1(r), \\ \beta_1(r) = \overline{Y_{01}}(r) + h_2(r), \\ \beta_2(r) = \overline{Y_{02}}(r) + k_2(r). \end{cases}$$

Now if we use the Taylor series of $F_1, F_2, G_1, G_2$ about $(\overline{X_{01}}, \overline{X_{02}}, \overline{Y_{01}}, \overline{Y_{02}})$, we have $\forall r \in [0, 1]$,

$$\begin{cases} F_1(\alpha_1,\alpha_2,\beta_1,\beta_2,r)=F_1(\Delta)+h_1F_{1_{\overline{X}_1}}(\Delta)+k_1F_{1_{\overline{X}_2}}(\Delta)+h_2F_{1_{\overline{Y}_1}}(\Delta)+\\ k_2F_{1_{\overline{Y}_2}}(\Delta)+O(\Gamma)=\overline{C}_1(r),\\ \\ F_2(\alpha_1,\alpha_2,\beta_1,\beta_2,r)=F_2(\Delta)+h_1F_{2_{\overline{X}_1}}(\Delta)+k_1F_{2_{\overline{X}_2}}(\Delta)+h_2F_{2_{\overline{Y}_1}}(\Delta)+\\ k_2F_{2_{\overline{Y}_2}}(\Delta)+O(\Gamma)=\overline{C}_2(r),\\ \\ G_1(\alpha_1,\alpha_2,\beta_1,\beta_2,r)=G_1(\Delta)+h_1G_{1_{\overline{X}_1}}(\Delta)+k_1G_{1_{\overline{X}_2}}(\Delta)+h_2G_{1_{\overline{Y}_1}}(\Delta)+\\ k_2G_{1_{\overline{Y}_2}}(\Delta)+O(\Gamma)=\overline{C}_1(r),\\ \\ G_2(\alpha_1,\alpha_2,\beta_1,\beta_2,r)=G_2(\Delta)+h_1G_{2_{\overline{X}_1}}(\Delta)+k_1G_{2_{\overline{X}_2}}(\Delta)+h_2G_{2_{\overline{Y}_1}}(\Delta)+\\ k_2G_{2_{\overline{Y}_2}}(\Delta)+O(\Gamma)=\overline{C}_2(r), \end{cases}$$

and hence $h_i(r)$ and $k_i(r)$; for $i=1,2$ are unknown quantities which can be obtained by solving the following equations, $\forall r\in[0,1]$,

$$J(\Delta)\begin{bmatrix} h_1(r)\\ k_1(r)\\ h_2(r)\\ k_2(r)\end{bmatrix}=\begin{bmatrix}\overline{C}_1(r)-F_1(\Delta)\\ \overline{C}_2(r)-F_2(\Delta)\\ \overline{D}_1(r)-G_1(\Delta)\\ \overline{D}_2(r)-G_2(\Delta)\end{bmatrix},$$

where

$$J(\Delta)=\begin{bmatrix} F_{1_{\overline{X}_1}} & F_{1_{\overline{X}_2}} & F_{1_{\overline{Y}_1}} & F_{1_{\overline{Y}_2}}\\ F_{2_{\overline{X}_1}} & F_{2_{\overline{X}_2}} & F_{2_{\overline{Y}_1}} & F_{2_{\overline{Y}_2}}\\ G_{1_{\overline{X}_1}} & G_{1_{\overline{X}_2}} & G_{1_{\overline{Y}_1}} & G_{1_{\overline{Y}_2}}\\ G_{2_{\overline{X}_1}} & G_{2_{\overline{X}_2}} & G_{2_{\overline{Y}_1}} & G_{2_{\overline{Y}_2}}\end{bmatrix}_{(\Delta)}.$$

Hence, the next approximations for $\overline{X}_1(r)$ , $\overline{X}_2(r)$ , $\overline{Y}_1(r)$ and $\overline{Y}_2(r)$ are as follows

$$\begin{cases} \overline{X_{1}}_1(r) = \overline{X_{0}}_1(r) + h_1(r), \\ \overline{X_{1}}_2(r) = \overline{X_{0}}_2(r) + k_1(r), \\ \overline{Y_{1}}_1(r) = \overline{Y_{0}}_1(r) + h_2(r), \\ \overline{Y_{1}}_2(r) = \overline{Y_{0}}_2(r) + k_2(r), \end{cases}$$

for all $r \in [0, 1]$.

We can obtain approximated solution, $\forall r \in [0, 1]$, by using the recursive scheme

$$\begin{cases} \overline{X_{n}}_1(r) = \overline{X_{n-1}}_1(r) + h_{1,n-1}(r), \\ \overline{X_{n}}_2(r) = \overline{X_{n-1}}_2(r) + k_{1,n-1}(r), \\ \overline{Y_{n}}_1(r) = \overline{Y_{n-1}}_1(r) + h_{2,n-1}(r), \\ \overline{Y_{n}}_2(r) = \overline{Y_{n-1}}_2(r) + k_{2,n-1}(r), \end{cases}$$

where $h_{i,0}(r) = h_i(r)$ and $k_{i,0}(r) = k_i(r)$; for $i = 1, 2$ and $n = 1, 2, \ldots$. For initial guess, one can use the fuzzy number

$$\begin{cases} \overline{X_0} = (\overline{X}_1(0), \overline{X}_1(1), \overline{X}_2(1)\overline{X}_2(0)), \\ \overline{Y_0} = (\overline{Y}_1(0), \overline{Y}_1(1), \overline{Y}_2(1)\overline{Y}_2(0)), \end{cases}$$

and in parametric form

$$\begin{cases} \overline{X_{0}}_1(r) = \overline{X}_1(1) + (\overline{X}_1(1) - \overline{X}_1(0))(r-1), \\ \overline{X_{0}}_2(r) = \overline{X}_2(1) + (\overline{X}_2(0) - \overline{X}_2(1))(1-r), \\ \overline{Y_{0}}_1(r) = \overline{Y}_1(1) + (\overline{Y}_1(1) - \overline{Y}_1(0))(r-1), \\ \overline{Y_{0}}_2(r) = \overline{Y}_2(1) + (\overline{Y}_2(0) - \overline{Y}_2(1))(1-r). \end{cases}$$

**Corollary 3.1** *Sequence* $\{(\overline{X}_{1_n}, \overline{X}_{2_n})\}_{n=0}^{\infty}$ *and* $\{(\overline{Y}_{1_n}, \overline{Y}_{2_n})\}_{n=0}^{\infty}$ *convergent to* $(\alpha_1, \alpha_2)$ *and* $(\beta_1, \beta_2)$*, respectively, if and only if* $\forall r \in [0, 1]$, $\lim_{n\to\infty} \overline{X}_{1_n}(r) = \alpha_1(r)$*,* $\lim_{n\to\infty} \overline{X}_{2_n}(r) = \alpha_2(r)$*,* $\lim_{n\to\infty} \overline{Y}_{1_n}(r) = \beta_1(r)$ *and* $\lim_{n\to\infty} \overline{Y}_{2_n}(r) = \beta_2(r)$*.*

**Theorem 3.2** *Let*

$$\begin{cases} F(\alpha_1, \alpha_2) = (\overline{C}_1, \overline{C}_2), \\ G(\beta_1, \beta_2) = (\overline{D}_1, \overline{D}_2), \end{cases}$$

*and if the sequence of* $\{(\overline{X}_{1_n}, \overline{X}_{2_n})\}_{n=0}^{\infty}$ *and* $\{(\overline{Y}_{1_n}, \overline{Y}_{2_n})\}_{n=0}^{\infty}$ *convergent to* $(\alpha_1, \alpha_2)$ *and* $(\beta_1, \beta_2)$*, respectively, according to Newton's method, then* $\lim_{n\to\infty} P_n = 0$, *where*

$$P_n = \sup_{0\leq r\leq 1} \max\{h_{1,n}(r), k_{1,n}(r), h_{2,n}(r), k_{2,n}(r)\}.$$

**Proof.** *It is obviously, because for all* $r \in [0, 1]$ *in convergent case for* $i = 1, 2$

$$\lim_{n\to\infty} h_{i,n}(r) = \lim_{n\to\infty} k_{i,n}(r) = 0.$$

# 4 Numerical application

Here we present two examples to illustrating the Newton's method for a dual fuzzy nonlinear systems.

**Example 4.1** *Consider a dual fuzzy nonlinear system*

$$\begin{cases} \overline{X}^2 + \overline{Y}^2 = \overline{X} + \overline{Y} + (4.4/5/7), \\ \overline{X}^2 + \overline{Y}^3 + (1/2/3) = \overline{X} + \overline{Y} + (8.6/11/17.2). \end{cases}$$

*Without any loss of generality, assume that* $\overline{X}$ *and* $\overline{Y}$ *are positive, then the parametric form of this system is as follows*

$$\begin{cases} \overline{X}_1^2(r) + \overline{Y}_1^2(r) = \overline{X}_1(r) + \overline{Y}_1(r) + (4.4 + 0.6r), \\ \overline{X}_2^2(r) + \overline{Y}_2^2(r) = \overline{X}_2(r) + \overline{Y}_2(r) + (7 - 0.2r), \\ \overline{X}_1^2(r) + \overline{Y}_1^3(r) + (1 + r) = \overline{X}_1(r) + \overline{Y}_1(r) + (8.6 + 2.4r), \\ \overline{X}_2^2(r) + \overline{Y}_2^3(r) + (3 - r) = \overline{X}_2(r) + \overline{Y}_2(r) + (17.2 - 6.2r). \end{cases}$$

*To obtain initial guess we use above system for* $r = 0$ *and* $r = 1$*, therefore*

$$\begin{cases} \overline{X}_1^2(0) & + & \overline{Y}_1^2(0) - \overline{X}_1(0) - \overline{Y}_1(0) & = & 4.4, \\ \overline{X}_2^2(0) & + & \overline{Y}_2^2(0) - \overline{X}_2(0) - \overline{Y}_2(0) & = & 7, \\ \overline{X}_1^2(0) & + & \overline{Y}_1^3(0) - \overline{X}_1(0) - \overline{Y}_1(0) & = & 7.6, \\ \overline{X}_2^3(0) & + & \overline{Y}_2^3(0) - \overline{X}_2(0) - \overline{Y}_2(0) & = & 14.2, \end{cases}$$

$$\begin{cases} \overline{X}_1^2(1) & + & \overline{Y}_1^2(1) - \overline{X}_1(1) - \overline{Y}_1(1) & = & 5, \\ \overline{X}_2^2(1) & + & \overline{Y}_2^2(1) - \overline{X}_2(1) - \overline{Y}_2(1) & = & 5, \\ \overline{X}_1^2(1) & + & \overline{Y}_1^3(1) - \overline{X}_1(1) - \overline{Y}_1(1) & = & 9, \\ \overline{X}_2^3(1) & + & \overline{Y}_2^3(1) - \overline{X}_2(1) - \overline{Y}_2(1) & = & 9. \end{cases}$$

*Consequently* $\overline{X}_1(0) = 2.22034$, $\overline{X}_2(0) = 2.539$, $\overline{Y}_1(0) = 1.893$, $\overline{Y}_2(0) = 2.32824$, $\overline{X}_1(1) = \overline{X}_2(1) = 2.30278$ *and* $\overline{Y}_1(1) = \overline{Y}_2(1) = 2$. *Therefore initial guess is*

$$\overline{X_0} = (2.22034/2.30278/2.539), \quad \overline{Y_0} = (1.893/2/2.32824).$$

*After 2 iterations, we obtain the solutions of* $\overline{X}$ *and* $\overline{Y}$ *which the maximum error would be less than* $10^{-8}$. *For more details see Figures 1 and 2. Now suppose* $\overline{X}$ *and* $\overline{Y}$ *are negative, we have*

$$\begin{cases} \overline{X}_2^2(r) + \overline{Y}_2^2(r) = \overline{X}_1(r) + \overline{Y}_1(r) + (4.4 + 0.6r), \\ \overline{X}_1^2(r) + \overline{Y}_1^2(r) = \overline{X}_2(r) + \overline{Y}_2(r) + (7 - 0.2r), \\ \overline{X}_2^2(r) + \overline{Y}_1^3(r) + (1 + r) = \overline{X}_1(r) + \overline{Y}_1(r) + (8.6 + 2.4r), \\ \overline{X}_1^2(r) + \overline{Y}_2^3(r) + (3 - r) = \overline{X}_2(r) + \overline{Y}_2(r) + (17.2 - 6.2r). \end{cases}$$

*For* $r = 0$, *we have*

$$\begin{cases} \overline{Y}_1^3(0) - \overline{Y}_2^2(0) = 3.2, \\ \overline{Y}_2^3(0) - \overline{Y}_1^2(0) = 7.2. \end{cases}$$

*The real solution of this system is* $\overline{Y}_1(0) = 2.01886$, $\overline{Y}_2(0) = 2.24241$. *We know that both* $\overline{Y}_1(0)$ *and* $\overline{Y}_2(0)$ *must be negative, therefore negative roots do not exit.*

**Example 4.2** *Consider a dual fuzzy nonlinear system*

$$\begin{cases} \overline{X}^2 + \overline{Y}^2 + 2\overline{XY} = \overline{Y}^3 + (1/1.682/4.194), \\ \overline{X}^2 + \frac{1}{2}\overline{Y}^3 + (1/2/3) = \overline{Y}^2 + \overline{XY} + (1/1.354/2.896). \end{cases}$$

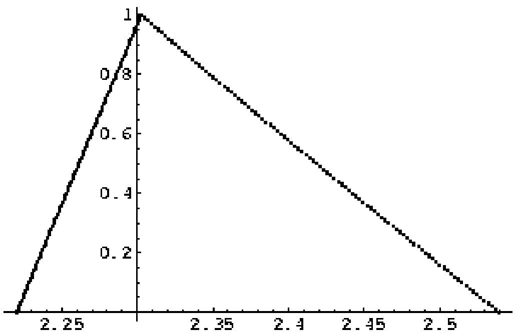


**Figure 1:** Newton's Method for $\overline{X}$

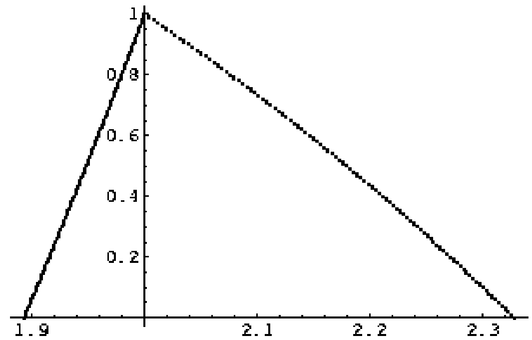


**Figure 2:** Newton's Method for $\overline{Y}$

*Without any loss of generality, assume that* $\overline{X}$ *and* $\overline{Y}$ *are positive, then parametric form of this system is as follows*

$$\begin{cases} \overline{X}_1^2(r) + \overline{Y}_1^2(r) + 2\overline{X}_1(r)\overline{Y}_1(r) = \overline{Y}_1^3(r) + (1 + 0.682r), \\ \overline{X}_2^2(r) + \overline{Y}_2^2(r) + 2\overline{X}_2(r)\overline{Y}_2(r) = \overline{Y}_2^3(r) + (4.194 - 2.512r), \\ \overline{X}_1^2(r) + \frac{1}{2}(r)\overline{Y}_1^3(r) + (1+r)\overline{Y}_1(r) = \overline{Y}_1^2(r) + \overline{X}_1(r)\overline{Y}_1(r) + (1 + 0.354r), \\ \overline{X}_2^2(r) + \frac{1}{2}(r)\overline{Y}_2^3(r) + (3-r)\overline{Y}_2(r) = \overline{Y}_2^2(r) + \overline{X}_2(r)\overline{Y}_2(r) + (2.898 - 1.744r). \end{cases}$$

*By solving the above system for* $r = 0$ *and* $r = 1$*, we obtain the initial guess* $\overline{X_0} = (1/1.1/1.5)$ *and* $\overline{Y_0} = (0/0.2/0.6)$*. If we apply two iterations from Newton's method, the maximum error would be about* $3.45 \times 10^{-6}$ *and* $1.34 \times 10^{-5}$*, respectively, see Figures 3 and 4.*

## 5 Conclusions

In this paper, we have suggested numerical method for solving a system of dual fuzzy nonlinear equations instead of standard analytical techniques which are not suitable everywhere. Initially we wrote fuzzy nonlinear system in parametric form and then solve them by Newton's method. Some examples were presented to illustrate the proposed method.

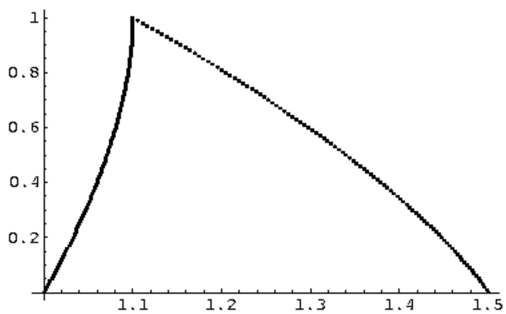


**Figure 3:** Newton's Method for $\overline{X}$

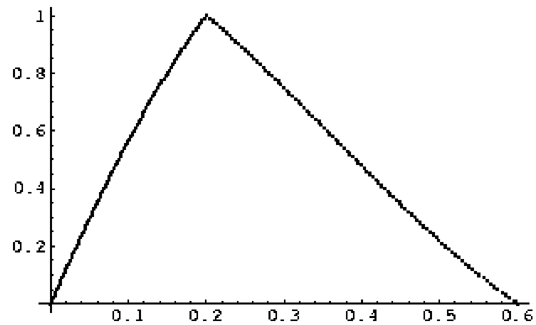


**Figure 4:** Newton's Method for $\overline{Y}$

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