March 1, 2012

Some Approaches for Using Stationary Iterative Methods to Linear Equations Generated from the Boundary Element Method

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ABSTRACT

For linear equations, there are numerous stationary iterative methods. However, these methods are not applicable in some important problems such as linear system arising from the boundary element method (BEM). In this paper, we proposed two approaches for using stationary iterative methods to linear equations arising from the BEM for the Laplace and convective diffusion with first-order chemical reaction problems. Our proposed methods are simple and graceful. Finally, numerical example is given to show the efficiency of our results.

Keywords: Stationary iterative methods, First-order chemical reaction, Boundary element method, FIM methods, Gauss-Seidel, Preconditioning methods.

1. INTRODUCTION

Consider the following dense linear system

\[ \mathbf{Ax} = \mathbf{b}, \]

where \( \mathbf{A} \in \mathbb{R}^{n \times n} \) nonsingular and nonsymmetric, \( \mathbf{b}, \mathbf{x} \in \mathbb{R}^{n} \).

If the coefficient matrix is diagonally dominant or positive definite the stationary iterative methods such as the Gauss-Seidel method, successive overrelaxation (SOR) method, accelerated overrelaxation (AOR) method and their modified methods are used; see, e.g., [1-10] and the references therein.

The coefficient matrix generated from the Boundary element method (BEM) analysis for a two-dimensional Laplace equation is not a diagonally dominant matrix, and is not a positive definite matrix. So the Gauss elimination method is used. However, for large \( n \) the total number of multiplication and divisions require \( O(n^3) \).

Recently, Sakakihara et al., in [11], By using the Gauss-Seidel and preconditioned method, proposed a method to transform a given matrix to the diagonally dominant matrix. However, for large \( n \) and big \textit{diagonally dominant ratio} this method is long and complicated.

In this paper, we proposed two approaches for using stationary iterative methods to linear equations arising from the boundary element method for the Laplace and convective diffusion with first-order reaction problems. Our proposed methods are simple and graceful. Finally, numerical example is given to show the efficiency of our results.

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with the boundary condition

\[ \nu = g \quad \text{on} \quad \partial \Omega, \]

where, \( \Omega = [0, 1] \times [0, 1] \) and

\[
g = \begin{cases} 
1, & 0 \leq x \leq 1, y = 0, \\
0, & \text{otherwise}.
\end{cases}
\]

\[ \xi_i \varphi_i = \sum_{j=1}^{N} \left\{ -v_j \int_{\partial \Omega} \frac{\partial \nu_j}{\partial n} (S_j, Q) d\Gamma + z_j \int_{\Omega} \nu^* (S_j, Q) d\Omega \right\} \quad i = 1, 2, \ldots, N, \]

where, \( S_j, Q \) are source point and a reference point, respectively.

For \( N = 8 \), we have the matrix as follows:

\[
A = \begin{pmatrix}
1.193 & 0.369 & 0.111 & -0.030 & -0.058 & -0.005 & 0.124 & 0.514 \\
0.369 & 1.193 & 0.514 & 0.124 & -0.005 & -0.058 & -0.030 & 0.111 \\
0.124 & 0.514 & 1.193 & 0.369 & 0.111 & -0.030 & -0.058 & -0.005 \\
-0.030 & 0.111 & 0.369 & 1.193 & 0.514 & 0.124 & -0.005 & -0.058 \\
-0.058 & -0.005 & 0.124 & 0.514 & 1.193 & 0.369 & 0.111 & -0.030 \\
-0.005 & -0.058 & -0.030 & 0.111 & 0.369 & 1.193 & 0.514 & 0.124 \\
0.111 & -0.030 & -0.058 & -0.005 & 0.124 & 0.514 & 1.193 & -0.369 \\
0.514 & 0.124 & -0.005 & -0.058 & -0.030 & 0.111 & 0.369 & 1.193
\end{pmatrix}
\]

Clearly, \( A \) is not \( SDDM \) or \( SPDM \). Therefore, we cannot guarantee the convergence of the stationary iterative methods for this problem. Now, we test our methods. By \( 3\)-steps \( FIM \) methods with different parameters, we have;

<table>
<thead>
<tr>
<th>( w )</th>
<th>( 0.2 )</th>
<th>( 0.7 )</th>
<th>( 1 )</th>
<th>( 1.5 )</th>
<th>( 2 )</th>
<th>( 2.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>iteration</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Furthermore, by transpose preconditioned method with Gauss–Seidel, we can obtain the exact solution after 39 iterations.

3. CONCLUSIONS

In this paper we have proposed some new methods for the linear system arising from the boundary element discretization for a convective diffusion problem with a first-order chemical reaction term. We have also studied how the stationary iterative methods for above systems are affected, if the system is constructed by our models. Numerical results show the influence of our methods.

REFERENCES

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