"WCFields": A Magnetic Rotating Stellar Wind Model from Wind Compression Theory.

R. Ignace, *East Tennessee State University*

J. P. Cassinelli

J. E. Bjorkman

Available at: https://works.bepress.com/richard_ignace/57/
“WCFIELDS”: A MAGNETIC ROTATING STELLAR WIND MODEL FROM WIND COMPRESSION THEORY

R. IGNACE, J. P. CASSINELLI, AND J. E. BJORKMAN

Received 1997 December 1; accepted 1998 May 7

ABSTRACT

A stellar wind model for a magnetic rotating star is presented. We use the semianalytic wind compression model that predicts the two-dimensional geometry of outflows from rotating stars and consider the addition of a magnetic field. In the limit of weak magnetic fields, in such a way that the fields are unimportant in accelerating the flow, the wind compression model can be used to predict the magnetic field distribution throughout the wind, which is shown to follow the mass flux distribution. A compression of field lines near the equator results as the flow of material from higher latitudes brings magnetic flux toward that region. As examples, wind compression models with magnetic fields (“WCFIELD” models) are computed for both a Wolf-Rayet star and a red supergiant star. In both cases an order of magnitude enhancement of the equatorial magnetic field can result within a few stellar radii for stellar rotation rates around 20% of critical. Such enhancements could have consequences for explaining (1) nonthermal emission observed from some Wolf-Rayet winds and (2) the ring structures observed in the ejecta of SN 1987a.

Subject headings: stars: magnetic fields — stars: mass loss — stars: rotation — stars: Wolf-Rayet — supernovae: individual (SN 1987A)

1. INTRODUCTION

Magnetic fields are important for stellar evolution and for understanding a wide variety of observational behavior of certain stars. The observed and theoretical connections between magnetic fields and stellar rotation plus the overall importance of magnetic fields in stellar astrophysics have led us to examine the consequences of wind compression (wc) effects for magnetic stellar winds.

Bjorkman & Cassinelli (1993; hereafter BC) introduced an analytic approximation for the calculation of the two-dimensional axisymmetric wind structure from a rotating star. Assuming that the wind-driving force is a central force, they found that fluid elements injected into the supersonic wind will orbit toward the equator, leading to an equatorially enhanced density. This equatorward flow in rotating winds is a consequence of angular momentum conservation. They found that for a wind that is strongly radially driven, there is little opportunity for the transport of wind material toward the equator. In contrast, for a wind that has a low terminal speed or that accelerates slowly to terminal speed, fluid elements will attempt to cross the equatorial plane, resulting in a shock-bounded equatorial disk with a large equator to pole density contrast of order 10² to 10³.

BC investigated the wind distortion arising from stellar rotation in the winds of O stars and B stars that are primarily line-driven. The O stars have strong winds with terminal speeds of about 2000 km s⁻¹. Except for stars that rotate close to the critical rate, the winds of single O stars are expected to be predominantly spherical. In contrast, the B stars have winds with considerably lower terminal speeds of about 900 km s⁻¹. BC found that disk formation can occur for stellar rotations of about 50% critical, which is relevant to the Be stars that have dense equatorial disks and relatively large v sin i values. In this context, the wind model of BC has been labeled the “wind compressed disk” (WCD) model. Soon after the WCD model was first proposed, the basic features of the model were confirmed with numerical hydrodynamic simulations by Owocki, Cranmer, & Blondin (1994).

In addition to Be stars, the winds from other stellar types may also be affected by equatorial wind compression. Although the wind compression model of BC was originally developed in the context of line-driven winds, the expressions governing the wind flow are kinematical and do not require the wind to be line-driven. Ignace, Cassinelli, & Bjorkman (1996; hereafter ICB) adopted the kinematical approach to investigate the sensitivity of the equatorial compression to the steepness (shallowness) of the wind velocity law. ICB found that the wind distortion became more sensitive to stellar rotation as the velocity gradient at the base of the wind was made smaller. When this occurs, the radial speed near the base of the wind remains small throughout an increasingly extended region, where the effects of rotation are greatest, so the wind compression at a fixed rotation speed tends to be intensified. The slope of the velocity law can be dominant in determining the equatorial compression, to the extent that significant equator-to-pole density contrasts can result at moderate to low stellar rotation rates of only 20% of critical. ICB elected to call wind models with significant wind density contrasts but no disk formation “wind compressed zone” (WCZ) models.

The results of the wc model have been applied in a variety of interacting wind scenarios to explain the occurrence of aspherical nebulae that are commonly observed among certain classes of stars, such as the planetary nebulae (Balick 1987; Frank & Mellema 1996), luminous blue variable stars (Nota et al. 1995; García-Segura 1997a; Frank 1997), and Wolf-Rayet (W-R) stars (García-Segura, Langer, & MacLow 1996; Brighenti & D’ Ercole 1997). However, the
effects of magnetic fields are ignored in these applications. Yet, magnetic fields are thought to dramatically affect the stellar winds of some stars. Many of the atmospheric phenomena observed in our own Sun are regulated by magnetic fields; for example, Alfvén wave deposition in the Solar atmosphere is important for producing the coronal holes that give rise to the fast Solar wind (Parker 1991). The chemically peculiar B and A stars are known from polarimetric observations to possess strong surface magnetic fields of several kilogauss (Babel, North, & Queloz 1995; Landstreet et al. 1989). In the Helium peculiar B stars, empirical models indicate that the strong magnetic fields suppress mass loss at the equator, constraining the wind to occur in polar jets only (Shore 1987). Another class of objects thought to be affected by magnetic fields are pre-main-sequence stars (Königl & Ruden 1993) that are observed to have bipolar jets and dense disks (Shu, Adams, & Lizano 1987). The magnetic fields are believed to play a role in the collimation of the jets and in facilitating accretion from the circumstellar disk to the protostar itself (Shu et al. 1994).

Evidence for interesting magnetic effects in circumstellar environments has motivated us to investigate magnetic fields in the context of wc models. In particular, García-Segura (1997b) has considered three-dimensional, MHD-interacting wind simulations based on wc theory plus a crude approximation for the toroidal magnetic field that might be present in the outflow. He finds that magnetic collimation leads to the appearance of jets and ansae in his planetary nebulae models that are consistent with observations (confirmation of predictions made by Różyczka & Franco 1996 based on two-dimensional models). García-Segura notes that the gross properties of his results are relatively insensitive to his choice of the toroidal field parametrization; however, we will show that the equatorial magnetic field can be enhanced above what he assumed, which will naturally have quantitative, if not qualitative, consequences.

Note that previous considerations of magnetic winds have often drawn on the seminal work of Weber & Davis (1967) for a thermally driven and magnetic equatorial flow. Their model was applied to the Sun and was able to explain the rather slow solar rotation. The one-dimensional Weber & Davis model was expanded by Sakurai (1985) to two-dimensional axisymmetric winds. The only detailed example discussed by Sakurai was a solar type flow, for which the equatorial flow was found to be fast and the polar flow to be slow and more dense; however, those results are in opposition to what is typically inferred for axisymmetric mass loss in proto–planetary nebulae, Be and B[e] stars, and luminous blue variables, whereas the wc model naturally explains equatorial density enhancements as arising from rotation and suggests similar enhancements for the magnetic fields, as will be shown.

A major assumption of the wc model is that the wind-driving force is radial. Recently, Owoczki, Cranmer, & Gayley (1996; hereafter OCG) have investigated this basic assumption in the case of hot star winds. The radiation force that drives the wind has nonradial components arising from three effects: (1) The absorption of stellar photons is direction-dependent, owing to the asymmetric velocity gradients of a rotating wind, in such a way as to produce a retarding azimuthal torque that reduces the rotation of the wind. (2) Similarly, the absorption asymmetry also produces a latitudinal component of the force directed away from the equator. (3) A rapidly rotating star is oblate and gravity-darkened. Although the pole is brighter than the equator, the increased solid angle subtended by the equator causes the net radiative flux vector to be directed away from the equator. All three of these effects produce a nonradial component of the force that reduces the amount of wind compression. Although these nonradial forces are much less than the radial component of the radiation force, the equatorward drift of wind material is likewise considerably smaller than the wind terminal speed. Consequently, OCG find that the nonradial forces produced by these three effects can inhibit the formation of a WCD in a line-driven wind.

OCG use the Castor, Abbott, & Klein (1975; hereafter CAK) parameterization of the line-driving force as modified by Abbott (1982). In OCG's calculation of the WCD inhibition, they assume that the CAK parameters (k, α, and δ) are constant throughout the wind and that the force is produced by unattenuated stellar radiation. However, an optically thick equatorial disk redirects the photoionizing radiation away from the equatorial regions producing lower ionization states. There is also observational evidence for latitudinal ionization gradients in the winds of rapidly rotating hot stars (Bjorkman et al. 1994). This position dependence of the ionization balance implies that the CAK force parameters are not constant throughout the wind, as assumed by OCG. Changes in the driving forces will change the structure of the wind, which in turn changes the magnitudes of the nonradial forces. There are also several additional sources of nonradial forces that were not included by OCG. For example, the line force was calculated using stellar continuum radiation. Owing to the stellar rotation and associated Doppler shifts, photospheric line structure will produce nonradial forces. Clumping in the wind will also change the direction and magnitudes of the radiation forces, because the Sobolev approximation (which was used for the CAK line-driving force) would not apply to the clumps. If any of these additional effects reverse the direction of the nonradial forces or decrease the outward acceleration, then the wind compression will be enhanced instead of inhibited by nonradial forces. Furthermore OCG's concerns only apply to CAK line-driven winds. To the extent that other mechanisms can drive stellar winds (e.g., dust driving), our kinematical wind compression models still apply as long as the forces in these models do not have large nonradial components. Therefore, given wc models as an example of axisymmetric rotating winds, we investigate the effects of the flow on the magnetic field structure.

The paper is structured as follows: § 2 begins with a review of the wind compression theory and ends with the development of a rotationally compressed and magnetized wind flow. Applications of this new magnetic wind model are presented in § 3, with results given for W-R stars and red supergiant (RSG) stars as examples. A brief summary and discussion is given in § 4.

2. A MAGNETIC WIND COMPRESSION MODEL

2.1. A Review of the Wind Compression Model

Only the basic expressions and results of the wc model are described here; a more detailed description of the wc effects can be found in BC and ICB. Before overviewing its derivation, the fundamental result of the WCD model is
that disk formation becomes more likely if (1) the star is rotating more rapidly or (2) the wind acceleration is small, in which case disk formation can occur at rotation rates that are slow relative to break-up. With regard to point 2, the radial acceleration of a steady-state wind is \( v_r (dv_r/dr) \), and it follows that the wind acceleration will be small if either the wind terminal speed \( v_\infty \) or the velocity gradient \( dv_r/dr \) of the wind is small.

The crucial assumptions of the wc model are (1) wind-driving forces are radial and (2) the subsonic flow is ignored. For a supersonic rotating wind driven by central forces, the motion of a fluid element injected into the wind at the sonic surface is confined to an orbital plane that is inclined to the equatorial plane. Angular momentum conservation allows for the calculation of the streamline locations in the orbital plane as a function of radius \( r \) and initial colatitude \( \theta_0 \). The derivation of streamline trajectories requires a known wind velocity distribution. The manner in which the streamlines converge (at low latitudes) or diverge (at high latitudes) determines the wind density. Here we reproduce the necessary expressions to determine the streamline locations and the properties of the wind flow from a rotating star.

The streamline trajectory of a fluid element in the orbital plane is given by \( \phi(r, \theta_0) \), where the coordinates \( (r, \phi) \) are standard polar coordinates. The differential equation for the trajectory is given by \( d\phi/dr = v_\phi/r v_r \), where \( v_\phi \) is the azimuthal speed. (Note that primes are not attached to \( \phi \) or \( \theta_0 \), which have both \( \theta \) and \( \phi \) components in the stellar system.)

The azimuthal velocity in the orbital plane is derived from the condition of angular momentum conservation, with the result being that

\[
v_\phi = v_\text{rot} \sin \theta_0 \left( \frac{R_*}{r} \right),
\]

where \( v_\text{rot} \) is the rotation speed at the surface of a star with radius \( R_* \). For the radial component of the wind velocity, we follow ICB in using the \( \beta \)-velocity law

\[
v_r = v_\infty + \left[ v_\infty(\theta_0) - v_\infty \right] \left( 1 - \frac{R_*}{r} \right)^\beta,
\]

where \( v_\infty \) is the isentropic sound speed, which for simplicity is assumed to occur at the stellar radius \( R_* \), and \( v_\infty \) is the terminal speed of the wind. The exponent \( \beta \) determines the shape of the radial velocity distribution, with larger \( \beta \)-values producing shallower velocity laws. The terminal speed is a function of \( \theta_0 \) through the relation

\[
v_\infty(\theta_0) = \zeta v_{\text{esc}}(1 - \omega \sin \theta_0)^\gamma.
\]

Here \( v_{\text{esc}} = \left[ 2GM_* (1 - \Gamma)/R_* \right]^{1/2} \) is the escape speed, and \( \omega = v_\text{rot} / v_{\text{esc}} \) is the stellar rotation rate, being a ratio of the rotation speed to the critical speed of break-up. The parameter \( \zeta \) sets the scale of the polar terminal speed in terms of \( v_{\text{esc}} \). Table 1 shows typical values of the velocity parameters for several classes of stars.

With the known velocity distributions, \( v_\phi \) and \( v_r \), we use the structure of the wc flow as derived in Appendix A of ICB. The streamline trajectory is

\[
\phi(r, \theta_0) = \frac{\omega v_{\text{esc}} \sin \theta_0}{\beta v_r} \left( \frac{v_r}{v_\infty - v_r} \right)^{1/\beta} B^\gamma \left( \frac{1}{\beta}, 1 - \frac{1}{\beta} \right),
\]

where \( B \) is the incomplete beta function, given by Abramowitz & Stegun (1972), and \( Y = 1 - v_r/v_r(r) \).

Knowledge of the streamline locations provides the wind density and velocity structure of the flow. The wind density follows from the equation of mass continuity, \( \nabla \cdot (\rho \mathbf{v}) = 0 \).

BC showed that the density is

\[
\rho = \frac{M_0}{4\pi r^2 v_r(\theta_0)} \left( \frac{d\mu}{d\mu_0} \right)^{-1},
\]

where \( \mu = \cos \theta, \mu_0 = \cos \theta_0 \), and \( d\mu/d\mu_0 \) is the wind compression factor. Spherical trigonometry is employed to relate \( \mu \) and \( \mu_0 \), giving the following coordinate transformation:

\[
\mu = \mu_0 \cos \phi'(r, \mu_0) .
\]

Equation (6) relates the orbital coordinates for the trajectory of a streamline to those of the stellar coordinate system. Differentiating this expression yields the wind compression factor,

\[
\frac{d\mu}{d\mu_0} = \cos \phi' + \phi' \sin \phi' \cot^2 \theta_0 \frac{d\ln \phi'}{d\ln \sin \theta_0} .
\]

The derivative appearing in the second term of equation (7) is a function of both \( r \) and \( \theta_0 \) (see Appendix B of ICB). For simplicity we choose the mass loss rate \( M_0 \) to be spherically symmetric at the stellar surface, but other angular mass loss distributions could easily be incorporated in the model.

The wind velocity structure is derived from the tangency of the flow vector velocity field to the known streamline trajectories. In the coordinates of the orbital plane, the wind velocity has only radial and azimuthal components; however, in the stellar coordinate system, all three components of the wind velocity exist, with \( \mathbf{v} = (v_r, v_\theta, v_\phi) \). BC derived the components of the velocity in the nonrotating stellar reference frame. The azimuthal component is given by

\[
v_\phi = \frac{v_\text{rot} R_* \sin^2 \theta_0}{r \sin \theta},
\]

and the latitudinal component is

\[
v_\theta = \frac{R_* \sin \theta_0 \cos \theta \sin \phi'}{\sin \theta}.
\]

Note that \( \theta \), or alternatively \( \mu \), is described by equation (6), hence \( \theta = \phi(r, \theta_0) \).

Combining the three velocity components, the magnitude of the velocity vector, \( |\mathbf{v}| \), can be expressed as

\[
|\mathbf{v}| = v_r \sqrt{1 + \left( \frac{v_\text{rot} R_*}{v_r} \right)^2 \sin^2 \theta_0} .
\]
Expression (10) will prove useful in understanding the results of the magnetic wind model that are described in the following section. Note that near the base of the wind, where $r = R_*$, the velocity will typically be dominated by $v_{\text{rot}}$ as long as $v_{\text{rot}} > v_\infty$ and $\theta_0$ is not too near zero; in contrast, $v \approx v_{\text{rot}} \cos \theta_0$ near the pole. At large radii, the velocity becomes $v \approx v_\infty$, because both $v_\theta$ and $v_\phi$ tend toward zero.

2.2. “WC Fields:” Wind Compression with Magnetic Fields

Rather than consider wind models with strong magnetic fields that would require numerical MHD simulations, we choose to model the weak field limit. As discussed in Cassinelli (1991), when the field in magnetic rotator theory is weak, the velocity structure is determined by the nonmagnetic wind-driving forces and the field itself is drawn out by the flow. Although we are now dealing with a three-dimensional model instead of just the one-dimensional equatorial solution of classical magnetic rotator theory, the same idea is employed to derive the magnetic topology from the conditions that the field is rooted on the star and is governed by the flow. From magnetic rotator theory, it is known that the specific energy per unit mass carried to infinity by a magnetized wind is

$$\delta = \frac{1}{2} v_\infty^2 + \frac{v_M^3}{v_\infty}$$

(Belcher & MacGregor 1976), where $v_M$ is the Michel velocity (Michel 1969). The Michel velocity depends on the stellar magnetic field and rotation speed via

$$v_M^3 = \frac{v_{\text{rot}}^3 R_*^2 B_r^2}{M}$$

where $B_{r,*}$ is the radial magnetic field strength at the stellar surface. In the subsonic expansion the density of material experiences an approximately exponential decrease. By mass conservation, the radial velocity must have a corresponding exponential growth through this region. Combined with the assumption of a frozen-in field (i.e., a nearly infinite magnetic conductivity), any azimuthal or latitudinal magnetic field components should decrease exponentially in the subsonic expansion, whereas the radial magnetic field will decrease only as $r^{-2}$. Assuming that the subsonic zone is relatively narrow in radial extent (i.e., compared to $R_*$), we approximate the sonic surface as being at $R_*$ and set the total surface magnetic field as $B_* \approx B_{r,*}$ at these points.

Now the second term of equation (11), which involves $v_M$, is important, because it relates to the Poynting flux energy that is carried to infinity by the wind. Magnetic effects are therefore considered negligible for accelerating the wind when the Poynting flux contribution to the energy equation is small. This condition requires that $v_M \ll 0.8 v_\infty$. Using equation (12), the weak field limit implies that

$$B_* \ll 2500 \frac{M^{1/2}}{R_*^{1/2}} \frac{v_{\text{rot}}^{3/2}}{v_\infty}$$

where $B_*$ is in gauss and the numeral subscripts indicate powers of ten normalizations, with $M$ in $M_\odot$ yr$^{-1}$, $v_\infty$, and $v_{\text{rot}}$ in km s$^{-1}$, and $R_*$ in $R_\odot$. Of course, equation (12), and therefore equation (13), are derived from considerations of the one-dimensional equatorial case. However, if the star rotates as a solid body, $v_{\text{rot}}$ will decrease with higher latitudes, indicating that if condition (13) is satisfied at the equator, it will be satisfied at all latitudes, assuming that $B_*$ does not increase toward the pole more rapidly than $v_{\text{rot}} \sin \theta_0$ decreases.

Assuming that equation (13) for the weak field limit is satisfied, we next consider the magnetic field distribution throughout the wind flow. The WCZ model predicts the streamline trajectories of fluid elements in the wind of a rotating star. We have argued that in the weak field limit, the presence of magnetic fields will not alter the wind structure; hence the streamline locations of the WCZ model remain unchanged. However, the fact that the magnetic fields are assumed to be hydrodynamically dominated does not mean that the field lines are collinear with the flow streamlines of the nonmagnetic case. Assuming a frozen-in field, the field lines that are rooted in the stellar envelope at the base of the wind will follow what are commonly called “streaklines” (Currie 1974).

As illustrated in Figure 1, a streamline is the locus of elements of matter that are released sequentially from the same point on the star, whereas a streamline is the track followed by a single element of matter. For example, in magnetic rotator models, the difference between streamlines and streaklines is often explained using the analogy of a rotating phonograph record. The flow of a single element of matter is like the nearly radial outward track of the needle, while the streaklines that determine the magnetic field direction are like the grooves in the record. Just as the grooves are solidly fixed in the record, so is the pattern of the magnetic field distribution fixed in the flow; there can be

![Figure 1](image.jpg)

**Fig. 1.—Contrast of streamlines and streaklines in rotating stellar winds.** The solid line shows a streamline trajectory in the equatorial plane of a fluid element that is injected into the wind from a rotating star. The direction of stellar rotation is counterclockwise, hence the streamline deviates from a radial trajectory (dashed line) by an angle of $\phi$. The streamline tracks the motion of an individual fluid element. In contrast, the streakline (dotted line) follows the motions of fluid elements that are sequentially released from a given point on the rotating star; thus the streakline is seen to wrap around the star in the clockwise direction. Whereas the outflow velocity field in the nonrotating frame is tangent to the streamlines, a weak magnetic field that is carried along in the flow will be tangent to the streaklines. Consequently, at large radii the magnetic field becomes predominantly toroidal in the direction opposite to the stellar rotation.
no crossing of the field lines. This fixed magnetic field pattern is a consequence of making the assumption of a frozen-in field.

Let us consider how to determine the three-dimensional field distribution in the wind. In a corotating frame of reference, the streaklines that determine the magnetic field geometry are identical to the streamlines, thus $\mathbf{B} \propto \mathbf{V}$ at every point in the flow, where $\mathbf{V} = (V_r, V_\theta, V_\phi)$ is the flow velocity field in the rotating frame. The velocity components in the radial and latitudinal directions are just $V_r = v_r$ and $V_\theta = v_\theta$. The component of velocity in the azimuthal direction is given by

$$V_\phi(r) = v_\phi(r) - r\Omega \sin \theta,$$  

(14)

where $\Omega$ is the angular rotation speed of the star and $\theta$ is the colatitude. It is straightforward to find the streaklines that determine the magnetic geometry once the velocity $V_\phi$ of the rotating frame is known, which comes directly from the WCZ model using the expressions in § 2.1.

To obtain the equation relating the vector magnetic field and the vector flow velocity, we make use of the continuity equation and the conservation of magnetic field flux. Consider a flow tube of variable cross-sectional area $dA$, with $dA$ a vector in the direction of the outward unit normal. In this flow tube, the conservation of mass takes the form

$$F_M = \rho \mathbf{V} \cdot dA = \text{constant}.$$  

In the corotating frame, the velocity $V$ is tangent to the streamline and therefore parallel to $dA$. Magnetic flux freezing applies the additional constraint that $\Phi_B$ constant. For a magnetic field that is radial at the lower boundary, $\mathbf{B}$ is initially parallel to $dA$ and by the conservation of magnetic flux must remain so throughout the flow. It may be concluded then that $\mathbf{V}$ and $\mathbf{B}$ are parallel vectors.

To relate the magnetic field to the flow velocity, we define the parameter $\alpha = F_M/\Phi_B$, which is known at the base of the flux tube, taken as the sonic surface with $r = R_*$. Then

$$F_M - \alpha \Phi_B = 0.$$  

(15)

By direct substitution for $F_M$ and $\Phi_B$,

$$(\rho \mathbf{V} - \alpha \mathbf{B}) \cdot dA = 0.$$  

(16)

Since $\mathbf{V}$ and $\mathbf{B}$ are parallel to $dA$, the solution requires

$$\mathbf{B} = \rho \mathbf{V} \frac{\Phi_B}{F_M},$$  

(17)

where $F_M$ and $\Phi_B$ were substituted for $\alpha$. This result, derived more generally by Mestel (1968), shows that $|\mathbf{B}|$ is proportional to the mass flux $\rho |\mathbf{V}|$, so the magnetic field also has an equatorial amplification.

Substituting the density of equation (5) and evaluating $F_M$ and $\Phi_B$, we find

$$\mathbf{B} = B_* \frac{R_*^2}{r^2} \left( \frac{d\mu}{d\mu_0} \right)^{-1} \mathbf{V}.$$  

(18)

Note that the wind compression factor $d\mu/d\mu_0 \leq 1$ at the equator, thus accounting for the enhancement of the equatorial magnetic field. For a convenient normalization scale, we observe that the magnetic field in the nonrotating case is purely radial with magnitude

$$B(\omega = 0) = B_* \frac{R_*^2}{r^2}.$$  

(19)

Using the velocities of equations (2), (8), (9), and (14), we obtain the magnetic field strength in the rotating case, namely,

$$B(r, \theta) = B(\omega = 0) \left( \frac{d\mu}{d\mu_0} \right)^{-1} \times \sqrt{1 + \frac{\Omega^2}{c^2} \left( \frac{R_*}{r} \sin \theta_0 - \frac{r}{R_*} \sin \theta \right)^2}.$$  

(20)

For simulations of interacting winds, such as the planetary nebulae, one requires the field at large radii. Recalling that the magnetic topology follows the streaklines of the wind, Figure 1 shows that the streaklines will wrap around the star, hence a magnetic field that is initially radial at the stellar surface eventually becomes toroidal at large distances. In this limit the asymptotic field reduces to

$$B(r \gg R_*, \theta) \approx B_0 = -B_* \sin \theta \frac{r_{\text{rot}}}{v_{\text{rot}}} \left( \frac{d\mu}{d\mu_0} \right)^{-1},$$  

(21)

which decreases only as $r^{-1}$. Equation (21) is valid near the equatorial region, but in the vicinity of the poles, where $\sin \theta$ is small, the asymptotic field will be $B(\omega = 0)(d\mu/d\mu_0)^{-1}$. Near the poles, $d\mu/d\mu_0 \geq 1$, so that the magnetic field strength is reduced relative to the case of no rotation.

Note that the toroidal field assumed in the models of García-Segura (1997b) consists of the equatorial result from magnetic rotator theory multiplied by the factor $\sin \theta_0$ to account for latitudinal variation. However, here we see from the more self-consistent derivation that $B_0$ can be significantly stronger at the equator than was assumed by García-Segura, owing to the higher densities. Of course, not only is the field stronger at the equator, but it is also weaker toward the poles. This result may have interesting consequences for the magnetic collimation investigated in MHD simulations. Although we do not pursue the topic of planetary nebulae here, the applicability of a wind compression plus magnetic field model is considered for the winds of massive stars in the following section.

3. A CASE STUDY: MASSIVE STAR WINDS

The results of the preceding section for weak magnetic fields in rotating winds are applied to two classes of massive stars: W-R stars and RSG stars.

3.1. Wolf-Rayet Stars

There is evidence that magnetic fields are important in W-R winds. First, Abbott et al. (1986) found that a significant fraction of OB and W-R stars show nonthermal radio emission. Second, particle acceleration by the first order Fermi mechanism has been investigated by Chen & White (1991), who predict that the W-R stars might be hard X-ray and $\gamma$-ray sources. Biermann & Cassinelli (1993) have argued for the existence of kilogauss surface fields on W-R stars based on the energy spectrum of cosmic ray particles. They suggest that the presence of kilogauss surface fields combined with the ultimate supernova explosion of W-R stars could accelerate cosmic rays up to energies of $3 \times 10^9$ GeV.

A long-standing theoretical problem of the W-R winds has been their large momentum fluxes, with performance
ratios $\eta = \frac{M_{eq}}{L_*}$ ranging from a few to about 70 (Willis 1991). To explain the large momentum fluxes of W-R winds, Lucy & Abbott (1993) investigated the effects of multiple scattering and were able to obtain performance factors of order 10 for typical W-R star parameters. Subsequent studies by Springmann (1994) and Gayley, Owocki, & Cranmer (1995) have confirmed the potential of this method for explaining the W-R winds. However, to be effective the photons must scatter at order $\eta^2 \sim 100$ times. The major theoretical difficulty is no longer one of imparting sufficient momentum to the winds but of achieving sufficient opacity coverage for many scatterings to occur; otherwise, photons will leak out of the wind having scattered only a few times.

The presence of asphericities will modify the momentum/opacity problem in two ways: (1) the “apparent momentum” of the wind as commonly deduced from observations of a W-R star and (2) the severity of the opacity problem. The apparent momentum (Poe, Friend, & Cassinelli 1989) is the product of the mass loss rate derived from radio observations times the terminal speed from UV resonance lines. For a two-component wind with a fast polar flow and a slower dense equatorial flow, the radio flux comes primarily from the equatorial region, while the absorption in the P Cygni line profiles is formed along lines-of-sight through the fast wind at higher latitudes of the star. If, by reinterpretation of the observations, the true momentum of W-R winds is reduced, so will the need for large opacities. On the other hand, asphericities might exacerbate the opacity problem by providing geometrical avenues of photon leakage.

One-dimensional magnetic rotator models for the equatorial flow have been investigated for the W-R winds by Poe et al. (1989). However, only at rapid rotations of nearly 90% critical could magnetic forces significantly enhance the equatorial mass loss to produce the large performance factors that are observed. In contrast, the WCZ models of ICB were found to produce significant deviations from spherical for the W-R-type winds at much smaller rotation rates of about 20% critical. At these low stellar rotations, the magnetic rotator mechanism is not expected to appreciably increase the equatorial mass loss.

Using the W-R star parameters listed in Table 2 for a WNS type star, Figures 2a and 2b show the effects of wind compression on the magnetic field along a sequence of streamlines as a function of radius. Shown are ten streamlines that originate at $10^5$ intervals on the star between the pole and equator. The field strength is normalized to what would be present in the absence of rotation. Note that for the weak limit to apply in our model, equation (13) requires that the surface field strength of the W-R star should be $B_\tau < 30,000$ G, where we have used the WNS5 star properties of Table 2 and a stellar rotation $v_{rot}/v_{crit} \sim 0.2$.

The upper and lower panels of Figure 2 are for $\omega = 0.5$ and 0.9 of the disk threshold, for which $\omega = 0.29$. In both cases a velocity law with $\beta = 2$ is used. The major result is that a rotationally induced wind compression can cause an order of magnitude enhancement of the magnetic field at the equator for a stellar rotation of about 25% critical. Note that for the faster rotating case, the increase of the equatorial field is accompanied by a similar enhancement (factor of 20) of the wind density at the equator.

The magnetic field along all of the streamlines appears to quickly obtain asymptotic values within about one stellar radius. Note the presence of “kinks” along midlatitude streamlines where the field strength increases sharply, peaks, and then falls off toward asymptotic values. The sharpness of the kinks appears milder for the slower rotating model than for the faster one. An explanation of these kink features will be discussed shortly in relation to the RSG wind models.

Whereas the previous figure shows the two-dimensional variation of the total magnetic field strength throughout the W-R wind, Figure 3 shows the field in its individual vector components. The plot is logarithmic with each field component normalized to $B_\tau$. The distribution is plotted versus $\log (r/R_\tau - 1)$ to more easily see the rapid evolution of the magnetic field structure throughout the inner wind. Note that the component $B_\phi$ peaks at midlatitudes, so the lines showing the radial distribution of $B_\phi$ cross one another.

Two types of lines are shown: solid for the field distribution along streamlines from latitudes $10^\circ$–$50^\circ$ and dashed for those originating at latitudes $60^\circ$–$80^\circ$. At both the pole and equator, $B_\phi = 0$ and thus are not shown in the logarithmic plot. To more clearly illustrate the latitudinal dependence of $B_\phi$, an inset graph shows $r^3B_\phi/R_\tau^3 B_\tau$ versus stellar latitude as computed for the largest radius of our wind model, at $r \approx 10R_*$. Unlike the latitudinal field component, the toroidal field increases steadily from zero at the pole toward midlatitudes followed by rapid growth in the vicinity of the equator to maximum value at $\theta = 90^\circ$.

3.2. Red Supergiant Stars

The effects of wind compression for magnetized winds of RSG stars have also been calculated. The RSG stars are of interest because stellar evolution theory predicts that some massive stars will evolve through a RSG phase before becoming a blue supergiant (BSG). In particular, the progenitor of SN 1987A that exploded as a BSG is thought to have been a single star that followed such an evolutionary track (Arnett et al. 1989; although see de Loore & Vanbeveren 1992 for a discussion of the binary hypothesis). The appearance of three axisymmetric rings surrounding SN 1987A (Wampler et al. 1990) has led to several models in an attempt to explain their existence (e.g., Blondin & Lundqvist 1993). Of interest to this work are the models of Chevalier & Luo (1994) and Washimi, Shibata, & Mori (1996), who consider the consequences of magnetic fields for producing the ring structures. In both of these papers, it is a toroidal magnetic field in the axisymmetric wind of a rotating RSG star that provides the mechanism for creating the asymmetry observed in the SN ejecta. The we results with magnetic fields, therefore, have relevance for interpreting the observations of the SN 1987A rings and deducing the
surface magnetic field strengths of the progenitor star while in its RSG phase. (Of related interest are the less explosive but similar models being considered for planetary nebulae, as noted previously.)

Parameters used for the RSG models are listed in Table 2 and were chosen to represent those expected of the SN 1987a progenitor during its RSG phase. For these parameters, the weak field assumption requires that the surface magnetic field be $B_0 < 1 \text{ G}$, assuming $v_{\text{rot}}/v_\infty \sim 0.1$. The results of the calculations for the magnetic field distribution in the RSG wind are shown in Figures 4a and 4b, again using a $\beta = 2$ radial velocity law. The threshold rotation speed of disk formation is $\omega_0 = 0.14$. The format of these figures is the same as for the W-R star case. Similarly, plots of the individual field components are given in Figure 5, again in the same format as the W-R case shown in Figure 3.

The radial and latitudinal distributions of the magnetic field strength for the RSG star differ significantly from that of the W-R wind. Note especially the lack of kinks and the failure of the field strength to obtain constant asymptotic values at the greatest radii shown (except at the pole), which occurs in the W-R models of Figures 2a and 2b. The latitudinal and toroidal field components also peak somewhat farther out in the RSG wind, at about $1.3R_*$ in contrast to $1.1R_*$ for the W-R case. At $r \approx 10R_*$, $B_\phi$ is roughly an order of magnitude greater in the RSG case compared to the W-R value. The differences between the W-R and RSG model results can be explained as follows.

Recall that Figures 2 and 4 are plots of the ratio $B/B(\omega = 0)$ of the magnetic field strength in a rotating equatorially compressed wind to that of a spherical nonrotating wind. At large radius, the magnetic field in the equatorial region becomes primarily toroidal, which decreases with radius as $B_\phi \sim r^{-1}$. But, the radial magnetic field decreases more rapidly as $r^{-2}$. Consequently, the ratio of the magnetic field strength from the weak field models to that of the nonrotating case will increase linearly with radius as $B_\omega/B_r \sim r^{-1}/r^{-2} = r$ for distances that are far from the star. More rigorously, this asymptotic behavior is understood from the
The distribution of the magnetic field components in the W-R wind for the case \( \omega = 0.9 \omega_0 = 0.26 \). As labeled, the three large panels are for the distribution of \( B_r \), \( B_\theta \), and \( B_\phi \) as normalized to \( B_\ast \) and plotted logarithmically with radial distance. As in Fig. 2, the field strength of a given component is shown for streamlines originating at 10\(^\circ\) intervals inclusive of the pole and equator. For the component \( B_r \), an inset is given, indicating the normalized latitudinal distribution at the largest radius of about 10\( R_\ast \) of the model calculation. \( B_\theta \) is zero at both pole and equator and peaks quite sharply just above and below the equator. The solid lines for the radial distribution of \( B_r \) are for streamlines with \( \theta_0 = 10^\circ - 50^\circ \); the dashed lines are for latitudes \( \theta_0 = 60^\circ - 80^\circ \). Naturally, the field strength at pole and equator, being zero, are not shown in the logarithmic plot. For the component \( B_\theta \), the field strength is zero at the pole (not shown) and increases steadily with latitude toward the equator.

Discussion of which is the exact expression for equation (20), the two-dimensional field distribution. Note that the wind compression factor in that expression is a smooth function along the streamline that quickly obtains constant values, so it contributes neither to the kink observed in the W-R models nor to the continual rise of field strengths observed in the RSG models.

We make three observations concerning equation (20) toward explaining the trends of the magnetic field distributions plotted in Figures 2 and 4:

1. We have explored the occurrence of the kink in the W-R wind models numerically. It arises from the fact that \( v_{\text{rot}}/v_\ast \) starts at the typically large value of \( v_{\text{rot}}/v_\ast \) and decreases to a much smaller value of \( v_{\text{rot}}/v_\ast \) with increasing radius. Defining \( \Pi = R_\ast \sin \theta_0/r - r \sin \theta/R_\ast \), we see that \( \Pi = 0 \) at the stellar surface but increases in magnitude with increasing radius. Consequently, the kink may or may not occur depending on the product of \( v_{\text{rot}}/v_\ast \) and \( \Pi \). At constant values of \( \omega/\omega_0 \), the wind structures are roughly the same for both the W-R and RSG wind models (a result discussed in ICB), and therefore the distribution of \( \Pi \) for the two types of stars must also be similar. At fixed values of \( \omega/\omega_0 \), it is the different ratios of \( v_{\text{rot}}/v_\ast \) for the two stars that accounts for the appearance of the kinks in the W-R case but not in the RSG case. In the W-R case, the kink is present because \( v_{\text{rot}}/v_\ast \) varies from about 10 at \( R_\ast \) to only 0.1 at large radii, whereas \( v_{\text{rot}}/v_\ast \) is more nearly constant in the case of the RSG star. Therefore, the kink does not appear.

2. Exactly at the pole, \( \theta_0 = \theta = 0 \), and the ratio of \( B/ B(\omega = 0) \) is just the inverse of \( d\mu/d\mu_0 \), true in both the RSG and W-R models. The wind compression factor at the pole is equal to unity at the stellar surface and increases with radius, hence the polar magnetic field in the rotating case is reduced relative to that for a nonrotating spherical wind, yet decreases asymptotically as \( r^{-2} \).

3. At large radius equation (20) reduces to equation (21) with \( B/B(\omega = 0) \sim r \). The reason that this limit is observed
4. EFFECTS OF THE WIND COMPRESSION ON THE MAGNETIC FIELD

in the RSG models and not the W-R models is that $v_{rot}/v_o$ is of order unity for the RSG star but only one-tenth for the W-R star. As a result, the RSG models obtain asymptotic tendencies at much smaller radii than in the W-R case. The W-R models do eventually show a linearly increasing trend with radius, but Figure 2 has not been plotted to sufficiently large radius to reveal it.

4. SUMMARY AND DISCUSSION

Assuming weak magnetic fields and the frozen-in condition, we have explored how $v_w$ effects may enhance the magnetic field in a stellar wind at regions near the equator. In this approximation the magnetic field strength is found to be $B \propto \rho V$, where $V$ is the wind velocity in the rotating frame of reference (Mestel 1968). Thus in a WCZ model, the field at the equator can increase by as much as an order of magnitude for fairly small stellar rotation rates of only 10% to 20% break-up (i.e., assuming a $\beta = 2$ velocity law).

The magnetic $v_w$ model was applied to the winds of massive stars, first for a W-R star and then for a RSG star. In the W-R case, the enhanced equatorial magnetic field has relevance for explaining nonthermal emissions observed among some of these objects at the radio and X-ray wavelengths (Abbott et al. 1986; White 1985; Leitherer, Chapman, & Koribalski 1997; Pollock 1987). In addition, radio measurements of radio supernovae provide indirect suggestions of strong magnetic fields in the previous W-R phase (Biermann & Cassinelli 1993).

In applying magnetic wind models to W-R stars, the works of White (1985) and Biermann & Cassinelli (1993) hinge on the toroidal magnetic field at the equator being the dominant component at large radius. In that context the advantage to be gained from the WCZ model is that $B_0$ at $r \gg R_*$ is to be replaced by $B_0(d\mu/d\mu_0)^{-1}$, where the wind compression leads to an amplified field strength at the equator, since $d\mu/d\mu_0 \leq 1$ at low latitudes. The major consequence is that the surface magnetic field necessary to explain the observations as predicted in these previous works is reduced by typical factors of $2-4$, resulting in subkilo gauss values, assuming mild wind compressions like the examples of § 3.1. In that section we also estimated that the surface field strength for our weak field approximation to remain valid should be much less than about 30,000 G for
W-R winds, and the inferred sub-kilogauss values fall comfortably below that limit.

In the RSG star case, WCField models were presented with stellar parameters that are representative of the RSG phase of the SN 1987a progenitor. The presence of a magnetic field may be important for understanding the observed triple ring system. Although we neglect the magnetic field for producing an equatorially enhanced density, it can still have a role in shaping the shocked interface between the inner fast BSG wind and the outer dense RSG wind (e.g., Chevalier & Luo 1994; García-Segura 1997b). To explain the presence of the equatorial ring seen around SN 1987a, Chevalier & Luo (1994) estimated that their model required a surface magnetic field $B_{\perp} \sim 1.3 \times v_{\perp}/v_{\text{rot}}$ (eq. [4.2] of that paper). For our model with $\omega = 0.9 \omega_{\text{rot}} = 0.13$, the ratio of terminal speed to rotation speed is $v_{\perp}/v_{\text{rot}} \sim 1.2$, and therefore $B_{\perp} \sim 1.5$ G. The asymptotic value of the wind compression factor at the equator is $d\mu/d\mu_0 \approx 0.375$, hence combining our WCField result with the formalism of Chevalier & Luo indicates $B_{\perp} \approx 0.5$ G.

For the ratio $v_{\perp}/v_{\text{rot}}$ of the model calculation, the weak field limit for our WCField model in the RSG phase of the SN 1987a progenitor requires surface fields below about 0.1 G. Our estimate of $B_{\perp}$ from the expression of Chevalier & Luo (1994) exceeds this limit by a factor of 5. So, in the case of the SN 1987a ring, a proper treatment of the magnetic forces for the wind flow is required. Better calculations may still reveal an enhancement of the magnetic flux at the equator. Other effects, such as those explored by Washimi et al. (1996), may also be found to explain the two rings located above and below the equator. For RSG stars with surface fields that do satisfy our weak limit, the WCField results will still have application. The formation of axisymmetric planetary nebulae is one such example.

It is a tremendous challenge to observationally test whether the WCField model is realized in stellar winds, because the Zeeman effect is not especially useful for directly measuring low magnetic fields in the sub-kilogauss regime that is predicted by the model (although see Carter et al. 1996, who use a method of coadded spectra). The reason is twofold. First, the Zeeman effect produces a splitting of the two circularly polarized components that can be used to infer the magnetic field strength in the line formation region from the broadening of the line, a technique common to solar magnetograms. However, in the case of stars with winds, the Zeeman splitting is much smaller than the wind broadening of the line, making such measurements difficult. Second, the magnetic properties of the wind could

![Figure 5](image-url)

**Fig. 5**—Distribution of the magnetic field components in the RSG wind for the case $\omega = 0.9 \omega_{\text{rot}} = 0.13$. This figure is like Fig. 3 in the W-R case, with similar overall global properties yet slightly different radial and latitudinal variations, as explained in the text.
also be inferred from observations of the circular polarization. The problem here is that the polarization induced by the magnetic field will likely suffer from (1) geometric cancellation effects owing to the fact that the observed polarization arises from a volume integral over the emitting region and (2) polarimetric cancellation from the incoherent superposition of the polarized Zeeman components owing to the overlap that results from the wind broadening. Both of these effects lead to small expected net polarizations, again making practical observations of magnetic fields formidable.

So the Zeeman effect will only be useful for rather large field strengths if present in low mass loss, low terminal speed winds (e.g., B and A stars). However, the much weaker magnetic fields, perhaps in the 1–100 G range, appear to have interesting effects for producing nonthermal radio emissions and shaping wind bubbles. What is needed, therefore, is a new magnetic diagnostic that is both direct and sensitive to weak fields. Ignace, Nordsieck, & Cassinelli (1997) have investigated a magnetic diagnostic that employs the Hanle effect. This diagnostic uses polarization arising from resonance line scattering to probe the magnetic fields in circumstellar envelopes. The Hanle effect is sensitive to fields in the sub-kilogauss regime as needed, and a technique that uses multiline polarimetric observations to determine the stellar magnetic properties appears promising. Observations to be obtained by the Far-Ultraviolet Spectro-Polarimeter (FUSP) of several hot stars that are thought to have rotationally compressed winds would provide an important test of the magnetic wind model presented here.

Andrew Conway (University of Glasgow) is credited for dubbing the magnetic version of the wind compression model as “WCFields.” We thank Wlodzimierz Kluzniak and Declan Diver for discussions regarding magnetic fields. We also acknowledge suggestions made by an anonymous referee leading to improvements in the manuscript. This research was supported through NASA grants NAG 5-2854 and NAGW 2210, the NSF grant AST 91-15375, and a UK PPARC Rolling Grant.