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Political Economy of a Public Corporation

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POLITICAL ECONOMY OF A PUBLIC CORPORATION

Pricing objectives of BART

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The revealed preference approach is used to identify the objectives implicit in the fare structure of a rail mass transit system. A model assuming bureaucratic aggrandizement provided a better predictor of actual fares than a majority rule or interest group political model. Bureaucratic objectives work to the advantage of commuters from distant suburbs who are relatively wealthy and young. The data is too weak to support firm conviction about these conclusions, but our methods contribute to quantitative political economy.

1. Introduction

Competitive prices are the result of impersonal forces, but administered prices are deliberate choices. The methodology for studying administered prices is underdeveloped and we hope to contribute to it in this paper. We shall use the revealed preference approach to analyze two distinct aspects of administered pricing. First, we ask what prices reveal about the objectives of the institution which sets them; second, we ask who benefits from the price structure. We show how to test econometrically some hypotheses of political economy. This paper is most like the studies of the pricing behavior of public utilities such as electricity, hospitals, or transportation. Our contribution resides more in the method of study than in specific conclusions about BART, because the data does not warrant firm conviction.

2. Possible BART objectives: Hypotheses

Rail transportation for travelers in the San Francisco Bay Area is managed by an entity whose popular acronym is BART. The entity is

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1For example, see McFadden (1975), Peltzman (1971), Meyer (1975), Harris (1977), Newhouse (1970), and Pauly and Redisch (1973).

2The full title is Bay Area Rapid Transit District.
curious from the viewpoint of economic theory because of its quasi-public nature. BART is unlike a regulated industry because private shareholders do not own it; in this respect it is like the post office. Policy decisions are made by the Board of Directors who are directly elected by voters in the counties and cities it serves; in this respect it is like local public schools. However, its product is sold and more narrowly defined than education; in this respect it is like a private firm.

It is impossible to form a precise notion of BART’s objectives by studying official pronouncements and company records. Consider the ‘Principles upon which BART fares are based’: (1) fare level sufficient to meet revenue need; (2) fares must be reasonable and in line with competition; (3) fares in proportion to value received, distance covered, and relative speed of trip; (4) fare structure to maximize ridership from all market segments for a given revenue level; and (5) all fares determined by application of a general formula rather than arbitrary judgements. These criteria are both inconsistent and vague and it is clear that no unique pricing policy is implicitly determined. However, we can use economic theory to reduce the number of possible objectives to a short list and then ask which one best explains observed choices. In this paper we investigate two different models, namely the managerial theory of the firm and economic theories of politics, to see which one provides a more useful framework for understanding BART’s objectives.

We assume that the BART Board is capable of consistent choice, so that an ordering can be imputed to it by observing its choices when faced with known constraints. The dimensions of possible choice include fare structure, service quality, and capital investment. Extending the BART system is not politically viable at this time, so major capital investment is not at issue. The real choices of the BART Board are short-run allocational decisions about fares and service quality, which entail little or no capital expenditure.

It would be desirable to treat the choice of fare structure and service quality as a simultaneous optimization problem which the BART Board solves. However, the complexities of the theory and limitations in the data make such an approach impossible. Our approach is to examine fares, to the exclusion of service quality. This partial equilibrium approach is restrictive, but experience suggests that it will be fruitful. Private firms must choose price and quality, but the strategy of examining their pricing behavior in isolation has proved successful; we hope the same will be true of BART.

Subsidies for public transportation are available from property and sales

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3 Since November 1974 the voters in each of the nine districts elect one director to the Board for a four-year term.
4 See BART (1974).
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The Metropolitan Transportation Commission allocates this pool of subsidies among transit operators in the Bay Area. The MTC does not impose detailed constraints upon BART operations, but BART’s operating losses are not to exceed the subsidy available to it. The ability of BART to influence the available subsidy is limited, especially in the short run. This fact justifies a conceptual convenience: we proceed as if the available subsidy imposes a strict constraint upon BART, analogous to a consumer’s budget constraint. This assumption simplifies the mathematical formulation of alternative pricing objectives.

Our first hypothesis is that BART’s objectives are bureaucratic: fare structure is chosen to maximize bureaucratic emoluments, which increase with the size of the enterprise. This hypothesis is suggested by the theories of the firm in which management pursues its own objectives subject to a profit constraint. These theories treat measures of the firm’s size or rate of change in size as proxies for managerial objectives. We shall consider the fare structure which would maximize BART’s size as measured in two different ways: (i) farebox revenues (FR) and (ii) total passenger-miles (PM) traveled on the BART system. There are other measures of size which are equally acceptable a priori; our choice is dictated by convenience and some preliminary calculations.

Our second hypothesis is that BART tries to maximize electoral support. According to this hypothesis BART’s prices are determined by the forces of competition among directors for citizens’ votes. We shall consider two versions of the electoral model: (i) Black’s median rule and (ii) an interest group theory.

Black’s median rule suggests that subsidies will be distributed to maximize the benefits enjoyed by the median voter. We use two different methods to identify the median voter. First, we restrict our attention to BART riders and specify a median person within this group. Second, we consider all voters in BART district elections, including nonriders, and specify a median person.

The median rule is derived by assuming that voters face two alternatives

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5 The total subsidy available to BART for the fiscal year 1975–76 was $32 million, of which local property taxes constituted $5 million, state sales tax another $21 million, and direct financial assistance (from the Transportation Development Act of 1971) $1.7 million. This subsidy revenue of $32 million did not cover the total deficit of operating costs minus farebox revenues in 1975–76. The shortfall was $5.1 million. Another budget category, ‘funded excess of expenses over revenues’, is needed to offset this difference.

6 As noted in footnote 5, BART’s operating deficit for 1975–76 exceeded its regularly funded subsidy by approximately 15 percent. The difference was made up by a budget category entitled ‘funded excess’. It is difficult to know whether BART administrators expect to receive the funded excess or treat it as a bail out for unforeseeable losses. Our inquiries led us to the conclusion that the best computation of the subsidy constraint should exclude the funded excess.

7 For example, see Baumol (1959).

8 See Black (1971).
on a single dimension of choice at each election. In other words there must be two candidates and a single political issue. These conditions are rare in politics. Nevertheless, the median rule is often used to motivate empirical studies, presumably because it makes a precise prediction about the central tendency in democracy. The question remains unanswered as to whether the median rule is a good predictor when there are several candidates and several dimensions of choice. We shall test the predictive power of the median rule under such conditions.

The median rule predicts the political equilibrium by assigning equal weight to every voter. An alternative approach assumes that political influence is unequally distributed among rival interest groups. Our interest group hypothesis is that subsidies are granted to various groups of riders according to their demographic or socioeconomic characteristics. These characteristics affect the ability of citizens to organize and apply pressure upon elected officials. If the interest group model is correct, then we should obtain significant coefficients by regressing subsidy upon demographic characteristics of the ridership.

3. Notation and data

BART reports most of its data in aggregate form by station, so it will be necessary to work with these averages. For example, BART gave questionnaires to passengers as they entered the BART system asking about their destination and demographic traits; the information was then aggregated by the passengers' station of origin. This kind of averaging makes sense for a system like BART which is essentially a commuter service. The BART system consists of 33 stations grouped along four arms radiating from the central business district (CBD). Over 80 percent of the journeys are to or from CBD stations; less than 20 percent are from one outlying station to another. In brief, BART is in the business of transporting people from their homes in outlying regions into the CBD, and back home again. Most passengers are commuters from outside the CBD who reflect the characteristics of the neighborhoods around each station. We shall use BART's aggregate data to construct a profile of the average passenger for each station and use these averages to test our hypotheses.

BART's 'Station of origin data summary' (1975–76) provides the following information about the passenger population during a typical day for each of the 33 stations:10

9 BART patronage data indicated that only 18 percent of the ridership travels to non-CBD destinations. See Merewitz (1975).
10 A day is considered typical if it is thought that the passenger population on that day is typical, i.e. a population which is not thought to be influenced by any special events or holidays. BART estimates that the fiscal year consists of the equivalent of 257 typical days.
\[ p_{ij} = \text{number of passengers embarking from station } j \text{ and disembarking at station } i; \]
\[ p_j = \sum_i p_{ij} = \text{number of passengers embarking at station } j; \]
\[ m_{ij} = \text{miles from station } i \text{ to } j; \] and
\[ f_{ij} = \text{fare from station } i \text{ to } j. \]

From this data we calculated a weighted average trip length \( t_j \) and fare \( f_j \) for passengers embarking from each station \( j \) on a typical day in 1975–76:
\[ t_j = \frac{\sum_i p_{ij} m_{ij}}{p_j}, \quad j = 1, \ldots, 33, \quad (1) \]
\[ f_j = \frac{\sum_i p_{ij} f_{ij}}{p_j}, \quad j = 1, \ldots, 33. \quad (2) \]

We shall call \( t_j \) and \( f_j \) the trip length and fare for the 'average' passenger embarking from station \( j \). Table 1 displays information on the average passenger for each station.

Demand elasticities must be known in order to quantify our hypotheses. Ideally, we would like a separate elasticity for the average passenger departing from each station. Two different calculations are available, both of which applied logit techniques to survey data. The first calculation aggregated over the entire BART system, yielding a single elasticity for all passengers. The second approach estimated demand as a function of trip length, but this study aggregated demand for BART with demand for bus. The first approach did not distinguish between BART riders according to the length of their journey, and the second approach did not distinguish between riders on different modes of public transportation.

The first approach requires us to assign the same elasticity of demand to all BART riders, whereas the second approach permits us to calculate a separate demand elasticity for each of the 33 average passengers, as shown in table 1. The first approach measures the ease with which private transportation or bus can be substituted for BART, and the second approach measures the ease with which private transportation can be substituted for BART or bus. We expect the second approach to yield lower elasticities, which is consistent with the data in table 1. Under the second method elasticity increases with trip length, because it is harder for people who live near the CBD to substitute private cars for public transportation. The estimate obtained by the first approach is more appropriate for the purpose of our study. However, we shall also report calculations using elasticities

\(^{11}\) A constant price elasticity of 0.3 for all trips was computed by Train (1976).
\(^{12}\) Elasticities were estimated for two-mile trip distance intervals from 0–2 miles through 48–50 miles. With few exceptions, the elasticities are greater the longer is the trip distance; the average elasticity is approximately 0.15. See Harvey (1979).
## Table 1

Data pertaining to the average passenger for each station.*

<table>
<thead>
<tr>
<th>Station no. and initials</th>
<th>Trip length $t_j$ (miles)</th>
<th>Fare $p_j$ (dollars)</th>
<th>Elasticity $\eta_j$ (constant)</th>
<th>Elasticity $\eta_j$ (variable)</th>
<th>Subsidy (lower bound on cost)</th>
<th>Subsidy (upper bound)</th>
<th>Subsidy (best)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 LM</td>
<td>8.99</td>
<td>0.586</td>
<td>0.3</td>
<td>0.14</td>
<td>0.75</td>
<td>0.32</td>
<td>0.51</td>
</tr>
<tr>
<td>2 FV</td>
<td>10</td>
<td>0.682</td>
<td>0.3</td>
<td>0.14</td>
<td>0.80</td>
<td>0.42</td>
<td>0.58</td>
</tr>
<tr>
<td>3 CL</td>
<td>10.6</td>
<td>0.665</td>
<td>0.3</td>
<td>0.13</td>
<td>0.90</td>
<td>0.56</td>
<td>0.70</td>
</tr>
<tr>
<td>4 SL</td>
<td>11.56</td>
<td>0.702</td>
<td>0.3</td>
<td>0.13</td>
<td>1.01</td>
<td>0.73</td>
<td>0.83</td>
</tr>
<tr>
<td>5 BF</td>
<td>13.98</td>
<td>0.785</td>
<td>0.3</td>
<td>0.15</td>
<td>1.28</td>
<td>1.26</td>
<td>1.20</td>
</tr>
<tr>
<td>6 H</td>
<td>15.33</td>
<td>0.804</td>
<td>0.3</td>
<td>0.19</td>
<td>1.47</td>
<td>1.63</td>
<td>1.45</td>
</tr>
<tr>
<td>7 SH</td>
<td>16.38</td>
<td>0.839</td>
<td>0.3</td>
<td>0.18</td>
<td>1.59</td>
<td>1.91</td>
<td>1.64</td>
</tr>
<tr>
<td>8 UC</td>
<td>21.21</td>
<td>1.003</td>
<td>0.3</td>
<td>0.20</td>
<td>2.14</td>
<td>3.49</td>
<td>2.62</td>
</tr>
<tr>
<td>9 FM</td>
<td>24.57</td>
<td>1.045</td>
<td>0.3</td>
<td>0.17</td>
<td>2.59</td>
<td>4.92</td>
<td>3.48</td>
</tr>
<tr>
<td>10 MA</td>
<td>10</td>
<td>0.675</td>
<td>0.3</td>
<td>0.14</td>
<td>0.81</td>
<td>0.43</td>
<td>0.59</td>
</tr>
<tr>
<td>11 19</td>
<td>11.07</td>
<td>0.68</td>
<td>0.3</td>
<td>0.13</td>
<td>0.96</td>
<td>0.65</td>
<td>0.76</td>
</tr>
<tr>
<td>12 12</td>
<td>10.62</td>
<td>0.683</td>
<td>0.3</td>
<td>0.13</td>
<td>0.89</td>
<td>0.55</td>
<td>0.68</td>
</tr>
<tr>
<td>13 OW</td>
<td>7.64</td>
<td>0.689</td>
<td>0.3</td>
<td>0.10</td>
<td>0.44</td>
<td>-0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>14 RR</td>
<td>10.83</td>
<td>0.775</td>
<td>0.3</td>
<td>0.13</td>
<td>0.83</td>
<td>0.50</td>
<td>0.63</td>
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<tr>
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<td>0.15</td>
<td>1.06</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>16 LA</td>
<td>15.91</td>
<td>0.956</td>
<td>0.3</td>
<td>0.19</td>
<td>1.40</td>
<td>1.65</td>
<td>1.43</td>
</tr>
<tr>
<td>17 WC</td>
<td>19.05</td>
<td>1.038</td>
<td>0.3</td>
<td>0.22</td>
<td>1.78</td>
<td>2.63</td>
<td>2.05</td>
</tr>
<tr>
<td>18 PH</td>
<td>21.63</td>
<td>1.139</td>
<td>0.3</td>
<td>0.20</td>
<td>2.06</td>
<td>3.53</td>
<td>2.59</td>
</tr>
<tr>
<td>19 CN</td>
<td>25.2</td>
<td>1.158</td>
<td>0.3</td>
<td>0.17</td>
<td>2.57</td>
<td>5.10</td>
<td>3.55</td>
</tr>
<tr>
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<td>9.03</td>
<td>0.541</td>
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<td>0.14</td>
<td>0.80</td>
<td>0.37</td>
<td>0.56</td>
</tr>
<tr>
<td>21 B</td>
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<td>0.593</td>
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<td>0.13</td>
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<td>0.70</td>
<td>0.82</td>
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<tr>
<td>22 NB</td>
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<td>0.3</td>
<td>0.22</td>
<td>2.26</td>
<td>3.09</td>
<td>2.53</td>
</tr>
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<td>0.3</td>
<td>0.14</td>
<td>0.89</td>
<td>0.50</td>
<td>0.66</td>
</tr>
<tr>
<td>24 EN</td>
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<td>0.13</td>
<td>1.08</td>
<td>0.78</td>
<td>0.89</td>
</tr>
<tr>
<td>25 RM</td>
<td>14.39</td>
<td>0.641</td>
<td>0.3</td>
<td>0.19</td>
<td>1.49</td>
<td>1.52</td>
<td>1.42</td>
</tr>
<tr>
<td>26 MT</td>
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<td>0.808</td>
<td>0.3</td>
<td>0.15</td>
<td>1.14</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>27 PL</td>
<td>10.46</td>
<td>0.699</td>
<td>0.3</td>
<td>0.13</td>
<td>0.85</td>
<td>0.50</td>
<td>0.64</td>
</tr>
<tr>
<td>28 CC</td>
<td>11.3</td>
<td>0.727</td>
<td>0.3</td>
<td>0.13</td>
<td>0.95</td>
<td>0.65</td>
<td>0.76</td>
</tr>
<tr>
<td>29 16</td>
<td>7.15</td>
<td>0.507</td>
<td>0.3</td>
<td>0.10</td>
<td>0.55</td>
<td>0.09</td>
<td>0.31</td>
</tr>
<tr>
<td>30 24</td>
<td>6.23</td>
<td>0.474</td>
<td>0.3</td>
<td>0.10</td>
<td>0.45</td>
<td>0.0</td>
<td>0.24</td>
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<tr>
<td>31 GP</td>
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<td>0.3</td>
<td>0.08</td>
<td>0.46</td>
<td>0.02</td>
<td>0.24</td>
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<tr>
<td>32 BP</td>
<td>7.15</td>
<td>0.401</td>
<td>0.3</td>
<td>0.10</td>
<td>0.66</td>
<td>0.20</td>
<td>0.42</td>
</tr>
<tr>
<td>33 DC</td>
<td>8.86</td>
<td>0.604</td>
<td>0.3</td>
<td>0.14</td>
<td>0.71</td>
<td>0.28</td>
<td>0.47</td>
</tr>
</tbody>
</table>

*Calculation of each column:

1. From eq. (1) in text, data from BART's 'Station of origin data summary'.
2. From eq. (2) in text.
4. See footnote 12 for reference and calculation.
5. See eq. (5) in text; lower bound: $b_1 = 0$, $b_0 = 0.148$.
6. See eq. (5) in text; upper bound: $b_1 = 0.0091$, $b_0 = 0.019$.
7. See eq. (5) in text; best case: $b_1 = 0.004$, $b_0 = 0.086$.

Stations 1-9 comprise those of the southernmost arm; 14-19, the easternmost; 20-25, the northernmost; 26-33, the westernmost; and 10-13, the centrally located on the BART system.
obtained by the second method in order to test the sensitivity of our conclusions to the inputed elasticity of demand.

In order to calculate the subsidy for the trip of the average passenger for each station $j$, it is necessary first to calculate the cost of that trip. The allocation of costs within a transportation network is a difficult conceptual problem; it has not been solved for BART despite intensive study. As a consequence, we shall proceed by a sensitivity analysis in which we offer a best method for allocating costs, plus two extreme methods; we then show that our conclusions are insensitive to the choice of method. The validity of our conclusions does not rest upon the accuracy of any one method. In order not to distract the reader from the main point, we shall offer a summary account of these cost calculations; a detailed discussion is in the appendix.

The number of stations, the placement of track, and the stock of cars is fixed in the short run. Let us use the phrase 'train costs' to describe the short-run variable costs such as fuel, wear and tear on trains, and maintenance of track. The physical characteristics of the tracks are not very different from one segment to another, so we would expect that the train costs per mile are about the same for each segment of track. However the loading, or the number of passengers riding the train, is very different from one segment to another. We have remarked that BART is primarily a commuter service: trains leave the CBD in the evening loaded near to capacity and the loading diminishes as the trains get further from the CBD. The obvious inference is that the average loading during a passenger's trip is lower if that trip is long.

Our task is to assign to each station a cost per passenger-mile $c_j$ for the trip of the average passenger embarking from it or disembarking at it. (Note that the cost of the whole trip per person is $c_j t_j$.) It is intuitively appealing to think that the cost per passenger mile of a trip is greater if there are fewer passengers on the train. This intuition suggests that $c_j$ is an increasing function of a passenger's average trip length $t_j$. In the appendix we argue that this intuition corresponds to an accurate concept of short-run marginal costs for BART. We shall assume, for simplicity, that the relationship is linear:

$$c_j = b_0 + b_1 t_j, \quad b_1 \geq 0.$$  \hspace{1cm} (3)

Estimates of $b_0$ and $b_1$ are constrained by the fact that the assigned costs must add up to BART's total operating costs; hence, a prescribed value for $b_1$ implicitly determines $b_0$. We do not know a uniquely correct way to determine $b_1$, so we shall proceed by identifying an upper and lower bound, and a best estimate.
We establish a lower bound for $b_1$ by assuming that cost per passenger-mile is independent of trip length, i.e. we ignore the effect of reduced loading on costs per passenger:

(1) lower bound: $b_1 = 0$; $c_j = b_0 = 0.148$, $j = 1, \ldots, 33$.

Our other two calculations employ a simple mechanism to capture the influence of loading upon cost. If we divide the train costs per mile by the loading we obtain the cost per passenger-mile for any segment of track between two adjacent stations. This calculation allocates cost equally among all passengers riding on the train for that segment of track. Furthermore, it implies that cost per passenger-mile is greater for segments of track further from the CBD since the loading decreases with distance. For example, consider a station which is the fifth stop away from the CBD. Presumably the loading is greater between the fourth and fifth stations than between the fifth and sixth, so the cost per passenger-mile is less for the former track segment than for the latter. We shall use this concept of cost per passenger-mile for each segment of track to find an upper bound and best estimate of the parameter $b_1$.

We estimate an upper bound for $b_1$ by assuming that the $c_j$ for passengers embarking from the $j$th station is the cost per passenger-mile for that segment of the journey which is furthest from the CBD; our earlier discussion implies that this segment is the most costly one. For example, consider again a station which is the fifth stop away from the CBD. Under this assumption the $c_j$ for trips embarking from this station is the cost per passenger-mile for that segment of track between it and the fourth station (which is more costly than that segment between the fourth and third, the third and second, etc.). Such an assignment of costs to stations overestimates the effect of reduced loading on costs. OLS applied to these assignments yielded:

(2) upper bound: $b_1 = 0.0091$; $c_j = 0.019 + 0.0091t_j$, $j = 1, \ldots, 33$.

Our best estimate of $b_1$ assumes that the $c_j$ for trips embarking from the $j$th station is the weighted average of the costs per passenger-mile of those segments which comprise the trip to the CBD (the weights being the respective lengths of these segments). For example, the $c_j$ for that fifth station would be the weighted average of the costs per passenger-mile of the segments between the fifth and fourth, fourth and third, third and second, etc. OLS applied to these assignments yielded:

(3) best estimate: $b_1 = 0.004$; $c_j = 0.086 + 0.004t_j$, $j = 1, \ldots, 33$.

In the appendix we include details of the cost calculations, the regressions, and the cost constraint. Furthermore, we defend our claim that the third
procedure is best, arguing that it corresponds to a concept of short-run marginal cost which is reasonable for BART.

4. Predictions

In section 2 we characterized BART as choosing a fare structure to maximize some objective. However, it is more convenient to think of BART as choosing the subsidy structure rather than the fare structure. The subsidy $s_j$ for the average passenger embarking from station $j$ is the difference between the cost $c_j t_j$ and the fare $f_j$:

$$s_j = c_j t_j - f_j, \quad j = 1, \ldots, 33.$$  

(4)

It is straightforward to calculate the subsidy structure in 1975–76 under each of three methods of estimating costs. Combining eqs. (3) and (4) gives

$$s_j = (b_0 + b_1 t_j) t_j - f_j, \quad j = 1, \ldots, 33.$$  

(5)

We reported $t_j$ and $f_j$ in table 1; the values of $b_0$ and $b_1$ are provided by the three methods of estimating costs. Thus, the subsidy for the $j$th station can be calculated from this information and the preceding equation. The values $s_1, s_2, \ldots, s_{33}$ calculated in this way will be called the 'actual subsidies' which are shown in table 1.\(^{13}\)

We want to compare the actual subsidy structure to the subsidy structure predicted by each hypothetical objective in order to find the best explanation of BART's behavior. There is a simple way to make this test. First, we combine the data points $s_1, s_2, \ldots, s_{33}$ in a linear regression model relating subsidy to trip length, plus exogenous variables to be considered later:

$$s_j = a_0 + a_1 t_j + \ldots, \quad j = 1, \ldots, 33.$$  

(6)

We shall show that the linear model achieves a good fit. The estimated coefficients $(\hat{a}_0, \hat{a}_1)$ will be called the 'estimated subsidy structure'. Second, we compute the values of $a_0$ and $a_1$ which are predicted by each hypothesis, which we call the 'predicted subsidy structure'. Third, we test to see whether the predicted structure is outside the confidence limits of the estimated structure.

The remainder of this section explains the methods used in step 2 to make

\(^{13}\)Owing to the category 'funded excess' (footnotes 5 and 6), the difference between costs and farebox revenues is equal to regular subsidy revenue plus funded excess. Hence the total of these calculated 'actual' subsidies exceeds the regular subsidy revenue by approximately 15 percent.
the predictions. We can write fare as a function of subsidy structure by combining eqs. (4) and (6):

$$f_j = c_j t_j - (a_0 + a_1 t_j) = f_j(a_0, a_1).$$

(7)

Demand for embarcations from station $j$, denoted $p_j$, depends upon fare and service quality. Using (7) and holding service quality constant, we write $p_j = p_j(a_0, a_1)$. These demand functions, $p_j$, can be estimated by employing Taylor’s method and expanding around the observed data points for 1975–76.\(^\text{14}\) We have estimated two sets of demand functions, each of which makes use of one of the two elasticity estimates. BART’s budget constraint requires that its projected losses in a given year should not exceed the subsidy revenue available from outside sources, denoted $k.$\(^\text{15}\)

$$\sum_j s_j p_j = \sum_j (a_0 + a_1 t_j) p_j(a_0, a_1) \leq k.$$  

(8)

If BART were pursuing bureaucratic goals and trying to maximize farebox revenues, then it would choose the subsidy structure which solves:

farebox revenues (FR): \[ \max_{a_0, a_1} \sum_j f_j p_j(a_0, a_1) \text{ subject to (8).} \]

If BART were pursuing bureaucratic goals and trying to maximize passenger-miles, then it would choose the subsidy structure which solves:

passenger-miles (PM): \[ \max_{a_0, a_1} \sum_j t_j p_j(a_0, a_1) \text{ subject to (8).} \]

Our next hypothesis is that BART chooses the subsidy structure $(a_0, a_1)$ which maximizes electoral support, subject to the budget constraint (8). Only one of the choice variables can be chosen freely, since the other will be determined by the budget constraint. There is only one dimension of free choice and each of the 33 average passengers has a preferred value for that variable. The rider whose trip length is long would prefer the subsidy to

\(^{14}\)For any station $j$ we have:

$$p(f) - p^0(f^0) = \frac{\partial p}{\partial f} (f - f^0) + \ldots \text{ higher order terms;}$$

therefore, $p(f) = (1 + \varepsilon) p^0 - \varepsilon (p^0/f^0) f$, where $(p^0, f^0)$ are the observed 1975–76 points and $\varepsilon$ is an elasticity estimate, $-(\partial p/\partial f) f^0/p^0$.

\(^{15}\)Since $p_j$ is defined as the number of passengers per day, then the constant $k$ must be interpreted as the amount of subsidy revenue available per day; this magnitude is just the total subsidy divided by the number of riding days for the 1975–76 BART year.
increase sharply with distance (large $a_1$, small $a_0$), whereas the rider whose trip length is short would prefer large fixed subsidies (large $a_0$, small $a_1$). We could set riders in order of preferred value of $a_1$ by setting them in order of trip length. In brief, there is a single dimension of choice along which preferences are single peaked. The first version of the electoral hypothesis assumes that BART maximizes electoral support among passengers by maximizing the subsidy of the passenger whose trip length is the median:

\[
\text{median passenger (MP): } \max_{a_0, a_1}a_0 + a_1 t_{\text{med}} \quad \text{subject to (8).}
\]

The second version of the electoral hypothesis assumes that BART chooses the subsidy structure which maximizes benefits to the median voter. In order to identify the median voter, we must determine the effect of BART subsidies upon nonriders. There is good reason to think that nonriders would prefer the same subsidy structure which is preferred by their nearest neighbors who are riders.\textsuperscript{17} In other words, nonriders probably prefer the same subsidy structure as the average rider embarking from the station nearest to the nonrider’s residence. We ranked electoral districts by trip length on BART to the CBD, and calculated the median, weighted by voter turnout in BART elections, denoted $t_{\text{med vot}}$.\textsuperscript{18} Our second version of the electoral hypothesis assumes that BART solves

\[
\text{median voter (MV): } \max_{a_0, a_1}a_0 + a_1 t_{\text{med vot}} \quad \text{subject to (8).}
\]

\textsuperscript{16}If we order the stations by trip length, and add, station by station, the number of passengers embarking at each respective station, then the median passenger is indicated when the subtotal reached is 50 percent of the daily total of embarking passengers. For each station $j$ we have calculated an average trip length ($t_j$), so we can determine the median passenger’s trip length. Of course, the ridership varies with the subsidy structure, so the passenger whose trip length is the median depends upon the subsidy structure which is chosen. Our calculations are based upon the median rider under the actual, existing fare structure. This difficulty does not arise when we consider the median voter, rather than the median rider.

\textsuperscript{17}The main benefit from BART which is enjoyed by nonriders is the reduction in street congestion and pollution from transferring commuters off the roads and onto BART. In a circular city, a commuter to the CBD is more likely to enjoy a large reduction in congestion from switching commuters from road to BART who reside in his or her immediate neighborhood, than from switching an equal number of commuters who reside elsewhere. We assume that nonriders want the subsidy structure to be most favorable to their immediate neighbors so that many of them will change their commuting mode from auto to BART.

\textsuperscript{18}For any one of the nine electoral districts: (1) a weighted average trip length was calculated for passengers who embark upon BART from stations in that district; and (2) data was collected of the number of votes cast in the last BART district election. With (1) and (2) it is possible to order the electoral districts by trip lengths and to identify the median over voters ($t_{\text{med vot}}$). Voters, relative to riders, are distributed more heavily in areas further away from the CBD; i.e. $t_{\text{med vot}} = 11.9 > 11.3 = t_{\text{med}}$. Similarly, each Board member can be associated with the average trip length of the passengers from his/her district. The median trip length over the nine-member Board was 11.8 miles.
Our third version of the electoral hypothesis is an interest group model, which predicts that the subsidy enjoyed by riders will depend upon their socioeconomic and demographic characteristics. BART's 'Station of origin data summary' provided information on some socioeconomic and demographic characteristics of the passenger population embarking at each station. Our choice of variables was dictated by questions which BART survey-takers saw fit to ask. The following variables were calculated for \( j = 1, 2, \ldots, 33 \):

- \( WAYP_j \) = weighted average income of passengers embarking at station \( j \);
- \( RCBDP_j \) = number of riders embarking at \( j \) and traveling to the CBD;
- \( POLDP_j \) = percentage of elderly passengers embarking at \( j \);
- \( PJOBP_j \) = percentage of passengers embarking at \( j \) and traveling to work;
- \( PWP_j \) = percentage of riders with incomes greater than $50,000 embarking at \( j \);
- \( PREGP_j \) = percentage of riders embarking at \( j \) who travel BART every weekday; and
- \( WEDP_j \) = weighted average education of passengers embarking at \( j \).

Besides these demographic or socioeconomic variables, we have the variable describing the trip length of the average passenger for each station:

\( t_j \) = trip length for the average passenger embarking at \( j \).

We combine these variables in a linear model:

\[
s_j = a_0 + a_1 t_j + a_2 WAYP_j + a_3 RCBDP_j + a_4 POLDP_j + a_5 PJOBP_j + a_6 PWP_j + a_7 WEDP_j + a_8 PREGP_j + u_j, \quad j = 1, \ldots, 33.
\]  

(9)

If political pressure from socioeconomic groups explains the actual subsidy structure, then the coefficients on the demographic variables should be statistically significant:

interest group (IG): \( \hat{a}_2, \hat{a}_3, \ldots, \hat{a}_8 \) significant.

This hypothesis implies that pressure groups form along socioeconomic lines, rather than on the basis of a common place of residence. Consequently, there is no prediction that \( \hat{a}_1 \) will be significant.

5. Test of hypotheses

The results of estimating the linear model (9) are given in table 2. The variables \( PWP, PJOBP, \) and \( RCBDP \) have significant coefficients in some
estimations, which indicates that subsidies tend to favor riders from stations with numerous commuters bound for work, and disfavor very high income riders. However, none of the socioeconomic variables is statistically significant under all of the cost assumptions. Support is not strong for the hypothesis that pressure groups are based upon socioeconomic or demographic variables which operate independent of place of residence. By contrast, trip length $t_j$ is a powerful predictor of subsidy, which is not predicted by the interest group model.

<table>
<thead>
<tr>
<th>R.H. variable</th>
<th>Cost assumption 1 (lower bound)</th>
<th>Cost assumption 2 (upper bound)</th>
<th>Cost assumption 3 (best)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ($a_0$)</td>
<td>-0.426</td>
<td>-2.195</td>
<td>-2.195</td>
</tr>
<tr>
<td>($a_1$)</td>
<td>0.117</td>
<td>32.304$^b$</td>
<td>0.267</td>
</tr>
<tr>
<td>$WAYP$</td>
<td>-0.705E-3</td>
<td>-1.593</td>
<td>-0.859E-3</td>
</tr>
<tr>
<td>$RCBDP$</td>
<td>-0.548E-5</td>
<td>-0.376</td>
<td>0.702E-4</td>
</tr>
<tr>
<td>$POLDP$</td>
<td>0.551E-2</td>
<td>1.118</td>
<td>0.670E-2</td>
</tr>
<tr>
<td>$PJOPB$</td>
<td>0.238E-2</td>
<td>1.326</td>
<td>0.263E-2</td>
</tr>
<tr>
<td>$PWP$</td>
<td>-0.232E-1</td>
<td>-2.558$^b$</td>
<td>-0.303E-1</td>
</tr>
<tr>
<td>$WEDP$</td>
<td>0.333E-3</td>
<td>0.850</td>
<td>0.623E-2</td>
</tr>
<tr>
<td>$PRGP$</td>
<td>0.205E-3</td>
<td>0.098</td>
<td>-0.316E-2</td>
</tr>
</tbody>
</table>

$N = 33 \quad R^2 = 0.98 \quad R^2 = 0.97 \quad R^2 = 0.98$

*The estimates of $a_0$ and $a_1$ are constrained to satisfy (8) in order to test the various maximizing hypotheses, which are similarly constrained.

All of the hypotheses, except the interest group model, assume that subsidy is a function of trip length. Our next task is to compare the estimated subsidy structure, as described by $\hat{a}_0$ and $\hat{a}_1$ in eq. (9), to the subsidy structure predicted by our hypotheses. A summary of the performance of each hypothesis is provided by fig. 1. The solid line represents the locus of all points which satisfy the subsidy constraint

$$\sum_j (a_0 + a_1 t_j) p_j = k$$

under our best assumption about costs and our constant elasticity assumption about demand. The solid line depicts the subsidy structures which are feasible for BART to choose.

The numbers in parentheses indicate the trip length of the passenger whose subsidy is maximized by the values of $(a_0, a_1)$ at that point. Notice that the
numbers in parentheses are monotonically increasing as we move to the northwest, indicating that the preferred $a_1$ is increasing with trip length. We have also labeled the predicted values of the various bureaucratic and electoral hypotheses. The revenue hypothesis (FR) is calculated twice: FR1 refers to the predicted values when the subsidy constraint (8) is binding as an equality, and FR2 when the weak inequality holds. Notice that the second calculation yields a point (FR2) inside the frontier, which implies that if the bureaucracy pursues revenue maximization it does so to the detriment of all riders.

It is apparent from visual inspection of fig. 1 that the actual OLS values are closer to the values predicted by the bureaucratic hypothesis than the electoral hypothesis. This observation suggests that the electoral model offers an inferior account of BART's behavior in comparison to the bureaucratic model. In table 3 we compare the difference between predicted and estimated subsidy structure under each assumption. A superscript 'b' indicates the hypotheses under which the predicted values are not significantly different from the estimated values. The evidence which is available suggests that the bureaucratic hypothesis is nearer to the truth than the electoral hypothesis, but there is no hypothesis which cannot be rejected under some set of assumptions. For this reason it is not possible to conclusively reject the electoral hypothesis and accept the bureaucratic hypothesis. Perhaps our inability to make a definitive choice is due to weaknesses in our data, or perhaps BART has vague objectives which do not correspond precisely to any of our hypotheses.

What kind of riders benefit from BART's departure from electoral objectives? In fig. 1 we see that OLS corresponds to a trip length of 13-14 miles; in other words, the observed subsidy structure maximizes the subsidy of passengers whose trip length is approximately 13.5 miles. However, the graph shows that the electoral hypothesis favors the subsidy which is best for
Table 3
Predicted and estimated subsidy structures.*

<table>
<thead>
<tr>
<th></th>
<th>Predicted</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FR1</td>
<td>FR2</td>
<td>PM</td>
<td>MV</td>
<td>MP</td>
<td>OLS</td>
<td></td>
</tr>
<tr>
<td>Cost assumption 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(lower bound)</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
</tr>
<tr>
<td></td>
<td>-0.553, 0.127</td>
<td>-0.552, 0.068</td>
<td>-0.554, 0.127</td>
<td>0.769, 0.021</td>
<td>1.373, -0.03</td>
<td>-0.426, 0.117</td>
<td></td>
</tr>
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<td></td>
<td>-2.314, 0.255</td>
<td>-2.30, 0.106</td>
<td>-2.31, 0.266</td>
<td>1.504, -0.039</td>
<td>2.43, -0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost assumption 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(upper bound)</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
</tr>
<tr>
<td></td>
<td>-2.79, 0.299</td>
<td>-2.240, 0.205</td>
<td>-1.397, 0.199</td>
<td>-0.044, 0.091</td>
<td>0.574, 0.038</td>
<td>-2.195, 0.267</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5.627, 0.493</td>
<td>-4.12, 0.253</td>
<td>-3.22, 0.331</td>
<td>0.626, 0.034</td>
<td>1.58, -0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost assumption 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(best)</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_0$</td>
</tr>
<tr>
<td></td>
<td>-1.663, 0.213</td>
<td>-1.362, 0.133</td>
<td>-0.958, 0.161</td>
<td>0.373, 0.055</td>
<td>0.982, 0.002</td>
<td>-1.275, 0.189</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-3.997, 0.377</td>
<td>-3.18, 0.177</td>
<td>-2.75, 0.291</td>
<td>1.07, -0.003</td>
<td>2.01, -0.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*For each 'cost assumption', the upper entry refers to the prediction under the constant demand elasticity estimate and the lower entry to the prediction under the varying elasticity estimates.

bNot statistically different from the OLS estimates of the actual subsidy structure; that is, $(\hat{a}_1 - a_{1p})/SE\hat{a}_1 < 2$, where $\hat{a}_1$ is the OLS estimate, $a_{1p}$ the predicted value of $a_1$ and $SE\hat{a}_1$ is the standard error.
trip lengths of 11–12 miles. Thus, the actual outcome favors travelers whose trips exceed the median preference over voters. Presumably, the favored travelers are those who live in the distant suburbs.

We can obtain a more accurate description of the favored riders by regressing trip length $t_j$ upon the socioeconomic or demographic variables in the linear model (9). We have also included a new variable $MTEM$ in this regression, which measures distance between station $j$ and the CBD for $j=1,\ldots,33$. The results of this regression appear in table 4. Income and education are positive and significant; the percentage of elderly and percentage traveling to work are both negative and significant.

<table>
<thead>
<tr>
<th>R.H. variable</th>
<th>Estimated coefficient</th>
<th>$t$-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MTEM$</td>
<td>0.566</td>
<td>10.023*</td>
</tr>
<tr>
<td>$WAYP$</td>
<td>0.209E-1</td>
<td>1.987*</td>
</tr>
<tr>
<td>$RCBDP$</td>
<td>0.628E-3</td>
<td>1.588</td>
</tr>
<tr>
<td>$POLDP$</td>
<td>-0.307</td>
<td>-2.531*</td>
</tr>
<tr>
<td>$FJOBP$</td>
<td>-0.149</td>
<td>-3.311*</td>
</tr>
<tr>
<td>$PWP$</td>
<td>-0.122</td>
<td>-0.566</td>
</tr>
<tr>
<td>$WEDP$</td>
<td>0.362E-1</td>
<td>2.089*</td>
</tr>
<tr>
<td>$PREGP$</td>
<td>-0.544E-1</td>
<td>-0.585</td>
</tr>
</tbody>
</table>

$R^2 = 0.87$; $N=33$.

*Significant at the 5 percent level.

6. Conclusion

We have shown how to use the revealed preference approach to study the political economy of administered prices. We compared two hypotheses about BART objectives: (i) fares are set by bureaucrats to maximize BART’s size, or (ii) fares are set by elected officials to maximize votes. The two hypotheses were tested by comparing their predictions to the actual fare structure.19

We tested three versions of the electoral hypothesis. The interest group model predicted that elected officials would set fares to appeal to socioeconomic groups. However, regression analysis pointed to the conclusion that subsidies are a function of trip length, not the socioeconomic

19The reader may wonder whether BART might be pursuing Ramsey pricing in order to achieve social efficiency. We have speculated that the socially efficient subsidy would have $a_q > 0$ and $a_t < 0$, which is dramatically different from the observed values. The reason why Ramsey pricing would favor short trip-takers is because automobiles would be removed from the most congested streets, which are nearest the CBD. However, a detailed argument would require data on externalities which is unavailable.
characteristics of the riders. The median rider model predicted that fares would be set to maximize the subsidy enjoyed by the median rider, and the median voter model predicted that fares would maximize benefits enjoyed by the median voter in BART elections. These two models predicted subsidies which were very different from the actual subsidy. Appeal to the median rider or voter would require altering the subsidy structure in a direction favorable to shorter trip-takers.

We tested two versions of the bureaucratic hypothesis. One version assumed that BART bureaucrats maximize farebox revenues, and the other assumed that they maximize passenger-miles traveled on BART. The bureaucratic hypotheses predicted subsidy structures roughly similar to the actual structure, although the predictions could not pass the usual statistical tests.

The estimated subsidy structure is more favorable to long trip-takers than the subsidy structure predicted by the electoral hypotheses. Whose interests are being served by the apparent freedom of BART from political discipline? To answer this question we regressed trip length upon demographic and socioeconomic characteristics of the ridership. Our results show that riders who take long trips tend to be better educated and of higher income than other riders. So apparently there is no conflict of interest between BART bureaucrats, who want to maximize the size of the enterprise, and young, high income commuters from distant suburbs.

We were able to overcome deficiencies in the data and reach these conclusions by a sensitivity analysis. The question remains unanswered whether our inability to reach stronger conclusions is due to inadequacies in our data or vagueness in BART’s objectives.

Appendix

First we explain the mechanics of allocating costs among passengers and then we defend our best method by reference to economic theory. Operating costs for BART are distinguished into three categories: transportation (25 percent), maintenance of equipment and stations (45 percent), and general and administrative (18 percent); hence, at least 70 percent of operating costs is directly related to the service enjoyed by specific passengers. The operating costs attributable to passengers from any station are the costs listed above which could be saved if the trips were not taken.

BART’s 1975–76 annual report supplies information on the operating cost per train-car-mile for the BART system. We shall use this figure as a baseline for calculating the cost savings from reducing service on segments of the system. The baseline cost saving is the reduction in train-car-miles multiplied by the operating cost per train-car-mile. The baseline is an inaccurate
assessment of costs if the track is heterogeneous. Track segments between stations are relatively homogeneous, at least in the short run, but the distance between stations and the complexity of stations differs.

Given the baseline (operating costs per train-car-mile), the number of cars operating along each of the four tracks in a day, and the distance from the furthest station to the CBD for each line, it is possible to estimate the cost of providing \('x'\) amount of car-miles of service per day along that line. The question becomes how to allocate that cost to passengers embarking from each station.

The trip length \(t_j\) of the average passenger from each station increases as the stations become further from the CBD. We assume that the cost per passenger-mile can be written as a linear function of trip length:

\[
c_j = b_0 + b_1 t_j, \quad j = 1, \ldots, 33. \tag{A1}
\]

Furthermore, the total operating costs for 1975–76 were $58.9 million (BART 1975–76 Annual Report). Hence the estimated \(c_j\) values must satisfy the constraint

\[
257 \sum_j (b_0 + b_1 t_j) t_j p_j = 58.9 \times 10^6. \tag{A2}
\]

Eq. (A2) implicitly defines a linear relationship between \(b_0\) and \(b_1\). Thus, we want to place bounds on \(b_1\), with \(b_0\) being determined by (A2).

First we consider the lower bound on \(b_1\). Trains are more crowded near to the CBD and, hence, less expensive to operate per passenger-mile. Since the average passenger’s trip length increases for stations successively further from the CBD, we might expect the cost per passenger-mile to be an increasing function of \(t_j\). This reasoning implies that \(b_1 \geq 0\), with lower bound

\[
(1) \quad b_1 = 0; \quad c_j = b_0 = \$0.148, \quad j = 1, \ldots, 33.
\]

There is an argument which suggests that \(b_1 = 0\) is not the lower bound. If cars are uncongested on track segments far from the CBD, and if every train must go to the end of the line, then there is no additional cost to carrying more passengers between distant stations. Under these conditions the lower bound on \(b_1\) must be a negative number. However, it is inaccurate to assume that BART decision-makers are constrained to run every train to the end of the line. It is possible to reverse the direction of a train at any point on a line by installing a switch connecting the inbound rails to the outbound rails. In view of this flexibility, \(b_1 = 0\) seems to be an appropriate lower bound.

Our second and third methods attempt to obtain an upper bound and a best estimate for \(b_1\). From the preceding paragraphs we see that it is possible to calculate the operating costs of providing \('x'\) amount of car-miles of
service per day along any of the four corridors of BART traffic. We attempt to allocate this cost to the different groups of passengers embarking from their respective stations, explicitly taking into account the reduced loading for travel further away from the CBD. Consider the outermost station and the segment of track between it and the immediately preceding station. If service on this segment were reduced by stopping some trains at the penultimate station, there would be a saving of operating costs which can be attributed to those particular passengers embarking from that outermost station (or, equivalently, disembarking there after their return trip from the CBD); hence, a cost per passenger-mile can be calculated for that segment of the track. Consider now the penultimate station and the segment of track between it and the next immediately preceding station. If service were reduced by stopping some trains short of this track segment, there would be a saving of operating costs which, again, is attributable to a particular group of passengers — those embarking from the two outermost stations.

This procedure can be repeated for each successive segment of track until the CBD is reached; hence, for each segment of track (between two adjacent stations) we have obtained a cost per passenger-mile for service over that segment. The second method of allocating costs sets an upper bound for \( b_1 \). For any station, we assign a \( c_j \) which is the cost per passenger-mile of the most expensive segment of those segments comprising the trip to the CBD. Since the loading continually decreases as one moves away from the CBD, this most expensive segment is the one farthest away from the CBD. Hence, the second method overestimates the effects of reduced loading on costs for passengers embarking from the outermost stations, whereas the first method underestimates them. The specification (A1) is estimated with OLS, with these assigned values for \( c_j \), subject to the constraint (A2). This procedure yielded the following result:

\[
(2) \quad b_1 = 0.0091; \quad c_j = 0.019 + 0.0091 t_j, \quad j = 1, \ldots, 33.
\]

The estimate of \( b_1 (0.0091) \) was significantly different from zero with 95 percent confidence; \( b_0 \) was not free to vary as explained above. \( R^2 = 0.20 \), which is low, but acceptable for computing an upper bound.

Our third method of allocating costs determines a best estimate of \( b_1 \). We shall make use of the cost per passenger-mile for each segment of track whose computation was explained in the preceding paragraph. For the trips originating from any station, we assign a \( c_j \) which is the weighted average of the costs per passenger-mile of those segments which comprise the trip to the CBD, the weights being the lengths of the respective segments. The specification (A1) is re-estimated with those new values for \( c_j \), again subject to the constraint (A2). This procedure yielded:

\[
(3) \quad b_1 = 0.004; \quad c_j = 0.086 + 0.004 t_j, \quad j = 1, \ldots, 33.
\]
The estimate of $b_1 (0.004)$ was significantly different from zero with 95 percent confidence; $b_0$ was not a free variable as explained above; $R^2 = 0.40$.

This best method of assigning cost can be justified because it corresponds to a concept of marginal cost which is reasonable given the flexibility of the BART system. The marginal cost of serving passengers embarking at any station is just the savings in cost which arises from terminating service to them. Service is terminated by stopping some trains short of the station in question and reversing direction. Marginal costs can be assigned to the average passenger at each station in the BART network because the order for terminating service is well defined. The correct order is to proceed from the most distant station on each line and work towards the CBD.

If BART were faced with a moderate cut in its outside subsidy $k$, then it might not respond by stopping some trains short of the outermost station on a line. A more likely response would be to raise fares and run trains less often on each line. However, our appendix concerns the allocation of costs, not the objectives of BART.

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