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Polarization from microlensing of spherical circumstellar envelopes by a point lens

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ABSTRACT
We discuss the flux and polarization signals obtained from the microlensing of stars with extended circumstellar envelopes by a single point-mass lens. A previous paper considered the case of main-sequence stars, and showed that microlensing of the stellar photosphere could produce a small net polarization (up to 0.1 per cent). In this paper, we show that stars with extensive envelopes will show a much higher level of variable polarization (up to 10 per cent), even if they are spherically symmetric. Since the stellar envelopes most likely to be lensed are produced by red giant winds, we also investigate the effects of a central cavity (representing the dust condensation radius) within the scattering envelope, as well as different radial density distributions. Our study has led to three major results. (i) For optically thin envelopes with an inner cavity, the lensing light curve for the polarization can be of longer duration than that for the total flux, the latter being dominated by the much brighter, unpolarized photospheric emission. If observed, the differing time-scales would provide direct evidence of a thin circumstellar envelope with a cavity. (ii) The combination of photometric and polarimetric light curves determines the lens impact parameter even if the event is not a photospheric transit. (iii) The variation of the polarization position angle determines both the magnitude and direction of the relative proper motion. Finally, we derive an asymptotic limit of the lensing polarization for large Einstein radii. This limit greatly simplifies the determination of the lensing parameters.

Key words: gravitational lensing – polarization – stars: atmospheres – circumstellar matter – stars: late-type.

1 INTRODUCTION
In the first microlensing surveys, the question of finite source size presented more of an inconvenience than a help in the search for galactic dark matter. If the impact parameter is medium to high, the effect of the finite size of the source is to reduce the amplification, and round off the light curve, which could lead to events being missed. Finite sources can also cause a deviation in the light curve from the ideal achromatic form for a point mass lensing a point source (Nemiroff & Wickramasinghe 1994; Witt & Mao 1994).

However, it was pointed out by a number of authors (cf. Gould 1994; Loeb & Sasselov 1995; Simmons, Newsam & Willis 1995b; Gould & Welch 1996) that the measurement of finite star effects also provides important additional information to constrain the lens properties. Thus if the angular radius of the star is known, the detection of finite effects provides a scale which can be used to determined the angular Einstein radius of the lens, placing constraints on the mass of the lens.

Indeed the implementation of the early warning systems has allowed for accurate spectrophotometric measurements during the lensing event itself. Thus microlensing provides a means to “gravitationally resolve” lensed sources at spatial scales that would generally be impossible with conventional observational techniques. A decade on from the inception of the first surveys (Paczynski 1986), a significant number of high amplification events have been identified as the lensing of an extended source either by a point mass or by a binary lens. For reviews see Gould (2001).

This situation was anticipated in the work of Valls-Gabaud (1996, 1998) and Loeb & Sasselov (1995) who calculated from stellar atmosphere models, how photospheric absorption line profiles and equivalent widths would change during lensing by a single point mass. Emphasis is given to red giant atmospheres by Heyrovsky, Sasselov & Loeb (2000b). Polarization induced during lensing has been discussed by Simmons, Willis & Newsam (1995a) and Simmons et al. (1995b) for point lenses, and for binary lenses by Agol (1996).

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Coleman (1998), Gray (2000), and Gray & Coleman (2001) have posed the time variable amplification of a limb darkened disc by a point-mass lens as an inverse problem, and using the Backus–Gilbert method, determined the levels of accuracy required for the retrieval of limb darkening from non-parametrical considerations. Furthermore, a number of works have demonstrated the potential of microlensing studies for measuring stellar sizes (Nemiroff & Wickramasinghe 1994), stellar rotation (Gould 1997), starspot activity (Heyrovsky & Sasselow 2000a; Hendry, Bryce & Valls-Gabaud 2002), and even stellar winds (Ignace & Hendry 1999).

On the observational side, there are now several reports, with impressive analyses, of finite source effects in lensing light curves. For example, Alcock et al. (1997b) evaluated limb darkening coefficients via the point-mass lensing event MACHO 95-BLG-30, which was the first detailed analysis of data exhibiting finite source effects. There have also been several dramatic binary lensing events, as described in Albrow et al. (1999, 2000, 2001a,b), Afonso et al. (2000, 2001), An et al. (2002), Castro et al. (2001). Studies of these events have provided important information about both the lens and source, through photometric and/or spectral monitoring.

Basically, finite source effects arise from the differential amplification of the source. They will be appreciable when the amplification function strongly varies over distance scales comparable with the source diameter, or the distance scale over which the particular feature (line width, limb darkening, polarization, etc) varies. Essentially this will arise when the impact parameter is small compared with the Einstein radius for a point-mass lens, or near caustic crossings for a binary or multiple lens. Such effects as variable line strengths and polarization are not only observable in microlensing in the local Group of galaxies, but could also be observable for objects at cosmological distances. Thus the X-ray emitting region of quasars (Wyithe et al. 2000) and active galaxies would also be likely candidates for microlensing by planet sized lenses (Wambsganss 2001), as well as gamma-ray bursts (Gaudi, Granot & Loeb 2001).

Since stars should exhibit limb polarization to some extent, microlensing of even main-sequence stars can in principle induce variable polarization, even when the source star is spherically symmetric. According to the Chandrasekhar (1960) electron scattering atmosphere, a spherical star should show a limb polarization of around 12 per cent. Corresponding calculations in Simmons et al. (1995b) demonstrated that a significant fraction of microlensed main-sequence stars would show a net variable polarization of up to 0.1 per cent. Such values would be measurable by a suitably large telescope during a typical alert event. Agol (1996) later discussed this in relation to binary lenses and found that although such events would be rarer, they could produce higher levels of polarization. Measurement of the variable broad-band polarization during a lens event has yet to be carried out; however, several events such as OGLE-1999-BUL-23 (Albrow et al. 2001a) should have provided a detectable polarimetric signal, as demonstrated by Agol, who simulated the polarization variation for OGLE 3, 6, 7 and other events.

It was pointed out in Simmons et al. (1995b) that the most likely candidates for polarization variability during microlensing would be hot young stars, on the one hand because they would have electron scattering atmospheres, and on the other because they would be sufficiently bright for such levels of polarization to be measured. Unfortunately, the more ubiquitous cooler stars with absorptive atmospheres should not display levels of polarization as high as that predicted by the Chandrasekhar model (Collins & Berger 1974). But paradoxically, many spectral types of stars in fact do display an intrinsic polarization of up to several per cent. One explanation of this is that these stars have very extended scattering atmospheres and winds, the latter certainly being the case for many giant stars (White, Shawl & Coyne 1984). For this reason, many classes of stars such as red giants with extended cool atmospheres could display high line-of-sight polarizations. Such evolved cool stars also constitute a significant fraction of the lensed sources (Alcock et al. 1997a; Popowski et al. 2002), making them prime candidates for observing variable polarization during microlensing events.

For an unlensed star to yield a net integrated polarization, the envelope cannot be spherically symmetric. Nevertheless even in spherically symmetric envelopes, the polarization along any given line of sight through the envelope (except through the centre of the disc) can be high (Cassinelli & Hummer 1971). Consequently, a polarized flux will be induced during microlensing by the preferential amplification of one part of the scattering envelope. In this paper, we show for stars with circumstellar envelopes that much higher polarimetric variability (up to 10 per cent) can be induced than was previously calculated in Simmons et al. (1995a,b) for stellar photospheres. One major advantage of considering the microlensing of circumstellar envelopes is that the probability of an event being a transit event increases, leading to a greater likelihood for observing finite source effects. This probability is given roughly by the ratio of the angular source size to the angular Einstein radius. For a bulge red giant of 15 R⊙ and a typical Einstein radius Re = 3 au (for a lens of Ml = 0.3 M⊙ located at Dh = 4 kpc and a Galactic bulge source at Ds = 8 kpc), the probability of an event being a transit is around (Rc/Dh)/(Rb/Dh) ~ 1 per cent. If this hypothetical star had a dust shell extending to several stellar radii, then the event would have a good chance of being a transit.

Of course, the precise behaviour of the polarization and flux variability depends on the relative extent of the scattering region, its distance from the central light source, the impact parameter of the lens, and the Einstein radius. On the other hand, the variation of the polarization position angle depends only on the orientation of the line of centres of the star and lens. The position angle has a form that depends only on one free parameter, the duration of the event. The change in position angle thus provides an ideal template for detecting microlensing events and providing unambiguous information about the direction of the projected velocity of the lens relative to the source star.

In this paper we adopt a simple model for the stellar geometry. We take the envelope to be spherically symmetric and optically thin. For this model to more realistically mimic a circumstellar envelope, we allow for the possibility that there is a region, contiguous with the photosphere, where dust grains and molecules cannot form. This central ‘cavity’ thus contains no scattering particles. In our model, we assume this cavity to be a spherical shell around the star. Finally, we adopt a power law for the density distribution, and for simplicity, we use dipole scattering, which for our purposes provides a sufficient approximation for the scattering by dust grains and molecules.

The structure of the paper is as follows. In Section 2, we construct formal expressions for the variable flux induced by microlensing of extended sources. We then calculate in Section 3 the level of polarized intensity along an arbitrary line-of-sight for stars with extended envelopes. These calculations are carried out semi-analytically for the optically thin case. Having established the polarimetric profile for the source star, we evaluate in Section 4 the variable polarization and flux that arise during the microlensing event. In Section 5, the practicalities of observing such microlensing events are discussed, and consideration is given to how such data might be combined with other types of measurement to yield useful information about the lensing object. The applicability of these
methods to other types of astrophysical objects, such as AGNs, is briefly considered.

2 MICROLENSES OF EXTENDED SOURCES

In this section, formal expressions are presented for the time-variable flux and polarization in terms of convolution integrals of the intensity field with the amplification function for a point-mass lens. The brightness and the degree of linear and circular polarization of the source can best be described by the Stokes parameters, denoted by \( I, Q, U, V \), where \( I \) is the total intensity, \( Q = I_x - I_y \) is the difference in intensity measured in two perpendicular directions, \( x \) and \( y \) (see Fig. 1), \( U \) is the difference in intensity measured in two perpendicular directions at 45° to the \( x \)- and \( y \)-axes, and \( V \) is the net circular polarization. For convenience, we shall sometimes use the Stokes vector notation \( \mathbf{I} = (I, Q, U, V)^T \) for the intensities, and similarly for the flux, \( \mathbf{F} = (F_I, F_Q, F_U, F_V)^T \).

The effect of a single point-like lens is to amplify the intensity from any part of the source by an amount given by an amplification function, \( A \), which is a function of projected distance from the lens position. For a Schwarzschild lens, it is natural to introduce a scale factor, \( \kappa \), from any part of the source by an amount given by an amplification function, \( f_i \), for the intensities, and similarly for the flux, \( F_i = (F_I, F_Q, F_U, F_V)^T \).

For computation of the underlying source model and microlensing by a point mass, several coordinate systems must be introduced. For the stellar envelope, the natural coordinate system is spherical polar coordinates \((r, \theta, \phi)\) centred on the star. To integrate the transfer equation and obtain fluxes, we use cylindrical coordinates \((p, \alpha, z)\) where the \( z \)-axis is oriented toward the observer (see Fig. 1). The cylindrical radius \( p \) and azimuth \( \alpha \) are the polar coordinates of the source plane. So, the projected position of the lens on the source plane is \((p_L, \alpha_L)\). The \( x \)- and \( y \)-axes are oriented so that the (relative) motion of the lens is in the positive \( x \)-direction, consequently the position of closest approach will be \((x_L, y_L, 0) = (0, p_0)\), or \((p_L, \alpha_L, 0) = (p_0, \pi/2)\).

![Figure 1. Lens geometry. A lensing object travelling with velocity \( v_\perp = \mu v_\parallel D_\parallel \) in the \( x \)-direction with impact parameter \( p_0 \) is shown at projected distance \( p_L \) from the centre of a spherically symmetric source of radius \( R_\perp \). The length \( d \) is the projected distance between the lens and an emitting source point located at polar coordinates \((p, \alpha)\).](image)

For the lens, we have already defined \( \theta_L \) as the angular Einstein radius. The angular source radius is \( \theta_\perp = F_\perp/(\pi L_\perp) \), which can generally taken to be an observable (van Belle 1999). The angular distance between the lens and any other direction is taken as \( \theta \). The Einstein radius at the source plane is \( \theta_E = D_\parallel \theta_L \) (Gould 2000).

With these definitions, the amplification at an arbitrary point \((p, \alpha)\) in the source plane is given by

\[
A(p, \alpha; p_L, \alpha_L) = \left( \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \right),
\]

where \( u = \frac{\theta}{\theta_E} = \frac{d}{D_\parallel} \).

and \( d = D_\parallel \theta \) is the linear projected distance to an emitting point in the source plane as given by

\[
d = \sqrt{p^2 + p_L^2 - 2pp_L\cos(\alpha - \alpha_L)},
\]

(see Fig. 1). We specify \( \theta_L = p_0/D_\parallel \) and \( u_L = \theta_L/\theta_E \) as respective separations between the lens and the source centre.

Having specified the geometry and associated amplification function, we next consider the effect of lensing on the observed flux and polarization. As a function of lens position, the observed Stokes flux will be given by

\[
F = \frac{1}{D_\parallel^2} \int_{source} A(p, \alpha; p_L, \alpha_L) I(p, \alpha) p \, dp \, d\alpha.
\]

We define the observed fractional polarization \( P \) and polarization position angle \( \psi \) to be

\[
P = \frac{\sqrt{F_Q^2 + F_U^2 + F_V^2}}{F_I},
\]

and

\[
\psi = \frac{1}{2} \tan^{-1} \frac{F_U}{F_Q}.
\]

The light curve, that is the flux as a function of time, \( t \), is obtained by inserting the time-dependent lens position into (5). Defining the lens impact parameter, \( p_0 \), and relative proper motion \( \mu_{\text{rel}} \), the Cartesian position of the lens is

\[
x_L = D_\parallel \mu_{\text{rel}} (t - t_0),
\]

and

\[
y_L = p_0.
\]

where \( t_0 \) is the time of closest approach of the lens to the source. Equivalently, the lens position in polar coordinates is

\[
p_L = \sqrt{p_0^2 + D_\parallel^2 \mu_{\text{rel}}^2 (t - t_0)^2},
\]

and

\[
\alpha_L = \tan^{-1} \left[ \frac{\theta_0}{\mu_{\text{rel}} (t - t_0)} \right],
\]

where the angular impact parameter of the lens from the source is \( \theta_0 = p_0/D_\parallel \).

3 STOKES PARAMETERS FOR EXTENDED SCATTERING ENVELOPES

As a means of exploring the various effects of lensing an extended stellar envelope, we shall assume a simple model for the stellar photosphere and associated circumstellar envelope as illustrated in Fig. 2. Both the star and the extended envelope are taken to be
spherically symmetric. It is further assumed that the photospheric emission is unpolarized with uniform brightness, hence no ‘spots’ or limb darkening are included at this point. The circumstellar envelope is treated as (i) being optically thin with a line-of-sight scattering optical depth \( \tau_{sc} \); (ii) possessing a power law radial density distribution of scatterers, and (iii) exhibiting dipole (Rayleigh) scattering (like that of free electrons). Although observationally, it is cold dusty circumstellar envelopes that will likely be the best candidates, we instead employ the Rayleigh scattering phase matrix (as opposed to Mie scattering, for example) because it is amenable to analytic integration. Our goal is to demonstrate the feasibility of observing a polarization signal during the microlensing of stars with circumstellar envelopes. In the future, we will conduct numerical simulations to study more general scattering properties appropriate for dust.

Under the above assumptions, the spherical symmetry ensures that the total polarization of the scattered light is not a function of \( \varphi_{\ell} \); it is only a function of the distance between the lens and source, \( p_{\ell} \). Since the scattering is optically thin, the expressions for the Stokes intensities arising from dipole scattering are reasonably straightforward. The major complication arises from the so-called ‘finite disc correction factor’, which accounts for the influence of the finite stellar size on the polarization of the scattered light (Cassinelli, Nordieck & Marison 1987; Brown, Carlaw & Cassinelli 1989). The Stokes intensities are obtained by integrating the polarization source functions (see equation 20 of Bjorkman & Bjorkman 1994) over the line-of-sight coordinate, \( z \), giving

\[
I(p, \alpha) = I_0 + \frac{3\sigma}{8} \int_{\tau_{sc}}^{\infty} n \left[ (3J - K) + (3K - J)(z/r)^2 \right] \left[ (3J - K)(p/r)^2 \cos(2\alpha) ight] \left( (3K - J)(p/r)^2 \sin(2\alpha) \right] dz, \tag{12}
\]

where

\[
I_0(p) = \begin{cases} (I_\ast, 0, 0, 0)^T & (p < R_s), \\ 0 & (p \geq R_s), \end{cases}
\]

and

\[
\tau_{sc}(p) = \begin{cases} \sqrt{R_s^2 - p^2} & (p < R_s), \\ -\infty & (p \geq R_s), \end{cases}
\]

with \( I_\ast \) being the intensity of the star, \( \sigma \) the scattering cross-section, \( n \) the number density of scatterers, \( J \) the mean intensity, and \( K \) the second moment of the intensity. Note also that the circular polarization is assumed to be zero. The intensity moments are

\[
J(r) = \frac{1}{2} \left[ 1 - (1 - \frac{R_s^2}{r^2})^{1/2} \right] I_\ast, \quad \text{and}
\]

\[
K(r) = \frac{1}{6} \left[ 1 - (1 - \frac{R_s^2}{r^2})^{3/2} \right] I_\ast. \tag{15}
\]

We describe the density using the power law

\[
n = \begin{cases} 0 & (r < R_h), \\ n_0(R_h/r)^\beta & (r \geq R_h), \end{cases}
\]

where \( r^2 = p^2 + z^2 \), and \( R_h \) is the radius of the central cavity (see Fig. 2).

Fig. 2. A generalized model for a spherically symmetric source and associated circumstellar envelope. The central source has radius \( R_s \) and is shown as hatched. The extended envelope is shown with inner boundary at radius \( R_h \) (hence a cavity exists between the envelope and the star) and outer boundary at radius \( R_{env} \). We take \( R_{env} \gg R_h \) and assume a power law density distribution \( n(r) = n_0(R_h/r)^\beta \) for the circumstellar envelope.

As mentioned earlier, the presence of a cavity allows us to generalize the calculation. Its relevance to actual astrophysical settings is envisioned to be for cool evolved stars. In some of these objects, dust grains are the primary source of scatterers, but the dust forms only beyond the condensation radius (occurring at a temperature \( \approx 1500 \) K). The condensation radius will not be coincident with the photospheric radius, since the photosphere is too hot (>2500 K), hence in such stars a cavity absent of scatterers is to be expected. This is an interesting case since a substantial fraction of sources (about one-quarter) in lensing events directed toward the Galactic bulge are clump red giants (Alcock et al. 1997a; Popowski et al. 2002).

Using the preceding expressions and allowing for the central cavity, the Stokes intensity becomes

\[
I(p, \alpha) = I_0(p) + \frac{3}{16} I_\ast(\beta - 1) \left( \frac{R_h}{p} \right)^{\beta - 1} \left( \frac{R_s}{p} \right)^2 \tau_{sc} \times g_0(p) \begin{pmatrix} G_I \\ G_P \cos(2\alpha) \\ -G_P \sin(2\alpha) \\ 0 \end{pmatrix}, \tag{17}
\]

where the total optical depth \( \tau_{sc} = n_0\sigma R_h/(\beta - 1) \), and the stellar occultation factor, \( g_0 \), is

\[
g_0(p) = \begin{cases} 1/2 & (p < R_s), \\ 1 & (p \geq R_s). \end{cases} \tag{18}
\]

The integral factors \( G_I \) and \( G_P \) are given by

\[
G_I = \int_0^{r_{max}} \sqrt{\frac{p}{p_{\ell}} - 1} \left[ \frac{1 - \frac{1}{4} s q}{1 - s} \right] \left( \frac{1}{s q} \right)^{1/2} \left( 1 - \frac{1}{4 s q} \right) - 1 \right] ds, \tag{19}
\]
and

\[ G_p = \int_0^{\tau_{\text{max}}} \frac{\sqrt{1 - sq}}{1 - s} ds, \]  

(20)

with \( s = (p/r)^2 \), \( q = (R_s/p)^2 \), \( s_{\text{max}} = (p/r_{\text{min}})^2 \), and

\[ r_{\text{min}} = \begin{cases} \frac{R_s}{p} & (p < R_s), \\ p & (p \geq R_s). \end{cases} \]

(21)

Having composed the equations that describe the emerging Stokes vector intensity for any line-of-sight through the circumstellar envelope, it remains only to incorporate this ‘intensity map’ into the convolution for the lensed flux, which is the task of the following section.

4 EXPRESSIONS FOR THE AMPLIFIED STOKES FLUXES

An equation describing how lensing modifies the net source flux is given in equation (5). Owing to the spherical symmetry, the amplified flux and polarization is independent of the lens position angle, \( \alpha_L \), so we may conveniently rotate the source plane integration variables from \((p, \alpha)\) to \((p, \alpha')\), where \( \alpha' = \alpha - \alpha_L \). Note that the amplification function is symmetric under reflection about the line connecting the source and lens; that is, \( A(\alpha') = A(-\alpha') \).

Using the expressions from the previous section, the total observed flux during a lensing event becomes

\[ F = \frac{F_0}{F_s} + \frac{3(\beta - 1)}{16\pi} R_s^\beta - \tau_L \int_0^{2\pi} \int_0^\infty d\alpha' A(p, \alpha') \]

\[ \times \left( \frac{G_1(p)}{p^{\beta+1}} \left( G_R(p) \cos(2\alpha') \cos(2\alpha_L) - G_L(p) \cos(2\alpha') \sin(2\alpha_L) \right) \right), \]  

(22)

where

\[ F_0 = \frac{1}{\pi R_s^2} \int_0^{2\pi} \int_0^R dp d\alpha' A(p, \alpha') I_0 \]  

(23)

is the lensed stellar flux, with \( F_s = \pi I_0 \theta_s^2 \). We define the following integral moments:

\[ H_s(p_L) = \frac{1}{\pi R_s^2} \int_0^{2\pi} \int_0^R dp d\alpha' A(p, \alpha') \]  

(24)

\[ H_L(p_L) = \frac{3(\beta - 1)}{16\pi} R_s^\beta - \tau_L \int_0^{2\pi} \int_0^\infty d\alpha' \]

\[ \times \int_0^\infty g_0(p) A(p, \alpha') G_1(p) dp. \]  

(25)

\[ H_R(p_L) = \frac{3(\beta - 1)}{16\pi} R_s^\beta - \tau_L \int_0^{2\pi} \int_0^\infty d\alpha' \cos(2\alpha') \]

\[ \times \int_0^\infty g_0(p) A(p, \alpha') G_R(p) dp. \]  

(26)

With these relations, the emergent Stokes flux from a lensing event can be rewritten in the compact vector form

\[ \frac{F}{F_s} = \left( \begin{array}{c} H_s(p_L) + \tau_L H_L(p_L) \\ \tau_L H_R(p_L) \cos(2\alpha_L) \\ -\tau_L H_R(p_L) \sin(2\alpha_L) \\ 0 \end{array} \right). \]  

(27)

Equation (27) shows that the flux amplification (magnification) by lensing is

\[ M(p_L) = H_s(p_L) + \tau_L H_L(p_L) \approx H_s(p_L), \]  

(28)

and the lensed polarization is

\[ P(p_L) = \frac{\tau_L H_R(p_L)}{H_s(p_L) + \tau_L H_L(p_L)} \approx \frac{\tau_L H_R(p_L)}{H_s(p_L)}. \]  

(29)

The approximations in the above expressions are valid for \( \tau_L H_L \ll H_s \). In this limit, the magnification \( M \) depends only on lensing of the stellar photosphere, and the polarization scales linearly with envelope optical depth, \( P \propto \tau_L \).

In addition to the degree of polarization, the polarization position angle is also measured. This angle is given by

\[ \psi = \pi - \alpha_L = \pi - \tan^{-1} \left( \frac{\theta_0}{\mu_{\text{rel}}(t - t_0)} \right). \]  

(30)

Note that the net polarization is orthogonal to the projected line joining the star’s centre and the point lens. Except for the pathological case \( \theta_0 = 0 \), the position angle will show a 180° rotation during the lensing event, beginning at \( \psi = 0° \) when \( t = -\infty \), increasing to \( \psi = 90° \) at closest approach \( (t = t_0) \), and ending at \( \psi = 180° \) when \( t = +\infty \). We conclude that observing a position angle rotation of 180° in the form given above is an unambiguous signal of microlensing.

A key point is that at closest approach (i.e. flux maximum), the direction of the polarization is parallel to the direction of travel of the lens. This implies that we can determine not only the magnitude of the relative proper motion but its vector direction as well. The time rate of change of the polarization position angle is

\[ \frac{d\psi}{dt} = \frac{\mu_{\text{rel}}/\theta_0}{1 + (\mu_{\text{rel}}/\theta_0)^2 (t - t_0)^2}, \]  

(31)

and the time-scale for the position angle change is

\[ t_p = \left( \frac{\mu_{\text{rel}}}{\theta_0} \right)^{-1} = \frac{\theta_0}{\mu_{\text{rel}}}. \]  

(32)

The above expression implies that the changes in the polarization position angle occur more rapidly for smaller values of the impact parameter \( \theta_0 \).

The information that can be gleaned from lensing polarization include \( t_p \) as just noted, \( \theta_0 \) from the peak value of \( P/\tau_L \) (see Section 5.1), and the vector direction of \( \mu_{\text{rel}} \) in the sky from the polarization position angle. Combining these elements, one obtains \( \mu_{\text{rel}} \) from the ratio \( \theta_0/t_p \), and \( \theta_0 \) from \( \theta_0 \) and the peak flux amplification, \( M_{\text{rel}} \).

Some of this information can be obtained from the lensing flux amplification light curve for photospheric transit events. However, Hendry et al. (1998) and Newcomb et al. (1998) indicate that polarization may better constrain \( \theta_0 \) and \( \theta_0 \). Moreover, polarization provides the additional constraint of the vector direction of the relative lens-source proper motion. The orientation of the relative motion provides an additional constraint when used in conjunction with the value of \( \mu_{\text{rel}} \) for assessing the likelihood of the lens object being at different locations and a member of different known populations. Finally, we note that the polarimetric determination of \( \theta_0 \) does not require a photospheric transit.

5 CALCULATED MICROLENSING PROFILES

Intensity and polarization variations with lens distance have been computed for several different cases. Fig. 3 shows the run of intensity
Figure 3. A plot of the scattered intensity $I_{sc}$ (left) and polarization $Q_{sc}$ (right) versus $p$ for the unlensed envelope with $\beta = 2$. Each curve corresponds to an envelope with an interior cavity of different radius, as indicated.

$I_{sc}/I_*$ and $Q_{sc}/I_*$ versus $p$ without lensing. The curves are for $\beta = 2$ and different cavity sizes $R_0$ as indicated. For the intensity plots, the amount of scattered light jumps discontinuously at $p = R_*$ (the limb of the star). This occurs because the column depth of scatterers just outside the limb is twice that just inside the limb, hence $I_{sc}/I_*$ increases by a factor of 2, and so does the polarization $Q_{sc}/I_*$. For cases with envelope cavities, the scattered light is significant, but the polarization is small for $p < R_0$, because most of the scattering is forward/backward scattering which produces only small polarizations. Not until $p \approx R_0$ and greater does scattering through right angles occur along the line-of-sight.

Using the models shown in Fig. 3 (including variations in $\beta$ not shown in that figure) as input intensity maps, we have computed the total flux and polarization of these envelopes when lensed by a point-mass object using the formalism described in the previous section. A discussion of the results is presented below, which is split into three parts: microlensing effects for a filled envelope ($R_0 = R_*$), the effects of a central cavity (varying $R_0$), and the influence of different radial distributions of scatterers (different $\beta$), with and without the central cavity.

5.1 ‘Filled’ envelopes

For this case there is no cavity, hence $R_0 = R_*$. An envelope density with $\beta = 2$ only (i.e. $n \propto r^{-2}$) is considered. The variation in the flux amplification $M$, equation (28), and polarization $p$, equation (29), are shown in Fig. 4 as a function of $\theta_0/\theta_*$. As indicated, the four curves are for different Einstein radii, $\theta_0/\theta_* = 1, 2, 4$, and 8. The curve is conveniently reflected to ‘negative’ $\theta_0$ values to emulate light curves in the case of a lens trajectory that crosses the stellar centre ($\theta_0 = 0$). The double-peaked polarized light curve, characteristic of the photospheric lensing investigated by Simmons et al. (1995a,b), is preserved in the case of extended envelopes. In the photospheric case, the value of the limb polarization was taken to be 3.5 per cent. With this value (which is likely an overestimate at optical wavelengths), Simmons et al. found that the peak polarization attained during a photospheric lensing event was only a small fraction of a per cent ($\sim 0.1$ per cent). In contrast, with an extended envelope, the achievable peak polarization values can be higher by one to two orders of magnitude.

For non-central transits ($\theta_0 \neq 0$), a particular light curve can be obtained by considering how the flux amplification and polarization varies with time and fixed $\theta_0$ as the lens position $\theta_L$ initially decreases to a minimum value, $\theta_L = \theta_0$, and then increases back to larger $\theta_0$. Owing to the symmetry of the envelope, the light curves will be time-symmetric with respect to the point of closest approach, which is the time of flux maximum since the flux amplification, $M$, increases monotonically as $\theta_0$ decreases.

One immediately obvious feature of these simulations is that the peak flux increases with larger values of $\theta_0$. Another is that
the width of the flux maximum is relatively narrow (unaffected by the envelope), while the polarization light curves are much broader. This is a clear signature of the presence of an extended circumstellar envelope. Because of the low optical depth of the envelope, and the rapid decrease of scattered light with distance $p$ from the star, most of the unpolarized flux is produced by the stellar photosphere, while the polarized light is dominated by the much fainter (and spatially extended) scattered light component.

If we examine in more detail the plots of the degree of polarization against the angular distance of the lens $\theta$, we find that the polarization attains a maximum value near the photospheric limb ($\theta = \theta_\text{E}$), while the total flux increases monotonically as $\theta \to 0$. The peak polarization must occur near the star because the scattered light is dominated by the innermost and densest regions of the envelope. However, a high degree of polarization also requires right angle scattering, hence the peak occurs near the stellar limb. The fact that it occurs slightly outside the limb is in part a consequence of the finite disc correction factor of Cassinelli et al. (1987). Scattering at the photospheric radius yields no polarized light owing to the isotropy of the incident radiation as seen by the scatterer, so the polarized flux increases rapidly reaching a maximum just outside the limb of the photosphere. Enhancing this effect is the amplification of the unpolarized flux from the star. Since the polarization, equation (29), is the ratio of polarized to unpolarized flux, the rapid amplification of the unpolarized stellar component inside the stellar radius ($\theta_\text{E} < \theta_\text{E}$) rapidly drives the polarization to zero as $\theta \to 0$. Note that while the flux light curve is always single peaked, the polarization light curve can be either double-peaked or single-peaked, depending on whether or not the distance of closest approach, $\theta_\text{E}$, is inside or outside the location of the polarization maximum.

Another interesting aspect of the polarization is that for fixed stellar radius and impact parameter, the degree of polarization increases as the Einstein radius increases, asymptotically approaching a constant for large values of $\theta_\text{E}$. Again noting that the degree of polarization, equation (29), is the ratio of polarized to unpolarized flux, we see that amplification by lensing occurs in both the numerator and denominator. When $\theta_\text{E} \gg \theta_\text{E}$, the amplification function becomes $A \approx \theta_\text{E}^{-1} = \frac{\theta_\text{E}}{d}$. Since $\theta_\text{E}$ is a constant in the flux integrands, occurring in both the numerator and denominator, the $\theta_\text{E}$ dependence cancels, and the polarization becomes a constant independent of $\theta_\text{E}$. An analogous effect was found by Heyrovsky et al. (2000b) for absorption line equivalent widths during lensing.

Based on the discussion in Section 1, the probability of a transit event is fairly small (a few per cent) because $\theta_\text{E} \gg \theta_\text{E}$. In the present context, this would imply that our asymptotic limit for the lensing polarization is the most likely case. In this case, the value of the impact parameter $\theta_0$ may be determined as follows. At flux maximum (i.e. time of closest approach), we have $\theta_0 = \theta_\text{E}$. From Fig. 4, the asymptotic polarization curve has two solutions for a given value of $(P/\tau_\text{sc})_\infty$. If the polarimetric light curve is double-peaked, $\theta_0/\theta_\text{E}$ is given by the smaller root, while if single-peaked, $\theta_0/\theta_\text{E}$ is given by the larger root. If in fact the asymptotic limit does not apply, then these two roots provide lower and upper limits to $\theta_0/\theta_\text{E}$. Furthermore, the observed value of $(P/\tau_\text{sc})_\infty$ sets a lower limit to $\theta_0/\theta_\text{E}$. Finally, we note that for the general case, detailed modelling of the polarization and flux amplification will provide a unique solution for $\theta_0$ and $\theta_\text{E}$ in terms of $\theta_\text{E}$.

5.2 Envelopes with a central cavity

In Fig. 5, the effect of a central cavity of radius $R_h = 5R_\odot$ is shown, with curves for different Einstein radii. Since the dominant light source is still the stellar photosphere, there is negligible change between Figs 4 and 5 for the flux amplification. In the case of the polarization, the curves are qualitatively similar. Note that by keeping $\tau_\text{sc}$ fixed for the different cases, the peak polarizations in Fig. 5 are much larger than if we had simply removed all the scatterers interior to $R_h$, which is why the polarization levels remain relatively high. However, the peak now occurs at $p_\text{E} \approx R_\odot$ instead of the stellar limb, because very little of the scattered light along rays for $p < R_\odot$ is substantially polarized. In addition, as the lens nears the photosphere, there is strong dilution of the polarization, so the polarization drops rapidly for values of $p_\text{E} < R_\odot$.

In Fig. 6, we show how the polarization curves vary with different cavity sizes ($R_h/R_\odot = 1, 3, 5,$ and 7) at a fixed value of the Einstein radius $\theta_0 = 8\theta_\text{E}$. The location of the peak polarization measures the hole radius $R_h$. We see now that the flux amplification is not only of shorter duration than the variable polarization, it also occurs well after the polarization peak. This delay between the polarization peak and the onset of the flux amplification is a straightforward indicator of the presence of a central cavity. The size of the cavity can be estimated from the ratio of the time of polarization maximum to the width of the flux amplification. This interesting result is discussed at more length in Section 6.

5.3 Variation of the density law

In varying the density law, we compare microlensing curves between the cases $\beta = 2$, $3$, and $4$. When changing the $\beta$ value, while
Figure 6. Total flux and polarization curves for lensing of a star and envelope with different sized cavities. Shown are the effects of varying the cavity radius $R_h$. In each case, the polarization peak occurs during ingress and egress of the lens as it crosses the cavity. Note that the flux amplification begins well after the polarization maximum, indicating the presence of the central cavity.

Figure 7. Polarization curves for envelopes without a central cavity ($R_h = 1 R_*$) having different radial power law exponents $\beta$ as indicated. Steepening the power law exponent and keeping $\tau_{sc}$ fixed redistributes envelope matter to smaller radii. This results in higher peak polarization values and narrower lensing curves. The width of the polarization light curves measures the radial extent of the envelope.

holding the envelope optical depth fixed, the density scale $n_0$ varies as $n_0 \propto (\beta - 1) R_h^{\beta - 1}$, so at $r = R_h$, the density $n(R_h) \propto (\beta - 1) R_h^{-1}$.

The point is that increasing $\beta$ for fixed $\tau_{sc}$ leads to a redistribution of envelope mass. For fixed $R_h$, the scatterers tend to accumulate near the inner edge of the scattering volume for increasing $\beta$. This leads to interesting behaviour in the polarized microlensing curves.

Figs 7 and 8 show polarization curves at the indicated values of $\beta$ with a fixed Einstein radius $\theta_E = 8 \theta_\ast$. Fig. 7 shows the case $R_h = 1 R_\ast$, and Fig. 8 shows the results for $R_h = 5 R_\ast$. As in the previous section, we find that the peak polarization occurs when $\rho_{\ast} = R_h$. One notable feature is how the peak value changes with $\beta$.

In Fig. 7 ($R_h = 1 R_\ast$), the increase in $\beta$ leads to a slight increase in the peak polarization, as one intuitively expects when the scatterers are redistributed to be closer to the illuminating source. In Fig. 8 ($R_h = 5 R_\ast$), the increase in peak polarization with $\beta$ is far more substantial as compared to that seen in Fig. 7. The reason is that the finite disc correction factor of Cassinelli et al. (1987) reduces the polarization for scatterers quite near the star, within a couple of tenths of $R_\ast$. Thus the depolarization factor limits the polarization increase seen in Fig. 7 because the absence of a cavity implies that the scatterers are redistributed to lie close to the stellar surface. In contrast, Fig. 8 shows the normal increase in polarization when the scatterers are redistributed to lie closer to the star, but still outside the range of the depolarization factor.

In addition to increasing the peak polarization, we find that the $\beta$ exponent alters the width of the polarization curves. We see that envelopes with larger $\beta$ have narrower temporal variations in the polarization. This is sensible because the scatterers are redistributed closer to the star, decreasing the radial extent of the envelope. Note that for optically thin envelopes, the width of the lensing curve is essentially independent of the envelope optical depth. This result is especially useful because it implies that the width of the polarization...
light curve can be used to measure the radial extent of the envelope. We shall briefly comment on optical depth effects in the following section.

6 SUMMARY AND DISCUSSION

This paper extends the analysis of Simmons et al. (1995a,b) for the polarization effects from microlensing of stellar photospheres by point-mass lenses to that of circumstellar envelopes. In these earlier papers, it was assumed that the Chandrasekhar (1960) electron scattering model could adequately describe the limb polarization of the stellar atmosphere, and it was shown that under such circumstances one could expect a typical polarization of around 0.1 per cent for stellar atmosphere, and it was shown that under such circumstances one could expect a typical polarization of around 0.1 per cent for circumstellar envelopes. In these earlier papers, it was assumed that the Chandrasekhar (1960) electron scattering model could adequately describe the limb polarization of the stellar atmosphere, and it was shown that under such circumstances one could expect a typical polarization of around 0.1 per cent for circumstellar envelopes. 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In the future it will be important to include a more detailed model for scattering by dust particles, with relevance to the circumstellar environments of evolved cool stars, and the case of optically thick envelopes. We can speculate that for spherically extended envelopes of large optical depth, multiple scattering will lead to depolarization of the light and may mimic the cavity effects discussed in Section 5.2. In other words, the emergent intensities from thick interior regions of the envelope will be largely unpolarized; however, there will always exist a kind of ‘halo’ region that can be treated as optically thin, and a net polarization will arise from lensing of light emerging from this ‘last scattering’ shell. Although there is no real cavity, large optical depths lead to an effective polarimetric ‘void’ in regions near the star, and so polarimetric light curves from lensing with cavities versus no cavity but high optical depth may appear somewhat similar. On the other hand, the thick region will also act like an extended photosphere, which should alter the wings and width of the peak of the total intensity light curve. These are issues that must be investigated quantitatively in future studies.

Also, about 10 per cent of recorded events appear to be microlensing by binaries (Udalski et al. 2000). Such microlensing should give rise to substantial polarization variation for the models we have discussed. The issue of binary versus single lenses will be addressed in a separate paper.

Although we have discussed only the stellar case here, many of the methods are equally applicable to the cosmological situation. Recently the possible microlensing of quasars and the continuum region of AGNs, with typical dimensions of 0.1 pc, by planet-mass objects has received considerable attention (see Wambsganss 2001 for references). Since these objects are at high redshift, the probability of microlensing must increase. Gravitationally lensed quasars and AGNs that are already observed to be macrolensed by foreground galaxies are also candidates for microlensing by stars within the foreground galaxy (Wozniak et al. 2000). Electron scattering in the accretion disc would provide a significant level of polarized light, whose signal during a microlensing event should take a characteristic form. We plan to address this topic in the future.

In conclusion, we stress that the polarimetric signal from microlensing of cool stars with extended circumstellar envelopes, such as red giants, should be measurable for high amplification events after alert. For microlensing by point-mass lenses, the position angle changes only with the variation of the line of centres of the source and lens, rotating through 180° over the course of the event. Such behaviour should be considered a primary (model independent) characteristic of the microlensing effect. Polarization also provides the direction of the proper motion, and as we have shown, analysis of these events will better constrain the lens mass, and velocity. Finally, when combined with the analysis of line profiles and spectral energy distributions, microlensing will also provide considerable insight into the atmospheres and envelopes of cool stars.

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