Bi-Level Technologies

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Comments On A Proposed Change For EuroDOCSIS Specifications

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Comments on a proposed ECR for EuroDRFI

Submission to DOCSIS DRFI Working Group

By Ron Katznelson

1. Introduction

This paper provides support and explanations to the recommendation that certain specifications adopted for European applications of CableLabs’® Downstream RF Interface specifications (“DRFI”) will maintain their conformity to the existing DRFI and to Annex F of the current DOCSIS RF Interface Specifications [1] (“DOCSIS RFI”). The following contains responses to an ECR offered by European proponents for changes in the DRFI by means of a new annex (Annex A) to the DRFI. This note addresses the in-channel (Active Output Channel) return loss requirements and the ‘Other Channels’ spurious and noise requirements.

2. The Active Output Channel Return Loss Requirement.

In Table A-2 of the ECR, proponents of the ECR propose that the output return loss of the EQAM device comply with the following specifications:

<table>
<thead>
<tr>
<th>Output Return Loss</th>
<th>&gt; 16 dB within an active output channel in the frequency range from 108 MHz to 862 MHz (Note 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return Loss</td>
<td>&gt; 12 dB in every inactive channel from 81 MHz to 862 MHz</td>
</tr>
<tr>
<td>Loss</td>
<td>&gt; 10 dB in every inactive channel above 862 MHz</td>
</tr>
</tbody>
</table>

Thus, the major change from the DRFI specification is the 16 dB vs. 14 dB requirement within the active output channel. The ECR proponents do not suggest that the requirement of 12 dB in every inactive channel be changed. In support of this proposed change, the ECR proponents have informally offered an explanation based on an asserted requirement that reflected signals coming back from the head-end combiner and subsequently returned back from the EQAM modulator (at a power level that is lower by the return loss) must be at a relative level no higher than –38 dB with respect to the signal. Their derivation of the requirement of –38 dB was explained as being based on “a theoretical limit on CNR” for 256 QAM of 33 dB and an additional 5 dB margin. Based on network combiner reflection specifications and a “critical length” of coax introducing a delay of 1 symbol time (presumably the worst case condition?) and its resultant loss, they arrive at the requirement of Active-Channel return loss of 16 dB. As will be shown below, the problem with their rationale for this requirement is that it lacks a sound theoretical basis and is flawed on several grounds as follows:

(a) It is wrongly focusing on In-channel reflections that are dwarfed by other significant reflection terms.

(b) It assumes that digital channel degradations from reflections of replicas of the signal are the same as that due to random noise with the same relative power level.

(c) It assumes a “critical length” of a symbol time delay due to reflection without any explanation of what is “critical” or special about that delay, as opposed to other delay values.

(d) It arbitrarily assumes a 5 dB margin without explaining where that margin may be absorbed or distributed.
We shall now address these items in order. To see why the In-Channel return loss is an insignificant factor in determining the In-Channel reflection effects in head-end combining, we turn to Figure 1 wherein several RF modulators connected to an \( N \)-Way combiner are shown. Upon examining the return loss and port isolation of passive \( N \)-Way combiners used in head-end applications, it can be appreciated that these values are roughly the same because the internal passive design is such that the leakage \( \eta_2 \) from Modulator 1’s input port to another port (that of Modulator 2, for example) is comparable in magnitude to the ‘leakage’ to its own port \( \eta_1 \). Hence, essentially all ports receive similar low-level reflections of Modulator 1’s signals at frequency \( f_1 \). Since the other modulators are active on distinct channels, their return loss on frequency \( f_1 \) is characterized by their inactive channel return loss, which is specified to be lower than their active channel return loss. Consequently, at the output port of the \( N \)-Way combiner in the example shown, one has a specification stating that \( \eta_2 \rho_2 > \eta_1 \rho_1 \) for a signal from port 1 on frequency \( f_1 \). The In-Channel return loss (or reflection magnitude \( \rho_1 \)) for this purpose is made even more insignificant due to the superposition of reflections \( \eta_j \rho_j \) from all other modulators \( j=2,3,...,N \), each likely having the specification \( \eta_j \rho_j > \eta_1 \rho_1 \). In some instances, fixed-channel modulators may even have output channel filters that are virtually reflective out-of channel, further dominating the composite reflected signals. The effective transfer function over the channel is thus given on the right side of Figure 1

\[
H(\omega) = 1 + \sum_j \eta_j \cdot \rho_j e^{j(\omega T_j + \phi_j)}
\]

\[
|H(\omega)|^2 \approx 1 + 2 \sum_j \eta_j \cdot \rho_j \cos(\omega T_j + \phi_j)
\]

For \( |\eta_j \cdot \rho_j| << 1 \)

Figure 1. Composite returns from head-end RF combiner’s port reflections and modulator output returns.

Clearly, the ECR proponents’ focus on changing only the Active-Channel return loss specification is inconsistent with their declared reasoning. If anything, they would have been more consistent with their stated goal by requiring changes in return loss across the band. The trouble is, that their logic would have required an improvement in the return loss magnitude by a factor of \( N \). Fortunately, their logic does not govern the reality of digital channel degradations due to reflections, because these effects are much more benign, as discussed below.
2.1. Digital channel effects.

The ECR proponents appear to assume that digital channel degradations from reflections of replicas of the signal are the same as that due to random noise with the same relative power level. That notion is patently wrong because, unlike random noise, the effects of head-end combining reflections that affect the effective channel frequency response are virtually eliminated by the built-in demodulators’ adaptive equalizers. While the engagement of equalization may result in “Noise Enhancement”, or “Pre-Whitening Noise Penalty”, it is shown in Appendix A that under the most extreme cases, an equivalent decoding SNR degradation of 0.3 dB and 0.1 dB are experienced under worst case single echo reflections of –15 dB and –20 dB respectively. Clearly, reflections of –38 dB are insignificant and are off-the chart. It is also shown in Appendix A that there is nothing “critical” in reflections having one symbol time delay, as the ECR proponents appear to suggest and that worst phase “Noise Enhancement” degradation monotonically increases to a worst-case echo delay of 0.7 symbol time and leveling off at delays that are longer than 2 symbol time. Furthermore, the critical length of 30 meters is also unlikely to be found in a rack of modulators, typically having the signal combiner serving the installed modulators within the same rack with, at most, a few meters of coaxial cable feeding the combiner.

Finally, the ECR proponents’ choice of an arbitrary additional ‘margin’ of 5 dB for reflection attenuation has no basis and no mechanism or named devices that would absorb such excess margin in the head-end. It is not explained why a 3 dB margin would be insufficient. Based on the proponents’ own rationale, no change in return loss specifications from the current DRFI would be required if a 3 dB margin is acceptable. The arbitrary 5 dB addition to an already flawed CNR based criteria for reflection only demonstrates that the 16 dB return loss specification was selected first and (unsuccessful) attempts to rationalize it followed later. To see the absurdity of requiring that reflections of magnitude \( r \) be at levels lower than –38 dB \((r = 0.0125)\), we use the single reflection example with a power transfer response given by

\[
|H(\omega)|^2 \cong 1 + 2 r \cos(\omega T + \phi)
\]

Thus, the maximum ripple resulting from such reflection is given by

\[
\pm 10 \log_{10}(1 + 2 r) = \pm 0.054 \text{dB} \quad \text{– Hardly a measurable quantity, let alone an impediment for a digital channel.}
\]

At this point, one might ask whether the digital channel performance ever becomes a limiting factor in setting return loss specifications on the active channel or on an inactive channel. Appendix A shows that for all practical purposes, it does not. Because such effects are relatively benign, other head-end combining requirements would usually emerge as the first limiting factors. Those are associated with co-channel noise leakage in Narrowcast applications, as described below.

2.2. Narrowcast co-channel isolation effects.

In an earlier submission to the DRFI Working Group [2] supporting the sufficiency of return loss specifications on inactive channels, this author has treated the impact of modulator return loss performance on reduced effective splitter isolation and its impact on co-channel interference among independent narrowcast sources operating on the same channel but directed to different serving nodes. It was shown that modulators that are configured in combination for narrowcast services, are not a factor and thus no implications for return loss requirements can be drawn. However, return loss of modulators that serve multiple nodes through a splitter, such as those
used in the broadcast tier, do have an impact to be considered in their specifications. However, even those requirements are shown to be modest. Under the worst-case conditions in two level combining, less than 0.6 dB degradation of broadcast splitter isolation will be incurred if all modulators exhibit a –10 dB return loss rather than –14 dB. That decrement goes up to 0.9 dB in a single level combining application. It is also argued that in all cases analyzed, the worst-case results are better than a major MSO’s broadcast splitter port-to-port isolation guideline of 20 dB.

2.3. Active-Channel return loss requirements.

Noteworthy is the fact that in all of the above considerations for return loss specifications, the distinction between the specification for Active Channel return loss and Inactive Channel return loss (the latter having a lower return loss specifications than the former), has not been invoked or called for based on network operational requirements. The reasons Active Channel return loss specifications are typically set higher than those for Inactive Channels are not due to significant operational reasons but rather due to the following:

(a) Because in RF upconverters and modulators, required band pass filtering in the active tuned channel is better matched to the input port of the output amplifier stage, the modest reverse isolation that is typical to output amplifiers presents a better return loss in-band than that of a reflective filter out-of-band. In other words, the specification in the active channel is better because by construction of the RF modulator it is better, without extra power and RF stages.

(b) Internal power detectors in RF modulators for power level reporting or for signal-loss alarm functions are more accurate and stable when tapping the well-matched source impedance. The performance of such detectors is only exhibited in the active channel, where RF power is launched.

(c) RF power measurements and channel frequency response measurements can be made more repeatable and accurate with laboratory instruments when output return loss is near nominal. Also, higher power is transferred to the load under such conditions. The performance of such power transfer attributes is only exhibited in the active channel, where RF power is launched.

Thus, requiring return loss specifications for the Active-Channel that are higher than that of the Inactive-Channels may be desirable, but it requires the balancing of only these positive factors enumerated above (and not any network performance factors) with other EQAM implementation factors, such as power consumption, density and cost. In this regard, it is shown in Appendix B that the latter implementation factors are very significant and that just the F connector’s return loss degradations alone necessitate a reduction of the overall return loss requirements at higher frequencies. Furthermore, from the cable operators’ point of view, items (b) and (c) above do not actually present an issue because power detector accuracy and maximum RF power delivered to nominal terminating impedance are guaranteed by express specifications for these items.

The current DRFI Active-Channel return loss is specified as 14 dB, declining to 13 dB above 750 MHz and to 12 dB above 870 MHz, up to 1 GHz. Among other considerations, a judicious balancing of factors (a)-(c) listed above and implementation considerations of the type described in Appendix B, led to the adoption of this DRFI compromise relaxation at the top of the band. In contrast, the proponents of the EuroDOCSIS ECR have proposed an Active-Channel return loss that deliberately deviates from the balance struck in the DRFI in that it ignores high frequency implementation considerations. Their proposal lacks any supporting evidence or rationale for
such deviation and they point to no system performance factors that they seek to protect by requiring that no relaxation at higher frequency be permitted in the Active-Channel return loss. As shown in this paper, in order to provide a credible rationale for their proposed deviation, the ECR proponents must show that improvements in the enumerated factors (a)-(c) above due to their proposed deviation are of greater consequence and value than the implementation factors that led to the current DRFI compromise.

3. “Noise in Other Channels” requirements.

In Table A-4 of the ECR, the Out-of-Band Noise and Spurious Emissions Requirements for adjacent channels and Other Channels are shown as having the same values as those in the DRFI, even though the channel bandwidth is increased from 6 MHz to 8 MHz. This deviates from the single channel specification in Annex F of the DOCSIS RFI by 1.25 dB, the channel bandwidth increase factor. Yet, the more demanding requirements offered in the ECR is apparently supported only by another reverse calculation aimed at arriving at a result rather than bottom-up requirements analysis. This can be seen in the spreadsheet of Table 1 as provided by the ECR proponents, along with their supplied comments.

Upon further consideration, it should be appreciated that within a fraction of a dB, the conditions for protecting the 525 line analog NTSC channels are virtually identical to those of protecting the 625 line analog PAL systems. This is due to the fact that the luminance noise dominates the subjective assessment and because the horizontal video line rates are nearly identical, mapping the same video frequency to horizontal spatial frequency that undergoes identical subjective weighting by viewers of both television systems. This well known property is documented in CCIR recommendation 654 [3], wherein the same subjective assessment is assigned to the same unweighted random noise levels in both TV systems. Subjective noise weighting in both TV systems were experimentally found to be very similar, giving rise to the recommendation for using a single value for video Signal-to-Noise Ratios in CCIR Recommendation 568 [4] by the application of the Unified Noise Weighting provided as a standard video low-pass filter in CCIR Recommendation 567 [5].

Because white input RF noise is decoded by a VSB receiver for both PAL and NTSC using identical decoding structures and gains, the relationship between pre-detection CNR and video SNR as computed by the results provided by Straus [6] apply equally to both television systems. In this regard, it is important to note that while the same noise density will produce a lower CNR for PAL over a channel bandwidth of 5 MHz, as compared to an NTSC channel bandwidth of 4 MHz, the subjective statement under these conditions are virtually identical because the weighted SNR will be virtually identical. This observation is supported by the fact that the Unified Noise Weighting filter attenuates the noise between 4 MHz and 5 MHz by approximately 13 dB, making the noise contribution in that extra 1 MHz subjectively irrelevant. The conclusion is therefore that while the CNR might be lower in PAL for the same conditions, the target CNR for the same subjective statement is also lower by approximately the same amount.
Minimum requirement is defined in CENELEC EN50083-7 Table 5 (1996 version) resp. Table 7 (2000 version) to be:
- PAL-I: 44 dB in 5.08 MHz
- PAL-B,G: 44 dB in 4.75 MHz
- PAL-L: 44 dB in 5.00 MHz

Appropriate values include a safety margin that is required b

<table>
<thead>
<tr>
<th>Parameters as basis for current requirement</th>
<th>Comment Provided</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Required CNR for analogue TV at the customer CPE [dB]</strong></td>
<td>46</td>
</tr>
<tr>
<td><strong>Carrier to noise of optical link [dB]</strong></td>
<td>50</td>
</tr>
<tr>
<td><strong>Carrier to noise of coaxial network [dB]</strong></td>
<td>50</td>
</tr>
<tr>
<td><strong>Carrier to noise at input OHE [dB]</strong></td>
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</tr>
<tr>
<td><strong>Overall safety margin per network design [dB]</strong></td>
<td>7</td>
</tr>
<tr>
<td><strong>Required analogue TV CNR for M-CMTS [dB]</strong></td>
<td>59.9</td>
</tr>
<tr>
<td><strong># QAM channels loading the system</strong></td>
<td>90</td>
</tr>
<tr>
<td><strong>Insertion loss of digital signals [dB]</strong></td>
<td>4</td>
</tr>
<tr>
<td><strong>Noise bandwidth for analog signals [MHz]</strong></td>
<td>4.75</td>
</tr>
<tr>
<td><strong>Noise bandwidth for specified parameter [MHz]</strong></td>
<td>8</td>
</tr>
<tr>
<td><strong>Out of band spurious spec (N=1) [dB]</strong></td>
<td>73.2</td>
</tr>
</tbody>
</table>

**Table 1.** Other Channel noise analysis provided by the EuroDOCSIS ECR Proponents

It is therefore argued that if, in addition to that lower CNR requirement, one considers the fact that the average European system can carry significantly fewer than 90 digital channels in the downstream, the actual noise levels imparted by QAM channels will likely contribute 3-4 dB less noise than in the DRFI reference condition of 119 channel. This should roughly account for the 1.25 dB allowed noise increase and the 2 dB insertion level noise penalty.

Furthermore, as explained in [7], if one aggregates multiple single channel devices that are compliant with Annex F of the DOCSIS RFI, one obtains the values that are 1.25 dB noisier than that of the proposal in Table 1. Do the ECR proponents also suggest modification of the single channel specifications in Annex F of the DOCSIS RFI?

However, the analysis provided in Table 1 offered by the ECR proponents, appears to introduce an arbitrary 7 dB of “Overall safety margin per network design” in order to “accommodate the different noise contributions from the various signal sources in the headend”. It is not clear what other sources the proponents of the ECR had in mind and why a 5.5 dB “Overall Safety Margin”, for example would not be adequate. With that margin, no deviation from the DRFI equivalent specifications would be required. Given the careful balance between power consumption and cost of EQAM devices on the one hand and the design to meet the DRFI specifications on the other, and given the fact that every dB of dynamic range increase requires large increments in RF component performance and power consumption, the ECR proponent’s treatment of an unaccounted 7 dB just as a “Safety Margin” is breathtaking.
Appendix A  Adaptive Equalizer “Noise Enhancement” due to reflections

The effect of linear channel distortions due to signal reflections will be analyzed here in the context of the degradation it imparts to the noise margin that is otherwise available in a QAM link such as that based on the ITU-J.83 QAM transmission standard [8]. We first show that a flat channel frequency response produces the maximal possible demodulated signal-to-noise ratio or received Modulation Error Ratio (“MER”) and we subsequently analyze the degradation in this reference MER measure due to deviations from response flatness caused by channel reflection distortions. The model used for this problem is shown in Figure 2, wherein the channel distortion due to signal reflections is included within the channel response model $C(\omega)$ and wherein such linear distortions are assumed to exist prior to the introduction of the additive noise. This latter assumption covers a wide range of scenarios including that in which the reflection distortion is introduced at the head-end in RF combining networks.

![Digital channel link model](image)

**Figure 2.** Digital channel link model. The reflection distortions are subsumed within the channel transfer function $C(\omega)$.

In the model, $G_T(\omega)$, and $G_R(\omega)$ are the transmit Root-Nyquist filter and its receiver matched filter respectively, wherein their product $G(\omega)$ is the Raised Cosine Nyquist transfer function having excess bandwidth factor $\alpha$ and defined as follows:

$$G_T(\omega)G_R(\omega) = G(\omega) = \begin{cases} T_S \cos^2 \left[ |\omega| T_S - \pi (1-\alpha)/4\alpha \right] & \text{for } |\omega| < \pi(1-\alpha)/T_S \\ 0 & \text{for } \pi(1-\alpha)/T_S < |\omega| < \pi(1+\alpha)/T_S \quad (1) \\ \end{cases}$$

where $T_S$ is the symbol time. The time domain impulse response of this Raised Cosine is given by its inverse Fourier transform, resulting in the well-known impulse response:

$$g(t) = \int_{-\infty}^{\infty} G(\omega) \exp(i\omega t) \frac{d\omega}{2\pi} = \frac{\sin(\pi T_S/\omega)}{(\pi T_S/\omega)} \left[ \cos(\alpha \pi T_S/\omega) \right]$$

where the $\cos(\omega t)$ term is due to $G(\omega)$ being an even function of frequency. Note also that in accordance with the definition above, $G(\omega)$ is normalized such that
A.1 Adaptive equalizer operation.

The adaptive equalizer in the receiver converges and forms the transfer characteristics \( E(\omega) \) in order to equalize the channel. A Zero Forcing ("ZF") equalizer adjusts its taps so as to force the Inter-Symbol Interference ("ISI") to zero, approaching the condition \( E(\omega) = 1/C(\omega) \). A Mean Squared Error ("MSE") equalizer drives the tap coefficients so as to minimize the mean square error, including ISI and noise and the equilibrium tap values generally depend on the signal-to-noise ratio. However, at large signal-to-noise ratios, say above 20 dB, both ZF equalizers and MSE equalizers are essentially equivalent in that they drive the equalizer tap coefficients to approach the ZF condition \( E(\omega) = 1/C(\omega) \). It is also assumed that the distortions in \( C(\omega) \) are such that its inverse exists within the tap range and the transversal span of the equalizer. Indeed the types of head-end signal reflections we are dealing with here would certainly fall within these practical ranges of consumer receivers’ adaptive equalizers.

It is the pre-detection spectral coloring of the noise due to the equalization that causes noise enhancement, and consequently degradation in decoding performance. Some of this noise enhancement is partially relieved by the use of Decision Feedback Equalization ("DFE"), an account of which is provided in [9]. Because DFE is universally applied, the actual noise enhancement penalty in ZF or MSE employing DFE would likely be less than that of an equalizer that strictly inverts the channel response, i.e. converging to a profile of \( 1/C(\omega) \).

Nevertheless, because the degradations are relatively small even without DFE, we derive this channel-inverting noise enhancement penalty below as an upper bound for the actual degradations in subscriber demodulators due to the deviation from a flat channel response associated with signal reflections.

Excluding the noise, the signal output of the receiver due to a transmitted complex symbol \( Z_k \) weighted by the impulse response of the transmit filter is given by:

\[
V_k = Z_k \int_{-\infty}^{\infty} G_r(\omega)C(\omega)E(\omega)G(\omega) e^{j\omega} \frac{d\omega}{2\pi} = Z_k \int_{-\infty}^{\infty} G_r(\omega)G(\omega) e^{j\omega} \frac{d\omega}{2\pi} = Z_k \int_{-\infty}^{\infty} G(\omega) e^{j\omega} \frac{d\omega}{2\pi} \quad (4)
\]

We have assumed, without loss of generality, that the carrier frequency is 0, because an appropriate change of variable offset in \( \omega \) can always be made. In addition, it is assumed in the expression of Equation 4 that correct carrier phase and symbol timing has been attained in the demodulator and that phase noise effects are not taken into account. Note also that the condition \( E(\omega) = 1/C(\omega) \) was assumed in the above equation, implying that any deviations from flat unity gain response in the channel, including a mere gain scale, is absorbed in the demodulator’s equalizer \( E(\omega) \) so that a unity gain chain results and the output signal is kept at a constant level.

For an input additive Gaussian noise having power spectral density \( S_n(\omega) \), the noise power at the sampled output of the detector is given by
\[
\sigma_n^2 = \int_{-\infty}^{\infty} S_n(\omega) \left| E(\omega)G_R(\omega) \right|^2 \frac{d\omega}{2\pi} = \int_{-\infty}^{\infty} S_n(\omega) |C(\omega)|^{-2} G(\omega) \frac{d\omega}{2\pi}
\] (5)

where we have used the fact that the receiver’s matched filter is a Root-Nyquist filter, i.e., \(|G_R(\omega)|^2 = G(\omega)|^2\), and that the equalizer compensates for the channel response \(E(\omega) = 1/C(\omega)\). In the following, we shall assume that the Gaussian noise is white with a one-sided spectral density given by \(N_0/2\). Without loss of generality, we take the signal sample in Equation 4 at time \(t_k = 0\) and by using Equation 5, we obtain the received MER due to the noise at the output of the demodulator, by averaging the symbol power over the ensemble of received symbols \(V_k\). We use the \(\overline{\cdot}\) notation to indicate the statistical averaging operation:

\[
MER = \frac{\overline{|V_k|^2}}{\sigma_n^2} = \frac{\overline{|V_0|^2}}{\sigma_n^2} = \frac{\left|Z_0\right|^2}{\int_{-\infty}^{\infty} S_n(\omega) |C(\omega)|^{-2} G(\omega) d\omega / 2\pi} = \frac{\left|Z_0\right|^2}{\int_{-\infty}^{\infty} S_n(\omega) |C(\omega)|^{-2} G(\omega) d\omega / 2\pi}
\] (6)

where we have used the normalization condition of Equation 3. As expressed in Equation 6, a larger channel transfer gain, or magnitude of \(C(\omega)\) (even by simple scaling), will result in a larger MER because we assume that the receiver adjusts its decoding gain by \(E(\omega) = 1/C(\omega)\) to arrive at a fixed signal output level. This larger magnitude in the channel transfer corresponds to an increased transmitted power to decoder devices, which, in turn, attenuate the noise accordingly, thereby producing larger MER values. In order to assess the impact of only the shape changes in \(C(\omega)\) over frequency but not its absolute scale, and because cable operators adjust their launched digital signals’ power to a fixed relative level at their plants’ output (after combining), we will treat the channel transfer function expressed in Equation 6 as being constrained to have constant transmitted power \(P_T\), which is given by:

\[
P_T = \left|Z_0\right|^2 \int_{-\infty}^{\infty} |C(\omega)|^2 |G_T(\omega)|^2 d\omega / 2\pi = \left|Z_0\right|^2 \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi
\] (7)

By substituting in Equation 6 the expression for \(\left|Z_0\right|^2\) extracted from Equation 7 in terms of the transmitted power \(P_T\), the MER obtained for a given transmit power \(P_T\) is given by:

\[
MER = \frac{\left|Z_0\right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |C(\omega)|^{-2} G(\omega) d\omega / 2\pi} = \frac{2P_T}{\frac{N_0}{2} \int_{-\infty}^{\infty} |C(\omega)|^{-2} G(\omega) d\omega / 2\pi \cdot \frac{N_0}{2} \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi}
\] (8)

Thus, the channel condition that maximizes the MER is that which minimizes the product of the two integrals in the denominator:

\[
\min_{C(\omega)} \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi \cdot \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi
\] (9)

Because the integrals in Equations 8 and 9 depend only on the magnitude of \(C\) and not on its
phase, phase angle distortions in the channel transfer can be equalized without affecting the MER and thus the minimum value attained for the integral product in Equation 9 by a solution $C(\omega)$ is unchanged if one considers instead the solution $C(\omega)\exp[i\phi(\omega)]$ with arbitrary phase profile $\phi(\omega)$. Hence, without loss of generality, we restrict ourselves to finding the minimum in Equations 9 for channel transfer functions $C(\omega)$ that are real functions of frequency.

To see that the minimum is attained when $C(\omega)$ is flat, we note that since $G(\omega)$ is strictly positive in the channel integration interval $[-\pi(1+\alpha)/T_s, \pi(1+\alpha)/T_s]$ and is thus a weighting function, both integrals in Equations 9 can be viewed as inner products in the Hilbert space of real valued square-integrable functions. The inner product in this Hilbert space for any two of its elements $h_1$ and $h_2$ can be defined by the $G$ weighted integral:

$$\left\langle h_1, h_2 \right\rangle \equiv \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} h_1(\omega)h_2(\omega)G(\omega)d\omega/2\pi$$

(10)

Thus, the two integrals in Equation 9 can be represented as $\left\langle C^{-1}, C^{-1} \right\rangle$ and $\left\langle C, C \right\rangle$ respectively and their product can be bounded from below using the Cauchy-Schwarz inequality for elements in the Hilbert space as follows:

$$\left\langle C^{-1}, C^{-1} \right\rangle\left\langle C, C \right\rangle \geq \left\langle C^{-1}, C \right\rangle^2 = \left| \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} C(\omega)C^{-1}(\omega)G(\omega)d\omega/2\pi \right|^2 = \left| \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} G(\omega)d\omega/2\pi \right|^2 = 1$$

(11)

By Cauchy-Schwarz, equality in Equation 11 is attained if and only if $C(\omega)$ is proportional to $C^{-1}(\omega)$, meaning that for a proportionality scalar $a$, the following must hold for equality (a minimum) to hold:

$$C(\omega) = a C^{-1}(\omega) \quad \Rightarrow \quad C^2(\omega) = a \quad \Rightarrow \quad C(\omega) = \text{constant}$$

(12)

The minimum value at the right-hand side of Equation 11 is therefore attained with $C(\omega)$ having a flat channel response. By inserting this minimum in Equations 8 we obtain the maximal possible value of the MER:

$$MER_{max} = \frac{2P_T}{N_0}$$

(13)

Two observations can now be made:

(a) The fact that a flat channel response maximizes the MER is not surprising because only under such condition, the total receiver filter chain $E(\omega)G_s(\omega) = C^{-1}(\omega)G_s(\omega)$ is matched to the total transmit filter chain $G_T(\omega)C(\omega)$. Any deviation in $C(\omega)$ from the flat response renders a mismatch between the total transmit and receive filters, thereby enhancing the pre-detection noise and degrading the MER below that achievable in Equation 13.

(b) As discussed above in connection with Equations 8 and 9, because channel phase profiles do not affect the maximum attainable MER value, channel distortions that are merely
group delay distortions (phase dispersion) without amplitude response distortions can be equalized without any degradation in MER performance. Because all consumer demodulators employ adaptive equalizers having sufficiently long transversal span to equalize group delay variations encountered in cable networks, for all practical purposes, these group delay variations are often considered benign linear distortions.

A.2 Noise degradation due to a single signal reflection.

As the channel transfer function deviates from the flat response and the equalizer adapts to force zero ISI, the MER experienced at the demodulator will degrade because it no longer has an optimal matched filter. As can be seen from Equations 8 and 11, the degradation relative to the non-distorted flat channel case is \( D = \langle C^{-1}, C^{-1} \rangle : \langle C, C \rangle \).

A channel with a single echo, or reflection, having a relative magnitude \( r \) has a transfer response given by

\[
C(\omega) = 1 + r \exp[-i(\omega \tau + \phi)], \text{ and thus } |C(\omega)|^2 = 1 + r^2 + 2r \cos(\omega \tau + \phi)
\]

(14)

The relative degradation factor \( D \) is thus given by

\[
D = \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi \cdot \int_{-\infty}^{\infty} |C(\omega)|^2 G(\omega) d\omega / 2\pi = I_2 \cdot I_1
\]

where

\[
I_1 = \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} [1 + r^2 + 2r \cos(\omega \tau + \phi)] G(\omega) d\omega / 2\pi \quad \text{and}
\]

\[
I_2 = \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} \frac{G(\omega)}{1 + r^2 + 2r \cos(\omega \tau + \phi)} d\omega / 2\pi
\]

(15)

We evaluate \( I_1 \) by using the identity \( \cos(\omega \tau + \phi) = \cos(\phi) \cos(\omega \tau) - \sin(\phi) \sin(\omega \tau) \) and by observing that the integration of the sinusoidal terms yields zero due to the even symmetry of \( G(\omega) \) and by using the known integrals of Equations 2 and 3:

\[
I_1 = \int_{-\pi(1+\alpha)/T_s}^{\pi(1+\alpha)/T_s} [1 + r^2 + 2r \cos(\omega \tau + \phi)] G(\omega) d\omega / 2\pi = 1 + r^2 + 2r \cos(\phi) g(\tau) =
\]

\[
= (1 + r^2) \left[ 1 + \frac{2r}{1 + r^2} \cos(\phi) g(\tau) \right]
\]

(16)

Because \( I_2 \) does not submit to a closed-form solution, we exploit the fact that in our application, \( r \) is sufficiently small compared to 1 and thus approximating the integrand in a power series up to second order in \( r \cos(\omega \tau + \phi) \) provides a very good approximation:

\[1 \quad \text{To a great extent, this is the reason that, unlike the authors of Annex A to ITU-T-J.83 who required that QAM transmitter group delay ripple be less than 0.1T_s, the Annex B authors have refrained from specifying any such requirement.} \]
\[
\frac{1}{1 + r^2 + 2r \cos(\omega \tau + \phi)} = (1 + r^2)^{-1} \frac{1}{1 + \frac{2r}{1 + r^2} \cos(\omega \tau + \phi)} = \\
(1 + r^2)^{-1} \left[ 1 - \frac{2r}{1 + r^2} \cos(\omega \tau + \phi) + \left( \frac{2r}{1 + r^2} \cos(\omega \tau + \phi) \right)^2 - \cdots \right] = \\
(1 + r^2)^{-1} \left[ 1 + \frac{2r^2}{(1 + r^2)^2} - \frac{2r}{1 + r^2} \cos(\omega \tau + \phi) + \frac{2r^2}{(1 + r^2)^2} \cos(2\omega \tau + 2\phi) - \cdots \right] 
\]

Using the above approximation and following the term by term integration method used to evaluate \( I_1 \) in Equation 16, we obtain the expression for \( I_2 \):

\[
I_2 = (1 + r^2)^{-1} \left[ \frac{2r}{1 + r^2} \cos(\phi) g(\tau) + \frac{2r^2}{(1 + r^2)^2} \cos(2\phi) g(2\tau) - \cdots \right] 
\]

Using Equations 16 and 18, we obtain the approximation for the noise degradation \( D \):

\[
D = I_1 I_2 \approx \left[ \frac{2r}{1 + r^2} \cos(\phi) g(\tau) \right] \left[ 1 + \frac{2r^2}{(1 + r^2)^2} - \frac{2r}{1 + r^2} \cos(\phi) g(\tau) + \frac{2r^2}{(1 + r^2)^2} \cos(2\phi) g(2\tau) \right] = \\
1 + \frac{2r^2}{(1 + r^2)^2} \left[ 1 + \cos(2\phi) g(2\tau) - 2 \cos^2(\phi) g^2(\tau) \right] + \text{terms of higher order in } r
\]

The above expression for \( D \) has a period of \( \pi \) in \( \phi \) and is shown in Figure 3 in dB relative to a flat channel \((r=0)\), over reflection delays \( \tau \) of up to three symbol times. As can be seen, the worst-case value of the combining phase \( \phi \) is \( \pi/2 \). This non-trivial extremum can be shown by setting the first derivative of \( D \) to zero and solving for \( \phi \):

\[
\frac{dD}{d\phi} = \frac{8r^2}{(1 + r^2)^2} \cos(\phi) \sin(\phi) \left[ g^2(\tau) - g(2\tau) \right] = 0
\]

The value of \( D \) under this worst-case combining phase \((\cos(\phi) = 0)\) is given by setting \( \phi = \pi/2 \) in Equation 19, thereby obtaining the value of \( D_{\text{max}} \) below:

\[
D_{\text{max}} = 1 + \frac{2r^2}{(1 + r^2)^2} \left[ 1 - g(2\tau) \right]
\]

It is plotted in Figure 4 for values of \( r \) corresponding to 20 log\((r) = -15\), -20 and -25 dB.
Figure 3. Noise degradation due to a single echo having a $-15$ dB relative level as a function of phase and delay for a QAM channel with excess bandwidth factor of $\alpha = 0.15$. (Based on Equation 19).

Figure 4. Noise degradation due to a single echo having worst case combining phase at the indicated relative reflection levels as a function delay for a QAM channel with excess bandwidth factor of $\alpha = 0.15$. (Based on Equation 21).
Appendix B  RF Output Connector’s effect on Output Return Loss

The effects of the RF connector’s own return loss is derived here in the context of a cascade of an RF device connected via the RF connector to other devices, as shown in Figure 5.

![Figure 5](image)

**Figure 5.** The RF connector cascade model for deriving the resultant return loss of an RF device. The single coaxial line depicting the input and output lines shown are two-port signal paths.

The RF device which, in our case, is an RF modulator or an upconverter within an EQAM chassis, has an output return loss characterized by the complex reflection coefficient \( \Gamma_0 \) and it is connected to a panel-mount connector characterized by the \( 2 \times 2 \) complex scattering matrix \( S \) having \( S_{ij} \), \( i,j=1,2 \) as its elements (S parameters). By definition of its S parameters, when the connector is connected to a nominal 75\( \Omega \) load instead of the RF Device, the return loss as seen at the output port would be characterized by \( \Gamma = S_{11} \). However, because the RF Device does not have such ideal absorptive output impedance but rather a non-zero reflection coefficient \( \Gamma_0 \), basic network theory provides for the general expression for the output reflection coefficient \( \Gamma \):

\[
\Gamma = S_{11} + \frac{S_{12}S_{21}\Gamma_0}{1-S_{22}\Gamma_0}
\]

(22)

In general, these reflection coefficients and the S parameters are complex functions of frequency. For example, they can be resolved into polar representation with real positive magnitudes as

\[
\Gamma = \rho(\omega)\exp[i\phi(\omega)], \text{ and } \Gamma_0 = \rho_0(\omega)\exp[i\phi_0(\omega)], \text{ and } S_{11} = \rho_1(\omega)\exp[i\phi_1(\omega)]
\]

(23)

The worst-case phase conditions that can be experienced in the system gives rise to the largest magnitude of the output reflection coefficient. By Equation 22, it is given by:

\[
\rho = |\Gamma'| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_0}{1-S_{22}\Gamma_0} \right| \leq |S_{11}| + \left| \frac{S_{12}}{1-|S_{22}|\Gamma_0} \right| |\Gamma_0| = \rho_1 + \frac{(1-|S_{11}|^2)\rho_0}{1-|S_{22}|\rho_0} = \rho_1 + \frac{(1-\rho_1^2)\rho_0}{1-\rho_1\rho_0}
\]

(24)

The last two steps in Equation 24 were taken based on (a) the assumption that the S parameters characterize a lossless network (i.e., \( S^T S^* = I \), meaning that losses through the connector are merely due to port reflections) and (b) the assumption that the network is reciprocal (i.e., \( S^T = S \)). The latter assumption is justified by noting that the connector’s characteristics is dominated by a distributed axial conductor surrounded by shunt coaxial capacitive reactance, essentially appearing symmetrically from the input and output ports. However, for small enough reflection coefficients, these assumptions are not even essential and Equation 24 practically means that...
\( \rho = \rho_1 + \rho_0 \). In other words, the effective worst-case reflection phasors add up as voltages to yield degraded return loss values.

Connectors are sometimes specified in terms of their steady-state Voltage Standing Wave Ratio (“VSWR”). They are measured under ideal termination conditions and the magnitude of the reflection coefficient and the return loss of the connector are expressed in terms of the VSWR as

\[
\rho_1 = \frac{\text{VSWR} + 1}{\text{VSWR} - 1} ; \quad \text{Return Loss} = -20\log_{10}(\rho_1)
\]  

(25)

However, modern network analysis tools typically do not employ steady-state type methods and F connector return loss measuring standards employ time-domain reflection methods [10,11].

As seen from the output port, the RF device’s return loss of \( -20\log_{10}(\rho_0) \) is degraded by the connector’s reflections as described above to produce the effective output return loss in accordance with Equation 24. The result is shown in Figure 6 below:

![Figure 6](image)

**Figure 6.** Effective output return loss degradation due to the output connector’s return loss having values of 14 dB, 20 dB, 30 dB and \( \infty \) dB (ideal). Based on Equation 24.

As can be seen in this figure, deviations from the ideal are more pronounced with decreasing connector return loss. Unfortunately, connectors’ return loss degrades with frequency, a fact recognized by the authors of EIA-550 [12], a standard for an improved 75 \( \Omega \) type F connector (the type FD connector). The requirements of EIA-550 as a function of frequency are shown in Figure 7. As can be seen in that figure, a connector meeting a 14 dB return loss above 600 MHz, complies with this standard. Fortunately, commercially available low profile F connectors have specifications up to 1 GHz that slightly surpass the EIA-550 requirements, as shown for four
examples in Figure 7. While low frequency performance is better than shown, these connectors are typically guaranteed to have return loss values in the order of 20 dB up to 1GHz. A more detailed account of the frequency degradation of F connectors’ return loss and the practical performance challenges is described in [13]. In this regard, it should be noted that unlike the EIA-550, none of the other F connector standards this author is aware of, specify requirements for return loss. Rather, they merely provide detailed mechanical specifications [14,15] that are substantially consistent with those in EIA-550.

The implications for product return losses can be seen in Figure 6: A 14 dB effective output return loss up to 1 GHz for EQAMs cannot be practically sustained by devices employing RF connectors that just meet the EIA-550 requirements. In that case, the RF devices would have to be ideal (infinite return loss) to meet this specification. Even a 20 dB return loss connector introduces great difficulties in meeting a 14 dB effective output return loss across the band, as it requires that the RF device’s own return loss be better than 20 dB. The only practical way this can be met is by doubling the power rating of the output amplifier, doubling its RF output level and using resistive output pads. Again, an outcome that clearly runs against the stated MSO goal of high-density EQAM solutions.

![Figure 7. F type RF connector return loss requirements and specification limits for some commercially available connectors. To the extent that VSWR values are specified, the equivalent return loss based on Equation 25 was used. Source: Reference [12] for EIA-550, AMP/Tyco and Amphenol product catalogs.](image)

In principle, RF connectors’ adverse return loss effects can be mitigated by designing a matching network at the output of the RF Device which employs an inductive reactance to compensate for the connectors’ predominantly capacitive reactance. However, this implies that the compensation will result in a resonance structure, meeting the return loss requirements only over a narrow band. This solution is unacceptable for EQAM devices that must be frequency agile.
over a decade and must present a reasonably good out-of-channel return loss in non-active channels. Thus, *traditional return loss compensation methods are inapplicable for EQAM devices.*

Alternatively, coaxial F connector receptacles having improved return loss specifications (25–30 dB up to 1 GHz) are also commercially available but they are more costly precision type connectors and, by necessity, they have higher profile construction than those shown in Figure 7 and would therefore impose nearly doubling of the height of RF modules within the EQAM. This outcome runs counter to the very goal of the DRFI of providing high density, space saving and cost effective solutions.

The current DRFI Active-Channel return loss is specified as 14 dB, declining to 13 dB above 750 MHz and to 12 dB above 870 MHz up to 1 GHz. Judicious balancing of requirements and implementation considerations of the type described above led to the adoption of the slight relaxation at the top of the band.

References


