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Shape selection of twist-nematic-elastomer ribbons

Yoshiki Sawa, Fangfu Ye, Kenji Urayama, Toshikazu Takigawa, Vianney Gimenez-Pinto, Robin L. B. Selinger, and Jonathan V. Selinger

Department of Material Chemistry, Kyoto University, Kyoto 615-8510, Japan; Liquid Crystal Institute, Kent State University, Kent, OH 44242; and Department of Physics and Institute for Condensed Matter Theory, University of Illinois, Urbana, IL 61801-3080

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How microscopic chirality is reflected in macroscopic scale to form various chiral shapes, such as straight helicoids and spiral ribbons, and how the degree of macroscopic chirality can be controlled are a focus of studies on the shape formation of many biomaterials and supramolecular systems. This article investigates both experimentally and theoretically how the chiral arrangement of liquid crystal mesogens in twist-nematic-elastomer films induces the formation of helicoids and spiral ribbons because of the coupling between the liquid crystalline order and the elasticity. It is also shown that the pitch of the formed ribbons can be tuned by temperature variation. The results of this study will facilitate the understanding of physics for the shape formation of chiral materials and the designing of new structures on basis of microscopic chirality.

Recent researches have revealed that chirality plays a critical role in controlling the shape of self-assembled supramolecular aggregates. A range of chiral shapes, including tubes with “barber-pole” markings, spiral ribbons (with cylindrical curvature and helical central line), and helicoids (with Gaussian saddle-like curvature and straight central line), have been observed in a rich variety of biological materials and their synthetic analogues. These materials include several amphiphiles (1–3), peptides (4–7), diacytelenic lipids (8, 9), gemini surfactants (10, 11), and multicomponent mixtures in bile (12, 13). Such aggregates often become bilayers, and the bilayer membranes form tubules, spiral ribbons, or helicoids in contrast to the normal expectation that the minimum energy state of the bilayers would be flat, or be large spherical vesicles with the minimum curvature. In addition, the correlation between material characteristics and the macroscopic shape of chiral aggregates is markedly complicated. For example, mixed bilayers of saturated and diacytelenic phospholipids change their shape between micron-scale cylindrical tubules, spiral ribbons, or nanometer-scale tubules, in response to temperature variation (14). Charged gemini surfactants with chiral counterions show a transition between spiral ribbons and helicoids as a function of molecular chain length (10). Many theoretical studies have been reported to explain what determines the size and shape of tubules, helicoids, and spiral ribbons (10, 15–18).

In this article, we will show, both experimentally and theoretically, how a flat twist-nematic-elastomer (TNE) film when subjected to temperature change can easily achieve the goal of shape selection between helicoids and spiral ribbons. Nematic elastomers (NEs) are a unique class of materials (19–23). Formed by cross-linking liquid crystalline polymers, NEs possess both the elastic properties of rubbers and the orientational properties of liquid crystals. The combination of these two properties makes the shape of NEs very sensitive to external stimuli. In this article, we will focus on NE films in which the nematic order orientation changes smoothly by 90° from the bottom surface to the top surface with the director at the midplane parallel to the long or short axis of the film as shown in Fig. 1. The director’s change in orientation through the film’s thickness leads to gradients in the induced strain when the film is heated or cooled, giving rise to complex changes in the sample’s overall shape. Broer and coworkers (24, 25) reported that temperature variation or light irradiation can turn a flat ribbon of densely cross-linked stiff liquid crystal networks with a similar twist orientation into a ribbon of saddle-like shape or spiral ribbon. Godinho et al. (26) showed that the self-winding of helices in jets and electropun fibers obtained from cellulosic mesophases result from an off-axis core line defect disclination. The NEs (i.e., loosely cross-linked elastomeric liquid crystal networks) possess considerably stronger coupling between mesogen orientation and network deformation than the materials in these studies (24–26). We will demonstrate that the NE films with the twist configuration mentioned above form helicoids or spiral ribbons in response to temperature variation. We will also show that these TNE films select the shape according to the value of their width-to-thickness ratio: the narrow TNE films twist around their central lines upon temperature change, forming helicoids; however, when the width-to-thickness ratio of the films becomes larger than a certain critical value, the central lines of the films curve into helices and the films themselves form spiral ribbons. In addition, we will also show how the helical/twist pitch and handedness change with temperature and the width-to-thickness ratio of the films.

In contrast to the chiral supramolecular aggregates of the order of nano- or micrometer scale, the shape selection and variation of the TNE films occur in the macroscopic scale of the order of milli- or centimeter, and the dimensional parameters of the TNE films can be experimentally varied as desired and the pitch of formed ribbons can thus be well controlled. These features facilitate to characterize the geometrical parameters of the shapes and to elucidate the dimensional effect on the shape selection. The results of the present study will provide an important basis of the understanding of the physics for the shape formation of chiral materials, and the designing of new structures on basis of microscopic chirality.


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1Y.S. and F.Y. contributed equally to this work.

2To whom correspondence should be addressed. E-mail: urayama@rheogate.polym.kyoto-u.ac.jp or jselinge@kent.edu.

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The strong long effect on the ribbon shape originates from a large change in local nematic order induced by $T$ variation. In fact, in the nematic phase of $T \leq T_{N}$ ($T_{N} = 367$ K is the nematic-isotropic transition temperature) with $T$-dependent nematic order, the structural parameters strongly depend on $T$, while they are independent of $T$ in the isotropic phase of $T > T_{N}$ with no nematic order (Figs. 3C and 4C). The ribbons become almost flat at a certain temperature ($T_{\text{flat}}$) around 353 K, as shown in Fig. 3A. It should be noted that $T_{\text{flat}}$ is different from the preparation temperature of the films ($T_{0} = 313$ K). The preparation state involving ca. 50 wt% solvent at $T_{0}$ corresponds to the initial flat state with twist alignment, but the subsequent volume reduction to the dry state causes a twist distortion due to a finite strain gradient driven by anisotropic shrinking. In general, a finite volume change in nematic gels results in a considerably anisotropic shape variation (30, 31). In fact, the ribbons at the temperatures slightly above $T_{0}$ show a considerable twist (Figs. 3C and 4C), although no structural data in the dry state at $T_{0}$ was obtained due to the glassy state: The glass transition temperature of the dry film was ca. 50 °C.

The direction of the director at the midplane relative to the long axis of the ribbons change the handedness of the helicoids and spiral ribbons: At the temperatures below $T_{\text{flat}}$, the helicoids and spiral ribbons with L-geometry are right-handed, while those with S-geometry are left-handed (Figs. 3 and 4). In other words, the cooling (that is, an increase in local nematic order) induces the right- and left-handed winding in L- and S-geometries, respectively.

The shape transition between helicoids and spiral ribbons driven by width variation is clearly observed in the high-temperature isotropic state where the structural parameters are substantially independent of $T$ (Fig. 5). For both L- and S-geometries, the shape transition occurs, but the critical width of the transition for L-geometry ($w_{c} \approx 0.3$ mm) is smaller than that for S-geometry ($w_{c} \approx 0.4$ mm). We also notice that the spiral ribbons with L- and S-geometries are not symmetric, and the spiral ribbon with L-geometry is wound a little more tightly than that with S-geometry: The values of $p_{T}$ and $d$ with L-geometry are slightly smaller than those with S-geometry. This trend is also observed in Fig. 4. In contrast, the helicoids with L- and S-geometries are almost symmetric, which can be seen in Fig. 3.

**Theory.** We develop a theoretic model to explain the experimental results. The two physical quantities we need are liquid crystal order tensor $Q_{ij}$ and nonlinear strain tensor $\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial x_{i}}{\partial u_{j}} + \frac{\partial x_{j}}{\partial u_{i}} - \delta_{ij} \frac{\partial x_{k}}{\partial u_{k}} \right)$, where the Einstein summation convention on repeated indices has been assumed. $x$ and $u$ represent, respectively, the position of a mass point before deformation and the corresponding displacement vector induced by the deformation. The elastic energy is assumed to take a simple form,

$$f = \mu (\epsilon_{ij} Q_{ij} - \alpha \epsilon_{ij}^{2} Q_{ij}).$$

![Fig. 3. Helicoids formed by the narrow TNE films with the thickness 25.2 μm. (A) Shape of the L-geometry ribbons: left-handed at 370 K; almost flat at 353 K; right-handed at 330 K. The ribbon width is 0.23 mm. (B) Shape of the S-geometry ribbons: right-handed at 378 K; left-handed at 334 K. The ribbon width is 0.22 mm. (C) Temperature dependence of the inverse of the twist pitch ($p_T$). The pitches of the left- and right-handed twist are defined to be positive and negative, respectively. The open and filled symbols represent the data obtained in the cooling and heating processes, respectively; and the (red) circles and the (blue) squares represent the data of the L-geometry and those of the S-geometry, respectively. The lines represent the theoretical predictions.](image)
where \( \mu \) is the shear modulus, \( \alpha \) the coupling constant, and \( \delta Q_{ij} \) the change of liquid crystal order tensor induced by external stimuli. Given the incompressibility constraint \( \epsilon_{ii} = 0 \) for small deformations, this simple energy form is then good enough to capture the main feature of the problem we study here. Note that \( \delta Q_{ij} \) is now defined in the body frame to ensure rotational invariance (32). We further assume that when subjected to external stimuli the change of the body-frame order tensor is mainly caused by the order parameter change \( \delta S \) and that the principle axes of the order tensor keep fixed in the body frame although they do rotate with the film in the laboratory frame, namely, we assume \( \delta Q_{ij} = \delta S(n_i n_j - \frac{1}{3} \delta_{ij}) \), where \( n \) is the orientation of the nematic director at \( T_{\text{flat}} \), twisting from the bottom surface to the top surface of the film. Integrating the above energy density along the direction perpendicular to the film plane and along the short axis of the film yields an effective 1D energy density (17) as a function of the curvature tensor \( C_{ij} \),

\[
f_{1D} = \mu dw [a_K K^2 + b C_{wy} + a_x C_{xx}^2 + a_y C_{yy}^2 + \sqrt{a_x a_y} (2 C_{xy}^2 + K)],
\]

[2]

where \( d \) and \( w \) are respectively the thickness and width of the film, \( K = C_{xx} C_{yy} - C_{xx} \) is the Gaussian curvature, and \( x \) and \( y \) represent, respectively, the directions parallel to the long and short axis of the film. The curvature tensor equals \( C_{ij} = \partial h / \partial x_i \partial x_j \), where \( h \) is the out-of-plane normal displacement. The linear term in \( C_{wy} \) is a chiral symmetry-breaking term, inducing the film to twist; the first term represents the in-plane stretch energy cost (33); and the other terms are the bending energy cost. The coefficients, which will be given in the Methods section, are determined by three parameters: the ratio between \( w \) and \( d \), the product of the coupling constant \( \alpha \) and the order parameter change \( \delta S \), and the twist angle span \( \theta_3 \).

The theoretical results can be obtained by minimizing \( f_{1D} \) with respect to \( C_{ij} \) for a given temperature change, narrow films form helicoids with the curvature of their central line \( C_{wy} \) being zero and the torsion of their central line \( C_{wx} \) determined by the equation \( b + 2 \sqrt{a_x a_y} C_{wy} + 4a_K C_{wy} = 0 \); wide films form spiral ribbons with \( C_{xx} = b^2 / (64 a_x a_y \sqrt{a_x a_y}) - 4a_x C_{wy} \) and \( C_{yy} = -b^2 (8/a_x a_y) \). The Gaussian curvature is \( K = (1/4) (a_x a_y C_{xx} + a_y C_{yy}) / (a_K C_{xx} + a_y) \). Note that \( K \) is always negative; only in the limit \( w/d \rightarrow \infty \), \( K \) tends to zero and the films form perfect spiral ribbons. (Ribbons of various curvatures are shown in Fig. S1). The critical width \( w_c \) that determines what type of ribbon to form is given by the condition \( a_x b^2 = 9 (a_x a_y)^{3/2} \). In the case of \( \theta_3 = \pi/2 \), this condition simplifies to

\[
(w_c / d)^2 = 3 \frac{40}{3} \left( 1 + \frac{1}{6} \right) \left( \frac{\alpha \cdot \delta S}{\alpha S} \right)^2 + \frac{1}{6} \frac{1}{\pi} \left[ \frac{1}{2} + \frac{1}{\pi} \right] \].

[3]

To compare the theoretical predictions with the experimental observations, we proceed to connect the product of the coupling constant and the order parameter \( \alpha S \) to temperature \( T \). For this purpose, we examine how much a corresponding nematic elastomer with planar orientation extends as temperature is lowered. It is easy to deduce from Eq. 1 that the extension along the nematic director in this case is \( \Lambda = \sqrt{1 + \frac{1}{3} \alpha S} \). By fitting the experimentally measured curve of \( \Lambda(T) \) (see Fig. S2), we obtain \( \alpha S = 3.3 (1.01 + T/T_N)^{2/3} \) as the temperature dependence of \( \alpha S \).

Given the relation between \( T \) and \( \alpha S \), we can then determine how \( C_{xx}, C_{wy} \), and the critical width \( w_c \) vary with temperature \( T \). The pitch and the diameter of the helicoids and spiral ribbons can thus be determined accordingly. The theoretical predictions of their change with temperature are plotted in Fig. 3C and Fig. 4C. In Fig. 4C, the thin dashed lines represent the pitch of helicoids; outside the thin-dashed-line region, the helicoids become spiral ribbons whose diameter first increases rapidly and then decreases slowly with temperature change (see Fig. S1 for an illustration of the shape change). Fig. 5 shows how the pitch and diameter of the ribbons vary with the width. From these figures we can find that the theoretical results agree well with the experimental observa-

![Fig. 4](image-url)  
Spiral ribbons formed by the wide TNE films with the thickness 35.2 \( \mu \)m. (A) Shape of the L-geometry ribbons: left-handed at 374 K; right-handed at 336 K. The ribbon width is 0.76 mm. (B) Shape of the S-geometry ribbons: right-handed at 374 K; left-handed at 330 K. The ribbon width is 0.83 mm. (C) Temperature dependence of the inverses of the helical pitch \( (p_h) \) and the diameter \( (d) \), where the lines represent the theoretical predictions. The pitches of the left- and right-handed twist are defined to be positive and negative, respectively. The inverse of the pitches is represented by the open symbols and the inverse of the diameters by the filled symbols. The (red) circles and (blue) squares represent, respectively, the data of the L-geometry and the data of the S-geometry. The thick (solid or dashed) lines represent the theoretical predictions for the structural parameters of spiral ribbons, and the thin dashed lines are the theoretical prediction of the pitch inverse of helicoids.

![Fig. 5](image-url)  
Width effect on the shape selection of the TNE ribbons with (A) L-geometry and (B) S-geometry. The ribbon thickness is 35.2 \( \mu \)m. The shape is observed at 378 K in the isotropic state. The shape transition occurs at ca. 0.3 mm and ca. 0.5 mm for L- and S-geometries, respectively. The (red) triangles represent the pitch inverse of the helicoids, and the open and filled (blue) squares represent the pitch inverse and diameter inverse of the spiral ribbons, respectively. The lines are the theoretical predictions.
to be $90^\circ$ in the experiments, our theory is also able to predict how

due to the saddle-like shape having no definite winding. There-

infinity as

spiral ribbons to helicoids occurring at

assessments of the theoretical prediction of the transition from

in Figs. 3

characterization of the shape at the temperatures near

induced by the chiral arrangement of achiral molecules rather

for the low temperature part of the L-geometry in Fig. 4

curvature but no bending; while, on the contrary, those with a

width-to-thickness ratio are easier to be stretched than to be

of the films depend on the competition between the bending

temperature change. The optimal shape and curvature (34, 35)

In summary, TNE films can form helicoids or spiral ribbons upon

behavior afterwards. A detailed analysis of

how the twist configuration along the normal direction of films
changes the ribbon shape will be presented elsewhere.

Methods

Materials. The monoacrylate mesogenic monomer A-6OCB (Fig. 1) was

synthesized by the method described in literature (36). 4-$\text{H}$-Hexyl oxy-$4'-$

methylphenyl 60CB (Sigma-Aldrich) and 1,6-hexanediol diacylate (Aldrich)

were employed as miscible nematic solvent and cross linker, respec-

tively, and they were used as received. The chiral dopant S-811 (Fig. 2) and

photoinitiator bis(cyclopentadienyl)bis[2, 6-difluoro-3-(1-pyrryl)phenyl]
tita-

nium (Irganacure 784®) were kindly supplied from Merck KGaA and Chiba

Specialty Chemicals Co., respectively. The mixing of A-6OCB with the nematic

solvent was required to broaden the temperature range of the nematic

ribbon, because the nematic phase of A-60CB is very narrow (only ca. 2°C)
due to the high crystallizability. The mixing ratio of A-60CB and nematic

solvent was 5:4 by weight. The cross-linker concentration was 7 mol%.

Sample Preparation. The chiral properties of the mixtures of A-60CB, 60CB,

and S-811 were characterized by the pitch of the resultant helicoidal struc-

corresponding to the distance of a $2\pi$ screw rotation of the molecular

packing. The pitch was evaluated by the Cano wedge method (37, 38). The

infrared spectra was almost the same as that of the mesogenic polymer.

The concentration of S-811 (0.06 wt%) was adjusted such that the total twist

angle could become $90^\circ$, the distance between the two substrates being 40 $\mu$m.

For cross-linking reaction, the glass cell was irradiated using a xenon lamp

with emission at a wavelength 526 nm for 30 min. The cross-linking tempera-

ture was $40^\circ\text{C}$ which is ca. 8°C lower than the transition temperature of the

mixture. The cell was immersed in dichloromethane for several days until the
cross-linked gel film detached from the glass substrates due to the swelling

pressure. The films were allowed to swell fully in dichloromethane in order to

wash out the unreacted materials and chiral dopants. The swollen gel

films were gradually deswollen by stepwise addition of methanol to dichlor-

omethane. The ribbon specimens with L- and S-geometries (Fig. 18) were
cut out from the dried film at a temperature where the film became flat

($T_{\text{flat}} = 353 K$), because the dried film at room temperature was considerably

curled and twisted. We confirmed that the dried film possessed a $90^\circ$ twist

rotation of director by polarized optical microscopy.

Measurements. The length and width of the ribbon specimens were measured

by optical microscope. The thickness of the ribbon was evaluated to be

35.2 $\mu$m using a laser displacement sensor LT-9500 and LT-9010M (Keyence)

at $T_{\text{flat}}$. The twist angle of the ribbons immersed in a custom-made temperature-

controllable bath of a silicone oil was observed with an optical microscope

with CCD camera. The silicone oil was a nonsolvent for the specimen. The

ribbons in the oil bath were subjected to no mechanical constraint. The twist

pitch, helical pitch, and diameter at each temperature were evaluated by

video analysis. The temperature was varied stepwise after confirming the

equilibration of the shape of the specimen at each temperature.

Derivation of the 1D Effective Energy. We use the energy given in Eq. 1. The

nematic director $n$ remains parallel to the film plane (or the xy-plane), and

therefore, $\partial Q_{xy} = \partial Q_{yx} = 0$. The angle $\theta$ between the director $n$ and

the long axis of the film (or the x-axis) changes linearly with the depth to the film

surfaces: for the L-geometry, we have $\theta = -\delta z/2$, where $z$ is the distance to the

midplane of the film, and $\delta z$ is the twist angle span, namely, the dif-

ference between the angle on the top surface ($z = d/2$) and the angle on the

bottom surface of the ribbon ($z = -d/2$) for the S-geometry, $\theta = -\pi/2 + z/2$. The

Frank energy has been ignored here because the typical length scale, i.e., the pitch

of the ribbons, is larger than the pitch. Given the traceless constraint and the condition that $\epsilon_{xx} = \epsilon_{yy} = 0$, the strain tensor has only three independent components, $\epsilon_{xx}$, $\epsilon_{yy}$, and $\epsilon_{xy}$. To reduce the full

$\theta$, affects the ribbon shape: approximately speaking, for fixed

width and temperature, when $\theta$ increases from zero, the pitch

and diameter of the formed ribbons first decrease, reaching a

minimum at somewhere between $\pi/2$ and $\pi$, and then show a

decaying oscillation behavior afterwards. A detailed analysis of

how the twist configuration along the normal direction of films
changes the ribbon shape will be presented elsewhere.

Discussion

In summary, TNE films can form helicoids or spiral ribbons upon

temperature change. The optimal shape and curvature (34, 35)

of the films depend on the competition between the bending

energy cost and the in-plane elastic energy cost. Films with a small

width-to-thickness ratio are easier to be stretched than to be

bended and thus form helicoids, which have a large Gaussian

curvature but no bending; while, on the contrary, those with a

large width-to-thickness ratio are easier to be bended than to

be stretched and thus form spiral ribbons, with a small Gaussian

curvature but a large bending. The pitch and handedness of the

formed ribbons are completely determined by only two para-

ceters: the temperature and the width-to-thickness ratio.

To obtain helicoids/spiral ribbons, a chiral symmetry-breaking

driving force is a must (otherwise temperature change would

cause only a uniform stretch or shrink of the whole sample).

In the systems we consider, the twist (chiral) arrangement of the

achiral nematogens from the bottom to the top surface of the film

was initially induced by the chiral dopants. After the removal of

these dopants, this twist (chiral) arrangement is memorized by the

polymer network; and it yields the required chiral driving

cforce, which when temperature change occurs and induces a tendency

of local shape change (stretch or shrink along the nematic

director). Therefore, the chiral symmetry breaking in TNE films is

induced by the chiral arrangement of achiral molecules rather

than by the chiral molecular-structure as in the case of supramo-

lecular aggregates.

On the whole, the experimental results agree well with the the-

oretical results. However, in the experiments, the precise charac-

terization of the shape at the temperatures near $T_{\text{flat}}$ was difficult

due to the saddle-like shape having no definite winding. There-

fore, the data of the structural parameters near $T_{\text{flat}}$ are lacking in Figs. 3C and 4C.

This difficulty precluded the experimental assessment of the theoretical prediction of the transition from

spiral ribbons to helicoids occurring at $T^*$ close to $T_{\text{flat}}$ for wide

TNE films. The reasons causing the relatively large difference between the experimental results and the theoretical results for the low temperature part of the L-geometry in Fig. 4C are not clear to us.

We also investigated samples in which the nematic director on

the midplane of the films does not align along the long or short

ribbon axis. In this case, straight helicoids are not allowed

because of the absence of the antisymmetry about the midplane.

The dependence of the shape of the formed spiral ribbons on the

width and temperature is similar to those of the L-geometry and

S-geometry described above. Although the span angle $\theta$ is fixed to

be $90^\circ$ in the experiments, our theory is also able to predict how

$\theta$, affects the ribbon shape: approximately speaking, for fixed

width and temperature, when $\theta$ increases from zero, the pitch

and diameter of the formed ribbons first decrease, reaching a

minimum at somewhere between $\pi/2$ and $\pi$, and then show a

decaying oscillation behavior afterwards. A detailed analysis of

how the twist configuration along the normal direction of films
changes the ribbon shape will be presented elsewhere.

Fig. 6. Theoretical Phase diagram of TNE ribbons in the width-temperature space. The ribbon thickness is 35.2 $\mu$m. The phase boundary of the L-geometry and the S-geometry is represented, respectively, by the red solid line and the blue dashed line. The handedness of the formed ribbons changes as the dotted line at $T_{\text{flat}}/T_{\text{flat}} = 0.962$ is crossed.
3D energy density to an effective 2D one, we then express the displacement vector \( u \) of points away from the midplane in terms of the displacement vector \( u^\alpha \) of points on the midplane (33). To the linear order of \( z \), we have \( u_x = u^\alpha_x z - \partial \alpha u^\alpha_x / \partial x \), \( u_y = u^\alpha_y z - \partial \alpha u^\alpha_y / \partial y \), and \( u_z = u^\alpha_z z - \partial \alpha u^\alpha_z / \partial y \). Substituting the expressions for \( u \) and \( \alpha \) into Eq. 1 and then integrating along the \( z \)-direction yield a 2D energy density including terms linear in \( \kappa \) and \( \epsilon_{\mu
u} \). Whose space reference is the equilibrium state of \( T_{0}\alpha \). To eliminate these linear terms, we change the reference space of the strain tensor to a new state, the equilibrium state of the case that the out-of-plane normal displacement is suppressed, which is acquired by a deformation from the flat geometry with the stretch along the long ribbon axis to be \( \Lambda_{\alpha\beta} = [1 + \chi \cdot \delta S_{\alpha\beta} + \sin \theta_{\alpha} \theta_{\beta}]^{1/2} \) and the stretch along the short axis to be \( \Lambda_{\mu\nu} = [1 + \chi \cdot \delta S_{\mu\nu} - \sin \theta_{\mu} \theta_{\nu}]^{1/2} \) for the L-geometry and \( \Lambda_{\alpha\beta} = [1 + \chi \cdot \delta S_{\alpha\beta} - \sin \theta_{\alpha} \theta_{\beta}]^{1/2} \) and \( \Lambda_{\mu\nu} = [1 + \chi \cdot \delta S_{\mu\nu} + \sin \theta_{\mu} \theta_{\nu}]^{1/2} \) for the S-geometry. We then express the in-plane elastic energy as a function of the Gaussian curvature (33). Assuming the curvature is nearly constant on the ribbon surface (17) and integrating the 2D energy density along the short ribbon axis, we thus obtain Eq. 2, in which the coefficients are \( a_{\alpha\beta} = \frac{1}{6} \delta_{\alpha\beta} \lambda_{\alpha\beta} \lambda_{\alpha\beta} \), \( a_{\alpha\beta} = \frac{1}{6} \delta_{\alpha\beta} \lambda_{\alpha\beta} \), \( a_{\mu\nu} = \frac{1}{6} \delta_{\mu\nu} \lambda_{\mu\nu} \), and \( b = \pm \chi \cdot \lambda_{\alpha\beta} \cdot \lambda_{\mu\nu} \cdot (\sin \theta_{\alpha} \theta_{\beta} + \cos \theta_{\mu} \theta_{\nu}) \) (* for the L-geometry and – for the S-geometry). The L-geometry and S-geometry are not exactly symmetric to each other because the \( \lambda_{\alpha\beta} \) and \( \lambda_{\mu\nu} \) of the L-geometry are different from those of the S-geometry.

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