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Mathematical literacy: are we there yet?

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Mathematical literacy – are we there yet?[1]

Introduction

As far as I can see, the term 'mathematical literacy' is a relatively new addition to the education lexicon. This phrase and the related terms numeracy, and quantitative literacy, are used in a variety of ways. The same words take different meanings in different contexts and have been used differently over time, and this has caused some difficulties in the debate about critical aspects of mathematical education. It is important therefore to understand what is intended when these words and phrases are used in presentations, debates and in the education literature.

This paper represents my struggle to clarify these matters as a lead-in to a discussion of the particular uses of the term within and related to the OECD's Programme for International Student Assessment (PISA), and the ways in which PISA's use of this and related terms has changed over the life of this programme.

Words, words, words …

Nowadays we seem to have every kind of literacy under the sun: scientific, mathematical, financial, environmental, digital, visual, computer, economic, statistical, arts, technology, information, multimedia, functional, rhetorical, and critical. And the list goes on. But where did all this come from?

At least some of the more recent proliferation may be associated with semiotics and the 'new literacies' movement.

The use of the phrase *scientific literacy* dates back at least to the early 1950s, and from the early 1960s it was used regularly in papers and discussions about science education, and specifically as part of the push for a broad school treatment of science and its implications for society. Roger Bybee (1997) has provided a brief discussion of this history. So at the time PISA was being initiated, the idea of *scientific literacy* was already well established.

Use of the term *mathematical literacy* appears to be newer, but still dates back to at least the late 1970s. Mogens yesterday gave us the results of his research into the origins of this term, which I found very interesting indeed (Niss, 2012). Mogens has traced it back to the mid 1940's, but he noted that no definitions were offered in the several early uses. The phrase is one of a handful of terms that are used in different places and by different people and groups to refer to similar or related things, and the particular term preferred would appear to depend on the audience, the impressions to be conveyed, and on how aspects of the educational debate have unfolded in different parts of the mathematics education world.

[1] Prepared as a presentation to Topic Study Group 6 (TSG6 – *Mathematical Literacy*) at the International Congress on Mathematical Education in Seoul, Korea (ICME-12)

The word ‘numeracy’ was coined in the Crowther Report in 1959, a report of the Central Advisory Council for Education (England) written to provide advice to the Minister of Education on the education of boys and girls between the ages of 15 and 18. The intended meaning of the term was quite broad. It was intended

> *… to represent the mirror image of literacy… On the one hand is an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand, there is the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind. Statistical ignorance and statistical fallacies are quite as widespread and quite as dangerous as the logical fallacies which come under the heading of illiteracy. The man who is innumerate is cut off from understanding some of the relatively new ways in which the human mind is now most busily at work. Numeracy has come to be an indispensable tool to the understanding and mastery of all phenomena … The educated man, therefore, needs to be numerate as well as literate.* (Crowther Report, 1959, pages 269-271).

Some 20 years later, the Cockcroft report noted that the meaning intended by Crowther had been replaced by a far narrower idea.

> *In none of the submissions which we have received are the words 'numeracy' or 'numerate' used in the sense in which the Crowther Report defines them. Indeed, we are in no doubt that the words, as commonly used, have changed their meaning considerably in the last twenty years. The association with science is no longer present and the level of mathematical understanding to which the words refer is much lower. …*
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> *…We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease..... Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation.* (Cockcroft 1982, chapter 2, pages 10-11).

It would appear that even in two of last century’s most important and influential studies of school mathematics in the UK, there is a tension between the intended meaning of words, and the ways in which people commonly use them. Yet there are strong and consistent statements there about an idea that needs to be captured better in words and implemented better in our school practice.

In Australia, the word *numeracy* is used very commonly, but also with different meanings in different places and in a way that has varied over time. The definition found in the current national curriculum document is this:

> *Students become numerate as they develop the knowledge and skills to use mathematics confidently across all learning areas at school and in their lives more broadly. Numeracy involves students in recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.* (ACARA 2012 – General Capabilities in the Australian Curriculum).

Australia's new national curriculum framework goes on to spell out the link with mathematics, identifying mathematics as something separate from numeracy, but that plays a central role in its development.

In the US, the term *quantitative literacy* has been preferred when writers want to refer to a broader notion of mathematics and its use to deal with real world contexts. But again, we see variations in the apparent definitions used that mirror the differences we see elsewhere in the way *numeracy* is used. Lynn Steen (Steen 2001) covers this territory well in his introductory essay to a work that has been widely quoted in this context.

But essentially, the same issue is posed when we see any of these terms used: where does mathematics begin and end and what else do we wish to value in our activity as mathematics educators? We can consider several ideas, behaviours and phenomena, and note that the ways they are grouped and labelled vary across individuals and groups in the mathematics education world and beyond. In the following table, a set of labels is given in the left column, and various ideas, behaviours and phenomena are grouped in columns to the right.

Labels	Ideas/behaviours/phenomena		
Mathematics Numeracy Quantitative literacy Mathematical literacy	Arithmetic Practical survival skills Number sense Spatial sense Using mathematics in context Historical and cultural appreciation	Mathematical knowledge and procedures Proofs, theorems, definitions School mathematics Abstraction, idealisation, generalisation	Thinking mathematically Doing mathematics Mathematical communication Mathematical representation Reasoning and argument Strategic thinking Modelling, mathematisation

I have begun to realise that each of these four labels can be used to mean some or other part of the set of ideas, behaviours and phenomena; and paradoxically each of the labels can also be used to mean all or most of them.

So some people use 'numeracy' or 'quantitative literacy' to refer mostly to the very practically oriented things in the left column, which they see as a small part of a much bigger thing that they would label as 'mathematics'.

On the other hand, others use ‘mathematics’ to refer mostly to the more technical things in the middle column, which they see as part of a much bigger thing that they would label as ‘quantitative or mathematical literacy’, or as ‘numeracy’.

It is through the OECD’s Programme for International Student Assessment (PISA) that the phrase *mathematical literacy* was given a definition and was adopted partly to try and deal with this terminology problem. We can trace the earliest documented use of the term *mathematical literacy* in the PISA context to March 1998, in the record of the first meeting of what was then called the ‘mathematics functional expert group’, a group of mathematics and education experts appointed to advise the contractor responsible for PISA development and implementation. The record states that PISA should be *“… a test of ‘mathematical literacy’, not of ‘school mathematics’ as such”*. The PISA developers identified a relatively broad meaning of the *literacy* notion as a suitable basis on which the PISA survey should be founded, and applied this idea to all of the PISA survey domains, including that of mathematics. This idea was adopted by the expert groups formed to steer PISA development, and was subsequently adopted by the committee of the OECD charged with directing PISA.

The phrase *mathematical literacy* was well established by the time the first PISA frameworks were published (OECD, 1999) and has been in common use within PISA and in contexts influenced by PISA ever since.

While it is clear that the intention in using the *literacy* language here was to provide a contrast with school mathematics, it is not completely clear that a consistent view was being presented as to whether *mathematics* should be seen as a relatively narrow or relatively broad concept. And here lies the key tension. Is mathematical literacy a superset that includes mathematics, or is mathematics the larger animal within which mathematical literacy is a part?

The development of ‘mathematical literacy’ within PISA

The PISA 2000 framework stated that the mathematical literacy domain is concerned with the capacity of students to draw upon their mathematical competencies to meet the challenges of the future. It is concerned with students’ capacities to analyse, reason, and communicate ideas effectively by posing, formulating and solving mathematical problems in a variety of domains and situations.

This by any definition reflects a broad understanding of the mathematical literacy domain. But it seems to imply that the mathematical skills drawn on in exercising one’s mathematical literacy could be seen either as a part of, or as separate from, that literacy.

The original formal definition of PISA mathematical literacy was as follows:

> *Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen* (OECD 1999, p. 41).

This definition aimed to establish a wide perspective on mathematics, incorporating broad appreciation, judgment, engagement and use of mathematics in all aspects of life and citizenship. The framework presented several so-called *big ideas* (such as chance, change and growth, quantitative reasoning, dependency and relationships, shape) that were clusters of connected mathematical concepts that occur in real situations and contexts; it identified mathematical curricular strands that represented the content of school mathematics as implemented in many school curricula and that would provide the knowledge context for PISA items; and it identified a set of context types in which questions would be formulated. Several mathematical competencies were identified, and three competency clusters were used as an organisational tool (these came to be referred to as *reproduction, connections*, and *reflection*). The processes associated with 'mathematisation' (or modelling) were also given prominence in the original PISA design.

Questions were then developed that fit into one or other of the big ideas, that were identified with one of the context types, that called for the use of relevant curricular knowledge, that demanded activation of (usually several of) the competencies but were associated primarily with one of the competency clusters, and that called on some part of the mathematisation cycle.

In the pool of items developed for the early PISA administrations, we see a wide range of contexts used – a small number of strictly mathematical contexts, and a preponderance of contexts from the surrounding world that we might expect 15-year-olds in participating countries to meet:

Some of the contexts used involved the layout of a room, arrangement of furniture, maps, blocks, bricks, number cubes, containers; data as presented in newspapers; objects in the environment such as a running track, garden beds, floor tiles, fences.

Examples of the mathematical content knowledge required were spatial reasoning, counting, developing numeric models, interpreting graphs, extracting data from a table or graph, arithmetic calculation, finding perimeter, area, or volume; formulating or interpreting algebraic expressions, interpreting or calculating a probability.

The early PISA results pointed very clearly to deficits in the school mathematics outcomes of 15-year-olds. The evidence we have at this stage, based on four PISA survey administrations, is that students around the world are seriously challenged by the demands of the kinds of test items so far used. One of the most striking is the poverty of student abilities to communicate their thinking and reasoning.

In the ensuing several years as PISA was implemented over successive survey administrations, as participating countries received the news (good and bad) about the PISA performance of their 15-year-old students, as many of the ideas underpinning the PISA mathematics study (and indeed the other PISA survey domains) gained more currency and were debated more widely, some criticisms bubbled up.

For the initial PISA administration in 2000, only two of the four broad content areas had been developed and assessed. Some tweaking of the framework was therefore required in the lead-up to PISA 2003 when mathematics was the major assessment domain for the first time, and when the full mathematics domain was first assessed. The changes for the 2003 framework (OECD, 2003) were largely to clarify some elements, and to provide a substantial set of items to exemplify the PISA approach. A particular issue requiring clarification related to the use of the word *authentic* and to clarify that PISA was not about everyday contexts and simple mathematics, but about realistic contexts where mathematics could be authentically used to solve problems.

A strong criticism was building from countries particularly in the Spanish speaking world, but in other places too, that the word 'literacy' had such an entrenched narrow meaning in their language that it was impossible to convey the broader meaning intended by PISA in their local and national educational debates. Literacy was interpreted to mean something rather basic – like minimal competence.

As one colleague has put it,

> *… the word for "literacy" in Spanish is "alfabetización". This concept leads to very basic reading and writing abilities. So, "alfabetización matemática" would be interpreted as knowing how to count and add, more or less, but no more than that.* (Prof. María Sánchez, personal communication).

The response in Uruguay, according to my correspondent, was to refer initially to "mathematical culture", "scientific culture" and "reading comprehension". More recently the concepts of "cognitive competency", "cognitive processes", "developing of competencies for life", have gained wider acceptance there, so they now refer to *Competency in Mathematics, in Science and in Reading*.

That led to pressure to modify the PISA language. So at the organisational level, the OECD has shifted its language towards referring to PISA as an assessment of mathematics, science and reading; and where reference to 'mathematics' is not sufficient, to refer to 'mathematical competence' which is intended to convey a broad notion similar to *mathematical literacy* but aiming to avoid the negative connotations of that term. Nevertheless, within each of those survey domains, the *literacy* reference has frequently been retained at least in English and in languages that do not have such a strong association of literacy with only a basic level of understanding.

A further criticism related to the level of *rigour* in the mathematics items used. This criticism included issues of mathematical correctness, as well as criticisms of the level and visibility of mathematical content. The most damaging part of this related to a view held by some that only low-level mathematics was needed to solve many of the problems; and that they related too much to everyday and simple common experiences that did not provide the opportunity for use of higher level mathematical knowledge.

You could take different views of this: on the one hand you could recognise that the knowledge students can readily bring to bear to solve a problem for which the solution and solution path are not immediately obvious is knowledge acquired a little time ago, that has had time to be consolidated in a variety of ways through experience, practice and reflection, and through the variety of contexts in which it has appeared and been applied. When knowledge has been well consolidated, it is available in a wider range of contexts, requiring fewer supporting cues than is the case with recently acquired knowledge.

On the other hand, you could say PISA only asks for baby maths and that is a bad thing, since it does not adequately reflect the standard school mathematics outcomes for 15-year-olds. For PISA to change this, problems would need to look much more like the kinds of tasks students would have seen in class. More scaffolding would be needed to give students the cues they would need. Contexts used would probably need to be less realistic, more carefully crafted to elicit the required responses. And difficult decisions would be required about exactly which higher level content could reasonably be tested, in light of the intersection between different curriculum arrangements in each country, and the time within the school year that different countries conduct their PISA survey.

Several items having demanding mathematical content have been proposed as part of previous test development processes; some of these have been used at field trial stage, and a very small number have appeared in main survey administrations. In the table, several of the most difficult items used in the field trial leading up to the PISA 2003 survey administration are listed. A brief summary of the context and of the mathematical knowledge needed is given, together with the percent of missing responses, and the percent correct for each item.

Item	Context	Mathematics	Missing %	Correct %
1	Selecting toppings for a pizza	Combinatorics – counting combinations	41	1.1
2	Tiling patterns	Combinatorics – counting geometric arrangements	21	4.5
3	Measurement of properties of tennis balls	Surface area of cylinder	42	5.5
4	Using playing cards to build a tower	Find formula for number sequence	35	5.8
5	Insect behaviour	Write an algebraic expression	35	7.2
6	Tiling around pool	Geometric/number pattern, algebraic thinking	14	8.2
7	Interest and repayments	Explain interest calculation	59	7.9
8	Number patterns	Formula for sum of sequence	49	8.8
9	Intra-mathematical	Side lengths of triangle	22	7.6

The high rate of missing responses and particularly the low percent correct figures for many of these more difficult items renders them of limited usefulness in an international survey – it is important that the items selected provide useful data on as many students as possible. Similarly, the number of very difficult items that can be selected for such a test should also be limited. Only two of these items were selected for inclusion in the 2003 main survey item pool.

Until now, by far the majority of PISA items have presented students with contextualised problems with the conscious aim of challenging students to scour their reservoir of knowledge and skills and find what they know that could be used; and to do this they need to activate certain mathematical competencies. Students need to analyse the problem situation, to think about what mathematics or what mathematical opportunity is represented in the stimulus provided, to apply various problem-solving skills such as simplifying, comparing to other similar situations confronted previously, to use trial and error or some other solution strategy, to apply specific mathematical procedural knowledge, and so on. This may involve use of parts of the modelling cycle. It can involve both receptive and expressive communication competencies.

We know that in the ‘real world’, including in daily life, in work situations, or while undertaking further study, mathematical problems usually present themselves in contexts, not as raw undisguised mathematical content. Fundamentally it is the ability of each individual to draw on mathematical competencies that facilitate the activation of relevant mathematical skills and knowledge in that problem situation that PISA promotes and surveys.

In preparing for the PISA 2012 survey, in which mathematics was the major domain for the second time, the OECD provided strong representations as to the desirability of refocussing certain aspects of the definition and operationalisation of the PISA mathematics domain. The use of the term *literacy* was not forbidden, but the message was clear that PISA should be much more an assessment of mathematical knowledge and skills than had been the case in earlier survey administrations.

Indeed we can summarise a number of issues and criticisms that had been identified with the way the mathematics part of PISA had been defined and developed, which were filtered through the political processes governing PISA and which were presented to the PISA developers as clear starting points for PISA 2012. Here is a summary of the major issues that were put forward for resolution[2]:

- PISA should assess <u>mathematics</u>, not some restricted 'everyday' version of it
- Mathematical content (school mathematics topics) should be more easily recognisable in the survey items used
- Authentic task contexts are desirable, but should not constrain what PISA assesses, nor obscure the mathematics involved
- Use of the *literacy* notion is questioned
- Recognise university-level mathematics classrooms as an important real-world context for adults, and the increasing demand for tertiary level mathematics education
- Ensure that task difficulty is driven by the mathematics involved, not by demands of the context used

So far, we might interpret the directions as pressure in a more or less coherent direction. But I continue…

- PISA should continue to go beyond reproduction of subject-based knowledge, towards assessing whether students can extrapolate from what they have learned and apply their knowledge in novel situations
- Recognise the increased economic importance of non-routine analytic and interactive capabilities
- Retain the 'mathematical modelling' emphasis

By now I am a little confused. These elements seem not to be entirely consistent with the first few. And now I go on…

- Seek validation of the importance of what is measured, particularly in relation to the future competencies needed by students, and to the various national standards in place or under development (avoid a 'PISA orthodoxy' aligned with narrow interests)
- Secure cross-cultural relevance and appropriateness (but avoid degrading PISA to 'lowest common denominator')
- Reporting scales should be based on competencies (rather than content)

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[2] These notes are derived from the record of the 28th meeting of the PISA Governing Board, in Istanbul, Turkey on 2-4 November 2009; and from a prior presentation of those views by Andreas Schleicher to the Mathematics Expert Group meeting in Offenbach, Germany, 15-17 October 2009.

- Balance innovation (seek state-of-the-art mathematics framework) with trends (retaining links to aspects measured the past)

So at the end of the day, perhaps a key thing that was required was to develop a stronger rationale and justification for the decisions taken in formulating the framework, to achieve greater buy-in from the participating countries. This is a fair point, and it is perhaps true that sometimes in our zeal as educational reformers we fail to do enough to bring others along with us, and this often leaves a political problem.

To some extent, these instructions continue to reflect the tension I have referred to between the different views about mathematics – whether mathematics is broad and inclusive, akin to the broadest definitions of numeracy, quantitative literacy and mathematical literacy; or whether it represents a narrower set of skills and knowledge that most people will recognise as mathematics, typically from their own school experience, and that can then be applied in a variety of settings.

The tension lies fundamentally in the extent to which the focus should be on the mathematics itself (however defined) or on the use of mathematics.

Following extensive deliberations about those issues, an updated definition was eventually developed and adopted for the PISA 2012 survey as follows:

> *Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.* OECD (2010).

In one of the regular lectures at this conference, Kaye Stacey has given a detailed analysis of the new definition and the new framework (Stacey, 2012).

We can see that while some aspects of the PISA definition of mathematical literacy have changed, many have remained constant. First, the intention to treat mathematics as a broad and encompassing concept is retained; as is the use of the 'literacy' label.

Second, the underpinnings of mathematical literacy have from the start been a set of what have been labelled competencies, now referred to as fundamental mathematical capabilities, which were originally articulated following work in the 1990s in Denmark by Mogens Niss, and his colleagues (Niss and Højgaard, 2011). The PISA frameworks have tweaked the definitions and labels of this set of competencies to some degree, but they remain central to what PISA mathematical literacy means in practice. Indeed working with PISA mathematics survey items and the data they generate has demonstrated even more strongly the fundamental importance of these competencies.

A focus on *competencies*

On the ground, those responsible for the development of PISA mathematics survey items have really taken that set of competencies to heart.

Members of the Mathematics Expert Group have carried out some interesting research involving the rating of mathematics items according to the extent to which they demand activation of a set of competencies based on those laid out in the PISA framework (Turner, Dossey, Blum and Niss, 2012; Turner, 2012). A scheme has been developed that defines levels of activation of each competency, and a process established for rating each item against each of the competencies. That research has shown that the PISA competencies are crucial determiners of the literacy demand of survey items, and that the level of demand for activation of the competencies is a powerful predictor of the difficulty and the cognitive complexity of those items (Turner & Adams, 2012).

As the pool of items was developed for the PISA 2012 administration, items were scrutinised carefully according to their demand for activation of each competency. This helped to ensure items were developed for PISA that varied in their difficulty, and that were centred on the competencies.

We can only hope that as this insight is more widely shared, a sharper focus will be placed on acquiring and developing those competencies in our mathematics classrooms.

So why bother? Why should we care about mathematical literacy?

Countless studies and reports, from many countries around the world, have concluded that the outcomes of our mathematical education systems are sub-optimal. Specifically, we see time and again statements to the effect that our students are not adequately prepared with the skills, knowledge and dispositions they will need as we progress further into the twenty-first century.

PISA has brought starkly into focus questions like: why are we so worried about now many kids know that sin(pi/2)=1 when they struggle to use their arithmetic skills effectively to solve real problems.

Moreover we see a persistence of the phenomenon of ‘mathematics terror’ and ‘mathematics avoidance’ in the general community which frequently has the effect of preventing ordinary people from even considering how their mathematical knowledge might be applied in their lives. And we see consistently in many countries a continuing decline in the numbers of students progressing through their school education to enter tertiary courses in the mathematical sciences.

Increasingly we have the outcomes of good analytic research into the mathematical needs of the modern workplace that show mathematical demands are expanding across all sectors of the workforce, and deepening to affect more and more kinds of work (for example, see Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002). But a widely held view is that educational outcomes are not adequately keeping up with those changed demands.

And we have observations of daily societal opportunities and demands with clear potential to make mathematical behaviour a routine part of everyday life for virtually everybody in today’s world. We see commonly expressed predictions that these demands will only increase as time goes on.

I think this is why we care: we love mathematics ourselves; we have some insight into its beauty and its utility. We believe that mathematics has something to offer people in our world. We think that people will be better off if they have a more positive attitude to the possibilities that mathematics offers, a willingness to bring their mathematical knowledge to bear on the situations they meet and the problems they want to solve. We suspect that a little success breeds a little more success, so that as people find ways to use their mathematical knowledge, their capacity to do so again in the future increases.

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