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A Review

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Abstract

A water distribution network consists of pipes, reservoirs, pumps, valves, and other hydraulic components and its purpose is to provide reliable service to the customers under various demand conditions. The least cost design of water distribution networks is an optimization problem, which has been solved using linear programming, nonlinear programming, dynamic programming, and heuristic based optimization methods. In this paper, we review the current status of the optimization models in the design of water distribution networks and present recommendations for future research.

CE Database Subject Headings: Water distribution systems; Water supply; Network design; Optimization; Algorithms; Reliability

Introduction

Safe drinking water supply is one of the basic human needs and is one of the primary goals of the International Decade for Action ‘Water for Life’ 2005-2015 (Water for Life: Making it happen, 2005). With the unprecedented population growth in cities, the stress on water infrastructure has increased phenomenally. The World Health Organization (WHO) has recognized that when infrastructure and services are lacking, cities are among the earth’s most threatening environments (World Water Development Report, 2003). This observation highlights the need for an efficient and safe water distribution system.

A water distribution system consists of network of pipes, reservoirs, pumps, valves, and other hydraulic elements. Its purpose is to supply good quality water to customers within specific pressure levels under various demand conditions. To analyze the interrelationships among various

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components, a water distribution system is transformed into a network representation called *water distribution network* (WDN) (Yang et al., 1996).

A WDN requires extensive planning at the design stage and maintenance during operations to ensure that good service is provided to the customers in the most reliable and economical way. The planning and maintenance of a WDN involves the determination of demand, the optimal layout of the WDN, the optimal dimensions of system components like pipes, pumps, valves etc. and the optimal operations. The infinite range of possible combinations of the pipe materials, routes, diameters, pumping station locations and capacities makes the task of the engineers and managers involved in the activity daunting. Adding to this complexity are the operational parameters such as pressure zone boundaries, control valve settings, and pump operating schedules which have significant impact on service quality and whole life costs. Bhave (1991) and Mays (2000) contain excellent description of the hydraulics of WDNs. A WDN can be designed using *conventional trial and error methods* or more intelligent *optimization methods*. Conventional trial and error methods for the design of WDNs are iterative techniques and hence can lead to inefficient solutions.

The task of decision making entails choosing between various alternatives. Optimization is central to any problem involving decision making, whether in engineering or in economics. The choice is governed by the desire to make the best decision. The measure of goodness of the alternatives is described by an objective function or performance index. Optimization theory and methods address the selection of the best alternative through the given objective function (Chong and Žak, 2001).

This paper discusses the optimal design problem of WDNs and it is organized as follows. The general mathematical formulation of the WDN optimal design problem will be discussed first. Next, literature pertaining to the optimal design of WDNs will be reviewed on the basis of type of the WDN configuration; namely branched WDN and looped WDN. However in actual practice, the optimization models are not used extensively for the design of a WDN (Mays, 2000). This may be primarily because the optimization models reduce the redundancy in design as they focus solely on minimizing cost, while practicing engineers want a good measure of redundancy for a reliable WDN. Thus, researchers later started incorporating additional complexities like multiple loadings, demand uncertainty, water quality, reliability etc. Considerations of all these complexities have made the optimization problem *intractable* to closed form optimization techniques. Therefore in recent times, researchers are focusing on heuristic based optimization methods like *genetic algorithms* (GA), *tabu search*, *simulated annealing*, etc. These meta-heuristic methods as applied for the design of WDNs are discussed in the final part of the paper. In conclusions, the advantages and disadvantages of all the optimization techniques and models used for designing WDNs are discussed along with some recommendations on future research directions.
Design of WDNs

The design of a WDN for a large water distribution system is a complex problem and involves decisions on pipe layout and sizes; location and capacity of tanks; location, types, capacity, and operating schedule of pumps; and location, types, and settings of different valves (Bhave, 2003). The designer’s problem is to determine the minimum cost system while satisfying the demands at the required pressure heads.

The cost of the system includes the initial investment for the components, such as pipes, tanks, valves and pumps, and the operating cost for pumping the water throughout the system (Mays and Tung, 1992). The main constraints are that the desired demands are supplied with adequate pressure heads at withdrawal locations. Also, the flow of water in a distribution network and the pressure heads at nodes must satisfy the governing laws of conservation of mass and energy. The WDN design problem can be stated as (Mays, 2000):

\[
\text{Minimize} \quad \text{Initial capital cost} + \text{operations cost}
\]

Subject to 1. Conservation laws of mass and energy

2. Water demand constraints

3. Meet nodal head requirements

In reality, additional complexities have also to be included for a good model. The other factors that can be included are:

Layout: In most distribution systems, pipes are restricted to be placed beneath roadways or in right-of-ways. However, in some cases, layout is not fixed a priori and can be determined optimally (Mays, 2000).

Multiple Loadings: The design of a WDN is based on peak hour demands (the average rate of usage during the maximum hour of usage). This assumes that the various demands (industrial, residential, commercial etc.) are simultaneously at their peaks at all the demands nodes of the network. In reality, the various demands have different peak values at different times of the day. Therefore, for proper design of a WDN, it is necessary to consider several loadings, including fire requirements, to provide reasonable flexibility to the network (Bhave, 2003). The low demand loadings also have to be considered to ensure the tanks are filled during low demand periods (Alperovits and Shamir, 1977).

Uncertainty: In general, two main types of uncertainty exist in a WDN: epistemic uncertainty and aleatory uncertainty (Kapelan et al., 2005). Epistemic uncertainty is mainly due to lack of information about some aspects of the problem being analyzed, e.g. the status of some valve in the WDN. This type of uncertainty is reduced by gathering the information about the missing parameter. Aleatory uncertainty is due to fluctuations that are intrinsic
to the system studied, e.g. the water demands at nodes. Both aleatory and epistemic uncertainty can have a potential impact on system reliability. Aleatory uncertainty contributes to unreliability through occurrence of extreme events (or accidents), i.e. excessive demand. Epistemic uncertainty contributes to unreliability through the imprecise measurement or the changes in the underlying phenomena after the measurement, i.e. imprecise estimation of variability in demand or changes in the demand pattern.

**Operations:** Operations of a WDN have been included in the formulation above. This alone is a significant problem in both defining the demands and in addressing large number of constraints and decisions associated with operations (Mays, 2000).

**Water Quality:** To meet pressure and flow requirements in a WDN, the components of the WDN are designed larger which results in longer residence times of the water. Water quality decays with time and smaller components are required to meet water quality constraints. Therefore, the objective of meeting water quality is in conflict with meeting pressure and flow requirements.

**Reliability:** Regardless of the optimization method, optimization reduces cost by reducing diameter of pipes or by completely eliminating the link between nodes. This makes the system unreliable, in case of component failures or demands that exceed design values (Methods et al., 2003). Therefore, concept of reliability becomes important in WDNs. Reliability is defined as the ability of a WDN to provide an adequate supply to the consumers, under both normal and abnormal operating conditions (Xu and Goultier, 1999). The adequacy of supply is measured in terms of requirements of pressure head, demands, and water quality. These requirements can not be fulfilled in the case of component failure (failure of pipes, pumps, valves etc.) and performance failure (demand on the system being greater than design value) (Mays, 2000). These two types of failures should not be considered separately for reliability assessment as they are strongly related (Ostfeld and Shamir, 1993).

**Rehabilitation:** A rehabilitation strategy (cleaning, relining, or replacement) of an existing network is necessary either for redevelopment or because of loss of carrying capacity and deterioration of the system. Rehabilitation allows a WDN to operate efficiently and economically within the defined operating requirements over an extended period (Engelhardt et al., 2000).

The overall optimal design problem of a WDN can be stated mathematically in terms of the nodal heads $H$ and the various design variables $D$ as follows (Mays, 2000):

**Objective**

\[
\min f(H, D)
\]  

**Subject to**
Conservation of flow and energy

\[ G(H, D) = 0 \]  \hspace{1cm} (2)

Constraints on nodal heads

\[ H_{\text{min}} \leq H \leq H_{\text{max}} \]  \hspace{1cm} (3)

Constraints related to design variables

\[ D_{\text{min}} \leq D \leq D_{\text{max}} \]  \hspace{1cm} (4)

Constraints related to design variables and nodal heads

\[ v_{\text{min}} \leq v(H, D) \leq v_{\text{max}} \]  \hspace{1cm} (5)

The objective function (Eq. 1) may be linear or nonlinear, depending upon the type of model formulation. Eq. 2 defines constraints \( G \) which are nonlinear equations based on the conservation laws of mass and energy. These constraints can be written for multiple loading conditions. The constraints \( H \) are the nodal heads at specified nodes in the WDN with \( H_{\text{min}} \) and \( H_{\text{max}} \) being their lower and upper bounds respectively (Eq. 3). The constraints \( D \) define the dimensions of each component in the WDN (Eq. 4). The design variables can be discrete or continuous. The discrete nature of the design variables, such as commercially available pipe sizes, makes the problem NP-hard. Essentially, the NP-hard result means that a rigorous algorithm to find an optimum design using discrete diameters is not a practical possibility. For a \( N \)-pipe system, the computational time required for a rigorous algorithm is at best an exponential function of \( N \) and is thus enormous even for relatively small water distribution networks (Yates et al., 1984).

Eq. 5 constrains parameters \( v \) which are functions of both the nodal pressure heads and the design variables. Examples of such constraints include pipe velocity constraints and reliability constraints.

The above general formulation incorporates most of the complexities in the design of WDNs. Due to the complexity of this problem, design of WDNs has been investigated by many researchers in the past 30 years and many optimization techniques have been applied to solve the problem. Traditional optimization techniques like linear programming (LP), nonlinear programming (NLP), dynamic programming (DP), integer programming; and meta-heuristic optimization methods have been commonly applied to the problem. Meta-heuristic optimization techniques include genetic algorithms (GA), tabu search, simulated annealing etc.

Due to large amount of literature available in optimal design of WDNs, the review is presented on the basis of network configurations. A WDN can have either a branched or a looped configuration (Figure 1). In branched systems, the water has only one possible path from the source to a particular customer while in looped systems there may be several paths from the source to a customer (Methods et al., 2003). Due to the difference in degree of complexity in solving both
the configurations, branched systems are reviewed first followed by looped systems.

**Branched WDNs**

Branched water distribution networks are commonly used for small communities and for industrial and agricultural supplies (Bhave, 2003). In such networks, starting from the end nodes and successively applying the law of conservation of mass, the discharges can be computed in all the links. Table 1 summarizes the optimization models used for designing branched WDNs.

Karmeli et al. (1968) use a LP model for optimal design of branched networks. In a branched network, the flow in each link is known, and therefore the effect of changing resistance of links on heads can be computed directly using Hazen-Williams Equation (Eq. 6).

\[
\frac{h_L}{L} = \frac{10.7Q^{1.852}}{C^{1.852}D^{4.87}}
\]  
(6)

where \( h_L \) (in m) is the head loss, \( Q \) (in \( m^3/s \)) is the flow rate, \( L \) (in m) is the length of the pipe, \( D \) (in m) is the diameter of the pipe, and \( C \) is the Hazen-Williams roughness coefficient. The value of \( C \) ranges from 140 for smooth pipe to 90 or 80 for old, unlined, tuberculated pipe (Mays, 2000). The cost of pipe is assumed to be a linear function of length. The resistance of pipe is also a linear function of pipe for a given diameter (Eq. 6). Therefore, the problem can be cast as a LP problem (Eq. 7) with lengths of the pipes with given diameters in each link as the decision variables. The set of diameters in each link is selected in advance. Constraints on minimum heads at nodes appear as linear inequalities imposed on the decision variables (Eq. 3).

The problem has been solved only for initial cost.

\[
\min_{L_{ij}} \sum_i \sum_{j \in M} c_j L_{ij}
\]

\[
s.t. \sum_{j \in M} L_{ij} = L_i \forall i
\]  
(7)

where \( c_j \) is the cost per unit length of pipe of diameter \( j \), \( L_{ij} \) is the length of pipe of diameter \( j \) in link \( i \), and \( L_i \) is the length of the link \( i \). The pipe diameters \( j \) are selected from the permissible pipe set \( M \).

Calhoun (1971) extends the model developed by Karmeli et al. (1968) to the pumped system with the head provided by pump as another decision variable alongside the lengths of pipes as decision variables. Robinson and Austin (1976) consider pressure ratings of the pipes and the location and settings of pressure reducing valves (PRV) in the preceding model of Calhoun (1971). Their approach allows the inclusion of maximum head constraints in the model (Eq. 3). The cost per unit head at source (pumped or tanked) is assumed to be a nonlinear function of head at source and computed iteratively. The above models consider only few pipe sizes for each link to
restrict the size of the models. Bhave (1979) presents a method based on the critical path concept for selection of the optimal pipe sizes for optimization of branching networks by LP.

Mandry (1967) considers the problem of economic design of pipe distribution systems for sprinkler projects. Sprinkler systems consist of pipes which are used to irrigate land under pressure. The cost per unit length $c$ of the pipe is considered as a power function of the diameter $D$ of the pipe with parameters $Y$ and $x$ (Eq. 8).

$$c = YD^x$$  \hspace{1cm} (8)

The cost includes the cost of pipe manufacturing, transportation to job site, excavation of pipe trench, placing the pipe in the trench, backfilling the trench, and other miscellaneous cost items such as clean-up, dewatering, etc. The costs vary with class of pipe and location of installation and are cost indexed for time of use. Initially, the author considers the case of cost increment if the flow is increased. It is assumed that the larger pipes required to carry the larger flows are more costly. Two flows with the same elevation of water source and delivery pressure are considered. The head losses for both the flows are same and from Eq. 6, a relation between the diameters to carry the given flows is determined. The cost of pipes is given as function of diameter (Eq. 8), therefore a relation between the costs of pipes to carry given flows is determined. The economic sizing of pipes in gravity and pumped systems is also presented. For the gravity system, cost of a pipe has been written as a function of flow, head loss, and length (using Eqs. 6 and 8). The objective is to minimize the total cost of pipes (Eq. 9)

$$\min C = \sum_i Yf(Q_i, h_{Li}, L_i)L_i$$  \hspace{1cm} (9)

Water flow in the pipes are assumed to be explicitly known. This model is based on the principle that the economic pipe sizes for a gravity system will be one set of sizes that uses all available head in friction when delivering the peak flow to hydraulic control points. Only one sequence of pipe diameters will produce the minimum total pipe cost. Since diameter of pipe is a function of friction head loss in that particular pipe, solving for economic hydraulic gradient (with given total head and flows) allows to compute the optimal pipe sizes. The author uses classical optimization techniques to solve for economic hydraulic gradient ($\frac{h}{L}$). For pumped system, the cost of pipe and cost of pump energy have been expressed in terms of the diameter of pipe. Costs of the pumping plant, operations, and maintenance are considered constant.

Swamee et al. (1973) extend the model of Mandry (1967) by including initial head required at the source as a decision variable. The cost of the pumping station is considered as a function of initial head and flow required, while the cost of overhead tank is considered as a function of the initial head. In a direct pumping system (no tanks included), the objective is the minimization of total cost of pipes, pumping plant, and pump energy with an equality constraint on head at the end node. The constrained problem is converted to an unconstrained problem by combining the
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objective function and constraint into a merit function (Eq. 10). Calculus based optimization techniques are applied on the merit function to compute the optimal initial head and diameter of pipes. The model has been extended to the case of continuous withdrawals.

\[
\min \sum_i Y D_i^2 L_i + A(Q_p h_p)^m + B Q_p h_p + \lambda(\sum_i K D_i^{-4.87} - (h_p - h_n))
\]  

(10)

where \(Q_p\) is the discharge through the pump, \(h_p\) is the head provided by the pump, \(\lambda\) is the Lagrangian multiplier, and \(h_n\) is the required head at the end node. \(A, B, K,\) and \(m\) are constants.

Appleyard (1975), Chiplunkar and Khanna (1983), and Young (1994) apply the Lagrange multiplier method to the constrained NLP problem, similar to Swamee et al. (1973). Appleyard (1975) considers the optimization of piping cost, for multiple branches, with equality constraints on the head at the end node for all the branches. Chiplunkar and Khanna (1983) examine a similar problem as Appleyard (1975). The essential departure from the latter is the inclusion of cost of pumping plant and pumping energy. Young (1994) considers the design of a branched WDN on an uneven terrain. Fujiwara and Dey (1988) design a branched network on flat terrain using both NLP and LP formulations. It is assumed that the network has a single source and minimum head requirements at all the end nodes are equal. The NLP formulation is similar to earlier models of Swamee et al. (1973), Appleyard (1975), and Chiplunkar and Khanna (1983). Lagrange multipliers are used to obtain optimal pipe sizes. The assumption of equal minimum heads at end nodes allows the computation of Lagrange multipliers using simple arithmetic operations. The computed pipe sizes are rounded off to commercially available pipe sizes using an analytical method. A LP model, similar to Karmeli et al. (1968), is then applied to compute optimal lengths of pipe for commercially available pipe sizes. An additional constraint on flow velocity is also considered (Eq. 5) in the LP model.

Liang (1971) has used a dynamic programming (DP) formulation to design a branched network. The system used is relatively simple with all pipe segments joined serially. At the end of each pipe segment, an outlet which discharges water is considered. Water is pumped into the upstream pipe at constant pressure. In this DP formulation, any segment is considered a stage. The annual cost of transporting water, which includes the cost of pipes, cost of energy, and cost of water wasted, is taken as the objective function. The head at the inlet of pipe (or stage) \(i\) \((X_i)\) is considered as the state variable and diameter of the pipe \((D_i)\) as the decision variable. The state transformation relation is obtained (Eq. 12) using Darcy-Weisbach equation (Eq. 11).

\[
h_L = f \frac{8LQ^2}{\pi^2 g D^5}
\]  

(11)

\[
X_{i-1} = X_i - h_{Li}
\]  

(12)

where, in Eq. 11, \(h_L\) (in m) is the head loss and \(f\) is the Darcy-Weisbach friction factor. \(f\) is a function of the flow rate, diameter of the pipe, density of the fluid, viscosity of the fluid, and
the internal roughness of the pipe. Possible values of $f$ range from 0.008 to 0.100 (Streeter and Wylie, 1983). In Eq. 12, $X_i$ is the head at the outlet of pipe $i$ and $h_{Li}$ is the head loss due to friction in pipe segment $i$. Figure 2 shows a stage in DP formulation where $r_i$ is the stage return function. In this case, the stage return function is the annual cost of transporting water. The discharge at the outlet of pipe $i$, $Q_i$, is dependent on $X_i$ and flows through pipes are related through the principle of conservation of mass. Constraints on design flow in each pipe are also considered in the formulation. The diameters are considered discrete in nature and the annual cost of transportation of water is minimized using backward recursion. Yang et al. (1975) extends the previous formulation of Liang (1971) to diverging branched network (nonserial system). Minor losses due to appurtenances can be significant, especially when the length of the pipe is short and are considered in the formulation. The annual cost of transportation of water is minimized using backward recursion.

Bhave and Lam (1983) consider optimal layout for a three node network with a single source. The problem is formulated as an unconstrained nonlinear problem and solved using calculus based optimization techniques. The objective is to minimize the total cost of the network. The procedure is then extended to multiple node network with several source and demand nodes. This problem is solved by treating the multiple node network as a Steiner problem. The Steiner problem is to find a layout connecting all given nodes such that the total length of the layout is minimal.

Deb (1973, 1974), Perold (1974), and Ingle and Raut (1985) discuss some analytical methods for the optimal design of a branched WDN. Deb (1973) develops a simple method for least cost design of a single branched system with an outlet at the end of each pipe. The total head loss is assumed to be known. The method has been developed by combining the Hazen-Williams equation (Eq. 6), pipe cost function (Eq. 8), and the least capital cost criteria. It is assumed that the ratio of head loss in single pipe to the total head loss is equal to the ratio of cost of the same pipe to the total cost (Eq. 13).

$$\frac{\Delta H_k}{\Delta H} = \frac{C_k}{C}$$

where $\Delta H_k$ is the hydraulic loss in the $k^{th}$, $\Delta H$ is the hydraulic loss in the total system, $C_k$ is the cost of the $k^{th}$ pipe, and $C$ is the cost of the total system. Deb (1974) extends this model to multiple branched networks. Initially, the total head loss is assumed as known and later this assumption is relaxed to consider the total head loss as variable. Perold (1974) describes a method for economical pipe sizes in gravity systems by comparing financial gains and losses resulting when pairs of changes in pipe sizes are made in various portions of the system in such a way that the additional pressure losses and gains balance. The application to time-variable branched flow is examined. Ingle and Raut (1985) discuss a method to design a branched system with two tanks and the objective is to minimize cost of the system along with the minimum pumping energy cost.

As mentioned earlier, branched systems are commonly used for rural and industrial water supply. However, looped systems are generally more desirable than branched systems, particularly
in urban environments, as they are more reliable in case of a failure.

**Looped WDNs**

From the mathematical perspective, a fundamental problem in the least-cost design of looped systems relative to branched systems occurs when assigning the flows in the individual pipes. In branched systems, a given demand pattern defines the flows in the pipes explicitly and uniquely. In a looped system, there are infinite number of distributions of flow in the network that can meet a specified demand pattern. The distribution of flows assumed in the pipes affects the optimal design of a WDN (Goulter, 1992). Also from the engineering perspective to provide reliable systems, minimizing cost in looped systems reduces redundancy and hence reliability of the system. This makes optimal design of a looped WDN more complex than a branched WDN. Looped WDNs are surveyed on the basis of optimization methods applied: traditional optimization methods and meta-heuristic optimization methods. Table 2 summarizes the optimization models used for designing looped WDNs.

**Traditional Optimization Methods**

Traditional optimization methods have been applied for the design of basic WDNs as well as design of WDNs considering additional design complexities. Design of a basic WDN mainly includes the sizing of the components like pipes, tanks, pumps, and valves. As discussed earlier, the additional complexities include layout, multiple loadings, uncertainty, operations, water quality, reliability, and rehabilitation.

**Design of Basic WDNs**

Jacoby (1968) formulates a nonlinear integer programming problem for the design of a looped WDN. The objective is to minimize the initial cost and the operating cost of the pumps. The decision variables are the diameters of the pipes \(D_i\) and the flows \(Q_i\). The conservation laws are taken as constraints. Initially, the problem is treated as continuous NLP problem and the computed values of diameters are rounded off to the nearest integer values. The constrained optimization problem is converted to unconstrained problem using a *merit function* (similar to Swamee et al. (1973)). The numerical gradient method, using *steepest descent*, is applied on the merit function to find the optimal diameters and the flows. Convexity of the merit function cannot be determined, therefore two other types of search directions are used to test and avoid local minima.

Watanatada (1973) augments the model of Jacoby (1968) by including constraints of specified demand flow rate and minimum allowable delivery pressure at each consumption node. The design variables in that case are the diameters of the pipes and the heads at the nodes. Shamir
(1974) extends the model of Watanatada (1973) by considering the design and operations of the system under multiple loading conditions. In a WDN, the flow related variables may be the head at nodes, the consumption at nodes, or the head loss due to friction. The number of variables that can be solved, using the law of conservation of mass, are equal to the number of nodes. In Shamir (1974), these variables are partitioned into basic and nonbasic variables. The nonbasic variables are a combination of heads at nodes, consumption at nodes, and head loss due to friction in pipes. Optimal solutions of nonbasic variables are obtained using a reduced gradient method with the equality constraint of conservation of mass. A computationally faster method of computing derivatives is applied, which uses the sparsity of the Jacobian matrix. A penalty method is used to deal with inequality constraints on the nonbasic variables.

Lansey and Mays (1989) formulate a general WDN design problem (Eqs. 1–5) and use a similar technique as Shamir (1974) for solving the nonlinear optimization problem. In that methodology, a network solver is linked with the optimization model so that the hydraulic constraints (Eq. 2) are removed from the optimization model. The decomposed problem is solved using generalized reduced gradient (GRG) method. The pressure heads \( \mathbf{H} \) are taken as basic variables and are expressed as functions of design parameters \( \mathbf{D} \), which are considered nonbasic variables. A set of decision variables \( \mathbf{D} \) is passed from optimizer to the network solver. The network solver solves the hydraulic equations (Eq. 2) and determine the values of the nodal pressure heads \( \mathbf{H} \). This information is then passed back to the optimizer and \( \mathbf{D} \) is modified. The process continues until the stopping criterion is met. The formulation includes sizing of the pipes, pumps, and tanks; and rehabilitation of the existing systems under multiple loading conditions. Varma et al. (1997) also use a network solver to reduce the size of the problem. Graph theoretic concepts are used to achieve the problem reduction. The reduced problem is solved using a successive quadratic programming (SQP) technique to iteratively modify the pipe diameters. The necessary derivatives required to set the reduced quadratic program (QP) at each iteration of SQP are obtained analytically. The formulation has been extended to include multiple sources, pump capital costs, and operating costs.

The above discussed formulations of looped WDNs require the use of a network solver, at each iteration of optimization, to solve for heads and flows and then using the hydraulic solutions to modify the design. Alperovits and Shamir (1977) discuss a linear programming gradient (LPG) method for the optimal design of a looped WDN, which does not require any assumptions about the hydraulic solutions of the network. Initially the link flows are assumed to be known and a LP formulation, similar to Karmeli et al. (1968), is used with constraints on heads at nodes (Eq. 3). The resulting optimal cost can be expressed as a function of \( \mathbf{Q} \) (Eq. 14), which is a vector of flows in all links.

\[
\text{cost} = LP(\mathbf{Q}) \tag{14}
\]

where LP denotes that the optimal cost that is computed using the linear programming formula-
tion. Using Eq. 14, a gradient method is applied for improving cost while systematically changing $Q$. The method for changing $Q$ is based on the use of the dual of the hydraulic constraint (Eq. 3). $\Delta Q$, a vector change in the flows in all links, is sought such that $LP(Q+\Delta Q) < LP(Q)$, and the move is made in the direction of negative gradient of cost. Figure 3 provides a description of the LPG method. The formulation has been extended to real networks, which contain pumps, valves, and reservoirs and which operate under multiple loadings. Quindry et al. (1981) propose a similar formulation as Alperovits and Shamir (1977), taking nodal heads as the decision variables in the gradient method for computing the minimum cost.

Fujiwara et al. (1987) apply a quasi-Newton search method instead of the steepest gradient direction, and the step size is determined by a backtracking line search method instead of a fixed step size. The convergence rate of this modified LPG algorithm is shown to be faster than the earlier LPG methods. Fujiwara and Khang (1990) use a nonlinear programming gradient (NLPG) method for the optimal design of a new looped WDN as well as for the parallel expansion of the existing ones. The nonlinear objective function includes capital and operating costs and is expressed as a function of pumping heads, flows, and head losses in links. The problem has been formulated with constraints on heads at nodes, diameters of pipes, and flows in links (Eqs. 2–4). In the first phase of the solution, the link flows and pumping heads are specified and solved for head losses in links using Lagrange multipliers. The gradient method, using optimal lagrange multipliers, is used to modify the flow distribution and pumping heads to achieve a reduction in system cost. It has been shown that this phase produces a local optimal solution. In the second phase, the local optimal solutions obtained in the first phase are improved by taking optimal link head losses computed as fixed and solving for link flows and pumping heads. This new optimum solution serve as an initial solution to restart the first phase to obtain an improved local optimal solution. The two-phase iterations continue until no further improvement can be achieved.

Eiger et al. (1994) discuss the complexity associated with the design of looped WDNs. It is mentioned that any formulation of the problem that is realistic enough to be useful is nonconvex and nonlinear. Due to the nonconvexity of the problem, a lower bound on the solution is required to check the quality of the solution. It is shown that Alperovits and Shamir (1977), and subsequent work based on the same formulation (Fujiwara and Khang, 1990; Quindry et al., 1981), ignore the fact that the gradient of the objective function does not always exist. Eiger et al. (1994) however consider the non-differentiability of the objective function and discuss a nonsmooth optimization algorithm. Nonsmoothness is handled by the bundle-trust algorithm and the bound on solution is computed using the duality theory. The overall design problem is solved globally by a branch and bound algorithm using nonsmooth optimization and duality theory. Sherali et al. (1998) provide an improved lower bound scheme compared to the earlier model of Eiger et al. (1994). The lower bounding scheme takes the advantage of the monotone convex-concave nature of the nonlinear constraints to develop tight linear relaxations via polyhedral outer approximations. Costa et al. (2001) also apply branch and bound algorithm for the WDN optimal design problem. Globally
optimal solutions are reached through the generation of convergent sequences of upper and lower bounds.

**Design of WDNs with Design Complexities**

All the above discussed formulations assume that the layout of the looped WDN is known. Rowell and Barnes (1982) and Morgan and Goulter (1985) address the problem of obtaining optimal layouts. Rowell and Barnes (1982) develop a two-level hierarchical model for the least cost layout of both single and multiple source WDNs. A least cost branched network is first determined using a NLP formulation. An integer programming model then chooses the links forming loops to provide a specified level of reliability. Morgan and Goulter (1985) propose a heuristic LP formulation, linked to a network solver, for the least cost layout, design, and expansion of a WDN under multiple loading conditions. The objective function includes only the initial cost of the pipes. The LP model is used to determine optimal replacement sizes for the pipes that are determined in the previous iteration and the network solver is used to modify the flows and pressure heads. The process continues until the best solution is found.

The issue of reliability is not discussed explicitly in any of the above models. Goulter and Coals (1986) develop and assess two quantitative approaches to the incorporation of reliability measures in the least cost design of looped WDNs. Both the approaches are based on LP formulation of Alperovits and Shamir (1977) with an additional constraint on number of breaks in a pipe (Eq.15).

\[
\sum_k r_{jk}X_{jk} \leq R_j \quad \forall j
\]  

(15)

where \( r_{jk} \) is the expected number of breaks per unit length per year for diameter \( k \) in link \( j \), \( X_{jk} \) is the length of the pipe of diameter \( k \) in link \( j \), and \( R_j \) is the maximum allowable number of failures per year in link \( j \). The probability of failure of individual links is modelled using Poisson probability distribution. The first approach is based on a node isolation approach where all the links connecting the node failed simultaneously. The probabilities of failure of each of the individual links connected to a node are considered independent and the probability of isolation of the node is expressed as

\[
\hat{P}_m = p_1 \cdot p_2 \cdot \ldots \cdot j \quad \forall j \in c(m)
\]  

(16)

where \( \hat{P}_m \) is the probability of isolation of node \( m \) and \( c(m) \) is the complete set of links connected to node \( m \). The probabilities of failure of all the links and isolation of all the nodes are computed, after the solution of the LP formulation. The worst node, namely, the node with the greatest probability of isolation is selected. The probability of its isolation is reduced by strengthening at least one of the links. The most suitable link to strengthen is selected which has the greatest effect on unacceptable \( \hat{P}_m \), at least cost. The second approach is based on a goal programming formulation. The objective is to minimize the deviations in reliability of individual links from the
uniform reliability of the links connected to the node, while satisfying all hydraulic constraints on
the system. It is assumed that each link has the capability of supplying the entire demand of the
node. In other words, they have similar capacities or similar reliability. The objective function is
written as
\[ \min G = \sum_j w_j^+ d_j^+ + w_j^- d_j^- \] (17)
where \( d_j^+ \) and \( d_j^- \) are the overachievement and underachievement of goal for link \( j \); and \( w_j^+ \) and
\( w_j^- \) are the weights associated with deviations.

Goulter and Bouchart (1990) extend the earlier model of Goulter and Coals (1986) by also
considering both the link failure and demand failure. The probability of pipe failure at each link
and the probability of demand exceeding the design values are combined into a single reliability
measure, the probability of no node failure. Bouchart and Goulter (1991) consider that the de-
mands are not lumped at the nodes, but are distributed along the links. The impacts of continuous
demands and the location of the valves on the reliability of the system are also discussed.

These earlier models do not consider pumps, tanks, and multiple loading conditions. Su et al.
(1987) propose a model to design the pipes, pumps, and tanks under multiple loading conditions,
considering reliability constraints for both system and demand nodes. The formulation is similar
to the formulation of Lansey and Mays (1989). The optimization model is linked to a network
solver and a reliability model. The reliability model, which is based on a minimum cut-set
method, determines the values of system and nodal reliability. A generalized reduced gradient
(GRG) method is used to solve the optimization model. Lansey et al. (1989) develop a chance
constrained NLP model with uncertainties in required demands, required pressure heads, and
pipe roughness coefficients, but it does not consider pipe failures. The model is solved using
the GRG technique. Xu and Goulter (1999) extend the model of Lansey et al. (1989) by jointly
considering the uncertainty in nodal demands and pipe hydraulic capacities as well as the effects
of mechanical failure of system components. Duan et al. (1990) augment the earlier models by
considering the reliability associated with pumping stations. In that study, it is assumed that
a failed component is repairable. Various reliability measures, such as failure frequency, cycle
time between failures, expected duration of the failures, and expected unserved demand are also
considered. The problem is formulated as mixed integer NLP problem that is solved using a
heuristic algorithm.

Cullinane et al. (1992) consider hydraulic availability as a measure of reliability. The measure
is defined as the relative time during which demand can be supplied at or above consumers’
demand. The availability is considered as a constraint in a NLP formulation shown in Eq. 18,
where \( AE \) represents availability at individual nodes.
\[ AE_{\min} \leq AE(H, D) \leq AE_{\max} \] (18)
Awumah et al. (1990) propose the use of Shannon’s informational entropy function as a surrogate measure for reliability of WDNs. A WDN must include some amount of redundancy (alternate supply paths to the nodes) to ensure the reliability of the network. In communication theory, entropy is a measure of information. It implies freedom of choice to select between alternative messages (Singh, 1997). Similarly, in a WDN entropy denotes alternate paths to the nodes. Therefore, more entropy means more redundancy and hence more reliable WDN. Awumah et al. (1990) express the reliability measure of a WDN, based on entropy, as

\[
S = -\sum_{j=1}^{N} \left[ \frac{Q_j}{Q_0} \sum_{i=1}^{n_j} \frac{q_{ij}}{Q_j} \ln \frac{q_{ij}}{Q_j} \right] - \sum_{j=1}^{N} \left[ \frac{Q_j}{Q_0} \ln \frac{Q_j}{Q_0} \right] \tag{19}
\]

where \(S\) is the overall measure of reliability, \(Q_0\) is the sum of flow in all links of the WDN, \(Q_j\) is the total flow to node \(j\), \(q_{ij}\) is the flow in link from node \(i\) to node \(j\), \(n_j\) is the number of nodes immediately upstream and connected to node \(j\), and \(N\) is the total number of nodes in the WDN. The first term on right hand side of Eq. 19 is the algebraic sum of the weighted reliability measure at each node and measures the distribution of flows in links upstream and connected to each node. The reliability of a node is maximum if all the links, upstream and connected to it, have equal flows (\(q_{ij}\)). The second term is the measure of reliability among \(N\) nodes and measures the distribution of flow to the nodes in the WDN. A network with equal nodal flows has the maximum internodal measure of reliability.

Ang and Jowitt (2003), Ang and Jowitt (2005), Awumah et al. (1991), Tanyimboh and Templeman (1993), and Tanyimboh and Templeman (2000) demonstrate the use of entropy to measure reliability of a WDN. Tanyimboh and Templeman (2000) integrate the entropy based reliability measure to the WDN optimal design problem. The objective in that study is to minimize the initial cost. The approach used in the paper has two steps. Firstly, the maximum value of the reliability measure is computed. The second step is to solve the optimal design problem with an extra constraint (Eq. 20) to ensure that the WDN has the desired reliability. The rest of the model is similar to Watanatada (1973).

\[
S \geq \hat{S}; \quad 0 \leq \hat{S} \leq S_{\text{max}} \tag{20}
\]

where \(S\) is the network reliability, \(\hat{S}\) is the specified reliability value, and \(S_{\text{max}}\) is the maximum reliability value of the WDN.

The other formulations of ensuring reliability in design of WDNs are based on graph theory, which considers the connectivity of each node to at least one source node. Ormsbee and Kessler (1990) present a graph theory based methodology for upgrading an existing single source WDN in order to sustain any single component failure (link or node). It is noted in that study that a reliable WDN must have topological redundancy (connectivity of each node to at least one source) and hydraulic redundancy (adequacy of pressure and flows under specified loading condition). The
method requires two major stages. In the first stage, the ‘two node connected’ network is divided into two overlapping spanning trees. These two trees assure a path from the source to each design node in case of a single one-link failure. The hydraulic redundancy is accomplished by solving a LP formulation that simultaneously considers individual head constraints for both trees. Kansal et al. (1995) propose the concept of appended spanning tree (AST) to compute the global reliability of a network that has large number of nodes and links.

Ostfeld and Shamir (1996) discuss a methodology which integrates the optimal design and reliability of multi-quality water supply system. In multi-quality water distribution systems, water of different qualities is taken from different sources, treated, conveyed, and supplied to consumers. The problem is formulated as a quadratic convex problem with constraints on water quality, flow, and pressure. The solution is achieved by decomposing the problem, as in Alperovits and Shamir (1977), into an outer non-smooth problem and an inner convex quadratic problem.

Kim and Mays (1994) and Luong and Nagarur (2001) describe the problem of optimal rehabilitation of existing system. In Kim and Mays (1994), the objective is to minimize the total rehabilitation and energy cost (Eq. ). The problem is formulated as a mixed-integer nonlinear programming problem with an additional decision constraint shown in Eq. . The constraint eliminates the possibility of simultaneously replacing and rehabilitating the same pipe.

\[
\begin{align*}
\text{min} & \quad f_1 + f_2 + f_3 + f_4 \\
\text{s.t.} & \quad N_j + R_j \leq 1
\end{align*}
\]  

(21)

where \( f_1 \) is the pipe replacement cost, \( f_2 \) is the pipe rehabilitation cost, \( f_3 \) is the expected repair cost, and \( f_4 \) is the energy cost. \( N_j \) and \( R_j \) are the binary variables indicating replacement or rehabilitation of pipe \( j \), respectively. The problem is solved by interfacing a branch and bound implicit enumeration procedure, to select \( N_j \) and \( R_j \), with a generalized reduced gradient (GRG) procedure. The GRG procedure is used to compute optimal pipe diameters and pump capacities.

In Luong and Nagarur (2001), a semi-Markov process is used to depict the behavior of the pipe. The state space is composed of all the states of the pipe: new, operating in early and late stages of deterioration, under repair, and under replacement. The objective is to maximize the long run availability of the pipe (Eq. 22) under budgetary constraints.

\[
\begin{align*}
\max & \quad A = \sum_{n=0}^{N} \phi_{2n} \\
\end{align*}
\]  

(22)

where \( A \) is the availability, \( \phi_{2n} \) is the proportion of the time the pipe stays in the operating state \( 2n \), and \( N \) is the maximum number of breaks allowed for a pipe. The replacement ages of the pipe in each of its deteriorating stages are taken as the decision variables.
More recently, meta-heuristic optimization methods have been applied to solve the optimal design problem of WDNs. Simpson et al. (1994) discuss genetic algorithms (GA) for the optimal design of a WDN with pipes and tanks. GA use concepts from population genetics and evolution theory to construct algorithms that try to optimize the fitness of a population of elements through recombination and mutation of their genes (Holland, 1975). Three loading conditions along with minimum pressure constraints at nodes are considered. The pipe diameters are selected from a set of discrete pipe sizes. The set of decision variables, the pipe diameters, are assigned a binary coding. The binary coding represents the options for each of the decision variables. A simple three-operator GA comprising reproduction, crossover, and mutation is used. The roulette-wheel scheme is used for reproduction (or selection to mating pool). The fitness of the coded string is determined by the total cost of the network which includes capital costs and penalty costs (where the minimum pressure requirements are violated). The strings with higher fitness (or lower costs) have a higher probability of being selected. The crossover probability and mutation probability are taken as 0.70 and 0.01, respectively. The results from the GA have been compared with the results from the complete enumeration and the nonlinear optimization. The GA is shown to find the global optimal solution for the test cases in a small number of evaluations compared to the size of search space.

Dandy et al. (1996) provide an improved GA for the pipe network optimization. The improved GA uses variable power scaling of the raw fitness function $f_i$ (Eq. 23).

$$f'_i = f_i^n$$

where the exponent $n$ is allowed to increase as the GA run develops. This is a crucial feature of the improved GA formulation and helps to maintain competitiveness throughout the GA search. An adjacency or creeping mutation operator, which mutates the selected complete decision variable substring to an adjacent decision variable substring up or down the list of design variable choices, is introduced. Also, Gray codes rather than binary codes are used to represent the set of decision variables so that similar coded strings represent designs nearby in the solution space. It is shown that the improved GA performs significantly better than the simple GA and all other traditional optimization techniques. Montesinos et al. (1999) also propose a modified GA where, in each generation, a constant number of solutions is eliminated. The selected ones are ranked for crossover and the new solutions are allowed to undergo at most one mutation. These modifications increase the algorithm convergence.

Halhal et al. (1997) apply structured messy genetic algorithms (SMGA) for the rehabilitation, replacement, and expansion of a WDN with pipes and reservoir. SMGA is based on the process of natural evolution of complex life-forms from single-cell organisms. The procedure starts with complete enumeration of all single elements to form an initial population. The elements from the
Optimal Design of Water Distribution Networks

The problem is formulated as a multi-objective optimization problem with capital cost and benefits are taken as dual objectives under budgetary constraints (Eq. 1).

\[
\begin{align*}
\max f(i) &= \sum_{t=1}^{N} w(t) \text{benefit}(i,t) \\
\min F(i) &= \sum_{t=1}^{N} PV(\text{cost}(i,t)) \quad (24)
\end{align*}
\]

where \( \text{benefit}(i,t) \) is the benefit of the solution \( i \) yielded in year \( t \), \( PV(\text{cost}(i,t)) \) is the present value of the cost of rehabilitation operations of solution \( i \) in year \( t \), budget is the total fund, \( N \) is the planning period in years. \( w(t) \) is a weighing factor which favors an early improvement to the system taking into consideration environmental and social aspects. The benefits considered are due to improvement in nodal pressures, reduction in the repair costs, increase in the flexibility, and quality improvement. The results obtained are compared with the traditional GA, and SMGA is shown to perform better than the traditional GA. Walters et al. (1999) extend the model of Halhal et al. (1997) by including sizing and operations of storage tanks and pumping installations as decision variables. Halhal et al. (1999) apply the SMGA for the optimal design and scheduling of investment for the rehabilitation of a WDN.

The above models used GA for small WDNs. Wu and Simpson (2001) apply GA for large-scale WDNs containing pipes, tanks, valves, and pumps. A messy GA is used for improving the efficiency of the optimization procedure. A network solver is integrated with the GA optimizer, as in Lansey and Mays (1989), to solve the hydraulic equations at each iteration. Keedwell and Khu (2005) provide a hybrid GA which uses a heuristic based local representative cellular automata approach to provide a good initial population to the genetic algorithm runs.

Tolson et al. (2004) present an approach that links GA with the first-order reliability method (FORM) for accurately estimating the network capacity reliability and identifying the most critical node in the network. FORM estimates the reliability of a system, \( \alpha \), by computing \( \beta \) which can be interpreted as the minimum distance between the mean-point of \( n \) random-variables and the failure surface. These \( n \) random-variables influence the load, the resistance, and the performance of the system. The limitation of the FORM is the repetitive calculations of the first-order derivatives and matrix inversions, which can be computationally demanding even for small WDNs. Uncertainty in nodal demands and pipe roughness conditions are considered.

Babayan et al. (2005) discuss the design of WDNs using a GA model with demand uncertainty. The randomness in the system in this model is due to the uncertain nature of nodal demands. The objective is to minimize initial cost with minimum nodal head constraints. The original
stochastic model is converted to a deterministic model with standard deviation as a natural measure of variability. The deterministic model is then coupled with a GA solver to find optimal solutions.

Kapelan et al. (2005) incorporate the uncertainty in nodal demands and pipe roughness conditions in the multi-objective optimization model for the design of a WDN. The objectives are to minimize the total design cost and maximize system robustness. System robustness is defined as the probability of simultaneously satisfying minimum pressure head constraints at all the nodes in the network. The problem is solved using robust nondominated sorting genetic algorithm II (RNSGAII). In RNSGAII, a small number of samples are used for each fitness evaluation compared to the full sampling approach. It is shown that RNSGAII is capable of identifying robust Pareto optimal solutions with reduced computational effort.

Other meta-heuristic based optimization methods used in the design of WDNs are simulated annealing (Cunha and Sousa (1999, 2001), and Loganathan et al. (1995)); tabu search (Cunha and Ribeiro (2004)); ant colony optimization (Maier et al. (2003) and Zecchin et al. (2005)); metamodels (Broad et al. (2005)); and shuffled frog leaping algorithm (Eusuff and Lansey (2003)). Loganathan et al. (1995) present a two stage outer search-inner optimization method for the globally optimal design of a WDN using a problem formulation similar to that of Karmeli et al. (1968). The outer search scheme chooses alternative flow configurations to find an optimal flow division among pipes. Multistart local search and simulated annealing are the two global search techniques used for the outer-search problem. Simulated annealing is based on the analogy with the physical annealing process in solids (Kirkpatrick et al., 1983). In the model, for each selected set of flows an inner linear program is used to optimize pipe diameters and pressure heads. Since the method uses only linear programs for the optimization, even large problems can be easily solved.

Cunha and Sousa (1999, 2001) propose a simulated annealing based heuristic for the least cost design of a WDN. The cost is defined as a function of pipe diameters and the objective is to minimize the initial capital cost with the constraints on nodal heads. In the proposed simulated annealing, cost is considered analogous to energy and the network configuration is defined by the pipe diameters. In each step of the algorithm, a change in configuration is produced and the cost is evaluated. The new configuration is then chosen in the neighborhood of the current configuration.

Cunha and Ribeiro (2004) develop a tabu search algorithm for the same formulation as Cunha and Sousa (1999). Tabu search is a higher level heuristic procedure based on human memory processes. It is designed to guide other methods to escape the trap of local optimality (Glover, 1990). Maier et al. (2003) and Zecchin et al. (2005) apply ant colony optimization for the WDN design problem. In these models, the cost is taken as a linear function of the length of the pipes. The pheromone information is modified using a pheromone penalty factor to discard choices that result in violation of hydraulic constraints.
Broad et al. (2005) use an artificial neural network (ANN) metamodeling approach, linked to a GA optimizer, to minimize the initial network cost with water quality constraints (Eq. 25). A metamodel is a surrogate for a complex simulation model.

\[ C_{j,\text{min}} \leq C_j \leq C_{j,\text{max}} \]  

(25)

where \( C_j \) is the residual chlorine concentration, \( C_{j,\text{min}} \) is the minimum allowable residual chlorine concentration, and \( C_{j,\text{max}} \) is the maximum allowable residual chlorine concentration at node \( j \).

For the ANN metamodel, the decision variables (pipe diameters and chlorine dosing rate) are taken at the input layer and the constrained variables (pressure and chlorine residual at node \( j \)) as the output layer. The data to train ANN is generated using uniform random sampling.

Shuffled frog leaping algorithm (SFLA) is used by Eusuff and Lansey (2003) to determine optimal discrete pipe sizes for a new WDN and for network expansions. SFLA is based on evolution of memes carried by interactive individuals and a global exchange of information among the population. Memes can be considered as the unit of cultural evolution. In SFLA, the frogs are seen as hosts for memes. SFLA and GA are subject to the same basic principles. However, in GA the genes can only be transmitted from parents to offspring while in SFLA the memes can be transmitted between any two individuals. Memotypes, the actual contents of a meme, are treated as the decision variables for the WDN optimal design problem. Each memotype of a meme is a numeric representation of a discrete pipe diameter. A WDN with \( n \) discrete pipe diameters has \( n \) memotypes in a meme.

Summary and Directions for Future Research

In this paper, the general optimal WDN design problem was presented first with all the additional complexities. A good model should be able to decide on pipe layout and sizes; location and capacity of tanks; location, types, capacity, and operating schedule of pumps; and location, types, and settings of different valves. A good model should also incorporate all the additional complexities like multiple loadings, demand uncertainty, water quality, reliability etc. In this paper, various successful models were reviewed but none of the existing models were found to be incorporating all the design requirements. The initial models developed for the design of a branched WDN use traditional optimization methods like linear programming (LP), nonlinear programming (NLP), and dynamic programming (DP), and focus primarily on the least cost design without including any additional complexities. These may be reasonable models for the design of branched WDNs as branched systems are mainly used for rural and industrial water supply.

However as mentioned earlier, looped WDNs are more common and more desirable than branched WDNs. Models by Jacoby (1968) and Watanatada (1973) use NLP formulations for solving the looped WDN design models. Shamir (1974), Lansey and Mays (1989), and Varma
et al. (1997) use a decomposition technique for solving the NLP formulation, with a network solver solving the hydraulic equations. Alperovits and Shamir (1977) propose the linear programming gradient (LPG) method for the optimal design of a looped WDN. The approach is similar to parametric optimization methods where the flow in network is considered as a parameter. Quindry et al. (1981), Fujiwara et al. (1987), and Fujiwara and Silva (1990) extend the model by Alperovits and Shamir (1977) by using different decision variables and search algorithms. These models require the differentiability of the objective function. However in reality, objective function may be nondifferentiable. This issue is addressed by Eiger et al. (1994).

Any formulation of the problem that is realistic enough to be useful is nonconvex and nonlinear. Nonconvexity of the objective function means that it is more difficult to find a global optimum. Eiger et al. (1994) and Sherali et al. (1998) consider the nonconvexity assumption and obtain a lower bound to check the quality of the solution. Further research should be pursued to develop efficient global optimization procedures for finding globally optimal solution for nonconvex programming problems with additional complexities. All these models consider the mathematical aspects of the optimization problem and do not consider many of the additional complexities which are required for the practical implementation of the models. Hence, later optimization models began to focus on the additional complexities.

The optimal layout in the design problem has been considered by Rowell and Barnes (1982) and Morgan and Goulter (1985). Goulter and Coals (1986), Goulter and Bouchart (1990), and Bouchart and Goulter (1991) address the reliability requirement under single loading condition in the design problem. Su et al. (1987) extends the reliability requirements under multiple loading conditions. The use of entropy as a measure of reliability has been demonstrated by many researchers. Awumah et al. (1990) propose the use of Shannon’s informational entropy function as a surrogate measure of reliability of WDNs.

Kim and Mays (1994) and Luong and Nagarur (2001) discuss the optimal rehabilitation of existing WDNs. The increasing complexity in the design models has made the solution of the optimization models using traditional optimization problems computationally difficult. This is primarily because, in real problems, it becomes extremely difficult to invert matrices (in LP) or compute gradients (in NLP). Also, the LP formulation, as in Alperovits and Shamir (1977), computes the solutions which consists of one or more pipe segments of different sizes between each pair of nodes. These are known as split-pipe solutions. This kind of solution is infeasible to implement in actual practice. Similarly, in NLP formulation, the differentiability requirements require that the pipe diameters should be continuous variables.

Recently, the focus of the research in this area has shifted to the meta-heuristic based optimization methods like genetic algorithms (GA), tabu search, simulated annealing, ant colony optimization, etc. Simpson et al. (1994) discuss GA for the the basic design problem under multiple loading conditions. Dandy et al. (1996) and Halhal et al. (1997) provide a better GA based approach for the design problem than Simpson et al. (1994). Wu and Simpson (2001) use a de-
composition technique, similar to Lansey and Mays (1989), to apply GA for large WDNs. Tolson et al. (2004) and Babayan et al. (2005) extend the previous GA models by incorporating demand uncertainty at nodes in the optimization models.

Kapelan et al. (2005) use minimum design cost and maximum robustness as the dual design objectives in a multi-objective design formulation. The robustness is defined as the probability that heads at all network nodes are simultaneously equal or above the corresponding minimum pressure requirements for that node. Other meta-heuristic optimization methods used in the design of WDNs are simulated annealing (Cunha and Sousa (1999, 2001), and Loganathan et al. (1995)): tabu search (Cunha and Ribeiro (2004)): ant colony optimization (Maier et al. (2003) and Zecchin et al. (2005)): metamodels (Broad et al. (2005)): and shuffled frog leaping algorithm (Eusuff and Lansey (2003)).

In general, when searching for an optimal solution with the heuristic optimization methods, the objective function is evaluated for a set of solutions. The new solutions are generated only on the basis of the value of the objective function. The process of evaluation and generation continues until a stopping criterion is met. As mentioned earlier, heuristic optimization methods only use objective function values to move to better solutions, they can easily handle discrete decision variables like pipe diameters. As only the objective function information is required, a large number of simulations are required to reach the optimal solutions. This is time consuming and limits the size of the problem that can be solved (Mays, 2000). The other disadvantage with these optimization methods is the adjustment of parameters. For example, in ant colony optimization: adjustment of number of ants, influence of pheromone trail, evaporation rate, etc. for finding optimal solutions.

In recent years, there has been a significant shift in the use of optimization models for the practical design of WDNs. Some optimization software developed for the design of WDNs are Darwin Designer, OptiDesigner, OPT DIS, Derceto, etc. All of these software use heuristic optimization methods. Further research in heuristic optimization methods should focus on hybrid methods, which combine the specific advantages of different approaches. Furthermore, the meta-heuristic methods orchestrate an interaction between local improvement procedures and higher level strategies to create a process capable of escaping from local optima and performing a robust search of a solution space (Glover and Kochenberger, 2003). However, many state-of-the-art meta-heuristics are too problem-specific or too knowledge-intensive; and this hampers their use in practice. In future, the research should include the use of hyper-heuristic models for designing WDNs. Hyper-heuristics leads to more general systems that can handle a wide range of problem domains rather than current meta-heuristic methods which tends to be customized to a particular problem or a narrow class of problems.

Walski (2001) argues that the objective of least cost design is an improper measure of effectiveness. Least cost optimization primarily reduces the size of the system components for reducing cost. However, additional system components allow better ability to deal with reliability. There-
fore, the optimal design model should provide a trade-off between the least cost and reliability. This can be achieved by using maximization of social welfare function as the objective, while designing water distribution networks. Since social welfare function includes producer’s (water utility) surplus; and cost minimization is fully implied by maximization of producer’s surplus, and hence cost minimization need not be explicitly considered. The standard social welfare functions are

**Utilitarian social welfare function**

\[ W(U_1, \ldots, U_I) = \sum_{i=1}^{I} U_i \]

**Bergsonian social welfare function**

\[ W(U_1, \ldots, U_I) = \sum_{i=1}^{I} \alpha_i U_i \]

**Rawlsian social welfare function**

\[ W(U_1, \ldots, U_I) = \min\{U_i, \forall i \in I\} \]

where \( U_i \) is the surplus (or net benefit) and \( \alpha_i \) is the weight assigned to a particular stake-holder \( i \in I \). In utilitarian social welfare function, all the stake-holders are given equal weight. In Bergsonian social welfare function, the stake-holders are assigned different weights. Rawlsian social welfare function is an extreme case of Bergsonian social welfare function, where the worst off stake-holder gets all the weight. There are many stake-holders in a WDN, and hence the choice of the social welfare function is a choice of distributional values (Jehle and Reny, 2001). The future research in this area should diverge from the objective of cost minimization to the maximization of the social welfare function to provide more practical design of WDNs. The biggest hurdle in this approach, however, is the estimation of stake-holders’ surpluses. One possible way of computing a consumer surplus (with respect to system reliability) is discussed in Howe and Smith (1994). Howe and Smith (1994) develop a methodology for incorporating water users’ valuation of reliability in system design. Using contingent valuation techniques, they have measured the benefits and costs of different reliability levels in terms of water users’ willingness to pay (WTP) for increases in reliability and in terms of their willingness to accept (WTA) compensation in the form of lower water bills for lower levels of reliability.

Also, current optimization models assume unconstrained water availability. However, many urban areas are already facing severe water stress (World Water Development Report, 2003).
Therefore in future, the design of WDNs should include *supply uncertainty* constraints for robust design of WDNs. Furthermore, the different models reviewed in this paper have quantified reliability differently. This is mainly because there still does not exist an universal measure of reliability. Hence, future research in this area should focus on the development of an universal measure of reliability or acceptable reliability measures for designing reliable WDNs. The literature in optimal design of WDNs has focussed primarily on aleatory uncertainty for reliability estimates. Hence, in future, the research should focus on the inclusion of epistemic uncertainty for making precise estimates of reliability. We hope that, in future, the research directions mentioned above will bridge the gap between the theory and practice and the optimization models for design of WDNs will have widespread acceptability among the practicing engineers.

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Table 1: Optimization in Design of Branched WDNs

<table>
<thead>
<tr>
<th>Objective</th>
<th>Constraints</th>
<th>Hydraulic Components</th>
<th>Decision Variables</th>
<th>Optimization Model(s)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum initial cost</td>
<td>Constraints on nodal heads</td>
<td>Pipes</td>
<td>Lengths of the pipes of given diameters</td>
<td>LP (Karmeli et al., 1968)</td>
<td>Operating cost can be included.</td>
</tr>
<tr>
<td>Minimum initial and operating cost</td>
<td>Constraints on nodal heads</td>
<td>Pipes</td>
<td>Diameters of pipes</td>
<td>DP (Liang, 1971; Yang et al., 1975)</td>
<td>Backward recursion is used. Yang et al. (1975) extends the model for multiple branches.</td>
</tr>
<tr>
<td></td>
<td>Constraints on nodal heads</td>
<td>Pipes and pumps</td>
<td>Diameters of Pipes</td>
<td>NLP (Swamee et al., 1973; Chiplunkar and Khanna, 1983)</td>
<td>Swamee et al. (1973) extends the model to the continuous withdrawals.</td>
</tr>
<tr>
<td></td>
<td>Constraints on nodal heads</td>
<td>Pipes and pumps/tanks</td>
<td>Length of pipes of given diameters and head at source</td>
<td>LP (Calhoun, 1971; Robinson and Austin, 1976)</td>
<td>Robinson and Austin (1976) assume cost per unit head at source as a nonlinear function of head at source and computed iteratively.</td>
</tr>
<tr>
<td>Objective</td>
<td>Constraints</td>
<td>Hydraulic Components</td>
<td>Decision Variables</td>
<td>Optimization Model(s)</td>
<td>Comments</td>
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<tr>
<td>Minimum initial and operating cost</td>
<td>Hydraulic conditions; constraints on nodal heads</td>
<td>Pipes and pumps</td>
<td>Diameter of pipes</td>
<td>INLP (Jacoby, 1968), NLP (Watanaatada, 1973; Lansey and Mays, 1989; Ostfeld and Shamir, 1996)</td>
<td>Ostfeld and Shamir (1996) consider additional constraints on flow and water quality; and achieve the solution by decomposing the problem into an outer non-smooth problem and an inner convex quadratic problem.</td>
</tr>
<tr>
<td>Minimum total rehabilitation and energy costs</td>
<td>Hydraulic conditions; constraints on nodal heads</td>
<td>Pipes and pumps</td>
<td>Diameter of pipes and pumps capacities</td>
<td>MINLP (Duan et al., 1990), NLP (Cullinane et al., 1992)</td>
<td>In Cullinane et al. (1992), availability is used as a measure of reliability.</td>
</tr>
<tr>
<td>Minimum initial costs and maximum benefits</td>
<td>Hydraulic conditions; constraints on nodal heads</td>
<td>Pipes, pumps, and tanks</td>
<td>Rehabilitation decisions; sizing and operations of pumps and tanks</td>
<td>SMGA (Walters et al., 1999)</td>
<td>Decision constraints are used to eliminate the possibility of simultaneously replacing and rehabilitating the same pipe. A network solver is integrated to the GA optimizer to solve hydraulic equations at each iteration.</td>
</tr>
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List of Figures

- Figure 1 Configurations of WDN
- Figure 2 Dynamic Programming Stage
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Figure 1: Configurations of WDN

(a) Branched WDN
(b) Looped WDN
Figure 2: Dynamic Programming Stage
Gradient
Compute the gradient of the cost using the result of LP

Linear Program
For the given link flows, formulate a LP problem and solve

New link flows
Dual of the hydraulic constraints

Figure 3: Linear Programming Gradient Method