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Film thinning in He II at temperatures above 1.1 K†

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We have made measurements of the velocity-dependent film thinning in saturated films of He II over the temperature range 1.1 < T < 1.6 K using a new technique. Results from several experimental runs which employ different geometries in general show quantitative agreement with predictions based on the Bernoulli equation made by Kontorovich. The effects of fluid acceleration encountered in these measurements are considered and shown to be small. A recently proposed vapor-replenishment mechanism appears inconsistent with our observations.

In 1970 Keller1 reported direct measurements of the thickness of a saturated He II film. In particular he found that the thickness was the same for a static film as it was for a film in motion. His observation appears to be in direct conflict with a prediction based on the Bernoulli equation made by Kontorovich2 and by Tilley.3 Kontorovich's2 result for the thickness δ of a superfluid film of velocity v, in terms of the thickness δ0 of the same film at rest,

\[ \delta^n(v) = \alpha \left( \frac{\alpha}{\delta_0^2} + \frac{1}{2} \frac{\rho_s v^2}{\rho g} \right)^{-1} \]

was derived under the assumption of steady-state isothermal conditions. Here α is the van der Waals coefficient, ρs/ρ is the superfluid fraction, and n is the van der Waals exponent (typically n = 3 or 4).

Since 1970 several experiments of various types have been conducted in an effort to confirm the validity of Keller's results. Specifically, van Sprossen et al.,4 have observed a thinning and suggest that the inconsistency between Keller's observations and the predicted thinning might be explained by a deposition mechanism whereby the film thickness is maintained by a replenishment process from the saturated vapor. In an experiment somewhat similar to Keller's,1 Banton, Hoffer, and Keller5 and also Williams and Packard6 have observed a thinning at temperatures below 0.9 K. The experiments of Banton et al.5 were carried out at temperatures as high as 1.6 K with thinning observed on machined but not on electropolished surfaces. In the experiment of Williams and Packard6 only a single flow surface was examined. Unstated films have also been studied using third-sound techniques by Telschow, Wang, and Rudnick7 with the conclusion that no thinning is observed as a function of flow velocity.

In an attempt to study this problem using a different technique8 we have made use of careful measurements of the frequency of film oscillations between two reservoirs. The oscillation in the levels of two reservoirs connected by the mobile He II film is damped at finite temperatures. The damping phenomenon has been shown to be in quantitative agreement9 with a theory based on the original ideas of Robinson.10 The frequency of oscillation of the levels of two reservoirs of He II is given11 under isothermal conditions by

\[ \omega^2 = \frac{\rho_s g}{\rho} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) \left( \int \frac{dl}{p(t)\delta(t)} \right)^{-1} \]

(2)

Here ρs/ρ is the superfluid density fraction, g is the gravitational acceleration, A1 and A2 are the cross sections of the two reservoirs, and dl is the incremental distance along the flow path which connects the two reservoirs. The quantity p(t) is the perimeter of the film flow path and δ(t) is the film thickness. Equation (2) was first obtained by Atkins.12 We have made the simple observation that for a damped sinusoid the average of the square of the velocity over a full period τ at the beginning of an oscillation is larger than that at any later time during the oscillation. Thus, the period average frequency observed during consecutive periods of the oscillation must be a monotonically increasing function of time if the film thins quadratically with velocity as predicted by Eq. (1).

The apparatus displayed in Fig. 1 is the fourth in a series of two-reservoir containers we have employed over the past year to look for changes in the frequency of oscillation due to velocity-dependent film thinning. Construction is from OFHC copper. In this case the film surfaces over which the film move have been electropolished.13 The annular measuring gap has a width14 of 0.005 cm. An inner annular gap of the same magnitude is present to avoid possible ambiguities in the calculated film profile from one He II surface to the other which arise from the effects of surface tension. The larger liquid surface inner reservoir is connected to the interior annular region by eight 1.6 mm holes below the liquid surface. This ensures that film flow across the interior lip is negligible. The results we shall present here are sim-
FIG. 1. Cross-section drawing of the film-flow apparatus. Helium is admitted through a superfluid valve and superfluid filter (not shown). The helium level is measured in the gap between slug A and barrel using a three-terminal capacitance technique. The ground shield which encloses the apparatus is not shown. The entire detector is immersed in He II in a sealed chamber. The film flows along slug A between the outer and inner annular regions. The aperture has a diameter of 0.022 cm. The major contribution to the frequency and frequency shift comes from the flow through the aperture. The insert plug serves to allow the helium to flow between fluid surfaces at the same level.

ilar to those we found earlier using less sophisticated versions of the present apparatus.

The helium level in the outer annulus is monitored with a General Radio 1615A capacitance bridge, which employs a Princeton Applied Research HR-8 as a null detector. Ultimate sensitivity of the level measurement technique is 200 Å, although this sensitivity is not required for the present measurements. Temperatures were measured to be stable to about 5 μK at any given temperature. Oscillations are generally induced by the removal of a dc voltage previously applied to the slug A. The approach (run-in) to the oscillations about equilibrium occurs at what has traditionally been referred to as the critical velocity. We expect that dissipation takes place in our aperture during this flow since it is the smallest perimeter in our apparatus. The oscillations themselves are subcritical after the first amplitude maximum. Typical peak oscillation amplitudes are 25 μm at T = 1.311 K. We have observed that our results are independent of the size of the run-in towards oscillation.

The oscillations are measured by recording the off-null bridge signal on a Hewlett-Packard 7004 A x-y recorder used in the time – y mode. The chart tracings are digitized using scanning machines, and the digitized data are fitted to a function of the form \( e^{-\gamma t} \sin(\Omega t + \phi) \). The fits are carried out using two cycles \( 2\pi \) of the data, and sequential fits are carried out by stepping the fits by \( \frac{1}{2} \tau \) for each subsequent fit. Our time axis for the chart recordings has been corrected for a reproducible anomaly in the x-y recorder of about \( \frac{2\pi}{20} \) which is always present.

The results we report here are selected from about 500 events observed in this apparatus over the temperature range from 1.1 to 1.6 K. Typically 10–15 events are observed at each level in the detector and at each temperature. In Fig. 2 we display the results of plotting \( \omega = (\gamma^2 + \Omega^2)^{1/2} \) vs time as determined for each time interval in the manner described above. (The factor \( \gamma \) only weakly modifies \( \Omega \). For example, at \( T = 1.311 \) K we find \( \gamma = 0.0085 \) sec\(^{-1} \), with \( \omega \approx 0.308 \) rad sec\(^{-1} \).) In order to portray accurately the general reproducibility of these results we have plotted a symbol for each frequency determination in each interval of time. The smooth curves drawn through the data are predictions for the rise in frequency given by Eq. (2). These predictions use the Kontorovich expression, Eq. (1), for the film thickness as a function of velocity. Quantitative agreement with our
observed asymptotic frequency is obtained by using the static film profile \( \delta = \xi / y^{1/3} \), where we have taken \( \xi = 3.24 \times 10^{-6} \) cm. Here \( y \) is the height of the film. Our calculations employ the macroscopic geometry of our flow surfaces. The effective value of \( \xi \) we have obtained indicates that our films are only about \( \frac{3}{5} \) as thick as Keller's \(^1\) thin films. Our films have an effective thickness in substantial agreement with those observed by Grimes and Jackson \(^29\) and Hemming. \(^31,32\)

Since the present measurements are non-steady state, the adequacy of the Kontorovich prediction which was obtained for nondissipative, steady-state, and isothermal conditions must be carefully examined for our case. The superfluid equation \(^33\)

\[
\frac{\partial \rho}{\partial t} + \nabla \left( \frac{1}{\rho} \frac{\partial \rho}{\partial y} \right) - \nabla T + \nabla (g y - \alpha/\varepsilon^2) = \nabla \times (\nabla \times \mathbf{v}) \tag{3}
\]

must be considered for the possible effect of time-dependent terms. Here \( \rho \) is the pressure, \( \rho \) is the density, \( \varepsilon \) is the entropy, \( y \) is the height of the film above the bulk surface, \( z \) is the perpendicular distance out from the flow surface, and \( g \) is the gravitational acceleration. Under the assumption that no vorticity is present along the flow path, we estimate that for our experimental conditions the largest possible correction comes from the term \( \int (g y - \alpha/\varepsilon^2) \cdot d\ell \) which in the worst case approaches \( \frac{1}{2} \% \) of the \( \varepsilon^2 \) term. Since this would produce a shift in \( \varepsilon \) of only \( 0.01\% \), it can be completely neglected. Our observations have been conducted in an open geometry in which the vapor was in direct proximity to the film. The presence of the predicted Kontorovich effect in this measurement suggests that the vapor-replenishment mechanism suggested by van Sproonen et al. \(^4\) does not operate. It is thus doubtful that the vapor mechanism is capable of explaining Keller's result.

In conclusion we summarize that the present observations are consistent with our earlier work, \(^6\) the observations \(^14\) of van Sproonen et al., \(^4\) and some of the work of Banton et al. \(^5\). If our interpretations of our observations are correct this work suggests that the claim that the Kontorovich thinning is observable only in the limit of vanishing vapor \(^5,6\) is incorrect and that calculations \(^8\) made on this assumption are premature. It thus seems that a resolution of the apparent inconsistency between our conclusions and those of Keller \(^1\) and Telschow et al. \(^7\) may require an additional term in the chemical potential for moving superfluid films. To be certain of this necessity requires a reproducible experiment over the broad range of \( 0.3 \, \text{K} < T < T_\text{c} \) so that possible ambiguities between the various experiments can be avoided. To this end we are presenting repeating the measurements of Keller\(^1\) over a wider temperature range.

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\(^7\)M. E. Banton, J. K. Hoffer, and W. E. Keller, Bull. Am. Phys. Soc. 19, 436 (1974); and private communications. Our measurements were initiated in an attempt to study this apparent surface effect with a different technique.


\(^13\)Qualitative agreement was obtained by J. K. Hoffer, J. C. Fraser, E. F. Hammel, L. J. Campbell, W. E. Keller, and R. H. Sherman, in Ref. 10.

\(^14\)F. Robinson, Phys. Rev. 52, 440 (1931).


\(^16\)Oxygen-free high conductivity.

\(^17\)Electropolishing was conducted by Electro-Glo, Inc., Chicago, III.

\(^18\)Precision measurements of the measuring gap were made by Westfield Gauge Co., Westfield, Mass. using an air gauge with an accuracy of 0.5 \( \mu \)m.

\(^19\)These scanning machines are routinely used to digitize bubble-chamber photographs and are accurate to 0.0025 cm over the 1-m square scanning table surface.

\(^20\)A precisely timed square wave of low amplitude was used to completely calibrate the time axis of our \( x-y \) recorder.

\(^21\)On occasion we see more or less frequency shift than presented here. We also sometimes observe more
structure than shown here. Slight changes in the reservoir levels brought about by adding or removing helium from the detector cause the structure to disappear and hence we associate it with visible imperfections on the outer wall of the inner annulus. In general our results for all four detectors for which we have data (one of which was not electropolished) show the trend depicted by the figures. We expect therefore that our qualitative conclusions are not strongly influenced by surface quality.


23Strictly speaking, this equation should also include a term $\kappa A v e^{-\Delta/v} b^T$ on the left-hand side to account for fluctuation nucleation effects [J. S. Langer and M. E. Fisher, Phys. Rev. Lett. 19, 560 (1967)]. Several experiments have been conducted in an attempt to measure $\nu$ and $E$ under the assumption that $E$ has the form $E - \epsilon_{\nu}/(\nu v)$. Somewhat conflicting values of $\nu$ and $\epsilon$ are reported [D. H. Liebenberg, Phys. Rev. Lett. 26, 744 (1971), and Ref. 11]. Using either set of published parameters we find that for our measurements such a term contributes $\sim 10^9$ of the $v^2$ term during the first cycle of oscillation.

24We estimate that the $\partial \nu/\partial t$ term in Eq. (3) could have contributed to the observations in Ref. 4. For some details on this point, see E. B. Flint and R. B. Hallock (unpublished).