November, 2006

Theoretical Constraints on the Price-Rent Ratio and Other Insights from a New Real Estate Model

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November 14th, 2006

Real Estate Model – Simplest Version

1. All people are risk neutral
2. All people are rational and there is no asymmetric information regarding returns and costs.
3. There are 2 kinds of people:
   i. Liquid – completely liquid. They can borrow as much as they want.
   ii. Illiquid – completely illiquid. They can't borrow at all.
4. There are 3 cities: warm, cold, and poor.
5. There are 5 investments:
   i. Homogeneous homes in the warm city – These have a net capital gain of c^w per year (where c^w is a percentage of the purchase cost of the home). They can be rented for a market bearing rate of ρ^w per year (where ρ^w is a percentage of the purchase cost of the home) but there is a (deadweight) cost to the landlord of d per year (where d is a percentage of the purchase cost of the home). This is due to moral hazard, transactions, and administrative costs, including possibly having the place sit empty while looking for new renters. Thus, for a landlord the total annual net return is: $r^w \equiv c^w + \rho^w - d$.
   ii. Homogeneous homes in the cold city – analogous to i. Denoted with c superscripts in place of w's. Thus: $r^c \equiv c^c + \rho^c - d$.
   iii. Homogeneous homes in the poor city –analogous to i. Denoted with p superscripts in place of w's. Thus: $r^p \equiv c^p + \rho^p - d$.

iv. Default free borrowing and lending at net interest rate i per year.

v. A stock which only pays a capital gain (during the period explored in the model). The net return on the stock is r^s per year.

6. There are no moving costs or transactions costs for buying or selling any assets.

7. Timing: The stock can be purchased only at the beginning of the year, and its net return is paid at the end of the year, or it can be sold at the end of the year. For example, if a person wishes to invest \$X in stock, then he will pay \$X on the first day of the year, and on the last day of the year he will receive a net return of \$Xr^s, that is he can sell the stock for \$X$(1 + r^s)$. Likewise, for the capital gain on a home, the person will pay \$Y to purchase the home on the first day of the year, and on the last day of the year, he can sell the home (in city j) if he wishes for \$Y$\left(1 + c^j\right)$. If a person rents a home that is worth \$Y, then rent is only paid on the last day of the year, so the person will pay nothing when he moves in and nothing throughout the year, but then on the last day of the year he will pay rent of \$Yρ^j, and the landlord will pay his deadweight loss on the last day of the year also, of \$Yd. Finally, interest on any money borrowed or lent is paid or received on the last day of the year, but the borrower is given the principle on the first day of the year.[1] These timing conventions make the model and its insights clearer, but do not affect its applicability or generality. Any timing scenario can be made economically equivalent to any other by just adjusting the returns or payment levels, for example a payment received earlier can be made economically equivalent to a payment received later just by adjusting its amount downward.

Lemma 1.1

$$E\left(\rho^i\right) \geq E\left(d\right), \forall i$$

Proof:

Otherwise, people won't rent out homes that they own, because the expected net profit from doing so will be negative. ■

Lemma 1.2

$$E\left(r^w\right) = E\left(r^c\right) = E\left(r^p\right) = E\left(r^s\right) = E\left(i\right)$$

[1] In a paper, some timeline diagrams might be nice to make this quick and clear

Proof:

This is a well known and obvious phenomena when investors are rational and risk neutral. It would be easy to provide a proof, or refer to one, if necessary in a paper. ■

Theorem 1.1

In equilibrium, all liquid people will own the homes they live in. Only illiquid people will rent.

Proof:

1. Suppose a liquid person has $X that he is considering allocating to housing in city j. If he were to *rent* a home that has purchase price $X and invests his wealth in the stock, or in lending, then *at the end of the year* his $X will generate in expectation:

$$XE\left(r^{s}\right) - XE\left(\rho^{j}\right) \tag{1}$$

   Remember item 7 of the model regarding timing. The person signs a contract to rent the home and moves in at the beginning of the year, but does not pay any rent until he pays $X\rho^{j}$ at the end of the year. Also, at the beginning of the year the person pays $\$X$ for stock, and at the end of the year receives a net return of Xr^{s}.

2. If he instead spent the $X purchasing his home instead of renting it, at the end of the year his $X would generate in expectation:

$$XE\left(c^{j}\right) = XE\left(r^{j} - \rho^{j} + d\right) = XE\left(r^{j}\right) - XE\left(\rho^{j}\right) + XE\left(d\right) \tag{2}$$

3. Given lemma 1.2, (2) – (1) is $XE\left(d\right)$ a positive number, so it always makes sense for some one who can afford to buy, to buy. Intuitively, as we would expect, by buying a person eliminates the deadweight cost and gets to pocket that savings. With buying there is not the moral hazard of the renter not taking optimal care of the home (or in one case that I heard of stealing everything – including the kitchen sink! and the toilet!), and there is not the cost of finding renters, managing the property, vacancy, etc.

4. The state space can be partitioned into two states. In state 1, the liquid person has the $X necessary to purchase. This state was just examined and we found that in this state the person will always purchase. In the second state, the person does not have the wealth to purchase, but being liquid, he could borrow it and then purchase,

or he could just rent. If the person rents, at the end of the year his expected net payment will be:

$$-XE\left(\rho^j\right) \tag{3}$$

5. If he instead borrows the \$X and purchases the home, his expected end of year payment will be:

$$\begin{aligned} -XE(i) + XE\left(c^j\right) &= -XE(i) + XE\left(r^j\right) - XE\left(\rho^j\right) + XE(d) \\ &= -XE\left(\rho^j\right) + XE(d) \end{aligned} \tag{4}$$

(4) − (3) is $XE(d)$. So again, it is more profitable to purchase than to rent, and the savings again is, as we would expect, equivalent to the deadweight loss. ■

Note that this theorem holds *no matter how low the cost of rent is.* If the market is in equilibrium, *it is always better for a liquid person to buy than to rent.*

Theorem 1.2

$$E\left(r^i\right) \geq E\left(c^j\right), \forall i, j$$

Proof:

Given lemma 1.2:

$$E\left(r^s\right) = E\left(r^w\right) = E\left(r^c\right) = E\left(r^p\right) \equiv E\left(c^j\right) + E\left(\rho^j\right) - E(d)$$

$$\Rightarrow E\left(r^s\right) = E\left(r^w\right) = E\left(r^c\right) = E\left(r^p\right) \geq c^j$$

because, from lemma 1.1, $E\left(\rho^j\right) - E(d) \geq 0$. ■

Thus, theorem 1.2 contradicts a belief held by many laymen that the return on an average house *a person lives in (and therefore does not receive rent payments from)* is better than the return on "intangible" assets like stock. Intuitively, in equilibrium, if investors are rational, the capital gain on a home must be worse, in a risk adjusted way, than the return on a stock, because otherwise the demand for stocks will go to zero, and all the demand will shift to housing as housing offers the same risk adjusted return without even counting the rent, and a higher risk adjusted return when the rent (or the benefit of enjoying the home if the person is living in it) is counted.

Theorem 1.3

The housing strategy that maximizes expected (monetary) wealth is to purchase the home *one lives in*, but to purchase as inexpensive a home *one lives in* as possible.

Proof:

This is a direct result of theorems 1.1 and 1.2[2] ■

Again, this contradicts a belief held by many laymen that, "You should buy the most expensive home you can afford", although in a more realistic model one could include the benefits of greater forced savings form purchasing a more expensive home, the costs of more furnishings, upkeep[3], and utilities, and tax costs and benefits. If it was worth pursuing, I would make richer versions of this basic model that included these things and much more. I think theorem 1.3, except for perhaps behavioral factors like forced savings[4], should hold even in a much richer model.

Housing is really an expense that decreases ones risk adjusted expected (monetary) wealth, but it may still optimize utility to splurge on a more expensive home than is absolutely necessary, given the daily added pleasure it can bring. I think a good way to really convince students of this is to give this explanation: Suppose the most expensive home you can just barely afford to make the payments on is \$400,000. What if instead of purchasing that home to live in, you instead purchased one that cost \$200,000 to live in, and a second home for \$200,000 to rent out. Then you would get the same amount of real estate appreciation – appreciation on \$400,000 worth of real estate, but by buying a less expensive home *to live in* you are able to collect rent on a \$200,000 home – in addition to getting the same amount of average appreciation.

Now suppose you spent only \$100,000 on the home *you live in*, then you can purchase a \$300,000 home to rent out and collect even more rent, while still getting the same \$400,000 worth of real estate appreciation that you would have gotten if you had purchased a \$400,000 home *to live in.* Thus, to maximize *monetary* wealth, one should purchase a home *to live in* that's as inexpensive as possible.

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[2] although in a paper I might present why in detail, but now, I just have too little time.

[3] It is important to keep in mind that homes age and decay. While the land doesn't depreciate, the structure on it usually does significantly, even with sometimes very costly maintanance, while intangible assets like stocks and bonds don't decay and require no maintanance expenditures.

[4] which unfortunately is now a lot less forced with all of these home equity loan predators flooding the airwaves.

Then, I would next say, regarding the last example, suppose instead of investing the \$300,000 in a home to rent what if you instead invested the \$300,000 in the S&P 500. Then, I would talk about market efficiency, to show that the risk adjusted return on the home can't be much different than the S&P 500. I would talk about CAPM and personal portfolio diversification, about how in an efficient market you will be compensated with a higher return for having to engage in time consuming property management. I would talk about leverage, volatility, and the research showing that still, all things considered, the risk adjusted performance of REITs has been no better, and possibly worse, than the S&P 500, etc., etc.

Theorem 1.4

$$\text{The noninclusive lower bound of } \textit{possible} \text{ values for } \frac{XE\left(\rho^j\right)}{XE\left(i\right)} \text{ is } 0. \tag{5}$$

where X, as earlier, is the purchase cost of a home in city j. This theorem deals with my original motivating question; in equilibrium, with rational investors, can the rent payment be much smaller than the mortgage payment? According to (5), it can be a tiny epsilon fraction of the mortgage payment!

Proof:

From lemma 1.1, the smallest $E\left(\rho^j\right)$ can be is $E\left(d\right)$, thus:

$$\frac{XE\left(\rho^j\right)}{XE\left(i\right)} = \frac{E\left(\rho^j\right)}{E\left(c^j\right) + E\left(\rho^j\right) - E\left(d\right)} \geq \frac{E\left(d\right)}{E\left(c^j\right)} \tag{6}$$

It is theoretically possible, that in a society that is extremely efficient regarding transactions and search costs, perhaps due to computer advances, and in a society whose culture is such that moral hazard is small, $E\left(d\right)$ may be small. At the same time, if that society is very productive, or if big future population flows into the city are expected, while laws limit new construction and the rent market is weak because there are few iliquid people in the city, $E\left(c^j\right)$ may be very large relative to $E\left(d\right)$, in which case (6) will be close to 0. Remember, in an efficient market it must only be the case that $E\left(r^s\right) = E\left(i\right) = E\left(r^i\right) \equiv E\left(c^j\right) + E\left(\rho^j\right) - E\left(d\right)$. This can hold true with $E\left(c^j\right)$ still being very large relative to $E\left(d\right)$. $\blacksquare$

Theorem 1.5

In a *"usual"* economy (one where $E\left(c^j\right) \geq E\left(d\right)$),

the upper bound of *possible* values for $\dfrac{XE\left(\rho^j\right)}{XE\left(i\right)}$ is 1.

Proof:

Given lemma 1.2:

$$E\left(i\right) = E\left(r^j\right) = E\left(\rho^j\right) + E\left(c^j\right) - E\left(d\right)$$

$$\Rightarrow E\left(i\right) \geq E\left(\rho^j\right)$$

$$\Rightarrow \max\left\{\frac{XE\left(\rho^j\right)}{XE\left(i\right)}\right\} = 1$$

■

This theorem gives us an economic limit on a rent payment compared to a mortgage payment in a "usual" economy. The expected rent payment can never exceed the expected mortgage payment on the same property. The intuition is if the rent payment were higher, then an investor could purchase the property and his rent would completely pay the mortgage payment, but in addition he would collect a capital gain on the property that was greater than his deadweight expense of renting.

Let's now consider an *"unusual"* economy, one where $E\left(c^j\right) < E\left(d\right)$. Given lemma 1.2:

$$\frac{XE\left(\rho^j\right)}{XE\left(i\right)} = \frac{E\left(\rho^j\right)}{E\left(\rho^j\right) + \left[E\left(c^j\right) - E\left(d\right)\right]} \tag{7}$$

Because $E\left(c^j\right) < E\left(d\right)$, $\left[E\left(c^j\right) - E\left(d\right)\right]$ is negative, so $\frac{XE\left(\rho^j\right)}{XE\left(i\right)}$ is maximized by making $E\left(\rho^j\right)$ as high as the market of illiquid people will bear. The highest they could possibly bear is $\rho^j = E\left(\rho^j\right) = 1$, because then the rent payment is equal to the cost of just buying the whole place. So, the limit on a rent payment in an unusual economy is just equal to the cost of purchasing the home outright. And looking at (7), by selecting 1 for $E\left(\rho^j\right)$ and selecting 1 for $E\left(d\right)$ (remember from lemma 1, $E\left(\rho^j\right) \geq E\left(d\right)$), and 0 for $E\left(c^j\right)$, $\frac{XE\left(\rho^j\right)}{XE\left(i\right)}$ is unbounded. But, rent far higher than a mortgage payment entails an unusual economy, and a highly transient and/or socially dysfunctional one.

What are some examples? One is a hotel, where the deadweight cost is extremely high, because people move so often. Another is a very bad neighborhood. A friend of a friend's father owns about 100 houses in such neighborhoods in Detroit that he rents out. Here $E(d)$ is very high because the turnover is high, evictions are common, there is a lot of theft and vandalism, and employees must be compensated for the danger of going into these neighborhoods. At the same time, the homes can be purchased for as little as $10,000, and population is expected to shrink steadily over the long term. Little capital gain is expected. However, there is a very high percentage of illiquid people in the area who need to rent. A $10,000 house can be sectioned into 3 apartments, which when rented generate much higher rent than the mortgage payment

Potential Additions for More Complicated Versions of the Model

- Risk aversion, volatility, and leverage – after adding this to the model, I could cite empirical work comparing REITs to the S&P 500, and perhaps do some original empirical work to go with this model, perhaps using some of the high tech econometrics, Bayesian methods, Bootstrapping, etc. that I have learned.
- Taxes
- More transient and less transient people
- Moving costs, maintenance costs, including more electricity to cool a bigger home, transactions costs
- Behavioral factors, including forced savings
- Bubbles
- Parametric utility functions
- Movement among cities
- Overall economic factors, like GNP, unemployment, technological change and shocks

Thank you very much for your time, and please let me know what you think. Assuming nothing too similar has already been done, I was concerned about how publishable a paper concerning real estate would be, given that the highest ranked real estate journal on our list from Alexandre is a C (*Journal of Real Estate Finance and Economics* and *Journal of Real Estate Research*), but I did do a quick jstor search for "real estate" in the full text and got back "more than 200" for papers from the 5 finance journals between 1990 and the most current. When searching for "real estate" in the title I got back 2 articles, both in *The Journal of finance*: "Liquidity and Liquidation: Evidence from Real Estate Investment Trusts" (in Shorter Papers) by David T. Brown of The University of Florida (2000) and "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets" by Steven Grenedier (1996). So, it looks like there's reasonable potential to publish an good real estate heavy article in a A or B journal. Thanks again for your time.

Notes for future work

1) Income – people want to live with people with as high an income as possible (usually), for prestige, safety, and schools. Assume that people will homogeneously spend X% of their income on their home expense, no matter how large their income grows (they will just want more prestige and better schools as their income grows), and that no new building is allowed (at least in the short run)due to zoning and lack of available land in a city, and people don't want to drive far to work. Now see what the equalibrium is. If tax laws and society and the economy changes so that the incomes of the rich go up a lot faster than the incomes of the poor and this is expected to continue for the next decades, then housing could apreciate much more in rich areas than poor, except that if this is expected why wouldn't the apreciation happen right away (maybe it would start to happen as soon as the changes in tax laws and economic dynamics takes place)? Because the higher price must be waited for and in the meantime you may not be able to collect much rent if iliquid people are much poorer than liquid ones, and at the same time while waiting you have to pay maintanance, management and depreciation. Still, everything should adjust as soon as the info about future change is available and perhaps it does, for example suppose the expected return on the stock is 12%, but there is little rent that can be charged in the rich city as a percentage of the homes' price because the homes are so expensive, and iliquid people, who are the only people who will rent just have incomes so much lower than the rich liquid people who buy in the city. Then, maybe all the market would bear for rent would be $E\left(\rho^j\right) = 2\%$, while $E(d) = 1\%$, then $E\left(c^j\right)$ could be a huge 11%.

$$12\% = E\left(r^s\right) = E\left(i\right) = E\left(r^i\right) \equiv E\left(c^j\right) + E\left(\rho^j\right) - E\left(d\right)$$

It apears that the ratio of rent payment to mortgage payment will be positively and largely correlated to the ratio of wealth of renters to wealth of owners. We should find that in wealthy citys the ratio is smaller than in less wealthy citys. also transientness and the size of $E(d)$ are factors that should be included. So, in future work this could be tested emperically.