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Integrating Mathematical Modeling for Undergraduate Pre-Service Science Education Learning and Instruction in Middle School Classrooms

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Computer-based mathematical modeling in physics is a process of constructing models of concepts and the relationships between them in the scientific characteristics of work. In this manner, computer-based modeling integrates the interactions of natural phenomenon through the use of models, which provide structure for theories and a base for experimentation. Utilizing this method, scientists construct knowledge, and in like manner, students in science construct their understandings in significant ways, addressing their preconceptions and their knowledge of concepts in physics. Project STIMMULIS (Science Teachers Integrating Mathematical Modeling in Undergraduate Learning for Instruction in Schools) provides a mathematical modeling context for pre-service mathematics and science teachers' conceptual and applied understanding of motion. Project STIMMULIS can serve as a prototype for teacher education departments by: (1) providing rich science and mathematics content through a scientific and mathematical modeling based curriculum; and (2) developing, implementing and demonstrating innovative constructivist practices (strategies) for teaching science and mathematics.

Keywords: mathematics, science, modeling, teachers, undergraduate, constructivism

Introduction

El Paso is a bicultural, bilingual community with a 76.6% Hispanic population. Combining El Paso's population with that of its sister city, Ciudad Juárez, Mexico, our community is the largest metropolitan area on any international border in the world. Students at the University of Texas at El Paso (UTEP) are mostly female, and 50% of the total student population is made up of first-generation college students. These demographics are accompanied by low socioeconomic and educational factors: 23.6% of local families live below the poverty level, compared to 12.5% nationally; 32.9% of 25-year-olds have not graduated from high school; 19% of local residents have less than a ninth grade education with only 15.8% holding a bachelor's degree or higher. Due to its location and quality of its academic programs, the UTEP ranked among top four-year colleges by enrollment and degrees to Hispanics (*Hispanic Outlook in Higher Education*, 2006), and the surrounding communities are in a unique position to contribute to the development of future Hispanic scientists, mathematicians, engineers and health professionals.

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We are in dire need of addressing the lack of student achievement in mastering fundamental science and mathematics knowledge. In the NAEP (National Association of Educational Progress) data of 2005 for science at the national level, there was no significant change in the gap between White and Hispanic achievement for grades 4 and 8, with the score gap being 32 points. In science at the state level of 8th grade, students scored poorly on Objective III, Structure of Matter (scoring 62%), and Objective IV, Motion, Force and Energy (scoring 63%). When the state tested, the TAKS (Texas Assessment of Knowledge and Skills) science data were disaggregated, only 61% of Hispanic students met the standard. In Texas, on the Nations Report Card of 2005, 47% of grade 8 students scored below basic level in science, and only 21% were considered proficient.

On the Nation's Report Card for mathematics published in 2005, in Texas, 28% of grade 8 students scored below the basic level of the NAEP assessment with an average score gap of 29.25, with only 25% scoring at the proficient level. On the mathematics TAKS test administered to grade 5-8 students, the topics they scored poorly on were geometry and spatial reasoning objectives, with only 54% of grade 7 students answering correctly on these objectives, and only 45% of grade 8 students answering correctly on objectives related to mathematical processes (patterns, relationships and algebraic thinking).

Based on our state certification test, the TExES (Texas Examinations of Educator Standards) results of 2007 show that, our pre-service teachers earning a 4-8 science/math certification performed poorly on key competencies including transformational geometry (Geometry and Measurement Domain), mathematical connections within and outside mathematics (Mathematical Processes Domain), number theory (Number Concepts Domain), and properties of matter (Physical Science Domain). Similarly, our pre-service teachers earning a 4-8 generalist certification performed poorly in the general mathematics domain on the following competencies: number theory, linear functions, foundations of calculus, measurement, transformational geometry and problem-solving.

As one of the 11 institutions participating in "Teachers for a New Era" grant sponsored by the Carnegie Foundation, we have a critical stake in teacher training. The relationship between UTEP and the local public school districts is often viewed as a "closed loop", since 70% of teachers currently working in local school districts are graduates of local schools and hold education degrees or certification from UTEP. According to the National Science Board (2006), educators need to learn effective pedagogical techniques to reach ELLs (English language learners). To increase the achievement level of all students, they need opportunities to learn science and math that focus on critical reasoning skills. The Board emphasized that these opportunities must occur at an early age. Our project provides intellectual resources to improve science and mathematical reasoning and establishes a relationship between scientific practice and sense-making through modeling (Brown, 1992; Lehrer & Schauble, 2000).

A major challenge for science and math teachers is to instruct students who come with diverse cultural and linguistic backgrounds. Often, they instruct their students from a mainstream perspective, not recognizing that students bring with them a rich set of linguistic and cultural resources that can enhance their science and math learning (Lee, 2002, 2003, 2005; Rosebery, Warren, & Conant, 1992). Based on the knowledge that there is limited attention given to science and math instruction within an English language and literacy development context (Lee, 2005), an important goal of our project is to establish a set of authentic learning experiences for future teachers to better serve their future ELLs.

Key Components of Project STIMMULIS

The key components of our project are as follows: (1) the development of an integrated mathematics and physics module (in English and Spanish) based on mathematical modeling, implementation research, and teaching and learning research that are aligned with state/national standards for science and mathematics, and the national standards for technology education, to include cross-curricular inquiry activities that aim to emphasize depth rather than breadth of content knowledge and reasoning skills; and (2) the improvement of pre-service science and math teaching and learning by refocusing methods course content to include inquiry-based activities and constructivist-based teaching to gain an appreciation of how science and math can be integrated in classroom instruction.

Component One: Development of an Integrated Mathematics and Physics Module

The focus of the curriculum module will be the integration of physics and mathematics content through mathematical modeling. Inquiry activities constitute the heart of this curriculum for undergraduates who will teach students in grades 6, 7 and 8. Inquiry activities will focus on mathematical modeling, which will unify key physics ideas (force, motion, and momentum) with reciprocal big ideas in math (measurement and estimation, ratio and proportion, functions, and transformations). Based on the recent National Research Council Report, *Learning to Think Spatially* (2006), the curriculum component aims to utilize relevant technologies that serve as resources to support the process of developing spatial reasoning in science and mathematics. The technology plays a key role in the project by focusing on the use of interactive software (e.g., spreadsheets, motion sensors, PASCO Data Studio) to learn science and mathematics at a more conceptual level.

Modeling was selected as the unifying concept/theme for our project, because it provided an opportunity to test the proportion/hypothesis/construct of “less is more”. As a unifying theme, pre-service teachers would have opportunities to approach teaching science and math, less from a coverage perspective and more from a depth perspective. A scientific model is a testable idea that tells a story or helps provide an explanation about what happens in nature. Such a definition has been widely accepted. By using modeling, teachers will have opportunities to teach a set of ideas that explain a process or occurrence which can predict how certain scientific phenomena will occur or behave while they are under observation. Teachers, for example, can examine the powerful ideas related to Sir Isaac Newton’s geometrical models of force and motion or the ideas that support pictorial and physical models of the solar system and apply them to their understanding of students’ learning and curriculum development. In practice, scientific models may include or consist entirely of mathematized descriptions of phenomena. A scientific model will become a mathematical model if the model represents a real-world situation with a mathematical construct involving mathematical concepts and tools. A mathematical model can embody foundational learning in such areas as algebra, geometry and statistics along with their algorithms and formulae.

Another reason for modeling being selected was that the significant problems faced modeling as an area of research in science and math education. Some of these problems identified by Niss (2001) include: (1) How students learn to critically and reflectively analyze and assess a given model: justification, behavior, mathematical properties and possible alternatives to it; (2) What difficulties students have in acquiring and consolidating these critical learning and analytical skills; and (3) How do students learn these skills which depend on the specific, scientific contexts in which the model is constructed. Though difficult, responding to these three questions about students’ learning through modeling is not only a fundamental goal of the Project, but also a goal of science and math education. Pollak (2003) cited other crucial problems related to using models and modeling in formal classroom settings, which include how to connect science and math to the "rest of the world". He claimed that, "What is usually missing is the understanding of the original situation, the process of deciding what to keep and what to throw away, and the verification that the results make sense in the real world" (Pollak, 2003, p. 650).

Component Two: Improvement of Pre-service STEM Teacher Preparation

The National Science Education Standards (1996) authored by the NRC (National Research Council) and *The Principles and Standards for School Mathematics* (2000) authored by the NCTM (National Council of Teachers of Mathematics) emphasize a critical need for students to study both science and mathematics in real-world contexts. Both documents stress the importance of inquiry-based learning when students examine problems and situations related to physical phenomena. The ability to represent situations verbally, numerically, graphically, geometrically or symbolically can be fostered (as cited in NCTM, p. 219) through inquiry approaches that integrate science and mathematics in classrooms and provide rich learning experiences for students (NRC, 2000).

Building on constructivist theories of scientific and mathematical knowledge (von Glasersfeld, 2001), a solid theoretical foundation for modeling as an inquiry-based approach encompasses the idea of representations (mathematical and non-mathematical), discourse, argumentation and negotiation and validation of models. Models are critical to the implementation of inquiry-based activities in classrooms. These ideas related to constructivist-driven inquiry have implications for both science and mathematics education in terms of understanding the individual and social construction of scientific and mathematical knowledge.

An important question asked by many teachers, mathematicians and scientists is "What do we want students to learn and know in mathematics and science?" Identifying what insight, knowledge and skills students need is a difficult task, especially because teachers, scientists and mathematicians may respond quite differently. Though difficult, identifying these elements of student learning is a fundamental goal of mathematics and science education. Although some critics of constructivism claimed that many scientific concepts or skills are not in the realm of student experience, von Glasersfeld (2000) made the distinction between conventional facts that students must possess permanently and concepts that are best constructed by a thinking, rational being. He wrote, "Whatever is conventional must be learned, so to speak, verbatim; what is based on rational operations, should be understood" (Glasersfeld, 2000, p. 2). One of the most intriguing aspects of constructivism is that it allows one to look more closely at the nature and importance of certain skills within given contexts and why those skills may or may not be important. For example, solving a proportion can be described as an important mathematical skill, but why this skill is important is a fundamental question teachers try to answer on a fairly routine basis. Key personnel in Project SMART will demonstrate such constructivist teaching practices when the curriculum modules are presented during the university methods courses.

Our approach used in the curriculum includes a number of hands-on, inquiry-based activities that lend themselves to constructivist-based implementation (inquiry) and are representative of ideas that are currently introduced in both science and mathematics classes, but in isolation. Most pre-service teachers rarely see how these ideas form a bridge between science and math. Furthermore, our approach provides an example of the kind of research recommended by the ICMI (International Commission on Mathematics Instruction) (2003) on teacher education.

Project Design

Each year, a total of 20 (60 over three years) pre-service science and math teachers of grade 4-8 in middle school will participate. These teachers are traditionally enrolled in and required to complete two courses in the final academic year prior to graduation. These courses are entitled "MSED 4310, Teaching Mathematics in the Intermediate and Middle Years" and "MSED 4311, Teaching Science in the Intermediate and Middle Years". These courses explore the methods of teaching mathematics and science, emphasizing the equity principle (mathematics and science for all) and development of conceptual understanding of critical mathematics and science topics. They also explore what it means to "know" and "do" mathematics and science by relying on constructivist principles for learning and teaching.

Measurement and Estimation

Some mathematics methods courses present activities that may or may not promote learning of estimation as anything other than a de-contextualized skill. Such activities are not necessarily based on an understanding of pre-conceived beliefs or pre-conceptions about estimation. We argue that learners' methods of estimation rely heavily on a pre-conceived belief that estimation is but a simple means or procedure to a far more important end—finding the "exact" or "true" answer. The belief that mathematics is an "exact" discipline (that mathematics does not allow for "error") prohibits students from understanding and appreciating the importance of estimation as proficiency, one we argue relies on important mathematical constructs such as measurement, ratio and percentage. The latter argument has an impact on the training of future math teachers. We argue that students typically rely on methods and/or procedures of estimation of discrete quantities for all situations regardless of context. This is typically due to overemphasis on very routine calculation problems (Kilpatrick, Swafford, & Findell, 2001).

Student estimation of continuous quantities (such as length or angle) usually relies on simple rounding techniques (as in whole number computation) and dismissal of "error" leading to limited understanding about precision and accuracy. Such reliance makes it difficult to determine or teach a uniform definition of estimation, which includes determining the appropriateness of the estimation (different for discrete and continuous quantities). Furthermore, it exposes the need for students to develop the connection between the continuity and the number (Goussinsky, 1959). The challenge for mathematics educators and those involved in the professional development of teachers is how to develop estimation proficiencies and skills of learners rather than teaching estimation as a separate mathematics topic or subject, devoid of context beyond its use in simple whole-number calculations.

Estimation is a mathematical strand requiring more emphasis on both elementary and middle school levels. However, we argue that estimation is a complex topic, necessitating the need to emphasize it as a means to determine an informed solution to a given problem. We agree with Kilpatrick et al. (2001, p. 216) who stated, "Estimation requires a flexibility of calculation that emphasizes adaptive reasoning and strategic competence, guided by children's conceptual understanding of both the problem situation and the mathematics underlying the calculation." Students could use varied approaches to finding a solution to a complex problem. Some have argued that what is involved in finding that solution, such as the use of multiple representations, has a great impact on how students understand and use the estimation (Ainsworth, Bibby, & Wood, 2002). Prior research has typically tried to address estimation (or has involved estimation) in the separate contexts of learning or using estimation with discrete quantities (Hogan & Brezinski, 2003; Montague & van Garderen, 2003) as well as continuous quantities (Clements & Burns, 2000; Keiser, 2004; Van den Heuvel-Panhuizen, 2003).

Unlike estimation, measurement as a mathematical concept has been duly emphasized and touted in both state and national standards. Mathematics methods courses for pre-service teachers typically focus on learning and teaching both formal and informal measurement (with and without standard units). Research studies in science education that focus on children's experiment design reveal how and why students use measurement during the process of inquiry (Petrosino, Lehrer, & Schauble, 2003). The activities involving measurement and estimation coupled with those involving discrete estimation should enhance learning estimation as proficiency rather than a de-contextualized skill. Such research has a significant impact on teachers' preparation in both content and pedagogy.

We further argue that estimation promotes mathematical modeling skills. It helps students connect mathematical structures and methods to real-life (real-world) situations, related to measurement error, experimental error and accuracy. In the realm of mathematical modeling, it is important to understand how students treat error when trying to solve a problem especially within the context of inquiry. Such understanding clarifies the role that estimation must play in determining a well-informed decision about a result and, furthermore, how students connect a more abstract view of mathematical structures and symbols to real-world situations. Table 1 provides the topics, research-based activities, and objectives that will be introduced into undergraduate math and science methods courses.

Table 1

Measurement and Estimation Activities and Objectives

Topic	Activity/Activities	Objective(s)
Length	Guess and measure	Estimating before using standard units
	Change of units	Understanding informal units; making rulers
	More than one way to measure	Different ways to measure the same length with one ruler
Area	Irregular figures	Estimation by decomposition into familiar shapes
Angles	Creating a protractor (creating unit angles)	Understanding unit angles and their attributes
Volume	Comparing tubes	Understanding 3-D measure
	Fill and compare	Understanding 3-D measure

Mathematical Modeling and Kinematics

Most undergraduate physics instruction provides curriculum materials in kinematics that involve some lab experiments with activities such as rolling a ball on a track, using a fan cart attached to a ticker-tape timer, and observing a fan belt attached to two pulleys. However, these experiments do not involve collection and analysis of real-time data as an integral part of the construction of the mathematical models for motion. Quantitative descriptions of position and time are only briefly discussed while more emphasis is placed on qualitative graphing (position-time and velocity-time graphs). Furthermore, the experiments can more aptly be described as demonstrations that are followed immediately by the introduction of formal (symbolic) mathematics, including precise definitions (e.g., instantaneous velocity) and procedures (e.g., finding the area under a graph). In these modules, the learner is given more guidance through the experiments, which require direct instruction from the facilitators or teachers. There is less emphasis on an inquiry process that might allow a learner to formulate his/her own mathematical models of the physical phenomena. The module on kinematics leaves several areas open for strengthening its inquiry-based approach to studying uniform and non-uniform motion.

The researchers are developing a kinematics unit based on: (1) activities that the Co-PI (Co-Principal Investigator) is using to engage middle school students and in-service teachers in learning physics content (Robertson, 2007); and (2) activities constructed and researches conducted by the PI on mathematical modeling of motion (Carrejo, 2004; Carrejo & Marshall, 2007; 2008). This curriculum facilitates a classroom environment for studying mathematical modeling from a constructivist point of view. Building on the assumption that the teachers' prior knowledge might be extremely limited or include only a procedural understanding of motion and associated mathematical models, the primary goal of the implementation is to facilitate a more conceptual understanding of both uniform and non-uniform motion equations. Table 2 provides the topics, research-based activities, and objectives that will be introduced into undergraduate math and science methods courses.

Table 2

Kinematics Activities and Objectives

Topic	Activity/Activities	Objective(s)
General motion	Invent and describe a motion	Identify critical concepts in describing motion (position and time)
		Differentiate position and distance, clock time and time of travel
		Understand that position can be predicted from a starting position and time and knowledge of how position is changing with time (velocity)
Constant velocity	Graph the relationship between the position of a rolling ball and elapsed time	Explain a procedure for finding the position of the object at some future time, using only a data table
		Use a graph to predict the position of an object at some future time
		Interpret the slope of a position-time graph as the velocity of a moving object
		Be able to draw a best fit line to represent a set of data. Be able to explain why a best fit line is a better representation of nature than the actual data points
		Derive an algebraic equation to represent an object moving with constant velocity
Accelerated motion	Acceleration with a spark timer	Create a position-time and velocity-time table for accelerated motion
		Find an average velocity for an accelerating object during successive small intervals
		Predict the future position and velocity of an accelerating object
		Create an equation for uniformly accelerated motion

Research

Researchers and graduate students will conduct qualitative analyses of videotaped teachers' and students' learning episodes based on the grounded theory (Cobb, Stephan, McClain, & Gravemeijer, 2001; Glaser & Strauss, 1967; Mann, 1993). The goal of such analyses is to identify learning episodes and characteristics of scientific and mathematical reasoning. Data collection will involve the whole class and group observations during the pre-service educators' methods courses, as well as in middle school classrooms. Qualitative notes, including researcher reflections, will be compiled from this analysis. Classroom artifacts, including representations from individual groups, as well as representations created from the whole class discussions will be analyzed.

The first phase of the analysis involves examining the video and transcripts chronologically to identify episodes. An episode is characterized as a segment in which a mathematical theme(s) is the focus of activity and/or discourse. Observations and conjectures are developed about reasoning and the context in which the reasoning takes place. As described by Cobb et al. (2001), "The result of this first phase of the analysis is a chain of conjectures, refutations, and revisions that is grounded in the details of the specific episodes" (p. 128). In grounded theory, three types of coding are typically involved in data analysis:

(1) Open coding (creating categories for data);

(2) Axial coding (using open codes and researchers' catalogue of data to determine characteristics or dimensions of categories and create a core category or categories);

(3) Selective coding (data collection and analysis focuses on the core category and supporting categories).

Through open and axial coding, patterns in thinking as well as emerging mathematical constructs will be identified throughout the implementation of the modules. Key episodes for the study will be utilized to indicate the scope and breadth of qualitative analysis. They need not be interpreted as isolated incidents to support certain claims; rather, they highlight the emergent patterns and constructs that are reflected throughout the data and reflect the thinking throughout the modeling process. Selective coding of the results will benefit the researchers for further study; given the creation of core categories from this study, we will attempt to identify these categories with other learners in different environments who are involved in the same implementation of the modules. Researchers will study two learning environments: university methods courses and classroom instruction.

The approach used to code data fits well with constructivist views on learning, whereby learners rely on prior knowledge or what pre-conceptions they may bring with themselves regarding certain phenomena. Learners in general, as with the participants in this study, have time and space to make sense of their experiences. In this sense, the "core" of a grounded theory will remain the same across classroom settings (university methods courses and middle school classrooms) while approaches to data collection and interpretation will reasonably change to not only reflect the setting but also be useful enough to apply to other classroom settings. Table 3 presents the major objectives that contribute to the institutionalization of the curricular component of our project, along with the research methodology, which will be used to collect and analyze the data.

The second phase of the analysis involves quantitative analysis of three basic sources of data: science, mathematics and technology inventories, teacher efficacy surveys, and content knowledge materials in mathematics and science for the appropriate content areas of the content modules. This research methodology for the study may be defined as the design, collection, and interpretation of data and information in order to understand the value of an instructional methodology. To measure the increases in student learning in science and technology, specific educational objectives will be tested and the results analyzed. Students participating in the research will be enrolled at UTEP and be participating in their new curriculum, which is purported to be theoretically more student-centered and technologically assisted. The main objective of the research project will be to determine if the new curriculum does increase student learning in objective areas as defined by the host institution.

The quantitative findings will be analyzed utilizing a MANOVA (multivariate analysis of variance) analysis, which by definition is useful for assessing differences among the dependent variables. It also allows the researcher the opportunity to look for significant effects from the independent variables on the dependent variables. Finally, it can be useful to determine the interaction effects from within subjects and within groups. Each one-way MANOVA will measure one main effect. For each of the focus areas, there will be a one-factor MANOVA test that will measure the differences in test scores over time. As the sample is further analyzed, other independent variables may be utilized in the analysis. The resulting two-factor MANOVAs can be further analyzed using the interactions and effects of the independent variable of test score by the separate independent variables of collected demographic information. This will be done in order to measure the differences among the sample in test scores over time for each additional independent variable.

Research Goals and Objectives

In order to fully implement such a program, the first step would be to design and develop a rich physics and mathematics content module that integrates mathematical modeling. For the work in El Paso, this would result in a cross-curricular science and math set of modules in English and Spanish using a collaborative team approach involving the key personnel from the Colleges of Education and teachers/administrators. In order to see the impact of this in classrooms, the research methodology would consist of a qualitative research analysis of data that would be collected during methods courses involving pre-service teachers. In cooperation with the information gathered and analyzed quantitatively, this mixed methods approach would in turn help to target, as well as identify, the specific learning episodes and characteristics of scientific and mathematical reasoning through the implementation of the cross-curricular science and math modules. The results from both qualitative and quantitative data analysis will provide support for evaluating the effectiveness and quality of the curriculum modules and determining if the goals of the curriculum were being met.

The second and important step would be to effectively plan and demonstrate the use of innovative constructivist practices and strategies in the implementation of the units within the pre-service middle school science and math methods coursework. As a goal, this idea would center on the implementation of the specific STEM content around which the cross curricular modules are designed. Then, during the methods courses for pre-service teachers, the instructor would use this content in practice and in effects that demonstrate the pedagogical skills for inquiry teaching and learning. At this stage, quantitative data would be collected and analyzed from pre-/post- science, mathematics, technology knowledge instruments and attitude inventories to calculate an effect size. Data would also be gathered and analyzed quantitatively from the efficacy surveys. Additionally, qualitative video analysis of pre-service teachers as they learn about and complete tasks in the modules would be done in order to examine the interactions of the content and method within the context of modeling. Finally, scores from previous the content and pedagogy mandated licensure exams (TExES) would be collected, analyzed, and compared with those from pre-service teachers participating in our project. There would be a set of matched control groups of pre-service teachers who did not participate, and whose scores will be compared on content knowledge and pedagogical competencies to measure the strengths of the relationship for each test.

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