Error Correcting Codes for Cooperative Broadcasting

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Error Correcting Codes for Cooperative Broadcasting

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Presented at the ISL Symposium
Stanford University
November 30, 2010
Outline

1. Motivation
2. Related work
3. Broadcast channels
4. Multilevel codes and the $|u|u+v|$ construction
   - Time sharing $|u|v$ versus $|u|u+v$ construction
   - Decoding for $|u|u+v$ construction: Two stages versus single state
5. Demapping and decoding performance
   - BPSK and QPSK modulations
   - 4-PAM modulation: Hierarchical mapping
6. Conclusions and future work
1. Motivation

- Background on unequal error protection (UEP) codes: *Ph.D. thesis on multilevel error-correcting codes*
- UEP codes are based on the idea of superposition coding proposed in Cover’s paper (1972) on broadcast channels
- The nodes of a wireless network (cooperative or not) always broadcast information (i.e., every node, in principle, receives this information)
- **Cooperative broadcasting** paper (Bergmans and Cover, 1974) “Superposition coding always outperforms orthogonal (time-division or frequency-division) schemes”
Broadcasting in a Wireless Network

1. Relay $\text{RN}_1$ has higher SNR compared to $\text{RN}_2 \quad C_1 > C_2$

2. “Shortest path” (smallest number of hops) not always most reliable
2. Related work

• Cover (1972): “Broadcast Channels”
  – Cloud structure of capacity-achieving superposition codes
• Bergmans and Cover (1974): “Cooperative Broadcasting”
  – Superposition codes always outperform orthogonal assignment
  – Amplify-and-forward, decode-and-forward, adaptive relaying, incremental relaying
• Stefanov and Erkip (2004): “Cooperative Coding for Wireless Networks”
  – Propose superposition coding. Do not refer to Cover!!?
  – Use Chase code-combining with hybrid ARQ
Related work (cont.)

  – Opportunistic coding

  – Feedback CSI and adapt constellation accordingly

• Li, Ge, Tang and Xiong (2008): Cooperative Diversity Based on Alamouti Space-Time Code
  – Multiple-access stage of relay nodes achieved with Alamouti’s scheme

  – Consider superposition coding performed at a single source, metrics

  – Constellation design for superposition (physical layer) network coding
3. Broadcast Channels

- Thomas Cover (1972) – Gaussian broadcast channel with one source broadcasting information to two users: User 1 has a larger signal-to-noise ratio (SNR) than User 2 ($N_1 < N_2$):

![Diagram of Gaussian broadcast channel]

**Fig. 16.** Gaussian broadcast channel.

Coding for broadcast channels: Cloud concept

- Cover showed that a channel code achieving capacity has a **cloud structure**, shown below for a binary symmetric broadcast channel:

![Diagram showing cloud structure for BSC]

**Fig. 4.** Space of codewords for BSC.


- A cloud is a set of codewords (or sequences) that is selected with the information bits (most important or MSB) to be transmitted to the high-noise user
Cooperitive Broadcasting (1974)

Fig. 3. Rates achievable by frequency division and variable power time sharing.

4. Multilevel LUEP codes and $|u|u+v|$ construction

- An LUEP code has subcode partition-chain $C \supset C_1 \supset \cdots \supset C_L$ with

$$G = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_L \end{pmatrix}, \quad G_2 = \begin{pmatrix} G'_2 \\ G'_2 \\ \vdots \\ G'_L \end{pmatrix}, \quad \ldots, \quad G_L,$$

and $d_1 > d_2 > \cdots > d_L$.

- Practical two-level LUEP codes can be constructed based on block, convolutional or LDPC codes and Plotkin’s (or $|u|u+v|$) construction:

$$G = \begin{pmatrix} 0 & G_1 \\ G'_2 & G_2 \end{pmatrix}$$

where $d_2 = 2d'_2 < d_1$. 

November 30, 2010
Time-sharing ($|u|v|$) versus Plotkin ($|u|u+v|$)

- **Time-sharing:**

  - **Transmitter:**
    - $M_1$ (MSB) → $C_1$ → BPSK mapper → $B_1$
    - $M_2$ (LSB) → $C_2$ → BPSK mapper → $B_2$

  - **Broadcast Receiver:**
    - $B_1$ → BPSK demapper → $L_c(Y)$ → Decoder 1
    - $B_2$ → BPSK demapper → $L_c(Y)$ → Decoder 2

- **Plotkin:**

  - **Transmitter:**
    - $M_1$ (MSB) → $C_1$ → BPSK mapper → C
    - $M_2$ (LSB) → $C_2$ → BPSK mapper → C

  - **Broadcast Receiver:**
    - C → BPSK demapper → $L_c(Y)$ → Two-stage decoder

Performance of $|u|v|$ vs. $|u|u+v|$: short LDPC codes

$C_1, C_2$: LDPC (96,50) codes of degrees (3,6) and (4,8)
A Plotkin $|u|u+v|$ coding scheme

- Multilevel codes *always improve the throughput* over any orthogonal (time- or frequency-division) approach.
- Follow Bergmans’ and Cover’s idea: Design an “*over-the-air*” $|u|u+v|$ (Plotkin) coding scheme:

\[
\bar{y}_1 = \alpha_{11} m_1(0 \mid \bar{v}_1) + \alpha_{21} m_2(\bar{v}_2 \mid \bar{v}_2) + \bar{n}_1
\]

\[
\bar{y}_2 = \alpha_{12} m_1(0 \mid \bar{v}_1) + \alpha_{22} m_2(\bar{v}_2 \mid \bar{v}_2) + \bar{n}_2
\]
Two sources with BPSK mapping

- **BPSK Mapping** $m_i$ from a bit to a signal set $M_i$, $i=1,2$. Assume $\alpha_{i1}=\alpha_{i2}$
- $M_1=\{s_{11}, s_{12}\}$ and $M_2=\{s_{21}, s_{22}\}$:

  $M_1=\{s_{11}, s_{12}\}$ and $M_2=\{s_{21}, s_{22}\}$:

  \[ s_{i1} \quad (B_i=0) \quad \text{and} \quad s_{i2} \quad (B_i=1) \]

  \[ \begin{array}{ccc}
  -\sqrt{E} & 0 & \sqrt{E} \\
  \end{array} \]

  ### At the receiver, the **direct-sum** $M_1+M_2$ is equal to a ternary signal set:

  \[ (B_1+B_2=0) \quad (B_1+B_2=1) \quad (B_1+B_2=0) \]

  \[ \begin{array}{ccc}
  -2\sqrt{E} & 0 & 2\sqrt{E} \\
  \end{array} \]
Single-source and cooperative broadcasting

- **Single source:**
  - Transmitter
    - $M_1$ (MSB)\[ C_1 \]
    - $M_2$ (LSB)\[ C_2 \]
  - Broadcast Receiver
    - Mapper\[ B_1 \]
    - Mapper\[ B_2 \]
    - AWGN channel
    - BPSK demapper
    - Decoder

- **Cooperative (two sources):**
  - Transmitter 1
    - $M_1$ (MSB)\[ C_1 \]
    - $M_2$ (LSB)\[ C_2 \]
  - Transmitter 2
    - $M_1$ (MSB)\[ C_1 \]
    - $M_2$ (LSB)\[ C_2 \]
  - Decoder
Decoding for $|u|u+v|$ construction

**Two stages (Kumar-Milenkovic, 2006)**

![Diagram of two-stage decoding](image)

**Single stage (SJSU, 2009)**

![Diagram of single-stage decoding](image)
$|u|u+v|$ decoding: Simulation results

$C_1$: Regular $(96,50)$ LDPC code with degrees $(3,6)$; $C_2$: Regular $(96,49)$ LDPC codes with degrees $(4,8)$
H matrix used in $|u|u+v|$ construction with LDPC codes of length 96
H matrix used in $|u|u+v|$ construction with LDPC codes of length 204
Systematic encoding

- Use Gaussian elimination to produce systematic generator matrices and a permutation:

Length 96:

Length 204:
**Single-source versus cooperative: BPSK Metrics**

### Single source

<table>
<thead>
<tr>
<th>B₁</th>
<th>B₂</th>
<th>C=B₁+B₂</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

### Cooperative (dual source)

<table>
<thead>
<tr>
<th>B₁</th>
<th>B₂</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Log-likelihood ratio (LLR) metric:**

\[
L_c(y) = \frac{\Pr\{c = 1|y\}}{\Pr\{c = 0|y\}}
\]
LLR metrics for cooperative broadcasting and BPSK modulation (E/N₀=10 dB)

<table>
<thead>
<tr>
<th>B</th>
<th>m(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Cooperative $|u|u+v|$: Lengths 96 and 204 AWGN channel
Cooperative $|u|u+v|$: Length 96 Flat Rayleigh fading channel
Cooperative $|u|u+v|$: Length 204

Flat Rayleigh fading channel

\[
\begin{align*}
\text{BER} & \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \\
E_b/N_0 (\text{dB}) & \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12
\end{align*}
\]
Results are applicable to QPSK modulation
QPSK = BPSK x BPSK: Subset mapping

- Two basis functions: $\phi_1(t), \phi_2(t)$.
- Basic idea: Quadrature multiplexing
- Source $i$: BPSK mapping with $\phi_i(t), i=1,2$
- Receiver processes two BPSK sequences in parallel branches
QPSK subset vs Cooperative $|u|u+v|$ BPSK

$C_1, C_2$: LDPC (204,102) codes of node degrees (3,6) and (5,10)
Cooperative $|u|u+v|$ with 4-PAM and natural mapping

- **4-PAM Mapping** $m_i$ from two bits to a signal set $M_i$, $i=1,2$. Assume $\alpha_{i1}=\alpha_{i2}$

- $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$ and $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$:

  \[
  s_{i1} (B_{i1}B_{i2}=00) \quad s_{i2} (01) \quad s_{i3} (10) \quad s_{i4} (11) \quad \phi(t)
  \]

  \[
  \begin{array}{cccc}
  -3\sqrt{E}/10 & -\sqrt{E}/10 & 0 & \sqrt{E}/10 & 3\sqrt{E}/10 \\
  \end{array}
  \]

- At the receiver, the **direct-sum** $M_1+M_2$ is equal to a 7-ary signal set:

  \[
  y/\sqrt{E}/10
  \]

  \[
  \begin{array}{cccccccc}
  -6 & -4 & -2 & 0 & 2 & 4 & 6 \\
  \end{array}
  \]

  \[
  \begin{array}{cccccccc}
  B_{11}+B_{12}= & 0 & 0 & 1,0,1 & 1 & 1,0,1 & 0 & 0 \\
  B_{21}+B_{22}= & 0 & 1 & 0,0,0 & 1 & 0,0,0 & 1 & 0 \\
  \end{array}
  \]

  No dichotomy for bit $B_2$
LLR metrics for cooperative broadcasting and 4-PAM modulation with **natural mapping** \((E/N_0=10)\)
Metrics for cooperative broadcasting and 4-PAM modulation with Gray mapping ($E/N_0=10$)
Cooperative |u|u+v| with 4-PAM and hierarchical mapping

- **4-PAM Mapping** $m_i$ from two bits to a signal set $M_i$, $i=1,2$. Assume $\alpha_{i1}=\alpha_{i2}$

- Again, two sets: $M_1=\{s_{11},s_{12},s_{13},s_{14}\}$ and $M_2=\{s_{21},s_{22},s_{23},s_{24}\}$ with two power levels

- At the receiver, the **direct-sum** $M_1+M_2$ is equal to a 28-ary signal set:

  ![Signal Set Diagram]

  \[ y / \sqrt{E/10} \]

This idea was proposed by L. Xiao, T.E. Fuja, J. Kliewer and D.J. Costello (2009)

Note: In their scheme, superposition takes place at the transmitter ...

- All dichotomies (i.e., metric $L_c=0$) are removed, in exchange for decreased error performance

- **Results are applicable to 16-QAM modulation**
Metrics for cooperative broadcasting with 4-PAM modulation and hierarchical mapping

![Graphs showing metrics for cooperative broadcasting with 4-PAM modulation and hierarchical mapping.](image)
Performance of 4-PAM with hierarchical mapping.
Length 96 LDPC codes
Performance of 4-PAM with hierarchical mapping. Length 204 LDPC codes
Conclusions and future work

• Proposed a coding scheme for two-user cooperative broadcasting ("over-the-air mixing"), based on Plotkin’s $|u|u+v|$ construction using BPSK, QPSK, 4-PAM and 16-QAM modulations

• **Cooperative broadcasting = Network coding over physical layer**

  MILCOM 2010 presentation, comment from Matthew C. Valenti: (His paper was on “Receiver Design for Noncoherent Digital Network Coding”)

• Future directions
  – Performance with **longer LDPC codes** (such as those used in WiMax)
  – Design rules based on LDPC code parameters (minimum distance, node distributions) versus proportion of MSB and performance
  – Use **a software radio** platform to study
    • Synchronization techniques for over-the-air mixing
    • Effect of channel estimation errors
    • Error performance over realistic (frequency-selective) wireless channels
    • Noncoherent modulation: Differential encoding, FSK (as in Valenti’s paper)
Supporting slides
Performance of WiMax codes

C1 = rate 3/4, n = 2304; C2 = rate 1/2, n = 2304

- Uncoded BPSK
- MSB
- LSB

BER vs. $E_b/N_0$ (dB)
Combining cooperative $|u|u+v|$ with Alamouti: Length 96 codes
Combining cooperative $|u|u+v|$ with Alamouti: Length 204 codes