Shape Selection of Twist-Nematic-Elastomer Ribbons

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How microscopic chirality is reflected in macroscopic scale to form various chiral shapes, such as straight helicoids and spiral ribbons, and how the degree of macroscopic chirality can be controlled are a focus of studies on the shape formation of many biomaterials and supramolecular systems. This article investigates both experimentally and theoretically how the chiral arrangement of liquid crystal mesogens in twist-nematic-elastomer films induces the formation of helicoids and spiral ribbons because of the coupling between the liquid crystalline order and the elasticity. It is also shown that the pitch of the formed ribbons can be tuned by temperature variation. The results of this study will facilitate the understanding of physics for the shape formation of chiral materials and the designing of new structures on basis of microscopic chirality.

Fig. 1. Schematics of the director configuration of TNE ribbons: (A) side view, (B) top view of the L- and S-geometry. Nematic director twists left-handed by 90° between the top and bottom surfaces with the director at the midplane parallel to the long or short axis of the ribbon.

In this article, we will show, both experimentally and theoretically, how a flat twist-nematic-elastomer (TNE) film when subjected to temperature change can easily achieve the goal of shape selection between helicoids and spiral ribbons. Nematic elastomers (NEs) are a unique class of materials (19–23). Formed by cross-linking liquid crystalline polymers, NEs possess both the elastic properties of rubbers and the orientational properties of liquid crystals. The combination of these two properties makes the shape of NEs very sensitive to external stimuli. In this article, we will focus on NE films in which the nematic order orientation changes smoothly by 90° from the bottom surface to the top surface with the director at the midplane parallel to the long or short axis of the film as shown in Fig. 1. The director’s change in orientation through the film’s thickness leads to gradients in the induced strain when the film is heated or cooled, giving rise to complex changes in the sample’s overall shape. Broer et al. (24, 25) reported that temperature variation or light irradiation can turn a flat ribbon of densely cross-linked stiff liquid crystal networks with a similar twist orientation into a ribbon of saddle-like shape or spiral ribbon. Godinho et al. (26) showed that the self-winding of helices in jets and electrosprun fibers obtained from cellulose mesophases results from an off-axis core line defect disclination. The NEs (i.e., loosely cross-linked elastomeric liquid crystal networks) possess considerably stronger coupling between mesogen orientation and network deformation than the materials in these studies (24–26). We will demonstrate that the NE films with the twist configuration mentioned above form helicoids or spiral ribbons in response to temperature variation. We will also show that these TNE films select the shape according to the value of their width-to-thickness ratio: the narrow TNE films twist around their central lines upon temperature change, forming helicoids; however, when the width-to-thickness ratio of the films becomes larger than a certain critical value, the central lines of the films curve into helices and the films themselves form spiral ribbons. In addition, we will also show how the helical/twist pitch and handedness change with temperature and the width-to-thickness ratio of the films.

In contrast to the chiral supramolecular aggregates of the order of nano- or micrometer scale, the shape selection and variation of the TNE films occur in the macroscopic scale of the order of milli- or centimeter, and the dimensional parameters of the TNE films can be experimentally varied as desired and the pitch of formed ribbons can thus be well controlled. These features facilitate to characterize the geometrical parameters of the shapes and to elucidate the dimensional effect on the shape selection. The results of the present study will provide an important basis of the understanding of the physics for the shape formation of chiral materials, and the designing of new structures on basis of microscopic chirality.
The strong long from the film sheet so that the director at the midplane is either director configuration even after the removal of the chiral dopant other. The polarized optical microscopy confirmed that the dried with uniaxially rubbed polyimide layer, and they were placed in glass substrates. The surfaces of the glass substrates were coated induces a 90° left-handed rotation of director between the two glass substrates. The surfaces of the glass substrates were coated with uniaxially rubbed polyimide layer, and they were placed in such a manner that the rubbing direction could cross with each other. The polarized optical microscopy confirmed that the dried elastomeric films with a thickness of 35.2 μm possessed a 90° twist director configuration even after the removal of the chiral dopant and nematic solvent. The long ribbon specimens were cut out from the film sheet so that the director at the midplane is either along the long axis or along the short axis of the ribbons (designated as L- and S-geometry, respectively, see Fig. 1). The ribbons with various widths (ca. 0.2 ~ 0.8 mm) were prepared, and the length of the ribbons (5 ~ 10 mm) was considerably longer than the widths. We observed the shape of the ribbons immersed in a temperature-controllable silicone oil bath by optical microscopy. The silicone oil is a nonsolvent for the TNEs, and the ribbons have no mechanical constraint in the oil bath.

We find that the TNE ribbons select a helicoid or spiral ribbon depending on the width. In the narrow case, the shape is a helicoid with Gaussian saddle-like curvature (Fig. 3), while the shape in the wide case is a spiral ribbon with cylindrical curvature (Fig. 4). In both cases, the temperature (T) variation greatly changes the structural parameters of the ribbons such as twist pitch (pT) (of helicoids), helical pitch (pH), and diameter (d) (of spiral ribbons), involving a reversal of handedness (Figs. 3 and 4). This shape change is thermally reversible (Fig. 3C). The strong T effect on the ribbon shape originates from a large change in local nematic order induced by T variation. In fact, in the nematic phase of T < TNI (TNI = 367 K is the nematic-isotropic transition temperature) with T-dependent nematic order, the structural parameters strongly depend on T, while they are independent of T in the isotropic phase of T > TNI with no nematic order (Figs. 3C and 4C). The ribbons become almost flat at a certain temperature (Tflat) around 353 K, as shown in Fig. 3A. It should be noted that Tflat is different from the preparation temperature of the films (Tc = 313 K). The preparation state involving ca. 50 wt% solvent at Tc corresponds to the initial flat state with twist alignment, but the subsequent volume reduction to the dry state causes a twist distortion due to a finite strain gradient driven by anisotropic shrinking. In general, a finite volume change in nematic gels results in a considerably anisotropic shape variation (30, 31). In fact, the ribbons at the temperatures slightly above Tc show a considerable twist (Figs. 3C and 4C), although no structural data in the dry state at Tc was obtained due to the glassy state: The glass transition temperature of the dry film was ca. 50°C.

The direction of the director at the midplane relative to the long axis of the ribbons change the handedness of the helicoids and spiral ribbons: At the temperatures below Tflat, the helicoids and spiral ribbons with L-geometry are right-handed, while those with S-geometry are left-handed (Figs. 3 and 4). In other words, the cooling (that is, an increase in local nematic order) induces the right- and left-handed winding in L- and S-geometries, respectively.

The shape transition between helicoids and spiral ribbons driven by width variation is clearly observed in the high-temperature isotropic state where the structural parameters are substantially independent of T (Fig. 5). For both L- and S-geometries, the shape transition occurs, but the critical width of the transition for L-geometry (wc ≈ 0.3 mm) is smaller than that for S-geometry (wc ≈ 0.4 mm). We also notice that the spiral ribbons with L- and S-geometries are not symmetric, and the spiral ribbons with L-geometry is wound a little more tightly than that with S-geometry. The values of pT and d with L-geometry are slightly smaller than those with S-geometry. This trend is also observed in Fig. 4. In contrast, the helicoids with L- and S-geometries are almost symmetric, which can be seen in Fig. 3.

Theory. We develop a theoretic model to explain the experimental results. The two physical quantities we need are liquid crystal order tensor Qij and nonlinear strain tensor εij = 1/2 (∂ui/∂xj + ∂uj/∂xi + ∂xj/∂ui/∂xj), where the Einstein summation convention on repeated indices has been assumed. x and u represent, respectively, the position of a mass point before deformation and the corresponding displacement vector induced by the deformation. The elastic energy is assumed to take a simple form,

\[ f = μ(εijε_\text{ij} - αε_\text{ij}δQ_\text{ij}). \]  

\[ \text{[1]} \]
where \( \mu \) is the shear modulus, \( \alpha \) the coupling constant, and \( \delta Q_{ij} \) the change of liquid crystal order tensor induced by external stimuli. Given the incompressibility constraint \( \epsilon_{ii} = 0 \) for small deformations, this simple energy form is then good enough to capture the main feature of the problem we study here. Note that \( \delta Q_{ij} \) is now defined in the body frame to ensure rotational invariance (32). We further assume that when subjected to external stimuli the change of the body-frame order tensor is mainly caused by the order parameter change \( \delta S \) and that the principle axes of the order tensor keep fixed in the body frame although they do rotate with the film in the laboratory frame, namely, we assume \( \delta Q_{ij} = \delta S(n_i n_j - \frac{1}{3} \delta_{ij}) \), where \( n \) is the orientation of the nematic director at \( T_{\text{flat}} \), twisting from the bottom surface to the top surface of the film. Integrating the above energy density along the direction perpendicular to the film plane and along the short axis of the film yields an effective 1D energy density (17) as a function of the curvature tensor \( C_{ij} \),

\[
f_{1D} = \mu dw |a_K K^2 + b C_w + a_x C_{xx} + a_y C_{yy} + \sqrt{a_x a_y} (2 C_{xy} + K)|,
\]

where \( d \) and \( w \) are respectively the thickness and width of the film, \( K = C_w C_{yy} - C_{xx} \) is the Gaussian curvature, and \( x \) and \( y \) represent, respectively, the directions parallel to the long and short axes of the film. The curvature tensor equals \( C_{ij} = \partial^2 h / \partial x_i \partial x_j \), where \( h \) is the out-of-plane normal displacement. The linear term in \( C_w \) is a chiral symmetry-breaking term, inducing the film to twist; the first term represents the in-plane stretch energy cost (33); and the other terms are the bending energy cost. The coefficients, which will be given in the Methods section, are determined by three parameters: the ratio between \( w \) and \( d \), the product of the coupling constant \( \alpha \) and the order parameter change \( \delta S \), and the twist angle span \( \theta_3 \).

The theoretical results can be obtained by minimizing \( f_{1D} \) with respect to \( C_{ij} \): for a given temperature change, narrow films form helicoids with the curvature of their central line \( C_0 \) being zero and the torsion of their central line \( C_T \) determined by the equation \( b + 2 \sqrt{a_x a_y} C_w + 4 a_K C_{ij} = 0 \); wide films form spiral ribbons with \( C_{xx} = b^2 / (6 \sqrt{a_x a_y} \sqrt{a_y} - a_y) \), \( C_{yy} = -b^2 / (8 \sqrt{a_y} a_x) \), \( C_{xy} = 0 \). The Gaussian curvature is \( K = -b^2 / (a_x a_y a_x a_x + a_y a_y) \). Note that \( K \) is always negative; only in the limit \( w / d \rightarrow \infty \), \( K \) tends to zero and the films form perfect spiral ribbons. (Ribbons of various curvatures are shown in Fig. S1). The critical width \( w_c \) that determines what type of ribbon to form is given by the condition \( a_K b^2 = 96 (a_x a_y)^{3/2} \). In the case of \( \theta_3 = \pi / 2 \), this condition simplifies to

\[
(w_c / d)^2 = \frac{4}{3} \pi^2 \sqrt{\frac{40}{3}} \left( (\alpha - \delta S)^{-2} + (\alpha - \delta S)^{-1} \left( \frac{1}{6} + \frac{1}{\pi^2} \right) \right).
\]

To compare the theoretical predictions with the experimental observations, we proceed to connect the product of the coupling constant and the order parameter \( (\alpha S) \) to temperature \( T \). For this purpose, we examine how much a corresponding nematic elastomer with planar orientation extends as temperature is lowered. It is easy to deduce from Eq. 1 that the extension along the nematic director in this case is \( \Lambda = \sqrt{1 + \frac{2}{3} \alpha S} \). By fitting the experimentally measured curve of \( \Lambda(T) \) (see Fig. S2), we obtain \( \alpha S = 3.3 (1.01 - T / T_{NI})^{2/3} \) as the temperature dependence of \( \alpha S \).

Given the relation between \( T \) and \( \alpha S \), we can then determine how \( C_{xx} \), \( C_T \), and the critical width \( w_c \) vary with temperature \( T \). The pitch and the diameter of the helicoids and spiral ribbons can thus be determined accordingly. The theoretical predictions of their change with temperature are plotted in Fig. 3C and 4C. In Fig. 4C, the thin dashed lines represent the pitch of helicoids; outside the thin-dashed-line region, the helicoids become spiral ribbons whose diameter first increases rapidly and then decreases slowly with temperature change (see Fig. S1 for an illustration of the shape change). Fig. 5 shows how the pitch and diameter of the ribbons vary with the width. From these figures we can find that the theoretical results agree well with the experimental observa-

Fig. 4. Spiral ribbons formed by the wide TNE films with the thickness 35.2 \( \mu \)m. (A) Shape of the L-geometry ribbons: left-handed at 374 K; right-handed at 336 K. The ribbon width is 0.76 mm. (B) Shape of the S-geometry ribbons: right-handed at 374 K; left-handed at 330 K. The ribbon width is 0.83 mm. (C) Temperature dependence of the inverses of the helical pitch \((\rho_u)\) and the diameter \((d)\), where the lines represent the theoretical predictions. The pitches of the left- and right-handed twist are defined to be positive and negative, respectively. The inverse of the pitches is represented by the open symbols and the inverse of the diameters by the filled symbols. The (red) circles and (blue) squares represent, respectively, the data of the L-geometry and the data of the S-geometry. The thick (solid or dashed) lines represent the theoretical predictions for the structural parameters of spiral ribbons, and the thin dashed lines are the theoretic prediction of the pitch inverse of helicoids.
Fig. 6. Theoretical Phase diagram of TNE ribbons in the width-temperature space. The ribbon thickness is 35.2 μm. The phase boundary of the L-geometry and the S-geometry is represented, respectively, by the red solid line and the blue dashed line. The handedness of the formed ribbons changes as the dotted line at $T_{\text{flat}}/T_{\text{NI}} = 0.962$ is crossed.

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3D energy density to an effective 2D one, we then express the displacement vector $u$ of points away from the midplane in terms of the displacement vector $u^m$ of points on the midplane (33). To the linear order of $z$, we have $u_1 = u_1^m - z \cdot \partial u_1^m/\partial x$, $u_2 = u_2^m - z \cdot \partial u_2^m/\partial y$, and $u_z = u_z^m - z \cdot (\partial u_z^m/\partial x + \partial u_z^m/\partial y)$. Substituting the expressions for $u$ and $u_z$ into Eq. 1 and then integrating along the $z$-direction yields a 2D energy density including terms linear in $\varepsilon_{xx}$ and $\varepsilon_{yy}$, whose reference space is the equilibrium state of $T_{1/2}$. To eliminate these linear terms, we change the reference space of the strain tensor to a new state, the equilibrium state of the case that the out-of-plane normal displacement is suppressed, which is acquired by a deformation from the flat geometry with the stretch along the long ribbon axis to be $\Lambda_{xx} = [1 + \frac{1}{2} \cdot \xi_S \sin \theta/\theta_0]^2$ and the stretch along the short axis to be $\Lambda_{yy} = [1 + \frac{1}{2} \cdot \xi_S \sin \theta/\theta_0]^2$ for the $\Lambda$-geometry and $\Lambda_{yy} = [1 + \frac{1}{2} \cdot \xi_S \sin \theta/\theta_0]^2$ for the $\Lambda$-geometry. We then express the in-plane elastic energy as a function of the Gaussian curvature (33). Assuming the curvature is nearly constant on the ribbon surface (17) and integrating the 2D energy density along the short ribbon axis, we thus obtain Eq. 2, in which the coefficients are $a_1 = \frac{1}{2} b^2 \Lambda_{xx}^2 \Lambda_{yy}^2$, $a_2 = \frac{1}{2} b^2 \Lambda_{xx}^2 \Lambda_{yy}$, $a_3 = \frac{1}{2} b^2 \Lambda_{xx} \Lambda_{yy}^2$, and $b = \pm \sqrt{\frac{2}{3}} S \cdot d \cdot \Lambda_{yy} \sin \theta/\theta_0$. For the L-geometry and $\Lambda$-geometry, the $\Lambda$-geometry and $\Lambda$-geometry are not exactly symmetric to each other because the $\lambda_{xx}$ and $\lambda_{yy}$ of the $\Lambda$-geometry are different from those of the $\Lambda$-geometry.

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