Business cycle and structural time series: warnings and hints

riccardo fiorito, University of Siena

Available at: https://works.bepress.com/riccardo_fiorito/18/
RICCARDO FIORITO

BUSINESS CYCLE AND STRUCTURAL TIME SERIES:
WARNINGS AND HINTS

Siena, giugno 1996
1. Introduction

Understanding business cycles is the bulk of current macroeconomics. Yet, several studies ascribe the sources of GNP fluctuations to other endogenous variables such as consumption, investment or interest rates that are certainly important in transmitting the shocks but that still need to be explained by some ultimate source (Cochrane, 1994).

The distinction between propagation mechanism and ultimate sources of shocks dates from the earlier work of Slutsky and Frisch but has been resurrected only recently by VAR paradigm (Sims, 1980). Indeed, the unrestricted VAR model is based on the lags of all the relevant variables ensuring a multivariate propagation mechanism while the solution of the model is based on the moving average representation expressing the history of the system in terms of all present and past innovations.

Though there are different approaches for analyzing business cycles, the search for exogenous shocks is perhaps the main reason why wide-sense VAR modeling became so popular in empirical macroeconomics. This success mostly occurred at the expenses of the traditional macroeconometric approach solving models in terms of exogenous (deterministic) variables. Hence, the only shocks in practical versions of the Cowles Commission approach are the disturbances appended to the equations. Indeed, these terms reflect uncertainty or idiosyncratic unpredictability rather than some fundamental force driving the economy at the business cycle frequencies.

The impossibility of interpreting in a structural way VAR innovations has been stressed by Cooley and Leroy (1985) who clarified how the Cholesky procedure used by Sims (1980) to make orthogonal the innovations was related to some implicit, though casual, identification scheme.

The next step was building structural VAR models, where identification could rest on a minimal set of restrictions suggested by some (simple) economic theory (Sims, 1986). These restrictions are placed in the contemporaneous part of the model, where some variables can affect other variables simultaneously or according to one-way causality as in the Cowles Commission approach. The first articulated models in this vein are due to Bernanke (1986) and Blanchard-Watson (1986) from which a large set of structural VAR models (SVAR) is initiated.

Instead of contemporaneous restrictions, Blanchard and Quah (1989) first utilized for identifying a VAR model the restrictions required by dynamic economic theory. In their application, Okun’s Law is expressed as a bivariate output/unemployment system where supply shocks are associated to the unemployment rate equation while demand shocks are associated to the real GNP equation. The assumptions that supply shocks have permanent unit-root dynamics while aggregate demand shocks have a zero long-run effect ensure econometric identification. Namely, this amounts to constraining to zero the sum of coefficients $c_i(1)$ relating output response to demand disturbances rather than the corresponding $c_i$ coefficient in
the contemporaneous part of the model. Different examples can be
drawn from neutrality propositions in monetary theory or by any
other model constraining long-run responses to particular shocks.
Joint usage of contemporaneous (Bernanke-Sims) and dynamic
restrictions is also becoming popular (Shapiro-Watson, 1988;
Gall, 1992) though one can be doubtful that the dynamic behavior
of the economy should be assumed rather than tested.

2. Identification is not enough

Whenever estimating a multivariate time-series model (VAR,
SVAR, State Space) several problems can be found in devising the
possible shocks affecting the economy: first, business cycle
components are not observable but must be extracted from
nonstationary macroeconomic variables.

After detrending, the business cycle components are
organized in a small VAR model (or alike) that is directly
estimated and possibly interpreted in some structural way once
econometric identification is provided. The resulting model is
usually a 'toy' model mainly devoted to transform observable
innovations into structural shocks. Similarly, the variance
decomposition, weighting over time the relative strength of each
innovation, is transformed into a structural shock decomposition.

Since time-series models cannot be too large, the
identification schemes are not simply 'plug in' advocated by
Sims (1986) but are often too simple for testing any articulated
macroeconomic theory. A related reason for drawing most
conclusions from SVAR models is that the identification scheme
is not obtained in most cases from a well-defined microeconomic
model where technology, preferences and the intertemporal budget
constraints dictate the specification of the structural model.1
As a result, the identification scheme is totally ad hoc although
its order condition is mechanically satisfied so as progress over
plain VAR could be more seeming than real.

Another difficulty in modeling SVAR models deals with
normalization, i.e. with the choice of the left-hand side
variable. In the bivariate supply/demand model, for example,
supply shocks have been either extracted by unemployment
(Blanchard-Quah, 1989) or price innovations (Bayoumi-Bichengreen,
1993), which seems to be also a legitimate normalization. Yet, still
we ignore if these structural shocks are actually
correlated as they should if the underlying models are
theoretically plausible.

Assuming that equilibrium holds at business cycles
frequencies may be realistic or not, depending on the market one
is investigating: in some cases adjustment is much faster than
in others, it being rather obvious than moving currency around

1 Among the exceptions, see Leeper-Sims (1994) and my own
work (Fiorito, 1996).
the world is easier than moving goods and especially people along a desired direction: hence, the assumption that in labor markets quantities clear at business cycle frequencies makes impossible to disentangle between demand and supply components although unemployment rate is substantial in most countries.

This raises again a normalization problem for which an immediate reference is Shapiro-Watson (1988). The motivation of this paper is demonstrating that labor supply is a major source of business cycles in the US. Meanwhile, equilibrium is assumed and labor quantities are interpreted as denoting labor supply. Actually, the empirical labor variable in the model is a variable (total hours) that at cyclical frequencies typically reflects the adjustment of labor input to changes in cyclical output, i.e. short-run changes in labor demand. All results confirm the interpretation that labor demand or productivity shocks are very important though this cannot by recognized by the reader because the relevant impulses are labeled by SW ‘labor supply’ rather than ‘labor demand’ shocks as they should.

Finally, it should be mentioned that the order condition used for identifying SVAR models can be satisfied by different models. While clearly an identification scheme should be preferred for theoretical reasons only, yet it can be interesting to evaluate how deeply impulse response functions can differ give the choice of the identification scheme. In a certain sense, this problem is not too different from comparing different results in the earlier VAR literature based on equally arbitrary Choleski decompositions. This is, however, just an apparent similarity since in the identified cases the question that one should address is the following: Is theory relevant in producing different results? Answers to this question are not easy since sensitivity analysis results are - at least to my knowledge - few and model-specific (Aoki, 1989) and cannot be generalized unless some theoretical result is formulated: yet, as in Choleski decomposition case, it seems that variance decomposition are more affected by the identification scheme than impulse responses are.

3. An example

As an example in this area of structural time-series model let me refer to my own work (Fiorito, 1996) where I follow some steps to evaluate the sources of business cycles in the US. Details are omitted for the sake of brevity. Yet, some steps I shall discuss here are quite general and are not confined to the SVAR class. In fact, I will use a state-space model both to detrend the data (first stage) and to estimate a cyclical model (second stage) whose formulation reflects the decision rules found through a small dynamic model.

The steps are the following:

ii) modeling of the relevant variable by an intertemporal optimizing model providing the relevant contemporaneous restrictions.

iii) estimate of the business cycle model, by using the second stage Aoki’s state space algorithm.

iv) evaluation of the structural response function by using the restriction implied by the theoretical model: these restrictions deal not only with the contemporaneous part of the endogenous variable but also with the covariance matrix of the structural shocks.

Detrending and state space algorithm

Detrending is always controversial and the choice among competing methods cannot simply rest on statistical criteria because the available tests have no power for deciding in this area. In the stylized facts literature and in calibration models the Hodrick-Prescott filter is widely used, partly to compare results with the body of the literature and partly because the idea that the growth component changes slowly over time is simple, attractive and easy to reproduce (Kydland-Prescott, 1990). Actually, the same idea can be pursued by using moving average filters that are typically adopted by national or official agencies to remove seasonality and/or trend components: in this case, however, is much more difficult for a single researcher to reproduce results that usually require several steps, frequent updating and judgemental corrections. While the risk of distortions mentioned in the theoretical literature on the HP-filter (King-Rebelo, 1993) are not eliminated by MA-type filters (Osborn, 1995), drawbacks are possibly amplifed in the latter case by the fact that the data released by official agencies are those used for policy discussions and policy decision making.

VAR models are typically used to make observable shocks once data are made stationary. A different possibility is using state space models by using Aoki’s 2-stage modeling algorithm; in the first stage the unobservable state vector is used to detect the stochastic trend. Residuals of the first stage are cyclical variables, i.e. variables that are both serially and cross correlated.

What makes attractive the use of this algorithm is that there is no necessity of estimating trends components for each individual series as it happens, for instance, when determinstic trends are used in VAR estimates. Meanwhile, unit root detrending is also avoided since differenced data do not remove the growth component only but erase also cyclical components as one can see by looking at autocorrelations decaying too quickly to be still ‘cyclical’ (Fiorito-Kollintzas, 1994).

In Aoki’s algorithm the roots of the transition matrix are not imposed to be one but are freely estimated so as the relevant
eigenvalues can be split in two classes: larger eigenvalues that must also be real and close to one refer to growth components while smaller eigenvalues - either real or complex - reflect business cycle dynamics. Further, the dimension of the state vector in the first stage should be small since growth components can be shared by several variables as outlined by balanced growth and cointegration literature; actually, one eigenvalue (common stochastic trend) or two eigenvalues should be sufficient to determine the long-run component of the data.

Modeling cyclical components

Cyclical components are estimated in the second stage that is successful as long as produces a white noise innovation vector. Second stage modeling usually implies a higher dimension of the state vector that should be selected jointly with the dimension of the autocorrelation matrix of the data (Hankel matrix). Aoki (1987, 1990) shows by several examples how to select the minimal dimension of the unobservable state vector given the dimension of the Hankel matrix to attain a white noise innovation vector and - possibly - a satisfactory fit for the data. Another criterion to evaluate the plausibility of the estimates could be looking at the length of cycles implied by the eigenvalues of the second stage state vector: in our example, since the selected dimension of the state vector was n=5, we obtained one real eigenvalue (0.497) and two couples of complex eigenvalues (0.936± 0.109i and 0.769± 0.208i) implying with quarterly data that cycles associated to the real root decay in about one year while cycles implied by complex roots last in one case perhaps too long, their duration being about 11 and 6 years respectively. Yet, this is not inconsistent with the main finding of the paper that productivity shocks are the main force driving business cycles in the US.

The theoretical model

Preferences

The representative agent maximizes over time the expected discounted utility function \( E_U(C_t, L_t) \), depending on consumption (c) and leisure (l). Time runs in discrete units (t=0,1,...), \( \beta \) is the constant discount factor. The instantaneous utility function yields a constant relative risk aversion parameter (\( \gamma \)):

\[
(1) \quad U(c_t, l_t) = \frac{(c_t^{\alpha} l_t^{\gamma-1})^\gamma - 1}{\gamma}, \quad 0<\alpha<1, \gamma<1.
\]

The representative agent will maximize over time equation (1) subject to the following constraints:
(2) \( l = h_t + l_t \) (time endowment)

(3) \( y_t = w_t h_t + \mu_m \) (sources of disposable income)

(4) \( y_t = c_t + s_t \) (used of disposable income)

(5) \( m_t = m_{t-1} + s_t \) (money holdings transition).

The time endowment refers to labor/leisure choices in a given time unit. Disposable income is generated by supplying labor services \((h_t)\) at the real wage \(w_t\) and by receiving interest \((r)\) payments on previous period money stock \((m_{t-1})\). Disposable income is allocated between consumption \((c_t)\) and savings \((s_t)\). All variables are in real terms. Real wages reflect serially correlated productivity shocks while \(r\) denotes a constant real interest rate.

By solving the following Lagrangean with respect to the relevant first-order conditions

\[
\mathcal{G} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\gamma c_t [(1-h_t)^{1-\gamma}]^\gamma}{\gamma} + \mu_t [w_t h_t - m_t (1+r)m_{t-1} - c_t] \right\}
\]

the decision rules for consumption and labor supply are

(7) \( c_t = \omega w_t - \alpha [m_t - (1+r)m_{t-1}] \)

(8) \( h_t = \alpha + (1 - \alpha) \left[ m_t - (1+r)m_{t-1} \right] / w_t \)

Both equations are function of real wage and asset holdings that are left exogenous since no solution can be found for the money stock variable. This is unsatisfactory for the single agent but can be reasonable in the aggregate model where money stock does not depend on individual's choices. Hence, aggregate consumption reflects labor income and an interest-bearing money stock definition \((M_2)\) which is estimated in nominal terms to assess the effects of nominal impulses on the economy. Finally, to avoid mismeasurement of labor supply in terms of observed worked hours, labor supply is approximated by labor force and is expressed in the aggregate model as in equation (8).

Technology

Aggregate output is produced according to a Cobb-Douglas technology where the technical progress level \((q)\) is assumed to be serially correlated:
(9) \( y_t = \theta \eta_t k_t^\beta n_t^{\gamma-\beta} \), \( 0 < \beta < 1 \),

(10) \( \theta_t = \theta_t \cdot e^{\gamma} \), \( |\gamma| < 1 \), \( \nu_t \sim i.i.d.(0, \sigma^2) \),

where \( K \) and \( \eta \) are the aggregate capital and employment stock, respectively. The AR(1) parameter \( \theta_t \) is expected to be positive and close to one since technology shocks should last for a long time.

If capital stock and the real interest rate are constant, the optimal demand for labor and aggregate supply of output will depend on real wages and on productivity shocks \( \nu_t \) only. Taking logs, we obtain the expressions used in the estimated time series model for employment and output:

(11) \( \ln(N_t) = \ln(\mu_n) - \frac{1}{\beta} \ln(w_t) + \frac{1}{\beta} \left( \frac{\nu_t}{1-\rho L} \right) \),

(12) \( \ln(Y_t) = \ln(\mu_y) + (\frac{\beta-1}{\beta}) \ln(w_t) + \frac{1}{\beta} \left( \frac{\nu_t}{1-\rho L} \right) \),

where \( \mu_n = K(1-\beta)^{1/\beta} \), \( \mu_y = [K(1-\beta)^{1/(1-\beta)/\beta}] \), and \( L^j x_t = x_{t-j} \) is the lag operator for any integer \( j \).

**Identification and Specification**

Let us express our \((p \times 1)\) vector of stationary business cycle variables as the VAR representation:

(13) \( \Gamma(L) z_t = e_t, \quad \Gamma_0 = L, \quad E(\varepsilon e') = \Omega, \)

where \( e_t = z_t - E[z_t|z_{t-1}, z_{t-2}, z_{t-3}, ...] \) is the one-step-ahead forecasting error conditional on all past information. Equation (13) is observationally equivalent to the reduced form of the structural model:

(14) \( \Lambda z_t = \Lambda_0(L) z_t + \eta_t, \quad E(\eta_t \eta') = \Sigma, \)

where \( \Lambda_0(L) = \Lambda_0(L) + A_{12}(L) \), and where \( \Lambda_0 \) is a nonsingular sparse matrix, displaying all ones on the main diagonal. Deterministic terms are omitted without loss of generality.

By
equating (13) and (14) one can see that:

\[ \eta_t = A_0 \eta_{t-1} \]

If structural disturbances are uncorrelated \( \Sigma \) is diagonal and we need to impose \((p(p-1))/2\) zero restrictions on the off-diagonal elements of \( A_0 \) to match the number of \((p(p+1))/2\) known terms in the estimated covariance matrix \( \Omega \). When \( \Sigma \) is not diagonal, the most general order condition requires that the number of zero restrictions on both \( A_0 \) and \( \Sigma \) is \( p(p-1) \) (Fiorito, 1996). Meanwhile, the dynamics of the time-series model will be unrestricted.

**Specification of the aggregate model**

The optimal decision rules dictate the specification for all cyclical variables in \( z \): Labor supply (L), Labor demand (N), Real wage (w), real GNP (y), nominal money stock (M2). The original nonstationary variables are quarterly, seasonally adjusted data for the US economy, ranging from 76Q1 to 90Q3.

Contemporaneously, labor supply depends on real wages and on the interest-bearing money stock definition. Labor demand also depends on real wages and on the technology shock. The same specification applies to the aggregate supply of goods which is normalized on the real wage. The structural shocks \( \eta_t, \eta_h \) in the covariance matrix \( \Sigma \) are cross-correlated as required by equations (12)-(11) and, in principle, should be also serially correlated. As in most SVAR models, aggregate demand is normalized on real GDP that depends - as in the theoretical model - on labor income \( w, L \) and on income earned by holding money assets \( M \). The money stock variable is exogenous in matrix \( A_0 \) because of informationally and decision lags in the reaction function (Sim, 1986). The complete model is:

\[
\begin{bmatrix}
1 & 0 & a_1 & 0 & a_2 \\
0 & 1 & a_3 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & a_4 & 0 & 1 & a_6 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
L_t \\
N_t \\
w_t \\
y_t \\
M_t
\end{bmatrix}
= \begin{bmatrix}
A_{11}(L) & A_{12}(L) & A_{13}(L) & A_{14}(L) & A_{15}(L) \\
A_{21}(L) & A_{22}(L) & A_{23}(L) & A_{24}(L) & A_{25}(L) \\
A_{31}(L) & A_{32}(L) & A_{33}(L) & A_{34}(L) & A_{35}(L) \\
A_{41}(L) & A_{42}(L) & A_{43}(L) & A_{44}(L) & A_{45}(L) \\
A_{51}(L) & A_{52}(L) & A_{53}(L) & A_{54}(L) & A_{55}(L)
\end{bmatrix}
\begin{bmatrix}
\eta_{t-1} \\
\eta_{t-2} \\
\eta_{t-3} \\
\eta_{t-4} \\
\eta_{t-5}
\end{bmatrix}
\]

The relevant Citicorp's Citibase codes are: LHC (L) Civilian Labor Force; LHEM (N): civilian employment; (GWY/LHEM)/GD = w, where GWY is the wages and salaries income and GD = GDP implicit price deflator; GNP82 (N) = GNP at 1982 prices; PM2 (M) = Money Stock (M2).
where $E(\eta_t\eta_t') = \Sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & 0 & \sigma_{14} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & 0 \\
0 & \sigma_{33} & 0 \\
0 & 0 & \sigma_{44} \\
0 & 0 & 0 & \sigma_{55}
\end{bmatrix}$

The unknown parameters in $A_o$ and in $\Sigma$ are not estimated as in the SVAR literature but are obtained by solving the following system of $(px(p+1))/2$ algebraic equations:

(17) $\Sigma = A_o G A_o'$

The structural shocks in $\Sigma$ are correlated because the model requires that both demand for labor and aggregate supply for goods depend on technology shocks. The specification is completed by inserting into $\Sigma$ three additional covariances necessary to achieve identification: namely, $\sigma_{13}$ recognizes that both employment and labor force reflect population changes, $\sigma_{14}$ accounts for the stylized fact that labor force in most industrialized countries is procyclical while the last covariance term ($\sigma_{35}$) accounts for the possibility that money shocks and productivity shocks can be correlated if Solow's residual is not truly exogenous (Evans, 1992).

Results of this model are evaluated both in terms of variance decomposition and of impulse response functions. Both are made structural by using as in (15) the identifying matrix $A_o$. Obviously, structural decompositions cannot disentangle between wages and productivity impulses because they are - as required - highly correlated. The responses of the model show that productivity shocks are the main source of fluctuations in the US economy, while aggregate demand and monetary shocks transmit minor impulses to real GNP. The same conclusion holds for labor supply shocks.
References


Kydland F. and E.C. Prescott (1990), Business Cycles: Real Facts and a Monetary Myth, "Federal Reserve Bank of Minneapolis Quarterly Review", Spring, 3-18


C.A. Sims (1980), Macroeconomics and Reality, "Econometrica",

C.A. Sims (1986), Are Forecasting Models Usable for Policy Analysis?, "Federal Reserve Bank of Minneapolis Quarterly Review", Winter, 2-16