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Exospheric models for the X-ray emission from single Wolf–Rayet stars

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Abstract

We review existing ROSAT detections of single Galactic Wolf–Rayet (WR) stars and develop wind models to interpret the X-ray emission. The ROSAT data, consisting of bandpass detections from the ROSAT All-Sky Survey (RASS) and some pointed observations, exhibit no correlations of the WR X-ray luminosity ($L_X$) with any star or wind parameters of interest (e.g. bolometric luminosity, mass-loss rate or wind kinetic energy), although the dispersion in the measurements is quite large. The lack of correlation between X-ray luminosity and wind parameters among the WR stars is unlike that of their progenitors, the O stars, which show trends with such parameters. In this paper we seek to (i) test by how much the X-ray properties of the WR stars differ from the O stars and (ii) place limits on the temperature $T_X$ and filling factor $f_X$ of the X-ray-emitting gas in the WR winds. Adopting empirically derived relationships for $T_X$ and $f_X$ from O-star winds, the predicted X-ray emission from WR stars is much smaller than observed with ROSAT. Abandoning the $T_X$ relation from O stars, we maximize the cooling from a single-temperature hot gas to derive lower limits for the filling factors in WR winds. Although these filling factors are considerably lower than those for O stars, we find that the data are consistent (albeit the data are noisy) with a trend of $f_X \propto (\dot{M}/v_\infty)^{-1}$ in WR stars, as is also the case for O stars.

Key words: stars: abundances – stars: early-type – stars: Wolf–Rayet – X-rays: stars.

1 Introduction

In 1867, Wolf & Rayet discovered three early-type stars with anomalously strong and broad emission bands. Today only about 200 of these hot ($\gtrsim 30\,000$ K), luminous (absolute magnitudes $M_V$ from $-4.5$ to $-6.5$) Wolf–Rayet stars are known in the Galaxy. They are characterized by high masses ($\approx 10-40$ $M_\odot$) with strong stellar winds. Helium-rich and hydrogen-deficient, nitrogen is prominent in some, the WN stars, whereas carbon is significant in the spectra of others, the WC stars. There is even a minority class of oxygen-rich WO stars. These unusual compositions suggest that WR stars are evolved phases of massive stars.

The O- and B-star winds are reasonably well described by the radiative line-driven wind theory of Castor, Abbott & Klein (1975, hereafter CAK), but a good understanding of how the dense Wolf–Rayet (WR) winds are driven remains somewhat elusive in spite of recent advances in theory and observations. It is well known that the momentum of WR winds $\dot{M}v_\infty$ typically exceeds the single-scattering limit $L_\text{sc}$ by an order of magnitude (e.g. Willis 1991). There have been numerous attempts to explain the large values of $\dot{M}v_\infty$, for example, considerations of wind clumping (Nugis, Crowther & Willis 1998), non-spherical geometries (Ignace, Cassinelli & Bjorkman 1996), magnetic fields (Poe, Friend & Cassinelli 1989; dos Santos, Jatenco-Pereira & Opher 1993) or super-Eddington winds (Kato & Iben 1992). The most promising model for accelerating the high-mass-loss WR winds derives from multiline scattering of photons (Lucy & Abbott 1993; Springmann 1994; Gayley, Owocki & Cranmer 1995). This theory is indeed fully capable of explaining the driving of WR winds, provided that the opacity is sufficient for photons to be scattered frequently ($\approx 100$ times) among different lines. Of especial relevance to this work, Gayley & Owocki (1995) have shown that even with multiple scattering, the instability mechanism that leads to shock formation in the lower mass-loss OB star winds should still operate in the WR winds, and so potentially provide a mechanism for producing the observed X-ray emission.

The first quantitative X-ray information on WR stars was obtained with Einstein by Seward & Chlebowski (1982), who detected WR25 (HD 93162). White & Long (1986) obtained observations of WR6 (EZ CMa, HD 50896). Both data sets were fitted with thermal bremsstrahlung models for hot gas around $10^7$ K and hydrogen column densities $N_H \approx 10^{22}$ cm$^{-2}$. Although Einstein’s spectral response of 0.2–4 keV had the potential of providing exciting results on WR winds, only WR25 and WR6

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had sufficient integration time to yield useful spectra. Pollock
(1987) has reviewed the passband detection of single and binary
WR stars by Einstein. He notes that single stars of the WN subclass
appear to be about 4 times brighter than single WC stars. He
suggests that this might be the result of very different abundances
between the two subclasses. Pollock, Haberl & Corcoran (1995) have
published a table of PSPC passband detections and upper limits for all Galactic WR stars from the
ROSAT All Sky Survey (RASS). In terms of ROSAT spectra,
Wessolowski et al. (1995) obtained nine pointed observations of
single WN stars, with enough signal to yield spectra for WR 1 and
WR 110.

ASCA has a higher energy response and greater spectral
resolution than ROSAT or Einstein, but has observed only four
WR stars: WR6, WR139, WR140 and WR147 (Koyama et al.
1994; Stevens et al. 1996; Skinner, Itoh & Nagase 1997; Maeda
et al. 1999). WR6 has an observed stable period of 3.766 d and
may be a binary. The other three are definite WR
similarly, only WR binaries are to be observed with
Chandra during its first cycle. However, at least a couple of single WR
targets will be observed with XMM. Overall, there has been
considerable activity in observing colliding wind binary systems
with WR components, but relatively little has been done with
recent X-ray satellites to study single-star WR envelopes.

None less, there have been some advances in studies of
X-rays from single WR stars. The RASS has provided PSPC
broad-band fluxes in the range 0.2–2.4 keV for nearly all Galactic
WR stars (Pollock et al. 1995). This data set has revealed that,
unlike their predecessors (the O stars), the X-ray luminosities
of single N-rich WR types (WN) are not correlated with
bolometric luminosity $L_{\text{bol}}$, wind momentum $M_{\text{w}}$, or wind kinetic
luminosity $M_{\text{w}}^2_{\text{e}}$, or WN subtype (Wessolowski 1996).

On the side of theory, Baum et al. (1992) presented model
results for X-ray spectra from single WR stars. The models took
account of the non-solar abundances in terms of the attenuation of
X-rays by the cool wind; however, the emission is based on purely
thermal bremsstrahlung only. It seems likely that cooling via line
emission of highly ionized species, as in the Raymond & Smith
(1977, hereafter RS) models for hot optically thin plasmas, will
be important, especially owing to the highly enhanced metal
abundances of WR winds. Ignace & Oskinova (1999, hereafter
Paper I) have sought to explain the trends (or rather the lack of
trends) found by Wessolowski (1996). The cool dense WR winds
are optically thick to X-rays for a broad range of energies, so that
observed X-ray emission can be thought of as forming exterior to an
`exosphere', a surface defined by optical depth unity in the cool
wind opacity. If the filling factor of hot gas (to be discussed
below) scales inversely with the ratio $M_{\text{w}}/v_{\text{w}}$ (i.e., the wind density
scale), all dependence on $M_{\text{w}}/v_{\text{w}}$ exactly cancels, and the $L_X$
values will show no correlations with such mass-loss, terminal speed, or
any combination thereof. Instead, there exists a dependence on
abundances, and although the dispersion of the ROSAT measure-
ments is relatively large, it was found in Paper I that the differences in abundances between the WC and WN classes may
be sufficient to explain why WN winds tend to be about 3–4 times
more X-ray-luminous than WC winds (confirming Pollock’s 1987
suggestion).

In this paper the analysis is taken one step further in an attempt
to assess the hot gas temperatures and filling factors. In Section 2
we expand on the emission model used in Paper I. We especially
elaborate on the effects of abundances for the wind attenuation. In
Section 3 we apply these models in several different ways, the
chief aim being to set limits on the hot gas filling factor and to test
the hypothesis that these filling factors vary inversely with the
ratio $M_{\text{w}}/v_{\text{w}}$. A discussion of these results is presented in Section 4.
Appendices detail some of the more technical aspects of the
emission modelling, and also the linear regression scheme in
fitting the data and model results.

2 SPECIFICATION OF THE MODEL

We consider a spherically symmetric and time-independent stellar
wind that is a homogeneous mix of `cool' and `hot' gas in
 dynamical equilibrium. The ambient stellar wind consists
predominantly of the cool gas component ($\approx 10^7$ K), whereas the
minor hot gas component ($\approx 10^8$ K) gives rise to the X-ray emission. This
hot gas emission is modelled as an optically thin hot plasma that is
characterized by a `filling factor', defined so that the emitted
power in X-rays from a differential volume element $dV$ is
$$\frac{dL_X(E)}{dV} = 4\pi j_s E = f_X n_\text{e} n_\text{i} \Lambda_s(T_X) dV,$$
(1)
where $j_s$ is the emissivity, $f_X$ is the filling factor, $n_\text{e}$ and $n_\text{i}$ are
the electron and ion densities of the cool or normal wind component,
$\Lambda_s$ is the cooling function, and $T_X$ is the temperature of the hot
gas. This definition for the filling factor is the same as that used by
Kudritzki et al. (1996), so that we may make reference to their
results at a later point. Note that, in general, $f_X$, $T_X$, $n_\text{e}$, and $n_\text{i}$
are potentially all functions of radius. However, observations of single
WR stars consist mainly of broad-band X-ray fluxes, so that in this
paper $f_X$ and $T_X$ will be treated as constants throughout the wind
flow, for simplicity. Without spectral information, there is little to
constrain any possible radial dependence of $f_X$ and $T_X$, if it exists.

The total specific luminosity emerging from the wind is given by
a volume integral over the observable envelope:
$$L_X(E) = \int_{V} f_X n\text{e} n_\text{i} \Lambda_s(T_X) e^{-\tau_e} dV,$$
(2)
where $\tau_e$ is the attenuation of X-rays by the wind. Self-absorption
by the hot plasma is ignored. The attenuation is therefore entirely
from the cool wind component intervening between the observer
and the point of emission. The wind optical depth is given by
$$\tau_e(p, z) = \int_{z}^\infty \kappa_w \rho \, dz,$$
(3)
with opacity $\kappa_w$ and density
$$\rho(r) = \frac{M}{4\pi r^2 v_1(r)}.$$
(4)
For the radial wind speed, it is standard to assume a $1/2$ velocity with
$$v_1(r) = v_\infty \left(1 - \frac{bR}{r}\right)^{\frac{1}{2}},$$
(5)
where the non-dimensional constant $b < 1$, and $R$ is the radius at
the wind base (taken to be the radius of the star). Including the
parameter $b$ ensures that the density is not singular at the lower
boundary. However, in our analysis it will be sufficient to assume
that the X-rays emerge from large radius only, where $v_1(r) = v_\infty$.

The dominant opacity at the X-ray energies is photo-absorption
by K-shell electrons. This opacity is
$$\kappa_w(E) = \frac{1}{\mu_\text{N} m_\text{H}} \sum_j \frac{n_j}{n_\text{N}} \sigma_j(E).$$
(6)
The opacity is a summation over cross-sections $\sigma_j$ presented by different atomic species $j$ and weighted by the relative abundance $n_j/n_N$, for $n_N$ the number density of nuclei. The factor of $\mu_N$ is the mean molecular weight per nucleus, but since there is essentially no neutral gas in hot-star winds, the number density of nuclei is the same as ions; hence $n_N = n_i$ and $\mu_N = \mu_i$.

Abundances can have an important effect on the emergent X-ray luminosity, both in terms of the cool wind attenuation and the emissivity. With the above expressions constituting our basic model for the X-ray emission from hot-star winds, we now address the consequences of the highly non-solar abundances of the WR stars for the various factors that determine the X-ray luminosity.

### 2.1 The effect of abundances for the wind opacity

For the wind attenuation of the X-rays, K-shell absorption by metals in the cool wind is the dominant opacity source. The contribution to the absorptive opacity can vary strongly with atomic species, as for example in the case of H-like atoms where the cross-section scales as the fourth power of the proton number. So even modest enhancements of metals from nuclear burning can dramatically alter the run of wind opacity with wavelength. Table 1 contrasts typical abundances of WN and WC stars (taken from van der Hucht, Cassinelli & Williams 1986) against cosmic abundances. The WN types are essentially helium stars with enhanced nitrogen and an underabundance of oxygen. The WC stars are essentially helium-carbon stars with substantial amounts of oxygen but essentially no nitrogen.

Fig. 1 displays the energy-dependent photoelectric cross-sections $\sigma_w(E) = \mu_n m_H \kappa_w$ in units of cm$^2$ per particle for stars of different metallicities and ionization states of hydrogen and helium. The curves were computed using codes made available by Balucinska-Church & McCammon (1992) that allow the abundances to be

<table>
<thead>
<tr>
<th>Table 1. Wolf–Rayet abundances (by number).</th>
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<tr>
<td><strong>Element</strong></td>
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<td>H</td>
</tr>
<tr>
<td>He</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>N</td>
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<tr>
<td>Mg</td>
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<td>Si</td>
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<td>P</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>Fe</td>
</tr>
</tbody>
</table>

**Figure 1.** Shown are plots of the absorption cross-section $\sigma_w$ in cm$^2$ of the cool wind to X-rays as a function of energy. The top, middle, and bottom panels are for cosmic, WN, and WC abundances. For the purposes of this work, it is adequate to treat O stars as having cosmic abundances. In the top panel, the solid line is for a gas with H$^+$, like the ISM, the dotted line is for an O star with H$^+$, and the dashed line is for an O star with H$^+$ and He$^+$. For both the WN and WC cases, hydrogen is assumed ionized or absent altogether. Solid, dotted, and dashed lines are then for neutral, once-ionized, and twice-ionized helium. In our models we always take helium to be once-ionized in the WR winds.
input parameters. Prominent edges can be seen at 0.28, 0.40 and 0.53 keV for the C, N and O atoms. Note that the Balucinska-Church & McCammon opacities are for neutral species. They comment that ionization of metals does not much affect the magnitude of the absorption cross-section, but it does shift the edge energy. Tabulations by Verner & Yakovlev (1995) indicate that the edge energy moves to increasingly large values for more highly ionized species. The change in edge energy between any two ions is just a few per cent, but the jump from a neutral atom to a hydrogenic atom is around 50 per cent or more (e.g., the edge for O is 0.53 keV, but that for O VIII is 0.87 keV). An element that is entirely ionized obviously makes no contribution to $\sigma_{\text{in}}$, which is relevant for hydrogen and possibly helium in the cool component of early-type winds. In our models we use edges for neutral metals.

In Fig. 1 the top panel shows three curves: solid for the standard cross-section appropriate to the ISM with cosmic abundances, dotted for an O star with cosmic abundances and completely ionized hydrogen, and dashed for the same O star but with helium completely ionized. The drop in the cross-section at low energies is similar to that found by Hillier et al. (1993) in their study of $\xi$ Puppis (O4f) which included the effects of helium ionization. The middle panel is for a hydrogen-deficient WN star, with a solid curve for He I, a dotted one for He II, and a dashed one for He III. Comparing the solid curves for an O and WN star, the cross-section for the latter is higher by about 0.5 dex at high energies and 0.8 dex at lower energies. Note, however, that $\mu_e(WN) \approx 3\mu_e(O)$, so that the opacity $\kappa_e$ is nearly the same for both O and WN stars. The greater attenuation of X-rays in WN winds relative to O stars is mostly a consequence of higher wind density.

The bottom panel of Fig. 1 is for a WC star with different ionizations of helium. The curves are relatively insensitive to helium ionization. The carbon edge is extremely prominent, and the overall cross-section is up by about 1.5 dex near 1 keV over that for a WN star (but this increment clearly varies strongly with energy). The ion mean molecular weight is greater in WC stars, being about 2 times that for WN stars and 6 times that for O stars, so in this case the opacity is actually significantly larger in WC stars than in other hot stars with less enhanced abundances.

### 2.2 The effect of abundances for the cooling function

For temperatures $T_X$ in which $\Lambda_e$ is dominated by line emission (in contrast to thermal bremsstrahlung that dominates for $T_X \geq 10^6$ K), the cooling function is roughly given by $\Lambda_e = \sum_k P_k n_k / n_i$, where $P_k$ is a factor relating to the emitted power in the line $k$ and will generally depend on density and temperature, and $n_k / n_i$ is the ratio of the number density population corresponding to the line $k$ to the total ion number density of the hot gas. For solar abundances, the RS cooling function is used, with $\Lambda_{\text{RS}} = \sum_k P_k(T)(n_k / n_i)_{\text{RS}}$, where $n_k$ is the ionized hydrogen density of the hot gas. Assuming that the $P_k$s vary weakly with density and temperature, and further that the ratio $n_k / (n_i)_{\text{RS}} = \tilde{A}$ is constant for every line $k$, a scaling correction to the known RS cooling function for non-solar abundances is (see Appendix A)

$$\Lambda_e(T_X) \approx \frac{\mu_{e,\odot}}{\mu_{e}} \tilde{A} \Lambda_{\text{RS}}(E, T_X), \tag{7}$$

where $\mu_{e,\odot}$ is the mean molecular weight per ionized hydrogen atom for solar abundances, which is the same for both the cool and the hot gas. In the case that $n_k / (n_i)_{\text{RS}}$ is not constant for every $k$, the overall average enhancement (or reduction) factor to the RS cooling function. This latter interpretation of $\tilde{A}$ is the most relevant to our case, since the ROSAT data that we will consider consists of bandpass fluxes, wherein the contributions of many lines are being summed together.

In Paper I we used an ion mean molecular weight for WN stars of $\mu_e = 4$ and for WC stars $\mu_e = 7.6$. We also argued for $\tilde{A}_{\text{WN}} \approx 1$, because the evolution from O stars to WN stars mostly results in converting hydrogen to helium, some enhancement of nitrogen, and a depletion of oxygen, elements that have relatively little consequence for the cooling function. On the other hand, further evolution to WC stars leads to substantial enhancements of carbon and oxygen, essentially the elimination of nitrogen, but also enhancements in neon and magnesium – changes with greater relevance for the relative intensity of some lines that appear at ROSAT energies. In this case $\tilde{A}_{\text{WC}} \approx 1$ is likely, with values of perhaps a few. For example, Koyama et al. (1994) require the abundance of neon to be about 100 times solar to explain the ASCA spectrum WR 140 (WC + O4–5). The enhanced neon is surely not from the O-star companion. The spectral feature they fit is at about 1.2 keV, which falls midway in the ROSAT band, so there is good reason to believe that $\tilde{A}_{\text{WC}}$ could be a few or greater. Taking $\mu_{e,\odot} \approx 1.5$, we estimate that $\tilde{A}_o/\tilde{A}_{\text{RS}} \approx 3$ for WN stars, and at least that for WC stars.

### 2.3 The effect of abundances for the filling factor

We assume the filling factor to be constant throughout the wind, with a value that can vary between different stars. First, it can vary with abundance as $f_X \propto (\mu_e n_e)/(\mu_e n_\odot) = (\mu_e n_e)/(\mu_e)_{\odot}$. Note that $(\mu_e n_e)/(\mu_e)_{\odot} \approx 2$ for reasonable assumptions about the ionization state in the cool and hot components, so this does not provide much variation in $f_X$ among different stars. The filling factor is also taken to vary inversely with the ratio $M/\epsilon_{\odot}$. For example, Kudritzki et al. (1996) has analysed ROSAT observations for 42 O stars, and empirically determined $f_X \propto (M/\epsilon_{\odot})^{-1}$. They attribute this result to the expectation that larger ratios of $M/\epsilon_{\odot}$ result in more efficient cooling, shorter cooling zones, and consequently smaller filling factors (see also discussion by Hillier et al. 1993). The end result is that the volume filling factor scales as

$$f_X \propto \left(\frac{\mu_e n_e}{\mu_e n_\odot}\right) \left(\frac{M}{\epsilon_{\odot}}\right)^{-1}. \tag{8}$$

Note that in the context of explaining the X-ray emission from O stars, Owocki & Cohen (1999) consider a filling factor that varies with radius as a power law. However, they do not consider how $f_X$ might vary from star to star. They are able to explain the observed relation between $L_X$ and $\tilde{A}_\text{RS}$ (which they identify as really being related to $M/\epsilon_{\odot}$) by adjusting the power-law exponent for the filling factor. Owing to the poorer data for single WR stars (no spectra and fairly large errors for bandpass measurements), it was assumed in Paper I that the filling factor of equation (8) is constant in the flow, but could vary from wind to wind. In Paper I the lack of correlations between $L_X$ and wind parameters could then be explained. However, if $f_X$ is not constant in the wind, an analysis like that of Owocki & Cohen will be needed to explain the observed lack of correlation. So the conclusion of Paper I is clearly model-dependent, but the assumptions adopted in Paper I do appear to be sufficient to explain the data.
2.4 The exospheric approximation

In their study of X-rays from OB stars, Owocki & Cohen (1999) presented a scaling analysis for the X-ray emission from hot-star winds based on an exospheric approximation. The observed X-ray emission arising from hot gas emerges only from radii exterior to the optical depth unity surface of radius \( r_1 \), with X-rays at smaller radii assumed to be completely attenuated. The extent of \( r_1 \) is energy-dependent, with

\[
    r_1(E) = \frac{M}{4\pi\overline{v}_e \kappa_0(E)}. \tag{9}
\]

Owocki & Cohen showed that for a constant expansion wind, the exospheric approximation overestimates \( L_X \) from an exact integration for the radiative transfer by a factor of only 2. Since \( r_1 \gg R_0 \) over a broad range of X-ray energies for the WR stars, a constant expansion wind is an excellent approximation. For the purposes of modelling the X-rays, we therefore assume a spherical wind with density \( \rho = M/4\pi\overline{v}_e r^2 \) for WR stars.

The emergent specific X-ray luminosity (including a factor of 2 reduction for the reasons just discussed) is thus given by

\[
    L_X(E) = 4\pi^2 \int_1^\infty \left( 1 + \frac{1 - \frac{r_1^2}{r^2}}{1} \right)^2 \, dr, \tag{10}
\]

where the parenthetical term accounts for geometric occultation by the spherical surface of radius \( r_1 \) (a minor 10 per cent effect that was ignored by Owocki & Cohen but which we choose to include). Substituting for the emissivity \( j_\nu \),

\[
    L_X(E) = \frac{1 + \pi/4}{16\pi} \frac{M^2}{\mu_e \mu_t m_H^2 c^2 v_0^2 r_1} \int_X A_\nu(T_X). \tag{11}
\]

Equations (10) and (11) are the same as those used in Paper I. Substituting for the factor \( r_1 \) yields

\[
    L_X(E) = \frac{1 + \pi/4}{4} \frac{M}{\mu_e \mu_t m_H^2 c^2 v_0 \kappa_0} \int_X A_\nu(T_X). \tag{12}
\]

The energy dependence of \( L_X(E) \) comes strictly from the ratio

\[
    \frac{L_X(E)}{L_X(E_\odot)} = \left( \frac{\overline{v}_e}{\overline{v}_e\odot} \right)^{3/2} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{r_1}{R_\odot} \right) \left( \frac{E}{E_\odot} \right)^{-1/2}. \tag{13}
\]
λ/κ_e(E). Also, note that L_X(E) appears to scale with the ratio \( M/v_\infty \); however, the hot gas filling factor \( f_X \) implicitly depends on \( (M/v_\infty)^{-1} \). Hence the scaling of X-ray luminosity should not scale with wind mass-loss or terminal speed. Although perhaps \( T_X \) may depend on these parameters in some way, it is not clear how this might affect \( L_X(E) \). Ignoring any such dependence between \( T_X \) and \( M \) or \( v_\infty \), the above expressions were used in Paper I to conclude that X-ray luminosities from WR winds will depend only on abundances.

3 APPLICATION TO THE ROSAT DATA

3.1 Description of the data and analysis

Having developed a model for the X-ray emission from WR winds, we now consider the existing ROSAT data. Although single and binary WR stars have been observed with several X-ray telescopes, the most ‘complete’ data set at present comes from the ROSAT All-Sky Survey. We have selected single WN and WC stars from the compilation of Pollock et al. (1995). We combine those ROSAT passband measurements with wind parameters derived by Hamann & Koesterke (1998) for WN stars and Koesterke & Hamann (1995) for WC stars. The merged data set is shown in Table 2 for WNs and Table 3 for WCs. Note that we have rescaled the X-ray luminosities according to distances from Hamann & Koesterke and Koesterke & Hamann versus those listed by Pollock et al. (1995) to obtain a more consistent data set, as was done by Wessolowski (1996). However, we have attempted no assessment of the distance estimates or corrections to the X-ray fluxes due to interstellar attenuation. We have simply taken these values from the literature, and so it should be borne in mind that errors in those values could affect our conclusions. Also, as noted by Wessolowski, we revise the count rate for WR 25 from 1960 to 194 k s^{-1} owing to a mistaken entry (presumably the standard deviation decreases by a factor of \( \sqrt{10} \), although this is not stated).

In Table 4 we list single stars that are neglected in our analysis:

<table>
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<tr>
<th>WR #</th>
<th>Subtype</th>
<th>( L_X/L_\odot )</th>
<th>( \sigma(L_X)/L_\odot )</th>
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<td>WN</td>
<td>1.28</td>
<td>0.67</td>
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<tr>
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<td>WN</td>
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<td>0.036</td>
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<tr>
<td>71</td>
<td>WN</td>
<td>0.052</td>
<td>0.043</td>
<td>Not analysed in Hamann &amp; Koesterke (1998)</td>
</tr>
<tr>
<td>91</td>
<td>WN</td>
<td>–</td>
<td>–</td>
<td>No conversion to ( L_X ) in Pollock et al. (1995)</td>
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<tr>
<td>94</td>
<td>WN</td>
<td>0.046</td>
<td>0.085</td>
<td>Not analysed in Hamann &amp; Koesterke (1998)</td>
</tr>
<tr>
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<td>WN</td>
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<td>–</td>
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<td>WN</td>
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<td>0.084</td>
<td>( \sigma(L_X)/L_X = 9 )</td>
</tr>
<tr>
<td>155</td>
<td>WN</td>
<td>0.0013</td>
<td>0.025</td>
<td>( \sigma(L_X)/L_X = 19 )</td>
</tr>
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<td>3σ upper limit</td>
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<tr>
<td>58</td>
<td>WN</td>
<td>&lt;0.18</td>
<td>0.060</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>61</td>
<td>WN</td>
<td>&lt;0.95</td>
<td>0.32</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>78</td>
<td>WN</td>
<td>&lt;0.021</td>
<td>0.0070</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>87</td>
<td>WN</td>
<td>&lt;0.17</td>
<td>0.056</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>107</td>
<td>WN</td>
<td>&lt;1.45</td>
<td>0.48</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>124</td>
<td>WN</td>
<td>&lt;0.23</td>
<td>0.078</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>33</td>
<td>WC</td>
<td>0.074</td>
<td></td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>52</td>
<td>WC</td>
<td>&lt;0.071</td>
<td>0.024</td>
<td>3σ upper limit</td>
</tr>
<tr>
<td>150</td>
<td>WC</td>
<td>&lt;0.14</td>
<td>0.047</td>
<td>3σ upper limit</td>
</tr>
</tbody>
</table>

(a) stars that have only upper limits and are therefore neglected in our analysis, (b) stars that have count rates listed in Pollock et al. (1995) but no conversion to \( L_X \), (c) stars that have extremely poor detections with \( \sigma / L_X \gtrsim 10 \), which we treat as upper limits, and (d) stars that have values of \( L_X \) given by Pollock et al. but no corresponding information for \( M \), etc. by Hamann & Koesterke or Koesterke & Hamann. In this last case, we do use the \( L_X \) values in computing mean WN and WC X-ray luminosities, but not in ensemble analyses that require knowledge of wind parameters.

Most of the data have 1\( \sigma \) or better detections, but we do use some with poorer detections. In the case of multiple detections, we take a straight average, but we give preference to pointed observations if the survey result is substantially worse. Our sample is supposed to be of single stars; however, some targets classified as `abs' systems (showing absorption features but not confirmed binaries) or single-lined spectroscopic binaries are included. It should be borne in mind that the sample is probably not free from binary contamination. Also, WR 25 is included, which has anomalously high X-ray flux and is a suspected binary, although attempts to find a companion have all been negative. We note that the detection rate among both WN and WC stars is around 80–85 per cent (see Table 5).

### Table 5. Summary of ROSAT detections.

<table>
<thead>
<tr>
<th></th>
<th>( \langle L_X \rangle / L_\odot )</th>
<th>( \sigma_X(\langle L_X \rangle) / L_\odot )</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>WN stars (detections)</td>
<td>0.11</td>
<td>±0.018</td>
<td>52/64</td>
</tr>
<tr>
<td>WN stars (3( \sigma ) upper limits*)</td>
<td>0.55</td>
<td>–</td>
<td>12/64</td>
</tr>
<tr>
<td>WC stars (detections)</td>
<td>0.038</td>
<td>±0.013</td>
<td>17/20</td>
</tr>
<tr>
<td>WC stars (3( \sigma ) upper limits)</td>
<td>0.14</td>
<td>–</td>
<td>3/20</td>
</tr>
</tbody>
</table>

*We include WR129 and WR155 among the upper limits (see Table 4).

Fig. 2 summarizes this data set as a plot of \( L_X \) versus \( L_\odot \) for single WN (circles) and WC (triangles) stars. This is the same figure as that shown in Paper I, except that the errorbars shown in that figure were not properly transformed and have been corrected here. The upper and lower horizontal lines indicate the weighted mean values for the WN subclass and WC subclass respectively. There is substantial scatter in the distribution of X-ray luminosities. Yet there appears to be no linear trend between \( L_X \) and \( L_\odot \) as is the case with O stars, neither for the whole ensemble nor for subsets of just the WN stars or just the WC stars. The only overall trend is that WN stars are about 3 times brighter than WC stars in the ROSAT band (Paper I and also Table 5), but even this is only a 1\( \sigma \) result.

We have considered a variety of weighted linear regressions to the data sample for \( L_X \) versus \( L_\odot \) and \( M / v_\infty \). The method is described in Appendix B, and a summary of the fits appear in Table 6. The weight for a single measurement \( i \) is given by \( w_i = 1 / (\sigma_1^2 + \sigma_0^2) \), where \( \sigma_1 \) is the measurement error, and \( \sigma_0 \) represents an additional dispersion present in the data. This additional spread is motivated by two facts. (a) A standard set of abundances are assumed for the WN types and the WC types, but of course the abundances of any given star will not exactly match the typical values. Variation in abundances among the WN and WC types respectively affects the emergent X-ray emission and introduces an additional dispersion in the data. (b) Likewise, the hot gas temperature is not known and may vary between stars. In all likelihood, it is not even isothermal, with each wind probably showing a range of temperatures in the hot component (e.g., as discussed by Feldmeier et al. 1997). This too introduces additional scatter into the sample. The data are of too poor quality, the spectral information too little (basically none), and abundances not sufficiently well known to account for these variations in each individual star. We therefore seek to account for the variations in a statistical manner through \( \sigma_0 \).

In practice, the most likely value of \( \sigma_0 \) comes from demanding that the reduced chi-square \( \chi^2_\nu \) be unity, where the number of degrees of freedom \( \nu = N - 2 \) for \( N \) data points and a two-parameter line fit. This means that \( \sigma_0 \) is adjusted until the weighted dispersion of the data yields the most probable fit by a straight line. The essential effect of \( \sigma_0 \) is to reduce the importance of those measurements with extremely good measurement errors in the fitting procedure. Again, this is motivated by the a priori realization that the poorly determined abundances and hot gas temperatures introduce an associated dispersion in the data that is unrelated to measurement errors. Only by allowing for this spread can we make a meaningful estimate of mean values or line fits.

The regressions allow for just the WN stars or just the WC stars, or the combined groups. For the WN stars, we show the fit parameters when WR 25 is included or not included, because of its uncertain nature. The case of the filling factors will be discussed later. For \( L_X \) versus \( L_\odot \), there seems to be no hint of a statistically significant linear relation; however, there is a suggestion that perhaps \( L_X \) varies with \( M / v_\infty \) with a power-law index of about 0.3–0.35.

How does one analyse such a data set, and exactly what are the goals of such an analysis, namely what physical parameters are to be constrained? In the context of our model, the fundamental properties relating the observed X-ray emission to the physics of the wind X-ray production are the filling factor, hot gas temperature, and abundances, with everything else taken as known. However, abundances are also not well-known for individual objects, so we will use typical values from Table 1 for all WN and WC stars. The desired result is then to empirically determine \( f_X \).
and $T_X$ that allow us to affirm, refine, or reject models for the wind driving and/or models for wind structure that leads to the existence of the X-ray-emitting gas.

Given the rather noisy character of the data set, we have selected two different approaches to study the data that each depend on ensemble properties in contrast to tailored fits to individual objects.

(i) First, the winds of O stars are for the most part successfully explained by CAK line-driven wind theory for non-overlapping lines. Lucy & Abbott (1993) find that multiple-scattering effects are probably important for driving the WR winds. Kudritzki et al. (1996) have determined empirical relationships for $T_X$ and $f_X$ based on the wind mass-loss rate and terminal speed. An immediate question is whether the X-ray properties of the WR winds are derivable from the empirical relations that seem to hold for O stars (modulo the effects of highly non-solar abundances for the cooling function and wind attenuation).

(ii) A different approach is to use the data set to place limits on the X-ray temperature or filling factor. We derive a lower limit to the filling factor by maximizing the X-ray emissivity (i.e., for isothermal shocks). This is accomplished by combining the cooling function, ROSAT responsivity, and typical wind attenuation dependence with energy to search for a temperature that maximizes the X-ray luminosity sampled in the ROSAT bandpass. Assuming this simple temperature to characterize the hot gas in and throughout every WR wind, the filling factor required to explain the observed X-ray emission is thereby minimized in each case.

### 3.2 Comparison of X-ray properties between O stars and WR stars

Based on figures presented in Kudritzki et al. (1996), we derived the following empirical relations for $T_X$ and $f_X$ for O stars from their figures:

\[
T_X^{\text{emp}} = 10^6 K \left( \frac{M_6 \varpi_{0.3}}{L_{\odot}} \right)^{0.8},
\]

and

\[
f_X^{\text{emp}} = 2.6 \times 10^{-3} \left( \frac{M_6 \varpi_{0.3}}{\varpi_{0.3}} \right)^{-1.0},
\]

Table 6. Results from linear regression analysis.

<table>
<thead>
<tr>
<th>Relation</th>
<th>$\chi^2$</th>
<th>$m$</th>
<th>$\sigma(m)$</th>
<th>$b$</th>
<th>$\sigma(b)$</th>
<th>$\sigma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log L_X / L_\odot$ vs $\log L_{\text{bol}} / L_\odot$: (WN only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with WR25</td>
<td>1.0</td>
<td>-0.24</td>
<td>0.28</td>
<td>0.44</td>
<td>1.60</td>
<td>0.50</td>
</tr>
<tr>
<td>no WR25</td>
<td>1.0</td>
<td>-0.17</td>
<td>-0.28</td>
<td>-0.02</td>
<td>1.56</td>
<td>0.48</td>
</tr>
<tr>
<td>(WC only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with WR25</td>
<td>1.0</td>
<td>0.39</td>
<td>0.81</td>
<td>-3.46</td>
<td>4.15</td>
<td>0.625</td>
</tr>
<tr>
<td>no WR25</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(WN and WC)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with WR25</td>
<td>1.0</td>
<td>0.33</td>
<td>0.21</td>
<td>-2.86</td>
<td>1.18</td>
<td>0.555</td>
</tr>
<tr>
<td>no WR25</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(WN and WC)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with WR25</td>
<td>1.0</td>
<td>0.35</td>
<td>0.17</td>
<td>1.53</td>
<td>1.27</td>
<td>0.545</td>
</tr>
<tr>
<td>no WR25</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above shows the results from linear regression analysis for the relation between X-ray luminosity and bolometric luminosity for O stars with and without WR25 stars.
where the numerical subscript stands for powers-of-ten normalization, with $M$ in $M_\odot$ yr$^{-1}$, $v_\infty$ in km s$^{-1}$, and $L_\odot$ in $L_\odot$. For a typical O star with $M = 10^{-6} M_\odot$ yr$^{-1}$, $T_{\text{emp}}^\text{emp} = 4 \times 10^8$ K or $kT_{\text{emp}}^\text{emp} = 0.4$ keV and $f_X^\text{emp} = 7 \times 10^{-3}$. In contrast, using these relations with typical WR star parameters of $M = 3 \times 10^{-5} M_\odot$ yr$^{-1}$, $v_\infty = 2000$ km s$^{-1}$, and $L_\odot = 3 \times 10^5 L_\odot$, the expected hot gas temperature is $T_X^\text{emp} = 10^8$ K and the filling factor $f_X^\text{emp} \approx 2 \times 10^{-3}$. If these relations hold for the WR stars, the WR wind should be comparatively much hotter with a far smaller filling factor.

To determine whether the O-star relations can be used with WR stars to explain the ROSAT observations, we have chosen to assume the Kudritzki et al. (1996) relation for $T_X^\text{emp}$ as applicable to the WR winds, and then to solve for the filling factor $f_X^\text{obs}$.

The results of this experiment are shown in Fig. 3(a). The inferred filling factor is consistently two orders of magnitude higher than that expected from the empirical relation.

Although not shown, typical error bars are about 0.3, but can be as high as 1.2. Every star has substantially larger $f_X^\text{obs}$ than expected from the O-star relation. The high WR filling factors are primarily a result of the high $T_X \sim 10^8$ K as predicted by the empirical relation. At this temperature, the emission is dominantly bremsstrahlung. Although the emission integrated over all energies increases as $T^{1/2}$ for bremsstrahlung, the emission in a fixed energy band decreases as $T^{-1/2}$. Consequently, extremely hot gas in the $10^8$ K regime cools much less efficiently at the energies of the ROSAT bandpass than does cooler gas of $10^6$–$10^7$ K. So it appears that the physics governing the production of X-rays in the WR winds can not be treated as merely a ‘scaled-up’ version of what operates in O-star winds.

### 3.3 Minimal X-ray filling factors for WR winds

In this section we consider the maximum possible emission to determine the minimum filling factor. Results from the previous section suggest that the empirical relations valid for O stars cannot simply be extended to include the WR stars. We make the hypothesis that the temperature relation for O stars almost certainly does not apply. Even in the colliding wind systems of WR binaries, there is little or no evidence for gas at $10^8$ K. Such hot gas may be present in small amounts, but the bulk of the X-rays appear to come from lower temperature ($10^6$–$30 \times 10^6$ K) gas. It seems unlikely then that single WR stars would have $10^8$ K gas.

On the other hand, the filling factor scales roughly as the inverse of the density. That is a somewhat more robust expectation, namely that cooling is more efficient for higher density material. This would appear to be insensitive to the details of the wind driving or shock formation mechanism(s). Perhaps the temperature relation of Kudritzki et al. (1996) fails miserably when applied to WR stars, but the filling factor scaling $f_X \propto \left(\frac{M}{v_\infty}\right)^{-1}$ may still be valid, an assumption that was made in the analysis of Paper I and which we seek to show a posteriori.

So, to set a lower limit on the filling factor, it is important to maximize the cooling function not in an absolute sense, but rather with respect to what ROSAT can detect. Fig. 4 summarizes the steps in doing this. The open boxes connected by the short-dashed curve plot the spectrum-integrated RS cooling function against temperature $T_X$. Note that it peaks around 200 000 K, with drops around $10^8$, $10^6$ and $10^4$ K. ROSAT is primarily sensitive to flux in the $0.2–2.4$ keV range. The triangles connected by the long-dashed curve are the integrated cooling function after first multiplying by the ROSAT response curve. ROSAT is insensitive
Figure 4. A figure to demonstrate where cooling by lines is maximized. The short-dashed line indicates how the energy integrated cooling function $\Lambda_{\text{tot}}$ varies with temperature $T_X$ for a single-temperature hot plasma. The long-dashed line includes the effect of the ROSAT sensitivity function. Finally, the solid line shows how the cooling varies when both the ROSAT sensitivity and wind attenuation are included. In this last case, a single prominent peak occurs around $10^7$ K. Note that the points are for individual calculations, and the curves have been individually normalized to their peak values, resulting in a relative ROSAT passband flux.

Figure 5. A plot of the filling factor $f_{\text{w}}^{X}$ against the wind density scale $M/v_{\infty}$. Circles are for WN stars, and triangles for WC stars. The errorbars reflect quoted measurement errors. Three linear regressions are shown as discussed in the text. The solid line is taken as our best fit, which is a weighted regression based on measurement errors and an additional but a priori unknown spread relating to variations in abundance and $T_X$ among the sample stars. This line has a power-law slope of $m = -0.9$ (see Table 6), consistent with the $m = -1$ slope derived for O stars. However, the data are indeed quite noisy, so that we can probably only conclude that the sample is not inconsistent with this slope.

3.4 Dependence of filling factors on $M/v_{\infty}$

Having derived filling factors, we now want to test empirically whether the filling factors scale like $(M/v_{\infty})^{-1}$ as assumed or not. In Fig. 5 we explicitly show the minimized filling factors as plotted against $M/v_{\infty}$. The expectation is that the points should fall along a straight line of slope $= -1$ in this log-log plot. The data are terribly noisy, so we have computed several weighted linear regressions, using the same methods as for comparing $L_X$ to $L_v$ and $M/v_{\infty}$. The results of the line fitting is summarized at the bottom of Table 6. Three lines are plotted in Fig. 5 for fits to the entire ensemble of points (WN and WC together), including WR 25. The first line is shown as short-dashed. In this case, weights $w_i = 1/\sigma_i^2$ based on measurement errors only were used. The line has a slope $m = -1.1$, somewhat steeper than desired. In fact, it is the rather large filling factor of WR 25, owing to unusually high $L_X$, combined with its small standard deviation, that is affecting this slope.

Two more fits were evaluated, this time with weights $w_i = 1/(\sigma_i^2 + \sigma_0^2)$, where $\sigma_0$ represents an additional dispersion in the data owing to variations in abundances and hot gas temperatures from what has been assumed in the model, as was previously discussed. The two lines are for $\sigma_0 = 0.69$ and 35. The two lines are almost indistinguishable. Since $\sigma_0 = 0.69$ is already about twice the typical measurement error, the weights for many points are dominated by $\sigma_0$, which tends to give equal significance to these points. Therefore it is not surprising that the two lines are so similar. The slope is $m = -0.9$, quite close to the expected value of $-1$, especially given the substantial dispersion in the data. A conservative conclusion is that the data are not inconsistent with the empirical relation $f_X \propto (M/v_{\infty})^{-1}$ as observed for O stars.
4 DISCUSSION AND CONCLUSIONS

The X-ray properties of single WR stars are in our opinion poorly studied both observationally and theoretically. Colliding wind binaries involving a WR star have naturally received more attention by virtue of being much brighter X-ray sources. Moreover, these systems are expected to show cyclic variations of X-ray emission with orbital phase that might straightforwardly be used to test theoretical models. Single WR stars present a greater challenge to observers, since they tend to be fainter sources and the production of the X-ray emission is less well-understood.

Using the RASS sample, a plot of $L_X$ versus $L_{\text{Bol}}$ for single WN and WC stars does indeed appear to be lacking correlation, as first pointed out by Wessolowski (1996). Compared to Paper I, we have rescaled the X-ray luminosities of Pollock et al. (1995) to the assumed distances from Koesterke & Hamann (1995) and Hamann & Koesterke (1998), to be consistent with wind parameters ($\dot{M}$) that we take from those papers. We note that due to the rescaling and the addition of sources that do not have wind parameters from the optical analysis but do have $L_X$ values from Pollock et al., we have recomputed weighted mean X-ray luminosities for the WN and WC subclass in the ROSAT band $0.2-2.4$ keV. The values are $L_X = 4.3 \pm 0.7 \times 10^{32}$ erg s$^{-1}$ for WN types and $L_X = 1.5 \pm 0.5 \times 10^{32}$ erg s$^{-1}$ for WC stars. These are only slightly larger ($\approx 5$ per cent) than the values quoted in Paper I. There may be some hint that $L_X$ for WN and WC stars increases with the ratio $\dot{M}/v_w$ as roughly the cube root (see Table 6), but it is not especially significant.

Using the RASS sample, we have considered two ‘experiments’. In the first we assumed that the empirical relations derived by Kudritzki et al. (1996) from ROSAT observations of O stars could be applied to WR stars. These relations predict typical hot gas temperatures of around $10^8$ K and filling factors of about $10^{-4}$. These values are not mutually consistent with the ROSAT data. If the temperature of the gas is truly around $10^8$ K, then our exospheric models demand filling factors about 2 dex larger than predicted.

The second experiment consisted of maximizing the cooling function (i.e., under the assumption of isothermal shocks), modulo the expected wind attenuation and the ROSAT response function, to derive lower limits for the hot gas filling factor. A rough analysis revealed that for a given filling factor, the X-ray emissivity is maximized for $T_X < 10^7$ K. Using this value for every WR star, the filling factors required to match the observations dropped by a full order of magnitude, yet remained larger than those predicted with the O-star relation by about 1 dex. Although the results of this second experiment seem more in line

![Figure 6](http://mnras.oxfordjournals.org/)

**Figure 6.** A comparison of instrumental sensitivities in terms of effective area against energy for XMM-EPIC (top; XMM Dahlem & Schartel 1999), Chandra HEG and MEG (middle; credit CXC/SAO), and ROSAT PSPC (bottom; Zimmermann et al. 1998; ESAS User’s Guide http://wave.xray.mpe.de/exsas/users-guide). The latest instruments have much superior collecting area, sensitivity to high X-ray energies, and spectral resolution (not shown).
with our expectations (i., somewhat more similar to the O-star results), we qualify our interpretation by noting that spectral data are truly needed to better constrain the X-ray temperatures and filling factors.

Finally, we considered linear regressions for the filling factor \( f_X \) versus the ratio \( M/v_{\infty} \). We find that the data appear to be broadly consistent with the assumption of \( f_X \sim (M/v_{\infty})^{-1} \), which is also found empirically for O stars. This seems to hold for WN stars alone, or for WN and WC stars combined. It does not hold for the WC stars alone, but they constitute a much smaller sample, so that the combination of relatively few points with large errors leads to a largely indeterminate fit. It is probably fair to say that \( f_X \) seems to decrease with \( M/v_{\infty} \) and is not inconsistent with a power-law index of \(-1\).

Is this result simply an artefact of our model? We assume the X-ray luminosity is of the form \( L_X = L_0 f_X \), with \( L_0 \propto M/v_{\infty} \) for optically thick winds, and we derive filling factors from data via \( f_X^{\text{obs}} = L_0^{\text{obs}} / L_0 \). Only if \( L_0^{\text{obs}} \) is essentially insensitive to \( M/v_{\infty} \) will \( f_X^{\text{obs}} \) vary inversely with \( M/v_{\infty} \). Clearly, if the observed X-ray luminosity had varied, say, linearly with \( M/v_{\infty} \), then we should have found a flat distribution for \( f_X^{\text{obs}} \), and we could have rejected the hypothesis of Paper I that \( f_X \propto (M/v_{\infty})^{-1} \). We concede that \( f_X \) does not hold for the WC stars alone, but they constitute a much smaller sample, so that the combination of relatively few points with large errors leads to a largely indeterminate fit. It is probably fair to say that \( f_X \) seems to decrease with \( M/v_{\infty} \) and is not inconsistent with a power-law index of \(-1\).

ACKNOWLEDGMENTS

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REFERENCES

XMM–PS–GM–14

APPENDIX A: COOLING FUNCTION FOR WOLF–RAYET ABUNDANCES

For the case when lines dominate the X-ray emission spectrum, the Raymond–Smith cooling function can be approximated in the compact form

\[ \Delta \kappa_T(E) \approx \sum_k \left( \frac{n_k}{n_\text{H}} \right) \kappa_k(T_k, E) \]

where \( n_k \) is the number density for the appropriate species, ion,
and level corresponding to the factor $P_0$ representing the various
emission processes contributing to the cooling function at energy
$E$. Correspondingly, the emissivity is then

$$j_\nu(E) = \frac{1}{4\pi n_e n_H} \Lambda_{\text{RS}}.$$  \hfill (A2)

However, this parametrization of the cooling function is difficult to
use for Wolf–Rayet winds where the hydrogen number density
approaches zero. Moreover, the Wolf–Rayet abundances are far
from solar. We therefore derive here a simplistic modification to the
classical Raymond–Smith cooling function for Wolf–Rayet winds.

We begin by defining our emissivity as

$$j_\nu(E) = \frac{1}{4\pi n_e n_\nu} \Lambda_\nu,$$  \hfill (A3)

with

$$\Lambda_\nu = \sum_k \left( \frac{n_k}{n_\nu} \right) P_k(E).$$  \hfill (A4)

Thus the problem reduces to relating $\Lambda_\nu$ to $\Lambda_{\text{RS}}$. This is done as follows:

$$\Lambda_\nu = \sum_k \left( \frac{n_k}{n_\nu} \right) \left( \frac{n_k}{n_H} \right) \circ \left( \frac{n_k}{n_H} \right) \circ P_k(E)$$ \hfill (A5)

$$= \left( \frac{n_H}{n_\nu} \right) \sum_k \left( \frac{n_k}{n_H} \circ \frac{n_k}{n_H} \right) \circ P_k(E).$$ \hfill (A6)

The ion number density and ionized hydrogen number density can
be extracted from the summation. Since they are both proportional
to mass density, their ratio becomes $n_{\text{H}\circ}/n_\nu = \mu_{\text{H}}/\mu_{\text{H}\circ}$. Further,
we define the parameter $\tilde{A} = (n_k)/(n_{\text{H}\circ})$. If this parameter is constant for every $k$, it too can be removed from the summation.
(Alternatively, $\tilde{A}$ could represent an appropriate ensemble mean
when the cooling function is sampled over a broad energy bandpass,
as is the case for ROSAT.) Making these substitutions, the expression becomes

$$\Lambda_\nu = \frac{\mu_{\text{H}}}{\mu_{\text{H}\circ}} \tilde{A} \Lambda_{\text{RS}}.$$  \hfill (A7)

**APPENDIX B: LINEAR REGRESSION**

**ANALYSIS OF THE ROSAT DATA AND MODEL RESULTS**

Here we briefly review the method of weighted linear regression
used in our analysis. The method is fairly standard. We adopt the
notation of Woan (2000).

For data consisting of $N$ points $\{x_i\}$ and $\{y_i\}$, we define a set of
weights $\{w_i\}$ with

$$w_i = \frac{1}{\sigma_i^2 + \sigma_0^2}.$$ \hfill (B1)

The standard deviations $\{\sigma_i\}$ are measurement errors for the
values $\{y_i\}$, whereas $\sigma_0$ is some other intrinsic spread to the data,
either known or unknown and possibly zero.

The data are assumed to be linear as $y = mx + b$. We make the
following convenient definitions:

$$d_i = y_i - mx_i - b,$$ \hfill (B2)

$$W = \sum w_i,$$ \hfill (B3)

$$\langle x, y \rangle = \frac{1}{W} \left( \sum w_i x_i \sum w_i y_i \right),$$ \hfill (B4)

$$D = \sum w_i (x_i - \langle x \rangle)^2.$$ \hfill (B5)

With these definitions, the slope and intercept of the best-fitting line to the data are

$$m = \frac{1}{D} \sum w_i (x_i - \langle x \rangle) y_i,$$ \hfill (B6)

$$\text{var}[m] = \frac{1}{D N - 2} \sum w_i d_i^2,$$ \hfill (B7)

$$b = \langle y \rangle - m \langle x \rangle,$$ \hfill (B8)

$$\text{var}[b] = \left( \frac{1}{W} - \frac{\langle x \rangle}{D} \right) \sum w_i d_i^2 / N - 2.$$ \hfill (B9)

The goodness of fit is determined by the reduced chi-square $\chi_r^2$, where $v$ is the number of degrees of freedom ($N - 2$ in this case). The goodness of fit is given by

$$\chi_r^2 = \frac{\sum w_i d_i^2}{N - 2}.$$ \hfill (B10)

For $\sigma_0 = 0$, the dispersion in the data is assumed to arise solely
from measurement errors. For $\sigma_0 > \sigma_i$ for all $i$, the dispersion
of the data is essentially unrelated to measurement errors. Note that
in this case, (a) $w_i$ is approximately constant for all $i$, so that each
point is treated as having equal weight in the regression, and (b)
the value of $\chi_r^2$ is driven toward zero, since the weights are
essentially all quite small (i.e., increasing values of $\sigma_0$ naturally
lead to an even better fit to the data). If $\sigma_0$ is a priori unknown, its
most likely value is found by requiring $\chi_r^2 = 1$.

This paper has been typeset from a \TeX/I\TeX file prepared by the author.